Adaptive Robust Stabilization of Dynamic Nonholonomic Chained Systems

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In this article, the stabilization problem is investigated for dynamic nonholonomic systems with unknown inertia parameters and disturbances. First, to facilitate control system design, the nonholonomic kinematic subsystem is transformed into a skew-symmetric form and the properties of the overall systems are discussed. Then, a robust adaptive controller is presented in which adaptive control techniques are used to compensate for the parametric uncertainties and sliding mode control is used to suppress the bounded disturbances. The controller guarantees the outputs of the dynamic subsystem (the inputs to the kinematic subsystem) to track some bounded auxiliary signals which subsequently drive the kinematic subsystem to the origin. In addition, it can also be shown all the signals in the closed loop are bounded. Simulation studies on the control of a unicycle wheeled mobile robot are used to show the effectiveness of the proposed scheme.

1. INTRODUCTION

In recent years, the control and stabilization of nonholonomic systems have been active research areas.[1] Due to Brockett's theorem,[2] it is well known that nonholonomic systems with restricted mobility cannot be stabilized to a desired configuration (or posture) via differentiable, or even continuous, pure-state feedback.[3] The design of stabilizing control laws for these systems is a challenging problem which has attracted ever increasing attention in the control community. A number of approaches have been proposed for the problem, which can be classified as (i) discontinuous time-invariant stabilization,[4,5] (ii) time-varying stabilization,[6] and (iii) hybrid stabilization.[1,7]

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Up to now, most research work on controller design for nonholonomic systems has been focused on the kinematic control problem, in which the systems are represented by their kinematic models and velocity acts as the control input. In practice, however, it is more realistic to formulate the nonholonomic system control problem at the dynamic level, where the torque and force are taken as the control inputs. Different researchers have investigated this problem. Sliding mode control was applied to guarantee the uniform ultimate boundedness of tracking error in ref. 8. In ref. 9, stable adaptive control was investigated for dynamic nonholonomic chained systems with uncertain constant parameters. Using geometric phase as a basis, control of Caplygin dynamical systems was studied in ref. 3, and the closed-loop system was proved to achieve the desired local asymptotic stabilization of a single equilibrium solution.

Sliding mode control receives much attention because of its simplicity, robustness to parametric uncertainties and disturbances, and guaranteed transient performance.[10] However, sliding mode control cannot discriminate the parametric uncertainties and disturbances. High gain feedback is usually needed, which will bring serious chattering in the sliding surface and excite neglected high frequency dynamics. Parametric uncertainties can be eliminated by adaptive control techniques; however, transient response of the system is usually difficult to specify. As a consequence, adaptive control and sliding mode control have been combined to preserve the advantages of the two methods: asymptotic stability of adaptive systems in the presence of parametric uncertainties and guaranteed transient performance of sliding mode control for bounded disturbances.[11,12]

In this article, the stabilization problem is considered for the stabilization of general dynamic nonholonomic systems in which the nonholonomic kinematic subsystem is transformed into the chained form and the dynamic subsystem has unknown constant inertia parameters and bounded disturbances. A robust adaptive controller is presented in which adaptive control techniques are used to compensate for parametric uncertainties and sliding mode control is used to suppress bounded disturbances. The controller guarantees the outputs of the dynamic subsystem (the inputs for the kinematic subsystem) to track some bounded auxiliary signals which subsequently drive the kinematic subsystem to the origin.

This article is organized as follows: the model and model transformations of the dynamic systems are presented in the Section 2. The adaptive robust control law and stability analysis are presented in Section 3. Simulation studies are presented in Section 4 to show the effectiveness of the proposed method. The conclusions are given in the Section 5.

2. DYNAMICS OF NONHOLONOMIC SYSTEMS

In general, a nonholonomic system having an n-dimensional configuration space with generalized coordinates $q = [q_1, \ldots, q_n]^T$ and subject to $n - m$ constraints can be described by[13]

$$J(q)\dot{q} = 0 \tag{1}$$

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + \tau_d = B(q)\tau + J^T(q)\lambda \tag{2}$$

where $M(q) \in R^{n\times n}$ is the inertia matrix which is symmetric positive definite, $C(q, \dot{q}) \in R^{n\times n}$ is the centripetal and coriolis matrix, $G(q) \in R^n$ is the gravitation force vector, $B(q) \in R^{n\times r}$ is the input transformation matrix, $\tau \in R^r$ is the input vector of forces and torques, $J(q) \in R^{(n-m)\times n}$ is the matrix associated with the constraint, $\lambda \in R^{n-m}$ is the vector of constraint forces, and $\tau_d \in R^n$ denotes bounded unknown disturbances including unstructured unmodeled dynamics. The dynamic system (2) has the following properties[14,15]:

Property 2.1. $M(q)$, $C(q, \dot{q})$, and $G(q)$ are bounded in the sense that there exist scalars μ_1 and μ_2 and functions $k_c(q)$ and $k_g(q)$ such that $\mu_1 I \le M(q) \le \mu_2 I$, $\|C(q, \dot{q})\| \le k_c(q)\|\dot{q}\|$, and $\|G(q)\| \le k_g(q)$.

Property 2.2. $\dot{M} - 2C$ is skew-symmetric; i.e., $x^T(\dot{M} - 2C)x = 0$, $\forall x \neq 0$.

Property 2.3. The dynamic system (2) can be expressed in the linear-in-parameters form; i.e., $M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \phi(q, \dot{q})\theta$.

Since $J(q) \in R^{(n-m)\times n}$, it is always possible to find an m rank matrix $S(q) \in R^{n\times m}$ formed by a set of smooth and linearly independent vector fields spanning the null space of $J(q)$; i.e.,

$$S^T(q)J^T(q) = 0 \tag{3}$$

Since $S(q) = [s_1(q), \ldots, s_m(q)]$ is formed by a set of smooth and linearly independent vector fields spanning the null space of $J(q)$, define an auxiliary time function $v = [v_1, \ldots, v_m]^T \in R^m$ such that

$$\dot{q} = S(q)v(t) = s_1(q)v_1 + \cdots + s_m(q)v_m \quad (4)$$

Equation (4) is the so-called kinematic model of nonholonomic systems in the literature. Differentiating Eq. (4) yields

$$\ddot{q} = \dot{S}(q)v + S(q)\dot{v} \quad (5)$$

Substituting (4) and (5) into Eq. (2), and then premultiplying by $S^T(q)$, the constraint matrix $J^T(q)\lambda$ in Eq. (2) can be eliminated by virtue of Eq. (3). As a consequence, we have the transformed nonholonomic system

$$\dot{q} = S(q)v = s_1(q)v_1 + \cdots + s_m(q)v_m \quad (6)$$

$$M_1(q)\dot{v} + C_1(q, \dot{q})v + G_1(q) + \tau_{d1} = B_1(q)\tau \quad (7)$$

where

$$M_1(q) = S^T M(q) S$$

$$C_1(q, \dot{q}) = S^T\left(M(q)\dot{S} + C(q, \dot{q})S\right)$$

$$G_1(q) = S^T G(q)$$

$$B_1(q) = S^T B(q)$$

$$\tau_{d1} = S^T \tau_d$$

which is more appropriate for controller design as the constraint λ has been eliminated from dynamic Eq. (7).

For ease of controller design in this article, the existing results for the control of nonholonomic canonical forms in the literature are exploited. In the following, the kinematic nonholonomic subsystem (6) is first converted into the chained canonical form and then to the skew-symmetric chained form for which a very nice controller structure[16] exists in the literature that can be utilized as will be detailed later. The nonholonomic chained system considered in this article is the m-input, $(m-1)$-chain, single-generator chained form given by Walsh and Bushnell[17]

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_{j,i} &= u_1 x_{j,i+1} \quad (2 \le i \le n_j - 1)(1 \le j \le m-1) \\ \dot{x}_{j,n_j} &= u_{j+1} \end{aligned} \quad (8)$$

Note that in Eq. (8), $X = [x_1, X_2, \ldots, X_m]^T \in R^n$ with $X_j = [x_{j-1,2}, \ldots, x_{j-1,n_{j-1}}](2 \le j \le m)$ are the states and $u = [u_1, u_2, \ldots, u_m]^T$ are the inputs of the kinematic subsystem.

The class of nonholonomic systems in chained form was first introduced in Murray and Sastry[18] and has been studied as a benchmark example in the literature. It is the most important canonical form that is commonly used in the study of nonholonomic control systems. It is well known that many mechanical systems with nonholonomic constraints can be locally, or globally, converted to the chained form under coordinate change and state feedback. Interesting examples of such mechanical systems include tricycle-type mobile robots, cars towing several trailers, the knife edge,[3] a vertical rolling wheel, and a rigid spacecraft with two torque actuators.[1,18,19] The necessary and sufficient conditions for transforming system (6) into the chained form are given in Murray.[20] Theoretical challenges and practical interests have provided substantial motivation for extensive study of nonholonomic systems in chained form. The following assumption is in order.

Assumption 2.1. The kinematic model of the nonholonomic system given by Eq. (6) can be converted into the chained form (8) by some diffeomorphic coordinate transformation $X = T_1(q)$ and state feedback $v = T_2(q)u$ where u is a new control input.

The existence and construction of these systems have been established in the literature.[17,21] For the notations on differential geometry used below, readers are referred to ref. 22.

Proposition 2.1: *Consider the drift-free nonholonomic system*

$$\dot{q} = s_1(q)v_1 + \cdots + s_m(q)v_m$$

where $s_i(q)$ are smooth, linearly independent input vector fields. There exist state transformation $X = T_1(q)$ and feedback $v = T_2(q)u$ on some open set $U \subset R^n$ to transform the system into an $(m-1)$-chain, single-generator chained form, if and only if there exists a basis $f_1, \ldots, f_m$ for $\Delta_0 := \mathrm{span}\{s_1, \ldots, s_m\}$ which has the form

$$f_1 = (\partial/\partial q_1) + \sum_{i=2}^{n} f_1^i(q)\partial/\partial q_i$$

$$f_j = \sum_{i=2}^{n} f_j^i(q)\partial/\partial q_i, 2 \le j \le m$$

such that the distributions

$$G_j = \text{span}\left\{\text{ad}^i_{f_1} f_2, \ldots, \text{ad}^i_{f_1} f_m : 0 \le i \le j\right\}, 0 \le j \le n-1$$

have constant dimension on U and are all involutive, and G_{n-1} has dimension $n-1$ on U.[17,21]

For a two-input controllable system, a constructive method was given in ref. 23. It is given here for completeness. Consider

$$\dot{q} = s_1(q)v_1 + s_2(q)v_2 \tag{9}$$

where $s_1(q)$, $s_2(q)$ are linearly independent and smooth, $q \in R^n$, and $v = [v_1, v_2]^T$.

Define

$$\Delta_0 := \text{span}\left\{s_1, s_2, \text{ad}_{s_1}s_2, \ldots, \text{ad}^{n-2}_{s_1}s_2\right\}$$
$$\Delta_1 := \text{span}\left\{s_2, \text{ad}_{s_1}s_2, \ldots, \text{ad}^{n-2}_{s_1}s_2\right\}$$
$$\Delta_2 := \text{span}\left\{s_2, \text{ad}_{s_1}s_2, \ldots, \text{ad}^{n-3}_{s_1}s_2\right\}$$

If $\Delta_0(q) = R^n\ \forall q \in U$ (where U is some open set of R^n), Δ_1 and Δ_2 are involutive on U, and $s_1(q)$ satisfies $[s_1, \Delta_1] \subset \Delta_1$, then there exist two independent functions $h_1: U \to R$ and $h_2: U \to R$ which satisfy the following relationships:

$$dh_1 \cdot \Delta_1 = 0, \qquad dh_1 \cdot s_1 = 1$$
$$dh_2 \cdot \Delta_2 = 0, \qquad dh_2 \cdot \text{ad}^{n-2}_{s_1}s_2 \neq 0$$

Let $T_1(q): q \to X$ as

$$\begin{aligned} x_1 &= h_1 \\ x_2 &= L^{n-2}_{s_1}h_2 \\ &\vdots \\ x_{n-1} &= L_{s_1}h_2 \\ x_n &= h_2 \end{aligned} \tag{10}$$

It may be verified that $T_1(q)$ is a valid change of coordinates by evaluating the Jacobian of $T_1(q)$ at the origin.

Since $L_{s_2}L^{n-2}_{s_1}h_2 \neq 0$, let $T_2(q): v \to u$ as

$$v_1 := u_1$$
$$v_2 := \frac{1}{L_{s_2}L^{n-2}_{s_1}h_2}\left(u_2 - \left(L^{n-1}_{s_1}h_2\right)u_1\right)$$

Then, local coordinate transformation $X = T_1(q)$ and state feedback $v = T_2(q)u$ render system (9) into the chained form (11)

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_2 \\ \dot{x}_3 &= x_2 u_1 \\ &\vdots \\ \dot{x}_n &= x_{n-1}u_1 \end{aligned} \tag{11}$$

Under Assumption 2.1, i.e., the existence of transformations $X = T_1(q)$ and $v = T_2(q)u$, the dynamic subsystem (7) is correspondingly converted into

$$M_2(X)\dot{u} + C_2(X, \dot{X})u + G_2(X) + \tau_{d2} = B_2(X)\tau \tag{12}$$

where

$$M_2(X) = T_2^T(q)M_1(q)T_2(q)\,|_{q=T_1^{-1}(X)}$$
$$C_2(X, \dot{X}) = T_2^T(q)\left(C_1(q, \dot{q})T_2 + M_1\dot{T}_2(q)\right)|_{q=T_1^{-1}(X)}$$
$$G_2(X) = T_2^T(q)G_1(q)\,|_{q=T_1^{-1}(X)}$$
$$B_2(X) = T_2^T(q)B_1(q)\,|_{q=T_1^{-1}(X)}$$
$$\tau_{d2} = T_2^T(q)\tau_{d1}\,|_{q=T_1^{-1}(X)}$$

Next, let us further transform the chained form into a skew-symmetric chained form for the convenience of controller design. This transformation is the simple extension of the transformation of the one-generation, two-inputs, single-chained system given by Samson.[16] As shown in ref. 16, by introducing the skew-symmetric chained form, via Lyapunov-like analysis, it is easier to design $U_2 = [u_2, \ldots, u_m]^T$ and a time-varying control u_1 to globally stabilize $[x_1, X_2, \ldots, X_m]^T$ of the whole system as will be detailed later.

The kinematic model of the chained form (8) can be equivalently written as

$$\dot{X} = h_1(X)u_1 + \sum_{j=2}^{m} h_{2,j}u_j = h_1(X)u_1 + h_2U_2 \tag{13}$$

where

$$h_1(X) = [1, x_{1,3}, \ldots, x_{1,n1}, 0, \ldots, x_{m-1,3}, \ldots, x_{m-1,n_{m-1}}, 0]^T$$
$$h_2 = [h_{2,2}, \ldots, h_{2,m}]^T$$

and $h_{2,j}$, $j=2,\ldots,m$ is an n-dimensional vector with the $1+\sum_{i=1}^{j}(n_i-1)$th element being 1 while other elements are zero.

Consider the following transformation of coordinates

$$\begin{aligned} z_1 &= x_1 \\ z_{j,2} &= x_{j,2} \\ z_{j,3} &= x_{j,3} \\ z_{j,i+3} &= \rho_{j,i} z_{j,i+1} + L_{h1} z_{j,i+2} \\ &\qquad (1 \le i \le n_j - 3)(1 \le j \le m-1) \end{aligned} \tag{14}$$

where $\rho_{j,i}$ are real positive numbers and $L_{h1} z_{j,i} = (\partial z_{j,i}/\partial X) h_1(X)$ are the Lie derivatives of $z_{j,i}$ along $h_1(X)$. This transformation can convert the original chained system into the skew-symmetric chained form.

Define $Z = [z_1, z_{1,2}, \ldots, z_{1,n_1}, \ldots, z_{m-1,2}, \ldots, z_{m-1,n_{m-1}}]^T \in R^n$. Coordinate transformation (14) can also be written in a matrix form as below

$$Z = \Psi X$$

where $\Psi = \mathrm{diag}[1, \Psi_1, \ldots, \Psi_{m-1}]^T$ with $\Psi_k = [\psi_{j,i}] \in R^{n_k-1 \times n_k-1}$ being

$$\begin{aligned} \psi_{j,j} &= 1 \qquad (j = 1,2,\ldots,n_k-1) \\ \psi_{j,i} &= 0 \qquad (j < i; i,j = 1,2,\ldots,n_k-1) \\ \psi_{j,i} &= 0 \qquad ((i+j) \bmod 2 \neq 0) \\ \psi_{j,i} &= \rho_{j,i-3}\psi_{j,i-2} + \psi_{j-1,i-1} \\ &\qquad (j = 3,4,\ldots,n_k-1; i = 1,2,\ldots,n_k-1) \end{aligned} \tag{15}$$

It is explicit that matrix Ψ is of full rank. Moreover, $L_{h_2} z_{j,i} U_2 = 0$ $(1 \le i \le n_j - 1)$, and $L_{h_2} z_{j,n_j} U_2 = u_{j+1}$. Taking the time derivative of $z_{j,i+3}$ and using (13), we have

$$\dot{z}_{j,i+3} = \frac{\partial z_{j,i+3}}{\partial X}\dot{X} = (L_{h_1} z_{j,i+3}) u_1 + (L_{h_2} z_{j,i+3}) U_2 \tag{16}$$

From (14), we know that for $0 \le i \le n_j - 4$, there is

$$L_{h_1} z_{j,i+3} = -\rho_{j,i+1} z_{j,i+2} + z_{j,i+4} \tag{17}$$

Hence, for $0 \le i \le n_j - 4$, Eq. (16) becomes

$$\dot{z}_{j,i+3} = -\rho_{j,i+1} u_1 z_{j,i+2} + u_1 z_{j,i+4} \qquad (1 \le j \le m-1) \tag{18}$$

while for $i = n_j - 3$

$$\dot{z}_{j,i+3} = L_{h_1} z_{j,n_j} u_1 + u_{j+1} \qquad (1 \le j \le m-1) \tag{19}$$

Thus the original chained system has been converted into the following skew-symmetric chained form

$$\begin{aligned} \dot{z}_1 &= u_1 \\ \dot{z}_{j,2} &= u_1 z_{j,3} \\ \dot{z}_{j,i+3} &= -\rho_{j,i+1} u_1 z_{j,i+2} + u_1 z_{j,i+4} \\ &\qquad (1 \le j \le m-1)(0 \le i \le n_j - 4) \\ \dot{z}_{j,n_j} &= L_{h1} z_{j,n_j} u_1 + u_{j+1} \end{aligned} \tag{20}$$

$$M_3(Z)\dot{u} + C_3(Z,\dot{Z})u + G_3(Z) + \tau_{d3} = B_3(Z)\tau \tag{21}$$

where

$$\begin{aligned} M_3(Z) &= M_2(X)\,|_{X=\Psi^{-1}(Z)} \\ C_3(Z,\dot{Z}) &= C_2(X,\dot{X})\,|_{X=\Psi^{-1}(Z)} \\ G_3(Z) &= G_2(X)\,|_{X=\Psi^{-1}(Z)} \\ B_3(Z) &= B_2(X)\,|_{X=\Psi^{-1}(Z)} \\ \tau_{d3} &= \tau_{d2} \end{aligned}$$

To facilitate controller design, the properties of the dynamic model (21) are listed below,

Property 2.4. M_3 is symmetric positive definite and bounded. The boundedness of M_3 means that there exist positive scalars μ_1 and μ_2 such that $\mu_1 I \le M_3 \le \mu_2 I$.

Property 2.5. $\dot{M}_3 - 2C_3$ is a skew-symmetric matrix. This property will be fully exploited for control system design.

Property 2.6. The dynamics can be expressed in the linear-in-parameters form

$$M_3(Z)\dot{\xi} + C_3(Z,\dot{Z})\xi + G_3(Z) = \Phi\left(Z,\dot{Z},\xi,\dot{\xi}\right)\theta \tag{22}$$

where $\Phi(Z,\dot{z},\xi,\dot{\xi}) \in R^{m\times m}$ is the known regressor matrix and $\theta \in R^m$ is the unknown parameters vector of system. For any physical system, we know that $\|\theta\|$ is always bounded.

Assumption 2.2. $\|\tau_{d3}\|$ is bounded by a known scalar; i.e., $\|\tau_{d3}\| < \tau_{\max}$.

Assumption 2.3. $B(Z)$ is assumed to be known because it is a function of fixed geometry of the system. Accordingly, $B_3(Z)$ is assumed to be known exactly for the subsequent discussion.

3. CONTROLLER DESIGN

Consider the nonholonomic systems described by Eqs. (20) and (21). An adaptive sliding mode controller is designed to stabilize the system states Z to the origin. Since $Z = \Psi X$ is of global diffeomorphism, the stabilization problem of X is the same as the stabilization problem of Z.

Define an auxiliary vector $u_d \in R^m$ as

$$u_d = \begin{bmatrix} -k_{u1} z_1 + h(Z_2, t) \\ -(\rho_{1,n1-2} z_{1,n1-1} + L_{h1} z_{1,n1}) u_{d1} - k_{u2} z_{1,n1} + b_2^T \Lambda e_w \\ \vdots \\ -(\rho_{m-1,n_{m-1}-2} z_{m-1,n_{m-1}-1} + L_{h1} z_{m-1,n_{m-1}}) u_{d1} - k_{um} z_{m-1,n_{m-1}} + b_m^T \Lambda e_w \end{bmatrix} \tag{23}$$

where $Z = [z_1, Z_2^T]^T$, $Z_2 = [z_{1,2}, \ldots, z_{1,n_1}, z_{2,2}, \ldots, z_{2,n_2}, \ldots, z_{m-1,2}, \ldots, z_{m-1,n_{m-1}}]^T$, k_{uj} $(1 \le j \le m)$, and ρ_{j,n_j-2} $(1 \le j \le m)$ are positive constants, Λ is a constant matrix and e_w is an error vector to be defined later, $b_j \in R^m$ $(2 \le j \le m)$ with its ith element defined as

$$b_{j,i} = \begin{cases} 1, & i = j \\ 0, & i \ne j \end{cases}$$

and $h(Z_2, t)$ satisfies the following assumption as given in ref. 22:

Assumption 3.1. $h(Z_2, t)$ is a function of class $C^{p+1}(p \ge 1)$, uniformly bounded with respect to t, with all successive partial derivatives also uniformly bounded with respect to t, and such that

(i) $h(0, t) = 0$, $\forall t$; and
(ii) There is a time-diverging sequence $\{t_i\}_{i \in N}$, and a positive continuous function $\alpha(\cdot)$, such that

$$\|Z_2\| \ge l > 0 \rightarrow \sum_{j=1}^{j=p} \left(\frac{\partial^j h}{\partial t^j}(Z_2, t_i) \right)^2 \ge \alpha(l) > 0, \forall i$$

where N denotes the set of natural numbers.

Remark 3.1. Note that it is not difficult to find $h(Z_2, t)$ satisfying the required conditions just as shown in the simulation. In fact, function $h(Z_2, t)$ is referred to as the heat function, and its primary role is to force the system in motion as long as the system has not reached the desired equilibrium point, thus preventing the system's state from converging to other equilibrium points. The conditions imposed upon the heat function in Assumption 3.1 are not severe and can easily be met. For example, the following three functions all satisfy the conditions[16]

$$h(Z_2, t) = \|Z_2\|^2 \sin(t)$$

$$h(Z_2, t) = \sum_{j=0}^{j=n-2} a_j \sin(\beta_j t) z_{2+j}$$

$$h(Z_2, t) = \sum_{j=0}^{j=n-2} a_j \frac{\exp(b_j z_{2+j}) - 1}{\exp(b_j z_{2+j}) + 1} \sin(\beta_j t)$$

with $a_j \ne 0$, $b_j \ne 0$, $\beta_j \ne 0$, and $\beta_i \ne \beta_j$ when $i \ne j$.

The control law for τ and the parameter adaptation law are designed later to make the outputs of the dynamic subsystem (the inputs of the kinematic system) u tend to the auxiliary signals u_d. As has been shown in ref. 16, when u_1 tends to u_{d1}, $u_1 Z_2$ and $\dot{Z}_2$ converge to zero, and the definition of $h(Z_2, t)$ will guarantee that Z goes to zero as well.

The actual derivation of u_d will be clear in the proof of the Lyapunov function V later.

For ease of stability analysis, define the following signals

$$e_u = u - u_d \tag{24}$$

$$\dot{e}_w = e_u \tag{25}$$

$$u_r = u_d - \Lambda e_w \tag{26}$$

$$s = \dot{e}_w + \Lambda e_w \tag{27}$$

where Λ is the constant matrix whose eigenvalues are all in the right half of the complex plane.

In the presence of disturbances, adaptive control alone cannot guarantee the kinematic subsystem to track the auxiliary vector u_d exactly; as a consequence, the stability of the whole system is not guaranteed.[9] To solve this problem, sliding mode control is introduced to suppress the bounded disturbance to guarantee the asymptotical stability of the closed-loop system. Because of the introduction of the switching function s, the parametric adaptation should be driven by the filter error s rather than by the output error e_u of the dynamic subsystem as in ref. 9; the designs of the auxiliary vector u_d and the controller τ are also different and more involved to compensate for the complex terms in stability proof of the dynamic nonholonomic system.

From Property 2.6, we have

$$M_3(Z)\dot{u}_r + C_3(Z,\dot{Z})u_r + G_3(Z) = \Phi\left(Z,\dot{Z},u_r,\dot{u}_r\right)\theta \tag{28}$$

where θ is the unknown parametric vector and Φ is the regressor matrix of known kinematic functions.

From Eqs. (25) and (26), we obtain

$$\dot{s} = \dot{u} - \dot{u}_r \tag{29}$$

Combining (21) and (28) and then substituting (29) in yields

$$M_3(Z)\dot{s} + C_3(Z,\dot{Z})s = B_3\tau - \tau_{d3} - \Phi\left(Z,\dot{Z},u_r,\dot{u}_r\right)\theta \tag{30}$$

Consider the robust control law defined by

$$\tau = B_3^+(Z) \times \left[\Phi\hat{\theta} - c\,\mathrm{sgn}(s) - \sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}|z_{j,n_j}|\mathrm{sgn}(s)\right] \tag{31}$$

where $\hat{\theta}$ is the estimate of θ, $c = \tau_{\max} + \varepsilon$ with design parameter $\varepsilon > 0$ and B_3^+ is the left inverse of B_3 defined as

$$B_3^+ = B_3^T(B_3B_3^T)^{-1}$$

Substituting (31) into (30), the closed-loop error equation becomes

$$M_3(Z)\dot{s} = \Phi\tilde{\theta} - c\,\mathrm{sgn}(s) - \sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}|z_{j,n_j}|\mathrm{sgn}(s) - C_3(Z,\dot{Z})s - \tau_{d3} \tag{32}$$

Remark 3.2. The term $\Phi\hat{\theta}$ is to solve the problem of parametric uncertainties using adaptive techniques. For the bounded disturbance τ_{d3}, it is suppressed by sliding mode control, $-c\,\mathrm{sgn}(s)$. The last term—$\sum_{j=1}^{m-1}(1/\prod_{i=1}^{n_j-2}\rho_{j,i})|z_{j,n_j}|\mathrm{sgn}(s)$—is necessary to compensate for the complex terms in the stability proof of the dynamic nonholonomic system. Note that the last two terms in (31) will bring in chattering to the stable system since discontinuous surfaces exist.

The closed-loop stability is summarized in Theorem 3.1.

Theorem 3.1: *For the nonholonomic system described by* (20) *and* (21) *and control law* (31), *if the parameter estimates are updated as*

$$\dot{\hat{\theta}} = -\Gamma^{-1}\Phi^T s \tag{33}$$

where Γ is a symmetric positive define constant matrix, then Z is globally asymptotically stabilizable at the origin $Z = 0$.

Proof: For the convenience of proof, define the following two functions:

$$V_1(s,\tilde{\theta}) = \frac{1}{2}s^TM_3(Z)s + \frac{1}{2}\tilde{\theta}^T\Gamma\tilde{\theta} \tag{34}$$

$$V_2(Z_2) = \sum_{j=1}^{m-1}\frac{1}{2}\left[z_{j,2}^2 + \frac{1}{\rho_{j,1}}z_{j,3}^2 + \cdots + \frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}z_{j,n_j}^2\right] \tag{35}$$

where $\tilde{\theta} = \hat{\theta} - \theta$. Since θ is a constant vector, we have $\dot{\hat{\theta}} = \dot{\tilde{\theta}}$.

The derivative of V_1 along Eq. (32) is given as

$$\begin{aligned}\dot{V}_1(s,\tilde{\theta}) &= s^T M_3(Z)\dot{s} + \frac{1}{2}s^T\dot{M}_3(Z)s + \tilde{\theta}^T\Gamma\dot{\tilde{\theta}} \\ &= s^T\Bigg(\Phi\tilde{\theta} - c\,\mathrm{sgn}(s) - \sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}} \\ &\quad \times|z_{j,n_j}|\mathrm{sgn}(s) - \tau_{d3} - C_3(Z,\dot{Z})s\Bigg) \\ &\quad + \frac{1}{2}s^T\dot{M}_3(Z)s + \tilde{\theta}^T\Gamma\dot{\tilde{\theta}} \\ &= s^T\Bigg(\Phi\tilde{\theta} - c\,\mathrm{sgn}(s) - \sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}} \\ &\quad \times|z_{j,n_j}|\mathrm{sgn}(s) - \tau_{d3}\Bigg) \\ &\quad + s^T\left(\frac{1}{2}\dot{M}_3(Z) - C_3(Z,\dot{Z})\right)s + \tilde{\theta}^T\Gamma\dot{\tilde{\theta}}\end{aligned} \tag{36}$$

Since $\dot{M}_3 - 2C_3$ is skew-symmetric (Property 2.5), we have

$$\begin{aligned}\dot{V}_1 &= s^T\Bigg(\Phi\tilde{\theta} - c\,\mathrm{sgn}(s) \\ &\quad - \sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}|z_{j,n_j}|\mathrm{sgn}(s) - \tau_{d3}\Bigg) + \tilde{\theta}^T\Gamma\dot{\tilde{\theta}}\end{aligned} \tag{37}$$

The time derivative of V_2 is given by

$$\begin{aligned}\dot{V}_2(Z_2) = \sum_{j=1}^{m-1}\Bigg[& z_{j,2}\dot{z}_{j,2} + \frac{1}{\rho_{j,1}}z_{j,3}\dot{z}_{j,3} + \cdots \\ & + \frac{1}{\prod_{i=1}^{nj-2}\rho_{j,i}}z_{j,n_j}\dot{z}_{j,n_j}\Bigg]\end{aligned} \tag{38}$$

Substituting (20) into (38), we have

$$\begin{aligned}\dot{V}_2 &= \sum_{j=1}^{m-1}\Bigg[z_{j,2}u_1 z_{j,3} - \frac{1}{\rho_{j,1}}z_{j,3}\,\rho_{j,1}u_1 z_{j,2} \\ &\quad + \frac{1}{\rho_{j,1}}z_{j,3}u_1 z_{j,4} + \cdots \\ &\quad - \frac{1}{\prod_{i=1}^{n_j-3}\rho_{j,i}}z_{j,n_j-1}\,\rho_{j,n_j-3}u_1 z_{j,n_j-2} \\ &\quad + \frac{1}{\prod_{i=1}^{nj-3}\rho_{j,i}}z_{j,n_j-1}u_1 z_{j,n_j} \\ &\quad + \frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}z_{j,n_j}\left(L_{h1}z_{j,n_j}u_1 + u_{j+1}\right)\Bigg] \\ &= \sum_{j=1}^{m-1}\Bigg[\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}z_{j,n_j} \\ &\quad \times\left(\left(\rho_{j,n_j-2}z_{j,n_j-1} + L_{h1}z_{j,n_j}\right)u_1 + u_{j+1}\right)\Bigg]\end{aligned} \tag{39}$$

For stability analysis, let us consider the following Lyapunov function candidate

$$\begin{aligned}V&\left(s,\tilde{\theta},Z_2\right) \\ &= V_1(s,\tilde{\theta}) + V_2(Z_2) \\ &= \frac{1}{2}s^T M_3 s + \tilde{\theta}^T\Gamma\tilde{\theta} + \sum_{j=1}^{m-1}\frac{1}{2} \\ &\quad \times\left[z_{j,2}^2 + \frac{1}{\rho_{j,1}}z_{j,3}^2 + \cdots + \frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}z_{j,n_j}^2\right]\end{aligned} \tag{40}$$

The derivative of V can be obtained by combining Eqs. (36) and (39)

$$\begin{aligned}\dot{V} &= s^T\Bigg(\Phi\tilde{\theta} - c\,\mathrm{sgn}(s) - \sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}} \\ &\quad \times|z_{j,n_j}|\mathrm{sgn}(s) - \tau_{d3}\Bigg) + \tilde{\theta}^T\Gamma\dot{\tilde{\theta}} \\ &\quad + \sum_{j=1}^{m-1}\Bigg[\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}z_{j,n_j} \\ &\quad \times\left(\left(\rho_{j,n_j-2}z_{j,n_j-1} + L_{h1}z_{j,n_j}\right)u_1 + u_{j+1}\right)\Bigg]\end{aligned} \tag{41}$$

By adding $u_{d_{j+1}} - u_{d_{j+1}}$, Eq. (41) becomes

$$\dot{V} = s^T\left(\Phi\tilde{\theta} - c\,\mathrm{sgn}(s) - \sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}} \times|z_{j,n_j}|\mathrm{sgn}(s) - \tau_{d3}\right) + \tilde{\theta}^T\Gamma\dot{\tilde{\theta}} + \sum_{j=1}^{m-1}\left[\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}z_{j,n_j} \times\left(\left(\rho_{j,n_j-2}z_{j,n_j-1} + L_{h1}z_{j,n_j}\right)u_1 + u_{d_{j-1}} + u_{j+1} - u_{d_{j-1}}\right)\right] \tag{42}$$

From Eqs. (24) and (25), we know that

$$\dot{e}_{w_{j+1}} = u_{j+1} - u_{d_{j-1}} \tag{43}$$

Substituting (23) and (43) into Eq. (42), we have

$$\begin{aligned}\dot{V} &= s^T\left[\Phi\tilde{\theta} - \sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}|z_{j,n_j}|\mathrm{sgn}(s) - c\,\mathrm{sgn}(s) - \tau_{d3}\right] + \tilde{\theta}^T\Gamma\dot{\tilde{\theta}} \\ &\quad + \sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}z_{j,n_j}\left[\dot{e}_{w_{j+1}} - k_{u_{j+1}}z_{j,n_j} + b_{j+1}^T\Lambda e_w\right] \\ &= \left(\tilde{\theta}^T\Phi^T s + \tilde{\theta}^T\Gamma\dot{\hat{\theta}}\right) - \sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}k_{u_{j+1}}z_{j,n_j}^2 - s^T(c\,\mathrm{sgn}(s) + \tau_{d3}) \\ &\quad - \sum_{j=1}^{m-1}s^T\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}|z_{j,n_j}|\mathrm{sgn}(s) + \sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}z_{j,n_j}\left(\dot{e}_{w_{j+1}} + b_{j+1}^T\Lambda e_w\right)\end{aligned} \tag{44}$$

Substituting adaptation law (33) into (44) yields

$$\begin{aligned}\dot{V} &= -\sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}k_{u_{j+1}}z_{j,n_j}^2 - s^T(c\,\mathrm{sgn}(s) + \tau_{d3}) - \sum_{j=1}^{m-1}s^T\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}|z_{j,n_j}|\mathrm{sgn}(s) \\ &\quad + \sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}z_{j,n_j}\left(\dot{e}_{w_{j+1}} + b_{j+1}^T\Lambda e_w\right)\end{aligned} \tag{45}$$

Since $b_{j+1} \in R^m$ is a vector with $j+1$ elements being 1 while others being 0, we have

$$\begin{aligned}s^T b_{j+1} &= (\dot{e}_\omega + \Lambda e_\omega)^T b_{j+1} = \dot{e}_\omega^T b_{j+1} + (\Lambda e_\omega)^T b_{j+1} \\ &= \dot{e}_{\omega_{j+1}} + b_{j+1}^T\Lambda e_\omega\end{aligned} \tag{46}$$

This leads to

$$\dot{V} = -\sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}k_{u_{j+1}}z_{j,n_j}^2 - s^T(c\,\mathrm{sgn}(s) + \tau_{d3}) + \sum_{j=1}^{m-1}s^T\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}\left(-|z_{j,n_j}|\mathrm{sgn}(s) + z_{j,n_j}b_{j+1}\right) \tag{47}$$

Since

$$\begin{aligned}&s^T\left(-|z_{j,n_j}|\mathrm{sgn}(s) + z_{j,n_j}b_{j+1}\right) \\ &\quad = -|z_{j,n_j}|s^T\,\mathrm{sgn}(s) + z_{j,n_j}s^T b_{j+1} \\ &\quad \le -|z_{j,n_j}|s^T\,\mathrm{sgn}(s) + |z_{j,n_j}|s^T\,\mathrm{sgn}(s)\|b_{j+1}\| \\ &\quad \le 0\end{aligned} \tag{48}$$

$$\begin{aligned}&-s^T(c\,\mathrm{sgn}(s) + \tau_{d3}) \\ &\quad = -(\tau_{\max}s^T\,\mathrm{sgn}(s) + s^T\tau_{d3}) - \varepsilon s^T\,\mathrm{sgn}(s) \\ &\quad \le -(\tau_{\max}s^T\,\mathrm{sgn}(s) - s^T\,\mathrm{sgn}(s)\|\tau_{d3}\|) - \varepsilon s^T\,\mathrm{sgn}(s) \\ &\quad \le -\varepsilon s^T\,\mathrm{sgn}(s) \\ &\quad \le 0\end{aligned} \tag{49}$$

We have $\dot{V} \le 0$. Accordingly, s and Z_2 are bounded in the sense of Lyapunov.

Furthermore, considering the derivative of $V_1(s, \tilde{\theta})$ along Eqs. (32) and (33), we have

$$\begin{aligned}\dot{V}_1(s, \tilde{\theta}) &= s^T(B_3\tau - \Phi\theta - \tau_{d3}) + \tilde{\theta}^T\Gamma\dot{\tilde{\theta}} \\ &= -s^T\left[\sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}|z_{j,n_j}|\mathrm{sgn}(s) + c\,\mathrm{sgn}(s) + \tau_{d3}\right] \\ &\le -\varepsilon s^T\,\mathrm{sgn}(s) \\ &\le 0\end{aligned} \tag{50}$$

Then $s \in L_1^m \cap L_\infty^m$. From Eq. (32), since $C_3 s$, $B_3\tau - \Psi\theta$, and τ_{d3} are all bounded, $\dot{s} \in L_\infty^m$. Using the fact that $s \in L_1^m \cap L_\infty^m$ and $\dot{s} \in L_\infty^m$, then from the corollary of Baralart's theory,[14] s tends to zero as $t \to \infty$. For all the eigenvalues of matrix Λ in the right-half complex plane, e_w and e_u will go to zero when $t \to \infty$. The input vector u of the kinematic subsystem will converge to the auxiliary vector u_d as $t \to \infty$.

Next, let us prove the asymptotic stability of Z. The first equation of the controlled system is

$$\dot{z}_1 = -k_{u1} z_1 + h(Z_2, t) + e_{u1} \tag{51}$$

From Assumption 3.1, we know that $h(Z_2, t)$ is uniformly bounded. In addition, with e_{u1} converging to zero, (51) is a stable linear system subjected to the bounded additive perturbation $h(Z_2, t) + e_{u1}$. Therefore, $z_1(t)$ is also bounded uniformly.

Because z_1 and $h(Z_2, t)$ are bounded, it is clear that u_{d1} is bounded from (23). Together with e_u and e_w converging to zero, u_1 is bounded. Since u_1 and Z_2 are bounded, e_u and e_w go to zero, and u_{dj} and u_j $(2 \le j \le m)$ are bounded. Under the condition that Z_2, u_1, and u_j $(2 \le j \le m)$ are bounded, $\dot{z}_{j,n_j}$ and $\dot{z}_{j,i}$ $(1 \le j \le m-1, 2 \le i \le n_j - 1)$, from (20), are bounded.

Since $\dot{V} \le 0$, V must be bounded. Because s tends to zero, the second, third, and fourth items of $\dot{V}$ in (45) will go to zero. From the extended version of Barbalat's lemma, we can conclude that $\dot{V} \to 0$ when $t \to \infty$; subsequently $z_{nj,j} \to 0$ $(1 \le j \le m-1)$ when $t \to \infty$.

In the following, let us show that $u_{d1} Z_2$ tends to zero. For $1 \le j \le m-1$, consider

$$\begin{aligned}\dot{z}_{j,n_j} &= L_{h1} z_{j,n_j} u_1 + u_{j+1} \\ &= L_{h1} z_{j,n_j} u_1 - \left(\rho_{j,n_j-2} z_{j,n_j-1} + L_{h1} z_{j,n_j}\right) u_1 \\ &\quad - k_{u2} z_{j,n_j} - b_{j+1}^T \Lambda e_w + e_{u_{j+1}} \\ &= -\rho_{j,n_j-2} z_{j,n_j-1} u_{d1} + \Big(-k_{u2} z_{j,n_j} - b_{j+1}^T \Lambda e_w \\ &\quad + e_{u_{j+1}} - \rho_{j,n_j-2} z_{j,n_j-1} e_{u1}\Big)\end{aligned} \tag{52}$$

where

$$\frac{d}{dt}(z_{j,n_j-1} u_{d1}) = \dot{z}_{j,n_j-1} u_{d1} + z_{j,n_j-1} \dot{u}_{d1}$$

Because $\dot{z}_{j,n_j-1}$, u_{d1}, z_{j,n_j-1}, and $\dot{u}_{d1}$ are all bounded, $z_{j,n_j-1} u_{d1}$ is uniform continuous. Since z_{j,n_j}, e_u, and e_w tend to zero, the remaining parts of $\dot{z}_{j,n_j}$ converge to zero. From the extended version of Barbalat's lemma, $u_{d1} z_{j,n_j}$ converges to zero.

For $3 \le i \le n_j - 1$ and $1 \le j \le m-1$, consider

$$\begin{aligned}\frac{d}{dt}\left(u_{d1}^2 z_{j,i}\right) &= u_{d1}^2(-\rho_{j,i-2} u_1 z_{j,i-1} + u_1 z_{j,i+1}) + z_{j,i} \frac{d}{dt}(u_{d1})^2 \\ &= -\rho_{j,i-2} u_{d1}^3 z_{j,i-1} \\ &\quad + \Big(-\rho_{j,i-2} u_{d1}^2 e_{u1} z_{j,i-1} + u_{d1}^2 u_1 z_{j,i+1} \\ &\quad + z_{j,i} \frac{d}{dt}(u_{d1})^2\Big)\end{aligned} \tag{53}$$

Because

$$\frac{d}{dt} u_{d1}^3 z_{j,i-1} = u_{d1}^3 \dot{z}_{j,i-1} + 3u_{d1}^2 \dot{u}_{d1} z_{j,i-1}$$

and $\dot{z}_{j,i-1}$, u_{d1}, $z_{j,i-1}$, and $\dot{u}_{d1}$ are all bounded, $u_{d1}^3 z_{j,i-1}$ is uniform continuous. Since e_{u1}, $z_{j,i+1}$, and $z_{j,i}$ tend to zero, the remaining items of $(d/dt)(u_{d1}^2 z_{j,i})$ in (53) go to zero. Due to the extended version of Barbalat's lemma, we have $u_{d1} z_{j,i} \to 0$ when $t \to \infty$.

From the above conclusion, $u_{d1} Z_2$ tends to zero. Because $u_{d1} Z_2$, z_{j,n_j}, e_u, and e_w converge to zero, u_{dj} $(2 \le j \le m)$ tends to zero, and together with $u_{d1} Z_2$ and e_u go to zero, and $\dot{Z}_2$ tends to zero as well.

Because e_u tends to zero, $\dot{z}_1$ goes to u_{d1}, and together with $u_{d1} Z_2$ and $\dot{Z}_2$ converge to zero, we conclude that $Z \to 0$, as $t \to \infty$.[16] The theorem is proved. Q.E.D.

Remark 3.3. In Theorem 3.1, it has been proven that Z is globally asymptotically stabilizable, and all the signals in the closed loop are bounded. Accordingly, we can only claim the boundedness of the estimated parameters, and no conclusion can be made on its convergence. In general, to guarantee the convergence of the parameter estimation errors, persistently exciting trajectories are needed,[14,24] which is hard to meet in practice. Therefore, for the globally asymptotical stability of Z, it is an advantage to remove the stringent requirement of persistent excitation conditions for parameter convergence in actual implementation.

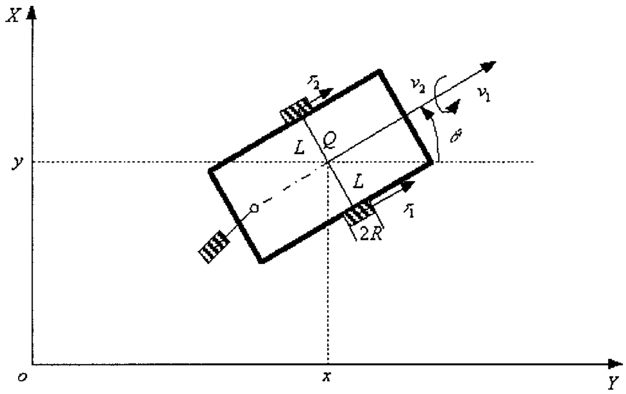

Figure 1. The unicycle wheeled mobile robot.

4. SIMULATION

Consider the unicycle wheeled mobile robot moving on a horizontal plane, as shown in Figure 1, which has three wheels; two are differential drive fixed wheels and one is a caster wheel and is characterized by the configuration $q = [x, y, \theta]^T$. We assume that the robot does not contain flexible parts, all steering axes are perpendicular to the ground, and the contact between wheels and the ground satisfies the condition of pure rolling and non-slipping.

The complete nonholonomic dynamic model of the unicycle robot is given by

$$J(q)\dot{q} = 0 \tag{54}$$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + \tau_d = B(q)\tau + J^T(q)\lambda \tag{55}$$

The constraint of the non-slipping condition can be written as

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0$$

From the constraint, we have

$$J(q) = [\sin\theta, -\cos\theta, 0]$$

which leads to

$$S(q) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}$$

The Lagrange formulation can be used to derive the dynamic equations of the mobile robot. Because the mobile base is constrained to the horizontal plane, its potential energy remains constant, and accordingly $G(q) = 0$. The kinematic energy K is given by[15]

$$K = \frac{1}{2}\dot{q}^T M(q)\dot{q}$$

where

$$M(q) = \begin{bmatrix} m_0 & 0 & 0 \\ 0 & m_0 & 0 \\ 0 & 0 & I_0 \end{bmatrix}$$

with m_0 being the mass of the mobile robot and I_0 being its inertia moment around the vertical axis at point Q. As a consequence, we obtain

$$C(q,\dot{q})\dot{q} = \dot{M}(q)\dot{q} - \frac{\partial K}{\partial q} = 0$$

From Figure 1, we have

$$B(q) = 1/R \begin{bmatrix} -\sin\theta & -\sin\theta \\ \cos\theta & \cos\theta \\ L & -L \end{bmatrix}$$

where R is the radius of the wheels and $2L$ is the length of the axis of the fixed wheels; τ_1 and τ_2 are the torques provided by the motors as shown in Figure 1.

Following the description in Section 2, the dynamics of the unicycle robot can be written as

$$\begin{aligned} \dot{x} &= v_1\cos\theta \\ \dot{y} &= v_1\sin\theta \\ \dot{\theta} &= v_2 \\ M_1(q)\dot{v} + C_1(q)v + G_1 + \tau_{d1} &= B_1\tau \end{aligned} \tag{56}$$

where

$$M_1 = \begin{bmatrix} m_0 & 0 \\ 0 & I_0 \end{bmatrix}, C_1 = 0, G_1 = 0, B_1 = 1/R\begin{bmatrix} 1 & 1 \\ L & -L \end{bmatrix},$$

and $v = [v_1, v_2]^T$ with v_1 and v_2 being the linear and angular velocities as shown in Figure 1.

Considering the coordinate transformation $X = T_1(q)$ and state feedback $u = T_2^{-1}(q)v$ given by[25]

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \sin\theta & -\cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

$$u_1 = v_2$$

$$u_2 = v_1 - v_2 x_2$$

together with the transform matrix $\Psi = I$ in this special case, system (56) is converted to the

$$\begin{aligned} \dot{z}_1 &= u_1 \\ \dot{z}_2 &= z_3 u_1 \\ \dot{z}_3 &= u_2 \end{aligned} \tag{57}$$

$$M_3(Z)\dot{u} + C_3(Z,\dot{Z})u + G_3(Z) + \tau_{d3} = B_3\tau$$

where

$$M_3(Z) = \begin{bmatrix} z_2^2 m_0 + I_0 & z_2 m_0 \\ z_2 m_0 & m_0 \end{bmatrix}$$

$$C_3(Z,\dot{Z}) = \begin{bmatrix} z_2 \dot{z}_2 m_0 & 0 \\ m_0 \dot{z}_2 & 0 \end{bmatrix}$$

$$B_3(Z) = 1/R \begin{bmatrix} z_2 + L & z_2 - L \\ 1 & 1 \end{bmatrix}$$

$$G_3 = 0$$

$$M_3(Z)\dot{u}_r + C_3(Z,\dot{Z})u_r = \Phi\left(Z,\dot{Z},u_r,\dot{u}_r\right)\theta$$

with the inertia parameters vector $\theta = [m_0, I_0]^T$ and

$$\Phi\left(Z,\dot{Z},u_r,\dot{u}_r\right) = \begin{bmatrix} z_2^2 \dot{u}_{r1} + z_2 \dot{u}_{r2} + z_2 \dot{z}_2 u_{r1} & \dot{u}_{r1} \\ z_2 \dot{u}_{r1} + \dot{u}_{r2} + \dot{z}_2 u_{r1} & 0 \end{bmatrix} \tag{58}$$

The auxiliary signal is

$$u_d = \begin{bmatrix} u_{d1} \\ u_{d2} \end{bmatrix} = \begin{bmatrix} -k_{u1} z_1 + h(Z_2,t) \\ -\rho_1 z_2 u_1 - k_{u2} z_3 + \Lambda_{21} e_{w1} + \Lambda_{22} e_{w2} \end{bmatrix}$$

where $h(Z_2,t)$ is chosen as

$$h(Z_2,t) = (z_2^2 + z_3^2)\sin t$$

It is easy to see that the selected $h(Z_2,t)$ satisfies Assumption 3.1.

In the simulation, the parameters of the system are assumed to be $m_0 = I_0 = 1.0$, $R = 0.1$, $L = 1.0$, $k_{u1} = 0.2$, $k_{u2} = 1.0$, $\rho_1 = 1.0$, $c = 0.5$, $\Lambda = I$, $\Gamma = 10I$, and disturbances τ_{d3_1} and τ_{d3_2} are random number in the range $[-0.1, 0.1]$. The initial estimate $\hat{\theta}(0) = [0.5, 0.5]^T$ is different from the true value.

Simulation results are shown in Figures 2 and 3. From Figure 2, we can see that the responses of states x_1, x_2, and x_3 of the chained form asymptotically tend to zero. From Figure 3, the control sequences τ_1 and τ_2 tend to zero as well. As shown in Figure 4, the estimates of the parameters m_0 and I_0 are all bounded. The results of the simulation verify the validity of proposed algorithm.

Remark 4.1. The proposed adaptive robust algorithm can guarantee the asymptotical stability of the system. The bounded disturbance can be completely suppressed by the sliding mode term. However, this may bring the chattering into the system. To

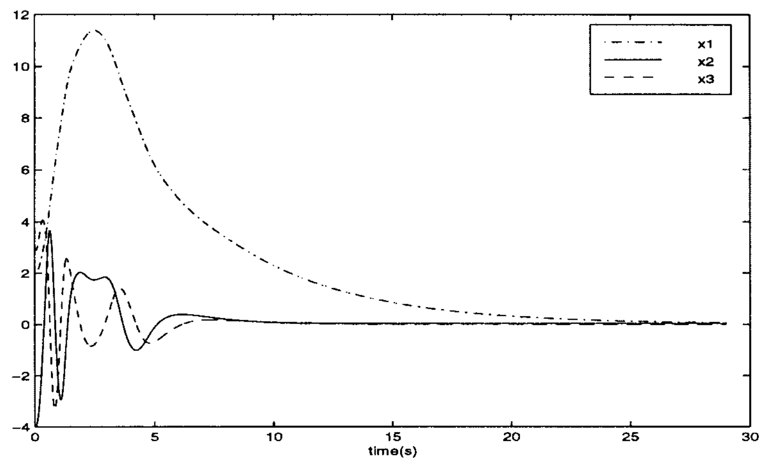

Figure 2. Responses of state x_1 (m), x_2 (m), and x_3 (rad).

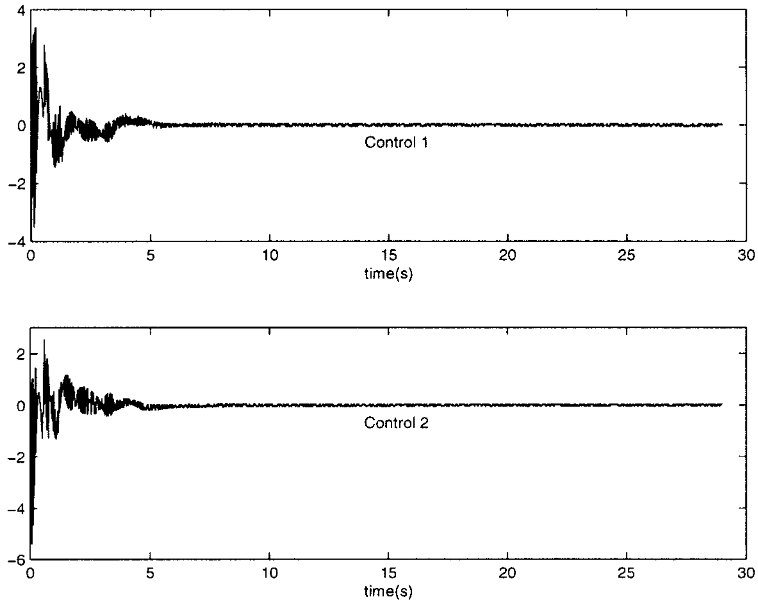

Figure 3. Control signals τ_1 and τ_2 (N).

alleviate this problem, many techniques have been developed, such as the boundary layer solution,[26] and the observer-based solution.[27] It should be mentioned that these modification may cause the estimated parameters to grow unboundedly because asymptotic tracking cannot be guaranteed. To deal with this problem, the σ-modification scheme or e-modification among others[28] can be used to modify the adaptive laws to guarantee the robustness of the closed-loop system in the presence of approximation errors. The drawback is that tracking errors may only be made arbitrarily small rather than zero.

5. CONCLUSION

In this article, stabilization of dynamic chained systems has been investigated with unknown constant inertia parameters and disturbances. For the conve-

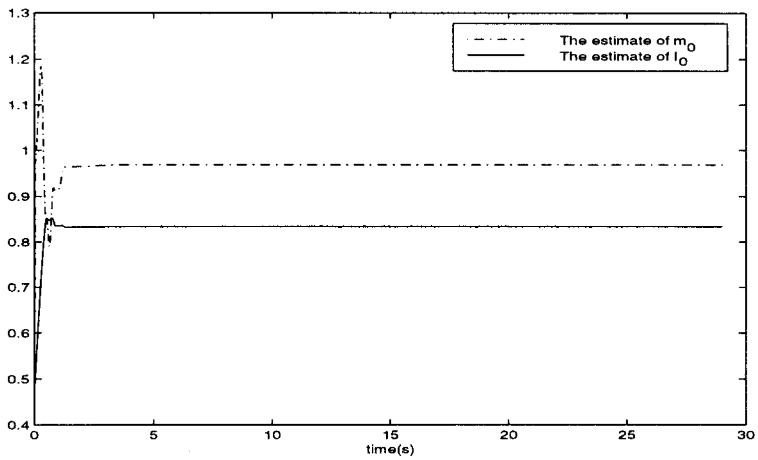

Figure 4. Response of the estimated parameters $\hat{\theta}$.

nience of controller design, the nonholonomic chained subsystems were firstly converted to the skew-symmetric chained form for ease of controller design. Then an adaptive robust control scheme was proposed where parametric uncertainties were compensated for by adaptive control techniques and disturbances were suppressed by sliding mode control. The controller forces the inputs of kinematic subsystem (the output of the dynamic subsystem) to tracking some auxiliary vector and makes the whole system asymptotically stable to the origin. Throughout this paper, feedback control design and stability analysis are performed via explicit Lyapunov techniques. Simulation studies on the stabilization of the unicycle wheeled mobile robot have been used to show the effectiveness of the proposed scheme.

APPENDIX

A. Corollary of Barbalat's Theory[14]

If $f(t)$, $\dot{f}(t) \in L_\infty$ and $f(t) \in L_p$ for some $p \in [1, \infty)$, then $f(t) \to 0$ as $t \to \infty$.

B. The Extended Version of Barbalat's Theorem[16]

If a differentiable function $f(t): R^+ \to R$ converges to a limit value as t tends to infinity and if its derivative $(d/dt)(f(t))$ is the sum of two terms, one being uniformly continuous and another tending to zero as t tends to infinity, then $(d/dt)(f(t))$ tends to zero when t tends to infinity.

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