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Position/force control of uncertain constrained flexible joint robots

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Abstract

This paper addresses the issue of adaptive position/force control of uncertain constrained flexible joint robots (FJRs). The controller is developed without the assumptions of weak robot joint flexibility as adopted in popular singular perturbation (SP) approaches. It relies on the feedback of joint state variables and avoids noisy joint torque measurements found in most controllers developed so far. In addition to the robot inertia parameters, the joint stiffness and the motor inertia are also assumed to be unknown. The proposed approach achieves the position tracking and the boundedness of force errors. The simulation study is conducted to verify the effectiveness of the approach.

Keywords: Constrained robots; Position/force control; Flexible joint robots

1. Introduction

Control of flexible joint robotic manipulators has been studied extensively in the last decades, almost parallel to that of rigid joint robotic manipulators. With singular perturbation (SP) analysis, controllers developed for rigid robotic manipulators, adaptive or non-adaptive, can be readily extended for the robots of *weak* joint flexibilities (the joint stiffness is very big) [1–6]. Feedback linearization method was also used in the controller design, but it required exact dynamic modeling of the robot and the measurements of its joint accelerations and jerks [7,8]. To deal with the uncertainties of robotic systems, many adaptive control schemes [9–11] were developed from the pioneering work on adaptive control of rigid robotic manipulators in [12]. To cope with the complexity of the system dynamics, the controller design required joint acceleration feedback, filtering of system dynamics and the calculation of the inverse inertia matrix of the robot. The controller was complex in structure and computationally intensive. Treating a flexible joint robotic manipulator as a cascading system, joint torque feedback and backstepping approaches were also applied in the controller design [13,14]. They required the measurement of joint torques and their derivatives which are normally very noisy.

Compared with those of free flexible joint robots, fewer research results were reported on controlling constrained flexible joint robotic manipulators. The challenge for controlling an uncertain constrained robot with arbitrary flexibility lies in the constraint force term which is in an algebraic relation to the system dynamics and is difficult to be handled in controller synthesis and stability analysis. Most researches focused on the robot systems with *known* parameters and *weak* joint flexibility [15,16]. In [17], a Cartesian-space robot model were used to develop the position control and the force control along certain curvilinear directions as proposed in [18]. The joint torque and its up to second-order derivatives are required in the controller design. In [19], a Cartesian impedance control of flexible joint robots was developed based on joint torque feedback. With computed torque control, the joint dynamics and the link dynamics were decoupled and the desired impedance

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was achieved. It needed an exact knowledge of the system dynamics and the noisy joint torque feedback.

In this paper, an adaptive position/force controller is developed for an uncertain constrained robot with flexible joints. It mainly relies on the feedbacks of *joint state variables* (joint positions and velocities) and thus avoids noisy joint torque feedback. The joint stiffness and the motor inertia are also assumed to be unknown in addition to the robot inertia parameters which were normally assumed to be known in most controllers developed so far. The controller is designed with a Lyapunov-based cascaded approach. In the first step, a desired joint position is found to make a Lyapunov function of link position tracking error and the constraint force error non-increasing. In the second step, the control input is derived to make the joint position to track its trajectory obtained in the first step. As the joint torque feedback is not used in the controller, the joint stiffness becomes an uncertain parameter scaling the control input of a subsystem of the whole controlled system. This additional challenge to the controller design was overcome with the results reported in [20]. The proposed approach achieves the position tracking and the boundedness of force errors. The simulation study is conducted to verify the effectiveness of the approach.

The paper is organized as follows. In Section 2, the system dynamics and problem formulation is described. In Section 3, the controller design and its stability analysis are provided. Section 4 presents the simulation results to verify the effectiveness of the controller. The conclusion is given in Section 5.

2. Dynamical model and properties

Consider the dynamic model of a constrained flexible joint robot as proposed in [15],

$$M(q_l)\ddot{q}_l + C(q_l,\dot{q}_l)\dot{q}_l + G(q_l) = K_s\theta + f, \tag{1}$$

$$J_m\ddot{q}_m + K_s\theta = \tau_m, \tag{2}$$

$$\theta \triangleq q_m - q_l, \tag{3}$$

where $q_l \in R^n$ and $q_m \in R^n$ are the positions of the robot links and the motor shafts, respectively, $M(q_l) \in R^{n\times n}$ is the inertia matrix of rigid links, $C(q_l,\dot{q}_l)$ is the Coriolis and centrifugal force matrix, $G(q_l)$ is the gravitational force, $J_m = \mathrm{diag}[j_{mi}] \in R^{n\times n}$ is the positive definite diagonal matrix of the moments of inertia of the motors, $K_s = \mathrm{diag}[k_{si}] \in R^{n\times n}$ is the positive definite diagonal matrix of the joint stiffness, $f \in R^n$ is the joint torque contributed by the constraint force and $\tau_m \in R^n$ is the input torque of the motors. j_{mi} and k_{si} $(i = 1,2,\ldots,n)$ are the inertia and the stiffness of ith joint. n is the degree of freedom of the robotic manipulator.

Assuming that the holonomic and frictionless constraint surface is described by

$$\Phi(q_l) = 0 \in R^m, \tag{4}$$

where $\Phi(q_l)$ is a set of independent equations and are twice continuously differentiable [21].

Constraint force in the joint space, f, can then be expressed by

$$f = J^{\mathrm{T}}(q_l)\lambda,$$
$$J(q_l) \triangleq \frac{\partial\Phi(q_l)}{\partial q_l} \in R^{m\times n}, \tag{5}$$

where $\lambda \in R^m$ is a generalized Lagrange multiplexer relating to the magnitude of the constraint force [21]. m is the dimension of the constraint surface and it is assumed that $m < n$.

Due to the m-dimension constraint, m degrees of freedom of the robot are lost. Partitioning the link position vector q_l to $q_l^1 \in R^{n-m}$ and $q_l^2 \in R^m$, we have

$$q_l = \begin{bmatrix} q_l^{1\mathrm{T}} & q_l^{2\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{6}$$

and accordingly, the Jacobian $J(q_l)$ is decomposed as

$$J(q_l) = [J_1(q_l) \quad J_2(q_l)],$$
$$J_1(q_l) \triangleq \frac{\partial\Phi(q_l)}{\partial q_l^1} \in R^{m\times(n-m)}, \tag{7}$$
$$J_2(q_l) \triangleq \frac{\partial\Phi(q_l)}{\partial q_l^2} \in R^{m\times m}.$$

As stated in [22], it is possible to have a partition such that $J_2^{-1}(q_l)$ exists and

$$\dot{q}_l = L(q_l)\dot{q}_l^1, \quad L(q) = \begin{bmatrix} I_{n-m} \\ -J_2^{-1}(q_l)J_1(q_l) \end{bmatrix}, \tag{8}$$

where I_{n-m} is an identity matrix of dimension $n-m$.

With the partition of the link position vector in Eq. (6), the position of the robot can be uniquely determined by q_l^1. The original dynamical model in Eqs. (1) and (2) is transformed to

$$M^1(q_l)\ddot{q}_l^1 + C^1(q_l,\dot{q}_l)\dot{q}_l^1 + G^1(q_l) = K_s\theta + J^{\mathrm{T}}(q_l)\lambda, \tag{9}$$
$$J_m\ddot{q}_m + K_s\theta = \tau_m, \tag{10}$$

where

$$M^1(q_l) = M(q_l)L(q_l) \in R^{n\times m},$$
$$C^1(q_l,\dot{q}_l) = M(q_l)\dot{L}(q_l) + C(q_l,\dot{q}_l)L(q_l) \in R^{n\times m},$$
$$G^1(q_l) = G(q_l) \in R^n.$$

Define $M_l(q_l) = L^{\mathrm{T}}(q_l)M^1(q_l) \in R^{m\times m}$, $C_l(q_l,\dot{q}_l) = L^{\mathrm{T}}(q_l)\times C^1(q_l,\dot{q}_l) \in R^{m\times m}$ and $G_l(q_l) = L^{\mathrm{T}}(q_l)G^1(q_l) \in R^m$. It can be proved that the dynamic models (1) and (9) have the following properties.

Property 1. $L^{\mathrm{T}}(q_l)J^{\mathrm{T}}(q_l) = 0$.

Property 2. *$M(q_l)$, $C(q_l,\dot{q}_l)$, $G(q_l)$, $M^1(q_l)$, $C^1(q_l,\dot{q}_l)$, $G^1(q_l)$, $M_l(q_l)$, $C_l(q_l,\dot{q}_l)$, $G_l(q_l)$, $L(q_l)\dot{L}(q_l)$, and $J(q_l)$ are uniformly bounded and continuous if q_l and $\dot{q}_l$ are uniformly bounded and continuous*; *$M(q_l)$ and $M_l(q_l)$ are symmetric positive definite* (*s.p.d.*).

Property 3. *$\dot{M}(q_l) - 2C(q_l,\dot{q}_l)$ and $\dot{M}_L(q_l) - 2C_L(q_l,\dot{q}_l)$ are skew-symmetric if $C(q_l,\dot{q}_l)$ is in the Christoffel form, i.e., $x_1^{\mathrm{T}}(\dot{M}(q_l) - 2C(q_l,\dot{q}_l))x_1 = 0$, $x_2^{\mathrm{T}}(\dot{M}_L(q_l) - 2C_L(q_l,\dot{q}_l))x_2 = 0$, $\forall x_1 \in R^n$ and $x_2 \in R^{n-m}$.*

Property 4. *The robot link dynamics described by Eq. (1) is linear in the robot link parameters, i.e., given an arbitrary vector $x \in R^n$*

$$M(q_l)\dot{x} + C(q_l,\dot{q}_l)x + G(q_l) = \Psi(\dot{x},x,\dot{q}_l,q_l)p, \tag{11}$$

where $p \in R^l$ is a vector of the lumped parameters of interest, $\Psi(\dot{x},x,\dot{q}_l,q_l) \in R^{n\times l}$ is a regressor matrix.

If the estimates of $M(q_l)$, $C(q_l,\dot{q}_l)$ and $G(q_l)$ are denoted by $\widehat{M}(q_l)$, $\widehat{C}(q_l,\dot{q}_l)$ and $\widehat{G}(q_l)$, then

$$\widehat{M}(q_l)\dot{x} + \widehat{C}(q_l,\dot{q}_l)x + \widehat{G}(q_l) = \Psi(\dot{x},x,\dot{q}_l,q_l)\hat{p}, \tag{12}$$

where $\hat{p}$ is the estimate of p.

Property 5 [20]. *If the regressor matrix $\Psi(\dot{x},x,\dot{q}_l,q_l)$ and the vector p in Eq. (11) are given in the following forms:*

$$\Psi(\dot{x},x,\dot{q}_l,q_l) = \begin{bmatrix} \psi_1^{\mathrm{T}}(\dot{x},x,\dot{q}_l,q_l) & 0 & \cdots & 0 \\ 0 & \psi_2^{\mathrm{T}}(\dot{x},x,\dot{q}_l,q_l) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \psi_n^{\mathrm{T}}(\dot{x},x,\dot{q}_l,q_l) \end{bmatrix} \in R^{n\times l}, \tag{13}$$

$$p = [p_1^{\mathrm{T}}\ p_2^{\mathrm{T}}\ \cdots\ p_n^{\mathrm{T}}]^{\mathrm{T}} \in R^l, \tag{14}$$

then

$$K_s\Psi(\dot{x},x,\dot{q}_l,q_l)p = \Psi(\dot{x},x,\dot{q}_l,q_l)p_s,$$

where

$$p_s \triangleq [k_{s1}p_1^{\mathrm{T}}\ \ k_{s2}p_2^{\mathrm{T}}\ \ \cdots\ \ k_{\mathrm{sn}}p_n^{\mathrm{T}}]^{\mathrm{T}} = \Lambda p \in R^l,$$

$$\Lambda \triangleq \begin{bmatrix} \mathrm{diag}[k_{s1}] & 0 & \cdots & 0 \\ 0 & \mathrm{diag}[k_{s2}] & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \mathrm{diag}[k_{\mathrm{sn}}] \end{bmatrix} \in R^{l\times l}$$

and $\psi_i(\dot{x},x,\dot{q}_l,q_l) \in R^{n_i}$, $p_i \in R^{n_i}$, $\mathrm{diag}[k_{\mathrm{si}}] \in R^{n_i\times n_i}$, $l = \sum_{i=1}^{n} n_i$, $i = 1,2,\ldots,n$.

Assume that K_s and J_m are unknown and their estimates are denoted as $\widehat{K}_s$, and $\widehat{J}_m$, respectively. Their estimate errors are defined as $\tilde{K}_s \triangleq K_s - \widehat{K}_s$ and $\tilde{J}_m \triangleq J_m - \widehat{J}_m$, respectively. For the controller design, the following assumptions are made for these terms.

Assumption 1. K_s and J_m are unknown and bounded.

Assumption 2. $\tilde{K}_s$ and $\tilde{J}_m$ are bounded such that $\|\tilde{K}_s\| \leqslant \delta_K$ and $\|\tilde{J}_m\| \leqslant \delta_J$, where δ_K and δ_J are known positive constants.

Assumption 3. $q_l(t)$, $q_m(t)$, $\dot{q}_l(t)$, $\dot{q}_m(t)$, $\ddot{q}_m$ and $\lambda(t)$ are all measurable.

Barbalat's lemma is important to the controller design and analysis [23,25]. It is listed below for an easy reference.

Lemma 1 (Barbalat). *If the differentiable function $f(t)$ has a finite limit as $t \to \infty$, and if $\dot{f}$ is uniformly continuous, then $\dot{f} \to 0$ as $t \to \infty$.*

As the boundedness of $\ddot{f}$ is a sufficient condition of uniform continuity of f, a practical corollary of Barbalat's lemma can be stated as

Corollary 1 (Barbalat). *If the differentiable function $f(t)$ has a finite limit as $t \to \infty$, and is such that $\ddot{f}$ exists and is bounded, then $\dot{f} \to 0$ as $t \to \infty$.*

3. Controller design

Let $q_{ld}(t)$ be the desired trajectory of the link position and $\lambda_d(t)$ be the desired magnitude of the constraint force. The control objective is to find a driving torque τ_m under which $q_l(t)$ tracks $q_{ld}(t)$ and the error between $\lambda(t)$ and $\lambda_d(t)$ is bounded. It is only necessary to make $q_l^1(t)$ track its desired trajectory $q_{ld}^1(t)$ since $q_l^1(t)$ completely determines $q_l(t)$.

From dynamic model represented by Eqs. (9) and (10), a cascaded Lyapunov based control design approach can be adopted to derive the control input τ_m. In the first step, a desired value of θ: θ_d, is determined aiming at making $q_l^1(t)$ track $q_{ld}^1(t)$ and the boundedness of $\lambda_d(t) - \lambda(t)$. Then, the control input τ_m is obtained to make θ to track θ_d.

Define the following variables which are related to the tracking errors of the link position and the constraint force:

$$e^1 \triangleq q_{ld}^1 - q_l^1, \quad e_\lambda \triangleq \lambda_d - \lambda, \tag{15}$$

$$r^1 \triangleq \dot{e}^1 + K_e e^1, \quad \dot{q}_r^1 \triangleq \dot{q}_{ld}^1 + K_e e^1, \tag{16}$$

where $K_e \in R^{(n-m)\times(n-m)}$ is a positive definite constant matrix.

With variables r^1 and q_r^1, the following variables are also defined:

$$\sigma \triangleq L(q_l)r^1 + \mu, \tag{17}$$

$$v \triangleq L(q_l)\dot{q}_r^1 + \mu, \tag{18}$$

where μ is a variable introduced to compensate the force error e_λ and it will be determined later.

From Eqs. (17) and (18), it is obvious that

$$r^1 = \dot{q}_r^1 - \dot{q}_l^1, \tag{19}$$

$$\sigma = v - L(q_l)\dot{q}_l^1, \tag{20}$$

$$\dot{\sigma} = \dot{v} - L(q_l)\ddot{q}_l^1 - \dot{L}(q_l)\dot{q}_l^1. \tag{21}$$

For the parameters update, the following vectors are constructed with the elements of diagonal matrices K_s, $\widehat{K}_s$ and $\widehat{K}_s^{-1}$.

$$k_s^{-1} \triangleq [k_{s1}^{-1}\ \ k_{s2}^{-1}\ \ \cdots\ \ k_{\mathrm{sn}}^{-1}]^{\mathrm{T}}, \tag{22}$$

$$\hat{k}_s^{-1} \triangleq [\hat{k}_{s1}^{-1}\ \ \hat{k}_{s2}^{-1}\ \ \cdots\ \ \hat{k}_{\mathrm{sn}}^{-1}]^{\mathrm{T}}, \tag{23}$$

$$\tilde{k}_s^{-1} = k_s^{-1} - \hat{k}_s^{-1}, \tag{24}$$

$$\varphi_1(K_s,\widehat{K}_s^{-1}) \triangleq K_s\hat{k}_s^{-1} = [k_{s1}\tilde{k}_{s1}^{-1}\ \ k_{s2}\tilde{k}_{s2}^{-1}\ \ \cdots\ \ k_{\mathrm{sn}}\tilde{k}_{\mathrm{sn}}^{-1}]^{\mathrm{T}}, \tag{25}$$

where k_{si} is the stiffness of ith joint and $\hat{k}_{si}^{-1}$ is the estimate of k_{si}^{-1}.

Note that $\widehat{K}_s^{-1}$ and $\hat{k}_s^{-1}$ here only serve as the estimates of K_s^{-1} and k_s^{-1}, respectively. They are NOT the inverses of $\widehat{K}_s$ and $\hat{k}_s$, respectively.

3.1. Step 1. Determination of θ_d: the desired value of θ

In this step, θ_d, the desired value of θ, is determined to make a Lyapunov function of e_1, e_λ and the system parameter estimation errors non-increasing.

Consider the following Lyapunov function:

$$V_1 = \frac{1}{2}\sigma^{\mathrm{T}} M(q_l)\sigma + \frac{1}{2}(p-\hat{p}_s)^{\mathrm{T}}\Gamma_1(p-\hat{p}_s) + \frac{1}{2}\mu^{\mathrm{T}}\mu + \frac{1}{2}\varphi_1^{\mathrm{T}}(K_s,\widehat{K}_s^{-1})\Gamma_2\varphi_1(K_s,\widehat{K}_s^{-1}),$$

where $\hat{p}_s$ is the estimate of p_s as defined in Property 5, $\Gamma_1 \in R^{l\times l}$ and $\Gamma_2 \in R^{n\times n}$ are positive definite diagonal matrices. Note that $p-\hat{p}_s$, instead of $p-\hat{p}$, is used in the above definition so that the unknown joint stiffness K_s lumped in p_s is also taken care of by the controller to be developed.

Differentiating V_1 with respect to time t and considering Properties 3 and 5, we have

$$\begin{aligned}\dot{V}_1 &= \sigma^{\mathrm{T}} M(q_l)\dot{\sigma} + \frac{1}{2}\sigma^{\mathrm{T}}\dot{M}(q_l)\sigma s - (p-\hat{p}_s)^{\mathrm{T}}\Gamma_1\dot{\hat{p}}_s \\ &\quad + \mu^{\mathrm{T}}\dot{\mu} + \varphi_1^{\mathrm{T}}(K_s,\widehat{K}_s^{-1})\Gamma_2\dot{\varphi}_1(K_s,\widehat{K}_s^{-1}) \\ &= \sigma^{\mathrm{T}}(M(q_l)\dot{\sigma} + C(q_l,\dot{q}_l)\sigma) - (p-\hat{p}_s)^{\mathrm{T}}\Gamma_1\dot{\hat{p}}_s + \mu^{\mathrm{T}}\dot{\mu} \\ &\quad + \varphi_1^{\mathrm{T}}(K_s,\widehat{K}_s^{-1})\Gamma_2\dot{\varphi}_1(K_s,\widehat{K}_s^{-1}). \end{aligned} \tag{26}$$

Substituting σ in Eq. (20) and $\dot{\sigma}$ in Eq. (21) into Eq. (26), we have

$$\begin{aligned}\dot{V}_1 &= \sigma^{\mathrm{T}}(M(q_l)\dot{v} + C(q_l,\dot{q}_l)v + G(q_l) - M^1(q_l)\ddot{q}_l^1 \\ &\quad - C^1(q_l,\dot{q}_l)\dot{q}_l^1 - G^1(q_l)), -(p-\hat{p}_s)^{\mathrm{T}}\Gamma_1\dot{\hat{p}}_s + \mu^{\mathrm{T}}\dot{\mu} \\ &\quad + \varphi_1^{\mathrm{T}}(K_s,\widehat{K}_s^{-1})\Gamma_2\dot{\varphi}_1(K_s,\widehat{K}_s^{-1}). \end{aligned} \tag{27}$$

Considering Eq. (9) and Property 4, Eq. (27) leads to

$$\begin{aligned}\dot{V}_1 &= \sigma^{\mathrm{T}}(\Psi(\dot{v},v,\dot{q}_l,q_l)p - K_s\theta - J^{\mathrm{T}}(q_l)\lambda) - (p-\hat{p}_s)^{\mathrm{T}}\Gamma_1\dot{\hat{p}}_s \\ &\quad + \mu^{\mathrm{T}}\dot{\mu}, +\varphi_1^{\mathrm{T}}(K_s,\widehat{K}_s^{-1})\Gamma_2\dot{\varphi}_1(K_s,\widehat{K}_s^{-1}). \end{aligned} \tag{28}$$

Our aim is to find θ_d, the desired value of θ to make $\dot{V}_1$ non-positive. Define the difference between θ and θ_d as

$$z \triangleq \theta_d - \theta$$

and Eq. (28) can be re-written as

$$\begin{aligned}\dot{V}_1 &= \sigma^{\mathrm{T}}(\Psi(\dot{v},v,\dot{q}_l,q_l)p - K_s\theta_d - J^{\mathrm{T}}(q_l)\lambda) - (p-\hat{p}_s)^{\mathrm{T}}\Gamma_1\dot{\hat{p}} \\ &\quad + \mu^{\mathrm{T}}\dot{\mu} + \varphi_1^{\mathrm{T}}(K_s,\widehat{K}_s^{-1})\Gamma_2\dot{\varphi}_1(K_s,\widehat{K}_s^{-1}) + \sigma^{\mathrm{T}}K_s z. \end{aligned} \tag{29}$$

Letting

$$\theta_d = K_\sigma\sigma - \widehat{K}_s^{-1}J^{\mathrm{T}}(q_l)\lambda_d + \Psi(\dot{v},v,\dot{q}_l,q_l)\hat{p}, \tag{30}$$

where $K_\sigma \in R^{n\times n}$ is positive definite and $\hat{p}$ is the estimate of p, and substituting it into Eq. (29), we have

$$\begin{aligned}\dot{V}_1 &= \sigma^{\mathrm{T}}(\Psi(\dot{v},v,\dot{q}_l,q_l)p - K_s\Psi(\dot{v},v,\dot{q}_l,q_l)\hat{p}) \\ &\quad - \sigma^{\mathrm{T}}K_sK_\sigma\sigma + \sigma^{\mathrm{T}}K_s\widehat{K}_s^{-1}J^{\mathrm{T}}(q_l)\lambda_d - \sigma^{\mathrm{T}}J^{\mathrm{T}}(q_l)\lambda \\ &\quad - (p-\hat{p}_s)^{\mathrm{T}}\Gamma_1\dot{\hat{p}}_s + \mu^{\mathrm{T}}\dot{\mu} \\ &\quad + \varphi_1^{\mathrm{T}}(K_s,\widehat{K}_s^{-1})\Gamma_2\dot{\varphi}_1(K_s,\widehat{K}_s^{-1}) + \sigma^{\mathrm{T}}K_s z. \end{aligned} \tag{31}$$

From the definition of σ in Eq. (17) and Property 1, it is obvious that

$$\sigma^{\mathrm{T}}J^{\mathrm{T}}(q_l)\lambda = \mu^{\mathrm{T}}J^{\mathrm{T}}(q_l)\lambda,$$

$$\sigma^{\mathrm{T}}K_s\widehat{K}_s^{-1}J^{\mathrm{T}}(q_l)\lambda_d = \mu^{\mathrm{T}}J^{\mathrm{T}}(q_l)\lambda_d - \sigma^{\mathrm{T}}K_s\tilde{K}_s^{-1}J^{\mathrm{T}}(q_l)\lambda_d.$$

Substituting the above terms into Eq. (31) and considering Property 5, we have

$$\begin{aligned}\dot{V}_1 &= (p-\hat{p}_s)^{\mathrm{T}}(\Psi^{\mathrm{T}}(\dot{v},v,\dot{q}_l,q_l)\sigma - \Gamma_1\dot{\hat{p}}_s) - \sigma^{\mathrm{T}}K_sK_\sigma\sigma \\ &\quad + \mu^{\mathrm{T}}J^{\mathrm{T}}(q_l)e_\lambda - \sigma^{\mathrm{T}}K_s\tilde{K}_s^{-1}J^{\mathrm{T}}(q_l)\lambda_d + \mu^{\mathrm{T}}\dot{\mu} \\ &\quad + \varphi_1^{\mathrm{T}}(K_s,\widehat{K}_s^{-1})\Gamma_2\dot{\varphi}_1(K_s,\widehat{K}_s^{-1}) + \sigma^{\mathrm{T}}K_s z. \end{aligned} \tag{32}$$

Letting μ evolves according to

$$\dot{\mu} + K_\mu\mu = -J^{\mathrm{T}}(q_l)e_\lambda, \tag{33}$$

where $K_u \in R^{n\times n}$ is a positive definite constant matrix, and substituting it into Eq. (32), we have

$$\begin{aligned}\dot{V}_1 &= (p-\hat{p}_s)^{\mathrm{T}}(\Psi^{\mathrm{T}}(\dot{v},v,\dot{q}_l,q_l)\sigma - \Gamma_1\dot{\hat{p}}_s) \\ &\quad - \sigma^{\mathrm{T}}K_s\tilde{K}_s^{-1}J^{\mathrm{T}}(q_l)\lambda_d - \sigma^{\mathrm{T}}K_sK_\sigma\sigma - \mu^{\mathrm{T}}K_\mu\mu \\ &\quad + \varphi_1^{\mathrm{T}}(K_s,\widehat{K}_s^{-1})\Gamma_2\dot{\varphi}_1(K_s,\widehat{K}_s^{-1}) + \sigma^{\mathrm{T}}K_s z. \end{aligned} \tag{34}$$

As both K_s and $\tilde{K}_s^{-1}$ are diagonal matrices, it is easy to verify that

$$\sigma^{\mathrm{T}}K_s\tilde{K}_s^{-1}J^{\mathrm{T}}(q_l)\lambda_d = \varphi_1^{\mathrm{T}}(K_s,\widehat{K}_s^{-1})\varphi_2(\sigma,q_l,\lambda_d), \tag{35}$$

where

$$\varphi_2(\sigma,q_l,\lambda_d) \triangleq [(J^{\mathrm{T}}(q_l)\lambda_d)_1\sigma_1\ (J^{\mathrm{T}}(q_l)\lambda_d)_2\sigma_2 \cdots (J^{\mathrm{T}}(q_l)\lambda_d)_n\sigma_n]^{\mathrm{T}} \tag{36}$$

and $(J^{\mathrm{T}}(q_l)\lambda_d)_i$ and σ_i are the ith elements of $J^{\mathrm{T}}(q_l)\lambda_d$ and σ, respectively. The vector $\varphi_1(K_s,\widehat{K}_s^{-1})$ is already defined in Eq. (25).

Substituting Eq. (35) into Eq. (34), we have

$$\begin{aligned}\dot{V}_1 &= (p-\hat{p}_s)^{\mathrm{T}}(\Psi^{\mathrm{T}}(\dot{v},v,\dot{q}_l,q_l)\sigma - \Gamma_1\dot{\hat{p}}_s) \\ &\quad - \varphi_1^{\mathrm{T}}(K_s,\widehat{K}_s^{-1})(\varphi_2(\sigma,q_l,\lambda_d) - \Gamma_2\dot{\varphi}_1(K_s,\widehat{K}_s^{-1})) \\ &\quad + \sigma^{\mathrm{T}}K_s z - \mu^{\mathrm{T}}K_\mu\mu - \sigma^{\mathrm{T}}K_sK_\sigma\sigma. \end{aligned} \tag{37}$$

Letting

$$\dot{\hat{p}}_s = \Gamma_1^{-1}\Psi^{\mathrm{T}}(\dot{v},v,\dot{q}_l,q_l)\sigma \tag{38}$$

$$\dot{\varphi}_1(K_s,\widehat{K}_s^{-1}) = \Gamma_2^{-1}\varphi_2(\sigma,q_l,\lambda_d) \tag{39}$$

and substituting them into Eq. (37), it follows that

$$\dot{V}_1 = \sigma^{\mathrm{T}} K_s z - \mu^{\mathrm{T}} K_\mu \mu - \sigma^{\mathrm{T}} K_s K_\sigma \sigma. \tag{40}$$

As K_μ and $K_s K_\sigma$ are all positive definite, it follows that

$$\dot{V}_1 \leqslant \sigma^{\mathrm{T}} K_s z \tag{41}$$

and when $z = 0$, that is, $\theta = \theta_d$, $\dot{V}_1 \leqslant 0$.

From Eq. (38) and the definition of p_s in Property 5, we have

$$\dot{\hat{p}} = \Gamma_p \Psi^{\mathrm{T}}(\dot{v}, v, \dot{q}_l, q_l)\sigma, \tag{42}$$

where $\Gamma_p \triangleq \Lambda^{-1}\Gamma_1^{-1}$.

From Eq. (39) and the definition of $\varphi_1(K_s, \widehat{K}_s^{-1})$ in Eq. (25), we have

$$\dot{\hat{k}}_s^{-1} = -\Gamma_k \varphi_2(\sigma, q_l, \lambda_d), \tag{43}$$

where $\Gamma_k \triangleq K_s^{-1}\Gamma_2^{-1}$.

Once the elements of $\hat{k}_s^{-1}$ are determined through adaptation law in Eq. (43), they are then used to form the diagonal matrix $\widehat{K}_s^{-1}$ needed for the calculation of θ_d in Eq. (30).

Remark 1. The uncertain terms Λ and K_s are "lumped" in the parameter adaptation gains Γ_p and Γ_k, respectively. Γ_p and Γ_k are always positive definite due to the fact that Λ, K_s, Γ_1 and Γ_2 are positive definite, and this meets the requirement for parameter adaptation.

3.2. Step 2. Determination of input torque: τ_m

In this step, the control input τ_m will be derived to make a Lyapunov function of z, e^1 and e_λ non-increasing.

Defining a Lyapunov function

$$\begin{aligned} V_2 &= V_1 + \frac{1}{2} z^{\mathrm{T}} K_s z \\ &= \frac{1}{2}\sigma^{\mathrm{T}} M(q_l)\sigma + \frac{1}{2}(p - \hat{p}_s)^{\mathrm{T}}\Gamma_1(p - \hat{p}_s) + \frac{1}{2}\mu^{\mathrm{T}}\mu \\ &\quad + \varphi_1^{\mathrm{T}}(K_s, \widehat{K}_s^{-1})\Gamma_2\varphi_1(K_s, \widehat{K}_s^{-1}) + \frac{1}{2} z^{\mathrm{T}} K_s z \end{aligned} \tag{44}$$

and differentiating it with respect to time t, we have

$$\dot{V}_2 = \dot{V}_1 + \dot{z}^{\mathrm{T}} K_s z. \tag{45}$$

Obviously, V_2 as well as V_1 is continuous, positive definite and radially unbounded.

With θ_d given in Eq. (30), $\hat{p}$ and $\hat{k}_s^{-1}$ adaptively tuned in Eqs. (42) and (43), respectively, it has been shown that

$$\dot{V}_1 = \sigma^{\mathrm{T}} K_s z - \mu^{\mathrm{T}} K_\mu \mu - \sigma^{\mathrm{T}} K_s K_\sigma \sigma \tag{46}$$

and, as a result,

$$\begin{aligned} \dot{V}_2 &= (\sigma + \dot{z})^{\mathrm{T}} K_s z - \mu^{\mathrm{T}} K_\mu \mu - \sigma^{\mathrm{T}} K_s K_\sigma \sigma \\ &= (\sigma + \dot{z})^{\mathrm{T}}(K_s\theta_d - K_s\theta) - \mu^{\mathrm{T}} K_\mu \mu - \sigma^{\mathrm{T}} K_s K_\sigma \sigma. \end{aligned} \tag{47}$$

Solving $K_s\theta$ in Eq. (2) and substituting it into Eq. (47), we have

$$\dot{V}_2 = (\sigma + \dot{z})^{\mathrm{T}}(K_s\theta_d - \tau_m + J_m\ddot{q}_m) - \mu^{\mathrm{T}} K_\mu \mu - \sigma^{\mathrm{T}} K_s K_\sigma \sigma. \tag{48}$$

Letting

$$\tau_m = \widehat{K}_s\theta_d + \widehat{J}_m\ddot{q}_m + k_\tau \mathrm{sgn}(\sigma + \dot{z}), \tag{49}$$

$$k_\tau \geqslant \delta_K\|\theta_d\| + \delta_J\|\ddot{q}_m\|, \tag{50}$$

where $\widehat{K}_s$ and $\widehat{J}_m$ are the estimates of K_s and J_m, respectively, as defined in Assumption 1, and, δ_K and δ_J are the bounds of $\tilde{K}_s$ and $\tilde{J}_m$, respectively, as defined in Assumption 2. $\mathrm{sgn}(\sigma + \dot{z})$ is a sign function applying on $\sigma + \dot{z}$ element wise such that 1 is returned when an element of $\sigma + \dot{z}$ is non-negative or -1 otherwise.

Substituting τ_m into Eq. (48), it turns out that

$$\begin{aligned} \dot{V}_2 &= (\sigma + \dot{z})^{\mathrm{T}}(\tilde{K}_s\theta_d + \tilde{J}_m\ddot{q}_m - k_\tau \mathrm{sgn}(\sigma + \dot{z})) - \mu^{\mathrm{T}} K_\mu \mu \\ &\quad - \sigma^{\mathrm{T}} K_s K_\sigma \sigma. \end{aligned} \tag{51}$$

Noting the definition of k_τ, we have

$$(\sigma + \dot{z})^{\mathrm{T}}(\tilde{K}_s\theta_d + \tilde{J}_m\ddot{q}_m - k_\tau \mathrm{sgn}(\sigma + \dot{z})) \leqslant 0 \tag{52}$$

and thus

$$\dot{V}_2 \leqslant -\mu^{\mathrm{T}} K_\mu \mu - \sigma^{\mathrm{T}} K_s K_\sigma \sigma \leqslant 0. \tag{53}$$

From here, we are ready to study the boundedness and the convergence of the closed-loop signals.

As $\dot{V}_2 \leqslant 0$ and V_2 is non-increasing, this guarantees the boundedness of σ, $p - \hat{p}_s$, μ, $\varphi_1(K_s, \widehat{K}_s^{-1})$ and z, consequently, v, $\varphi_2(\sigma, q_l, \lambda_d)$ and r^1 are also bounded from the definition. This means that all of the signals of the closed-loop system are bounded on $[0, t_f]$ where t_f defines the maximum time interval for the existence of these signals. It is obvious that the boundedness of the signals is independent of t_f, and thus all the closed loop signals are globally uniformly bounded on $[0, \infty]$.

To analyze the force response, first $\Psi(\dot{v}, v, \dot{q}_l, q_l)\hat{p}$ is expanded from Property 4,

$$\Psi(\dot{v}, v, \dot{q}_l, q_l)\hat{p} = \widehat{M}(q_l)\dot{v} + \widehat{C}(q_l, \dot{q}_l)v + \widehat{G}(q_l). \tag{54}$$

Substituting v defined in Eq. (18) into Eq. (54) and considering Eq. (33), we have

$$\begin{aligned} \Psi(\dot{v}, v, \dot{q}_l, q_l)\hat{p} &= \widehat{M}^1(q_l)\ddot{q}_r^1 + \widehat{C}^1(q_l, \dot{q}_l)\dot{q}_r^1 \\ &\quad + \widehat{G}^1(q_l), -(\widehat{M}(q_l)K_\mu - \widehat{C}(q_l, \dot{q}_l))\mu \\ &\quad - \widehat{M}(q_l)J^{\mathrm{T}}(q_l)e_\lambda, \end{aligned} \tag{55}$$

thus, θ_d in Eq. (30) is re-written as

$$\begin{aligned} \theta_d &= K_\sigma\sigma - \widehat{K}_s^{-1}J^{\mathrm{T}}(q_l)\lambda_d + \widehat{M}^1(q_l)\ddot{q}_r^1 + \widehat{C}^1(q_l, \dot{q}_l)\dot{q}_r^1 \\ &\quad + \widehat{G}^1(q_l) - (\widehat{M}(q_l)K_\mu - \widehat{C}(q_l, \dot{q}_l))\mu - \widehat{M}(q_l)J^{\mathrm{T}}(q_l)e_\lambda. \end{aligned} \tag{56}$$

Noting that $\theta = \theta_d - z$ and substituting it into Eq. (10), the following closed-loop system dynamics is obtained after some manipulations on the terms in the equation

$$(K_s\widehat{M}(q_l) + I_{n-m})e_\lambda = M^1(q_l)\dot{r}^1 + C^1(q_l,\dot{q}_l)r^1 + K_s\tilde{K}_s^{-1}J^{\mathrm{T}}(q_l)\lambda_d - K_s z - \beta, \quad (57)$$

where

$$\beta \triangleq (M(q_l) - K_s\widehat{M}^1(q_l))\ddot{q}_r^1 + (C(q_l,\dot{q}_l) - K_s\widehat{C}^1(q_l,\dot{q}_l))\dot{q}_r^1 + G(q_l) - K_s\widehat{G}^1(q_l) \quad (58)$$

and $\tilde{K}_s^{-1} = K_s^{-1} - \widehat{K}_s^{-1}$.

Multiplying both sides of Eq. (57) by $J(q_l)M^{-1}(q_l)$ and noting that $J(q_l)M(q_l)M^1(q_l) = J(q_l)L(q_l) = 0$, we have

$$J(q_l)M^{-1}(q_l)(K_s\widehat{M}(q_l) + I_{n-m})e_\lambda = J(q_l)M^{-1}(q_l)(C^1(q_l,\dot{q}_l)r^1 + K_s\tilde{K}_s^{-1}J^{\mathrm{T}}(q_l)\lambda_d - K_s z - \beta), \quad (59)$$

β is bounded as $\ddot{q}_r^1 = \dot{q}_{ld}^1 + K_e\dot{e}^1$ and the other terms in Eq. (58) are bounded. With β and the rest terms in Eq. (59) being bounded and $J(q_l)M^{-1}(q_l)(K_s\widehat{M}(q_l) + I_{n-m})$ being non-singular, it is concluded that e_λ is bounded.

Now we have shown that σ, $p - \hat{p}_s$, μ, $\varphi_1(K_s, \widehat{K}_s^{-1})$, z, v, $\varphi_2(\sigma,q_l,\lambda_d)$, r^1 and e_λ are bounded. From the definitions of r^1 and σ in Eqs. (16) and (17), respectively, it follows that r^1, e^1 and $\dot{e}^1$ are all bounded. The boundedness of $q_l(q_l^1)$, $\dot{q}_l(\dot{q}_l^1)$, λ and θ_d are guaranteed from their relations to e^1, $\dot{e}^1$, e_λ and planned q_{ld}^1, $\dot{q}_{ld}$ and λ_d which are all bounded. As θ_d and z are bounded, θ is bounded too. Studying Eq. (9) and multiplying both of its sides by $L^{\mathrm{T}}(q_l)$, we have

$$M_l(q_l)\ddot{q}_l^1 = -C_l(q_l,\dot{q}_l)\dot{q}_l^1 - G_l(q_l) + L^{\mathrm{T}}(q_l)K_s\theta. \quad (60)$$

Note the Property 1 is used in the above derivation.

From Property 2, $M_1(q_l)$ is symmetric positive definite and other matric terms in the righthand equation are bounded. As proved, the terms q_l^1, $\dot{q}_l^1$ and θ are also bounded. It is obvious from Eq. (60) that $\ddot{q}_l^1$ is bounded. It follows that $\ddot{e}^1 = \ddot{q}_{ld}^1 - \ddot{q}_l^1$ is also bounded. Now that e^1 and $\ddot{e}^1$ are bounded, we can conclude that $\dot{e}^1 \to 0$ when $t \to \infty$ from Corollary 1 (Barbalat).

From Eqs. (16) and (19), $\dot{r}^1 = \ddot{q}_{ld}^1 + K_e\ddot{e}^1 - \ddot{q}_l^1$ which is bounded due to the boundedness of all the terms in its definition. As r^1 is also bounded, we have $r^1 \to 0$ when $t \to \infty$ from Corollary 1 (Barbalat). From the definition of r^1 in Eq. (16), it can be concluded that $e^1 \to 0$, or equivalently, $q_l^1 \to q_{ld}^1$ when $t \to \infty$. As q_l is uniquely determined by q_l^1, it can be concluded that $q_l \to q_{ld}$ when $t \to \infty$.

Summarizing what discussed in Steps 1 and 2, we have the following theorem.

Theorem 1. *For a constrained flexible joint robotic manipulator modeled by Eqs. (1) and (2), the robot position q_l converges to its desired value q_{ld} and the force tracking error $\lambda_d - \lambda$ is uniformly bounded if*

$$\tau_m = \widehat{K}_s\theta_d + \widehat{J}_m\ddot{q}_m + k_\tau\mathrm{sgn}(\sigma + \dot{z}), \quad (61)$$

$$k_\tau \geqslant \delta_K\|\theta_d\| + \delta_J\|\ddot{q}_m\|, \quad (62)$$

where

$$z = \theta_d - \theta,$$

$$\theta_d = K_\sigma\sigma - \widehat{K}_s^{-1}J^{\mathrm{T}}(q_l)\lambda_d + \Psi(\dot{v},v,\dot{q}_l,q_l)\hat{p},$$

$$\theta = q_m - q_l,$$

$$\dot{\hat{p}} = \Gamma_p\Psi^{\mathrm{T}}(\dot{v},v,\dot{q}_l,q_l)\sigma,$$

$$\dot{\hat{k}}_s^{-1} = -\Gamma_k\varphi_2(\sigma,q_l,\lambda_d),$$

$$\hat{k}_s^{-1} = [k_{s1}^{-1}\ \ k_{s2}^{-1}\ \ \cdots\ \ k_{sn}^{-1}]^{\mathrm{T}},$$

$$\widehat{K}_s^{-1} = \mathrm{diag}[k_{si}^{-1}],\ i = 1,2,\ldots,n,$$

$$\sigma = L(q_l)r^1 + \mu,$$

$$v = L(q_l)\dot{q}_r^1 + \mu,$$

$$\dot{\mu} + K_\mu\mu = -J^{\mathrm{T}}(q_l)e_\lambda,$$

$$r^1 = \dot{e}^1 + K_e e^1,$$

$$e^1 = q_{ld}^1 - q_l^1,$$

$$e_\lambda = \lambda_d - \lambda,$$

$$\dot{q}_r^1 = \dot{q}_{ld}^1 + K_e e^1,$$

$K_\sigma > 0$, $\Gamma_p > 0$ and $\Gamma_k > 0$ are control parameters, $\widehat{K}_s$ and $\widehat{J}_m$ are the estimates of K_s and J_m, respectively. Regress matrix $\Psi(\dot{v},v,\dot{q}_l,q_l)$, δ_K and δ_J are defined in Property 4 and Assumption 2, respectively, and $\varphi_2(\sigma,q_l,\lambda_d)$ is defined in Eq. (36). K_σ, K_e and K_μ are all positive definite control gain matrices. Γ_p and Γ_k are positive definite parameter adaptation gain matrices.

Remark 2. The calculation of $\dot{z}$ in Eq. (61) is cumbersome. To avoid it, a new variable s_z is defined such that

$$s_z = \int_0^t \sigma\,\mathrm{d}t + z. \quad (63)$$

Obviously

$$\mathrm{sgn}(\sigma + \dot{z}) = \mathrm{sgn}(\dot{s}_z) \quad (64)$$

and we only need to check the changing trend of s_z to get the sign of $\dot{s}_z$ or $\sigma + \dot{z}$. In implementation, this check can be made by comparing the value of s_z in consecutive sampling times. If s_z is non-decreasing, the sign should be positive, or it should be negative.

Remark 3. It is well known that the sign function in the controller cause chattering problem which can be eliminated with the boundary layer approach [24]. In this approach, the sign function $\mathrm{sgn}(s)$ is replaced by s/Δ when $\|s\| < \Delta$. $\Delta > 0$ is defined as a boundary layer.

Remark 4. The measurement of joint acceleration $\ddot{q}_m$ is required in the controller. In practice, it is not difficult to directly measure the joint acceleration. It can also be obtained by taking the difference of the joint velocity in consecutive sampling times.

Remark 5. Though theoretically the controller can guarantee the convergence of the link position q_l to its desired value, and the boundedness of the force tracking error e_λ without special requirement of the joint stiffness, a large joint stiffness (weak flexibility) increases the sensitivity of the control system to the variations of the signals in the control loops through the term $K_s\theta$. This may cause the system response divergent. In this case, the singular perturbation based controller [5] is more suitable.

4. Simulation

The simulation example is schematically shown in Fig. 1. In this example, the end effector of a flexible joint manipulator moves along a part of the constraint surface and exerts a force on it at the same time. The length, inertia and the mass of each link of the manipulator are $l_i = 0.3$ m, $I_i = 0.3$ kg m^2 and $m_i = 0.1$ kg, respectively $(i = 1,2)$. The half of the link length is $d_i = \frac{l_i}{2} = 0.15$ m $(i = 1,2)$. The mass center of each link is assumed to be in the middle of the link.

In Fig. 1, the world coordinates is denoted by oxy. The constraint surface is described by

$$\Phi(r_d) = x_d - y_d + 0.35 = 0. \tag{65}$$

It is planned that the end effector moves along the following trajectory on the constraint surface

$$x_d(t) = -\frac{1}{10}\cos(2t),$$
$$y_d(t) = 0.35 - \frac{1}{10}\cos(2t),$$

while the magnitude of the constraint force is kept at $\lambda_d = 2$ N.

Let the link position $q_l = [\theta_1\ \theta_2]^{\mathrm{T}}$ and its partitions are $q_l^1 = \theta_1$ and $q_l^2 = \theta_2$, respectively. The desired link positions $q_{ld} = [\theta_{1d}\theta_{2d}]^{\mathrm{T}}$ can be obtained from $x_d(t)$ and $y_d(t)$ such that

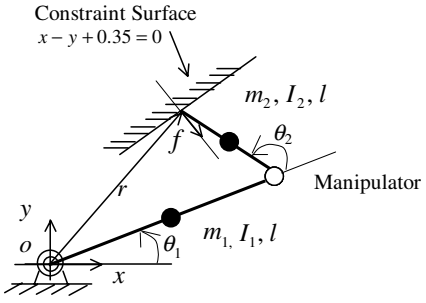

Fig. 1. Simulation example.

$$\cos(\theta_{2d}(t)) = \frac{x_d^2(t) + y_d^2(t) - d_1^2 - d_2^2}{2d_1d_2},$$
$$\sin(\theta_{2d}(t)) = -\sqrt{1 - \cos^2(\theta_{2d}(t))},$$
$$\cos(\theta_{1d}(t)) = \frac{(d_1 + d_2\cos(\theta_{2d}(t)))x_d(t) + d_2\sin(\theta_{2d}(t)y_d(t))}{x_d^2(t) + y_d^2(t)},$$
$$\sin(\theta_{1d}(t)) = \frac{(d_1 + d_2\cos(\theta_{2d}(t)))y_d(t) - d_2\sin(\theta_{2d}(t)x_d(t))}{x_d^2(t) + y_d^2(t)}.$$

Jacobians $J(q_l)$ and $L(q_l^1)$ are obtained as follows:

$$J(q_l) = \begin{bmatrix} -d_1\sin(\theta_1) - d_2\sin(\theta_1+\theta_2) & -d_2\sin(\theta_1+\theta_2) \\ d_1\cos(\theta_1) + d_2\cos(\theta_1+\theta_2) & d_2\cos(\theta_1+\theta_2) \end{bmatrix}, \tag{66}$$

$$L(q_l^1) = \left[1 - 1 - \frac{d_1(\sin(\theta_1) + \cos(\theta_1))}{d_2\sin(\theta_1+\theta_2)\cos(\theta_1+\theta_2)}\right] \tag{67}$$

with $J(q_l)$, the desired link velocity $\dot{q}_{ld}$ is obtained through $\dot{q}_{ld} = J(q_{ld})[\dot{x}_d\dot{y}_d]^{\mathrm{T}}$.

Choosing the system parameter vector p as

$$p = [p_1\ p_2\ p_3\ p_4\ p_5\ p_1\ p_2\ p_3\ p_4\ p_5],$$

where $p_1 = I_1 + m_1l_1^2 + I_2 + m_2(d_1^2 + l_2^2)$, $p_2 = m_2d_1l_2$, $p_3 = I_2 + m_2l_2^2$, $p_4 = (m_1l_1 + m_2d_1)g$, $p_5 = m_2l_2g$ and $g = 9.8$ m/s^2 is the gravitational acceleration.

The regressor matrix $\Psi(\dot{v}, v, \dot{q}_l, q_l)$ is expressed in a required form in Eq. (13) such that

$$\Psi(\dot{v},v,\dot{q}_l,q_l) = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Psi_{21} & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} \end{bmatrix},$$

where

$$\Psi_{11} = \dot{v}_1,$$
$$\Psi_{12} = 2\cos(\theta_2)\dot{v}_1 + \cos(\theta_2)\dot{v}_2 - \sin(\theta_2)(\dot{\theta}_1 + \dot{\theta}_2)v_2 - \sin(\theta_2)\dot{\theta}_2v_1,$$
$$\Psi_{13} = \dot{v}_2,$$
$$\Psi_{14} = \cos(\theta_1),$$
$$\Psi_{15} = \cos(\theta_1 + \theta_2),$$
$$\Psi_{21} = 0,$$
$$\Psi_{22} = \cos(\theta_2)\dot{v}_1 + \sin(\theta_2)\dot{\theta}_1v_1,$$
$$\Psi_{23} = \dot{v}_2 + \dot{v}_1,$$
$$\Psi_{24} = 0,$$
$$\Psi_{25} = \cos(\theta_1 + \theta_2)$$

with the given physical parameters, it can be calculated that $p_1 = 0.6135$, $p_2 = 0.0045$, $p_3 = 0.30225$, $p_4 = 0.441$ and $p_5 = 0.147$. For the simulation purpose, the initial values of their estimates are set as $\hat{p}_1(0) = 0.092$, $\hat{p}_2(0) = 0.0007$, $\hat{p}_3(0) = 0.045$, $\hat{p}_4(0) = 0.066$ and $\hat{p}_5(0) = 0.022$. Assume that the robotic manipulator is initially at rest with $q_l(0) = [2.85\quad -1.77]^{\mathrm{T}}$ (rad), $\dot{q}_l(0) = [0\quad 0]^{\mathrm{T}}$ and $\lambda(0) = 2.0$ N.

To show the effectiveness of the controller, the simulations are done for the robot with different joint flexibilities.

4.1. Simulation in the case when the joint flexibility is strong (stiffness is small)

In this case, the joint stiffness are set as $k_{s1} = k_{s2} = 10.0$ N m. The moments of inertia of the motors are set as $j_{m1} = j_{m2} = 0.5$ N m s^2. The estimates of J_m and K_s are set to be $\widehat{K}_s = \mathrm{diag}[8.0] \in R^{2\times2}$ and $\widehat{J}_m = \mathrm{diag}[0.4] \in R^{2\times2}$, respectively. The up bounds of the norms of $\tilde{K}_s$ and $\tilde{J}_m$ are thus chosen as $\delta_K = 4$ and $\delta_{J_1} = 2$, respectively. The initial value of the estimate $\widehat{K}_s^{-1}$ is set as $\widehat{K}_s^{-1}(0) = \mathrm{diag}[0.125] \in R^{2\times2}$. The control gains and parameter adaptation gains are chosen as $K_\sigma = \mathrm{diag}[1.0] \in R^{2\times2}$, $K_e = \mathrm{diag}[3.5] \in R^{2\times2}$, $K_\mu = \mathrm{diag}[1.5] \in R^{2\times2}$, $\Gamma_p = \Gamma_K = \mathrm{diag}[0.08] \in R^{2\times2}$. The width of the boundary layer is chosen to be $\Delta = 0.002$.

The position tracking performances of the robot and the force tracking performances are plotted in Figs. 2 and 3, respectively. The control torques for the manipulators are given in Fig. 4. The performance of parameter adaptation, represented by those of p_1 to p_4, and k_{s1} to k_{s2}, are plotted in Figs. 5 and 6 respectively. It can be seen that under the proposed controller, the link positions of the robot converge to its desired values and the force tracking error is bounded. The control torques demonstrate big fluctuation at the beginning, but are within a reasonable range after some time. The parameter estimation are also stabilized and bounded. Due to the introduction of boundary layer, the chatterings in torque and force signals are smoothed out.

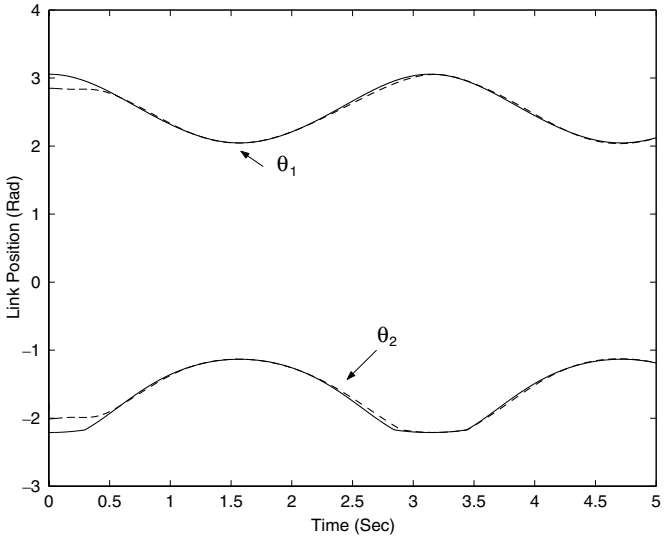

Fig. 2. Position tracking when $K_s = \mathrm{diag}[10.0] \in R^{2\times2}$ (solid: desired position, dashed: actual position).

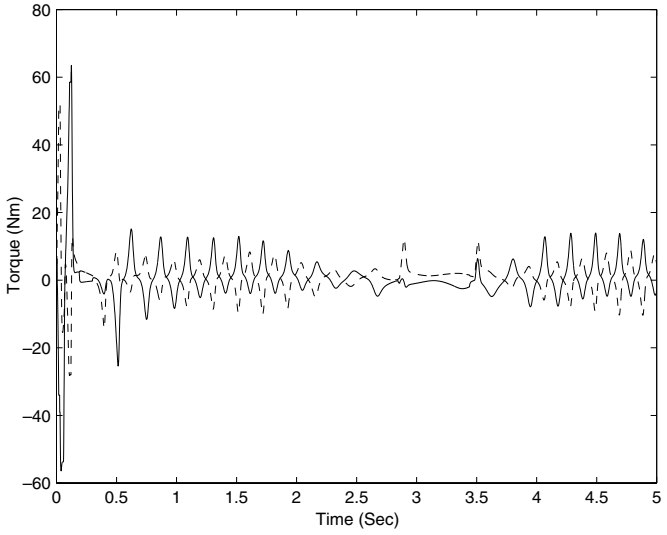

Fig. 4. Joint torques when $K_s = \mathrm{diag}[10.0] \in R^{2\times2}$ (solid: joint 1 torque, dashed: joint 2 torque).

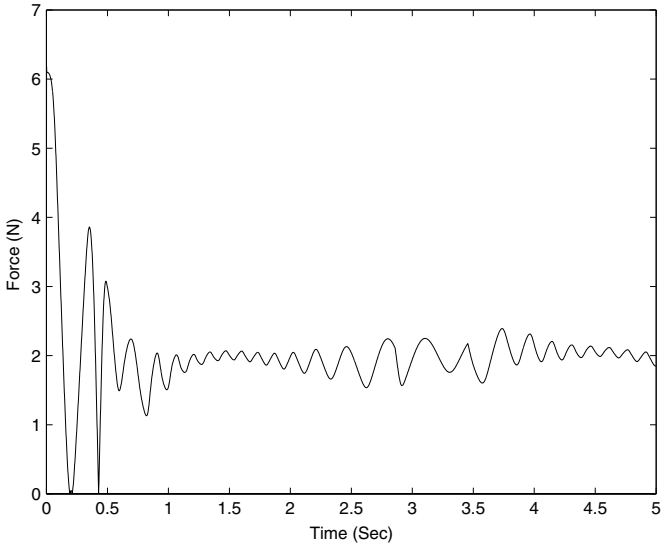

Fig. 3. Force tracking when $K_s = \mathrm{diag}[10.0] \in R^{2\times2}$ (solid: desired force, dashed: actual force).

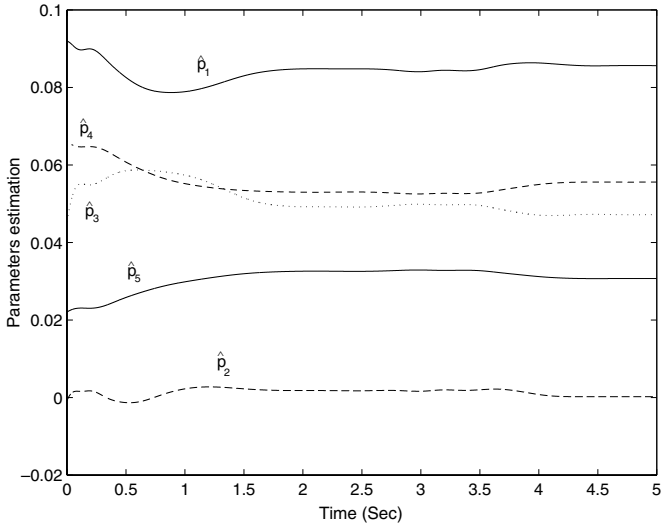

Fig. 5. Parameter estimations ($\hat{p}_1$, $\hat{p}_2$, $\hat{p}_3$, $\hat{p}_4$ and $\hat{p}_5$) when $K_s = \mathrm{diag}[10.0] \in R^{2\times2}$.

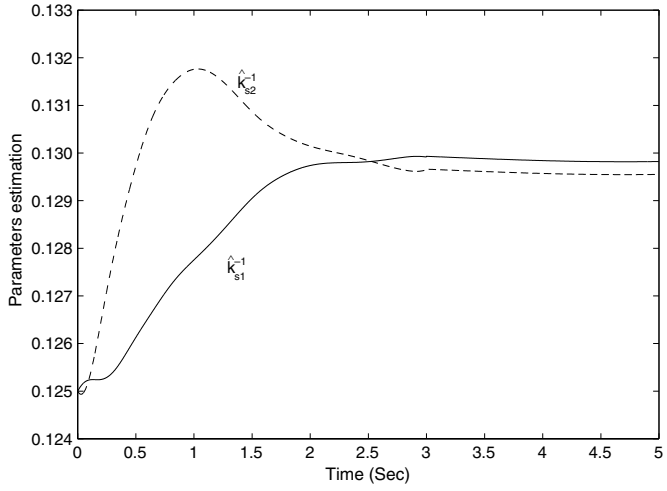

Fig. 6. Parameter estimations ($\hat{k}_{s1}^{-1}$ and $\hat{k}_{s2}^{-1}$) when $K_s = \text{diag}[10.0] \in R^{2\times2}$.

4.2. Simulation in the case when the joint flexibility is weak (stiffness is large)

In this case, the joint stiffness are set to be a bigger value: $k_{s1} = k_{s2} = 50.0Nm$. The initial value of the estimate $\hat{K}_s^{-1}$ is set as $\hat{K}_s^{-1}(0) = diag[0.02] \in R^{2\times2}$. The rest parameters of the controller remain the same as in the case when the joint flexibility is strong. The simulation results are plotted from Figs. 7–11. It can be observed that though the position tracking is almost as good as that in the case when the joint flexibility is strong, the responses of the constraint force and the control inputs demonstrate bigger fluctuations and chatterings. The magnitudes of the joint torques also increase sharply. The simulation is also done for the case when K_s becomes very big (e.g. $K_s = \text{diag}[100.0] \in R^{2\times2}$). The responses of the control system become divergent. As explained in Remark 5, in this case, the singular perturbation based controller is more suitable, and this will be presented in a separate paper.

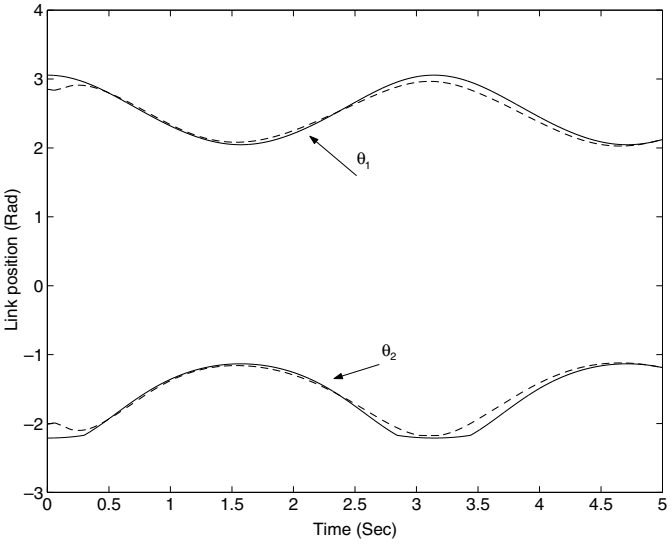

Fig. 7. Position tracking when $K_s = \text{diag}[50.0] \in R^{2\times2}$ (solid: desired position, dashed: actual position).

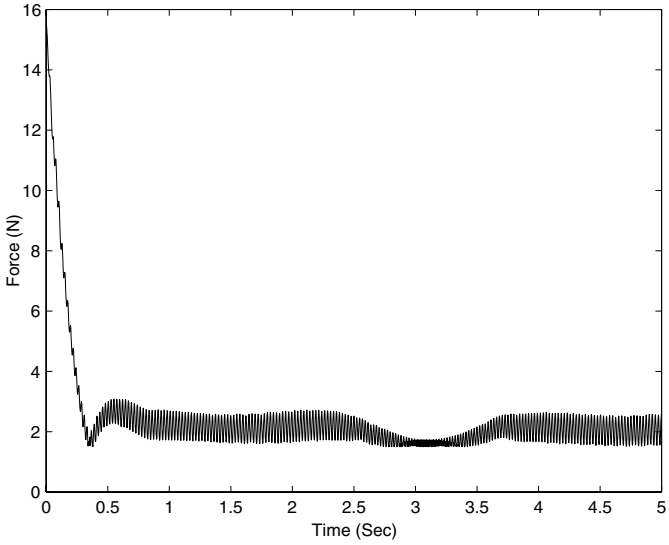

Fig. 8. Force tracking when $K_s = \text{diag}[50.0] \in R^{2\times2}$ (solid: desired force, dashed: actual force).

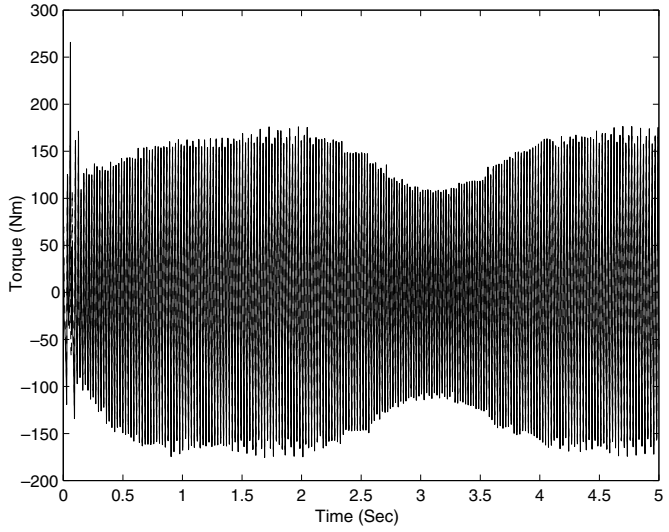

Fig. 9. Joint torques when $K_s = \text{diag}[50.0] \in R^{2\times2}$ (solid: joint 1 torque, dashed: joint 2 torque).

5. Conclusion

This paper addressed the issue of adaptive position/force control of an uncertain constrained flexible joint robots (FJRs) using singular perturbation approach. The fast variables and the slow variables were defined by combing the force and position signals and the controller relied on the feedback of joint state variables (position, velocity and force) and avoided the noisy joint torque feedbacks found in many controllers developed. The proposed approach achieves the position tracking and the

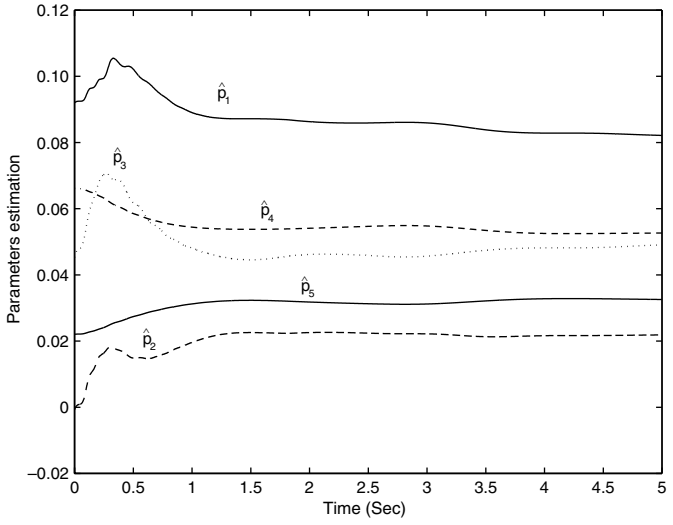

Fig. 10. Parameter estimations ($\hat{p}_1$, $\hat{p}_2$, $\hat{p}_3$, $\hat{p}_4$ and $\hat{p}_5$) when $K_s = \text{diag}[50.0] \in R^{2\times 2}$.

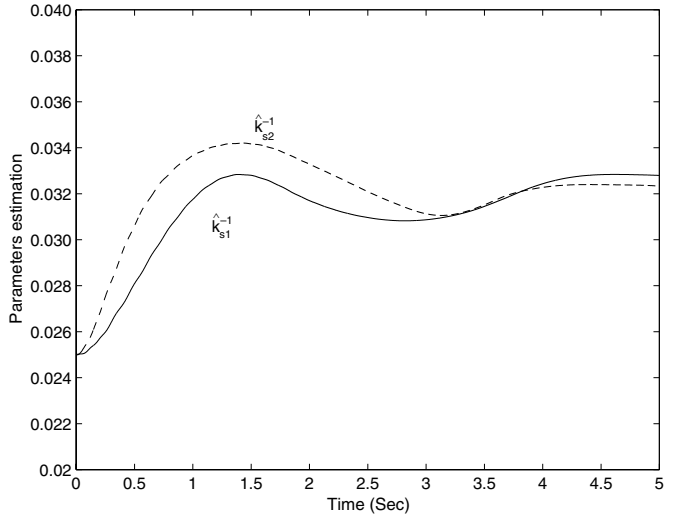

Fig. 11. Parameter estimations ($\hat{k}_{s1}^{-1}$ and $\hat{k}_{s2}^{-1}$) when $K_s = \text{diag}[50.0] \in R^{2\times 2}$.

boundedness of force errors for the constrained robot with weak joint flexibility (large joint stiffness).

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