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ADAPTIVE CONTROL OF ROBOTS HAVING BOTH DYNAMICAL PARAMETER UNCERTAINTIES AND UNKNOWN INPUT SCALINGS

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Abstract—The control problems of robots having both dynamical parameter uncertainties and unknown input scalings are addressed. By appropriately defining the parameters of interest and the corresponding regressor, an adaptive control law is presented by exploiting the structural properties of the resulting dynamical models. It is shown that the tracking errors converge to zero and all the signals in the closed-loop system are bounded. Even though the input scalings are unknown, there is no need to adapt them because of the particular controller structure introduced here. Due to the introduction of the input scalings, uncertainties in the transformation mechanisms of robots are allowed. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

Intensive studies have been carried out in the literature on rigid body robots described by a set of second order differential equations, characterising the dynamic behaviour of robots. However, most of the research work has been carried out under the assumption that the system is subject to external force/torque of the form $\Lambda\tau$, $\Lambda = I$ (unit matrix.) This is hardly the case in reality owing to uncertainties in power transmission mechanisms, such as gear boxes. It is true that input scalings can be determined by off-line estimation. However, no matter how accurate they are, they are still not the true values theoretically. Most importantly, because of aging, electronics drifting, wearing out and deterioration, Λ will inevitably differ from the initial assembly values as time goes by. It becomes very inconvenient to estimate Λ from time to time. An adaptive control method is proposed here to solve this problem. As a result, accurate off-line estimation for Λ is not necessary, and periodic off-line estimations are eliminated by the proposed adaptive controller. The most important advantage of the current problem formulation is that there is no need to estimate the input scalings off-line in practice any more. Because of the introduction of the input scalings, uncertainties in the transformation mechanisms of robots are allowed.

When Λ is unknown, the control problem is not trivial. The presence of unknown Λ makes the control problem very much different from that in which $\Lambda = I$. The well known computed torque method is not acceptable, at least theoretically, when Λ is unknown. It is true that the unknown input scaling Λ can be removed from the input side and transformed into the unknown parameters of linear-in-the-parameters dynamics by pre-multiplying both sides of the equations by Λ^{-1}. However, $\Lambda^{-1}D(q) \neq D(q)\Lambda^{-1}$ in general, even though $D(q) = D(q)^T$ and $\Lambda^{-1} = [\Lambda^{-1}]^T$. Here, $D(q)$ is the inertia matrix of a robot. Thus, the adaptive control laws relying on the property of $D(q)$ cannot be applied directly [1–6].

By appropriately defining the parameters of interest and the corresponding regressor, an adaptive control law is shown to be very effective in solving both parameter uncertainties and unknown input scalings. The control law can be taken as the generalisation of the control law for robots with unit input scaling matrices [1]. In other words, with very few amendments, the adaptive control law for robots with unit input scaling matrices [1] can be applied to the case when Λ is unknown.

Suppose $h(t)$ and $r(t)$ are matrix and vector functions of time, respectively, and $h \times r$ denotes the convolution product of h and r. Let $H(s)$ be the Laplace transform of matrix $h(t)$, if it exists.

Lemma 1.1. Let $e(t) = h \times r$, where $h = L^{-1}(H(s))$ and $H(s)$ is an $n \times n$ strictly proper, exponentially stable transfer function. Then $r \in L_n^2 \Rightarrow e \in L_n^2 \cap L_n^\infty$, $\dot{e} \in L_n^2$, e is continuous and $e \to 0$ as $t \to \infty$. If, in addition, $r \to 0$ as $t \to \infty$, then $\dot{e} \to 0$ [7]. □

In the context of this paper, let $q_d(t) \in C^2$, a twice differentiable vector, and let q_d, $\dot{q}_d$ and $\ddot{q}_d$ denote the desired position, velocity and acceleration vectors of the desired trajectories. Define the tracking error as

$$e(t) = q_d - q \tag{1}$$

and introduce the following notation:

$$r(t) = h^{-1} \times e(t) \tag{2}$$

$$\ddot{q}_r = r(t) + \ddot{q}, \tag{3}$$

where $h^{-1} = L^{-1}(H(s)^{-1})$ and $H(s)$ is an $n \times n$ strictly proper, exponentially stable transfer function of the form $H(s) = \mathrm{diag}[(s^m + n_i(s))/(s^{m+1} + d_i(s))]$, where $n_i(s)$ and $d_i(s)$ are the remaining polynomial terms of s for the ith entry. This in turn guarantees $\ddot{q}_r$ is a vector without $\dot{q}$ explicitly in it. For more detailed discussion on the definition for $H(s)$, see [8].

2. DYNAMIC MODELLING OF ROBOTS

In this paper, the dynamics of rigid body robots are described by the following general second-order differential equation:

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \Lambda\tau, \tag{4}$$

where

q is a vector of the link positions

$D(q)$ is the symmetric positive definite inertia matrix of robots

$C(q, \dot{q})\dot{q}$ represents the Coriolis and centrifugal forces
$G(q)$ is a vector which represents the gravitational forces
$\Lambda\tau$ is a vector of the torques (or forces) acting on the joints, with Λ being the constant diagonal input scaling matrix, and τ the control input.

For $\Lambda = I$, Eqn (4) becomes the conventional equation of robot dynamics. However, owing to the uncertainties in the power transmission mechanisms, such as gear boxes, Λ may be unknown. The presence of Λ is to highlight this situation. Without loss of generality, Λ is assumed to be positive definite here.

To facilitate controller design, the following properties are presented.

Property 1. $D(q)$ is symmetric and positive definite
In the proof of the closed-loop stability, the symmetric, and positive definite nature of the inertia matrix $D(q)$ is thoroughly exploited.

Property 2. $\dot{D}(q) - 2C(q, \dot{q})$ is skew-symmetric
That is, the identity $s^T[\dot{D}(q) - 2C(q, \dot{q})]s = 0$ holds, $\forall s \in R^n$ if the matrix $C(q, \dot{q})$ is defined by the so-called Christoffel symbols [4].

Property 3. Linear-in-the-parameters dynamics of Eqn (4) enables us to write [4]

$$D(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + G(q) = \Psi(q, \dot{q}, \dot{q}_r, \ddot{q}_r)P, \tag{5}$$

where $P \in R^l$ is the vector of dynamical parameters of interest and $\Psi(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \in R^{n \times l}$ is the corresponding known function regressor.

In order to solve the problem of unknown input scalings, Ψ and P have to take the following form:

$$\Psi(q, \dot{q}, \dot{q}_r, \ddot{q}_r)P = \begin{bmatrix} \psi_1^T p_1 \\ \psi_2^T p_2 \\ \cdots \\ \psi_n^T p_n \end{bmatrix}, \tag{6}$$

where

$$P = \begin{bmatrix} p_1 \\ p_2 \\ \cdots \\ p_n \end{bmatrix}, \quad \Psi = \begin{bmatrix} \psi_1^T & 0 & \cdots & 0 \\ 0 & \psi_2^T & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \psi_n^T \end{bmatrix} \tag{7}$$

with $p_i \in R^{n_i}$ being the ith vector of parameters of interest for the ith degree-of-freedom and $\psi_i \in R^{n_i}$ being the so-called corresponding regressor vector.

The definitions for Ψ and P are not unique in general. However, they should be defined as in Eqn (7) to facilitate the adaptive controller design later in the paper. For $\Lambda = \mathrm{diag}[\lambda_i]$, $\lambda_i \in R$, we have

$$\Lambda\Psi(q, \dot{q}, \dot{q}_r, \ddot{q}_r)P = \Lambda \begin{bmatrix} \psi_1^T p_1 \\ \psi_2^T p_2 \\ \cdots \\ \psi_n^T p_n \end{bmatrix} = \Psi(q, \dot{q}, \dot{q}_r, \ddot{q}_r)P_\lambda, \tag{8}$$

where

$$P_\lambda := \begin{bmatrix} \lambda_1 p_1 \\ \lambda_2 p_2 \\ \cdots \\ \lambda_n p_n \end{bmatrix} = M(\Lambda)P \tag{9}$$

$$M(\Lambda) := \operatorname{diag}[U_i(\lambda_i)], \qquad U_i(\lambda_i) = \operatorname{diag}[\lambda_i] \in R^{n_i \times n_i}. \tag{10}$$

Remarks

(1) Equation (4) can be rewritten as

$$\Lambda^{-1}[D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q)] = \tau, \tag{11}$$

which leads to

$$\Lambda^{-1}\Psi(q, \dot{q}, \dot{q}, \ddot{q})P = \tau. \tag{12}$$

From Eqn (8), we have

$$\Psi(q, \dot{q}, \dot{q}, \ddot{q})P_1 = \tau, \qquad P_1 = M(\Lambda^{-1})P, \tag{13}$$

where $M(\cdot)$ is defined by Eqn (10). It is clear that the unknown scaling matrix Λ in system (4) has been successfully transformed into parameters of linear-in-the-parameters dynamics as shown in Eqn (13). However, $\Lambda^{-1}D(q)$ is not symmetric in general even though both Λ^{-1} and $D(q)$ are symmetric. Thus, the adaptive control laws developed using the symmetry of $D(q)$ cannot be applied directly to Eqn (13) in this case.

(2) Closed-loop stability analysis is difficult for Computed Torque Method even if $D(q)$, $C(q, \dot{q})$ and $G(q)$ are known exactly. Consider the control law

$$\tau = \bar{\Lambda}^{-1}[D(q)(\ddot{q}_d + K_v\dot{e} + K_p e) + C(q, \dot{q})\dot{q} + G(q)], \tag{14}$$

where $K_v > 0$, $K_p > 0$ and $\bar{\Lambda}$ is the estimate of Λ. Suppose that $\Lambda\bar{\Lambda}^{-1} = I + \alpha$, we have an error equation of the form

$$D(q)(\ddot{e} + K_v\dot{e} + K_p e) = \alpha[D(q)(\ddot{q}_d + K_v\dot{e} + K_p e) + C(q, \dot{q})\dot{q} + G(q)].$$

When $\alpha = 0$, i.e. $\bar{\Lambda} = \Lambda$, it is easy to see that the system is stable. However, when $\alpha \neq 0$, i.e. $\bar{\Lambda} \neq \Lambda$, closed-loop stability cannot be established easily.

3. ADAPTIVE CONTROLLER DESIGN

In this section, an adaptive controller is introduced and overall closed-loop stability is guaranteed. Let $(\hat{\cdot})$ be the estimate of $(\cdot)$, and define $(\tilde{\cdot}) = (\cdot) - (\hat{\cdot})$. Further, let $\hat{D}(q)$, $\hat{C}(q, \dot{q})$ and $\hat{G}(q)$ correspond to the estimates of $D(q)$, $C(q, \dot{q})$ and $G(q)$, respectively, obtained by substituting the true parameter vector P by the estimated parameter vector $\hat{P}$. Then, we have

$$\hat{D}(q)\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{G}(q) = \Psi\hat{P}. \tag{15}$$

Since $\dot{q} = \dot{q}_r - r(t)$ and $\ddot{q} = \ddot{q}_r - \dot{r}(t)$ by Eqn (3), the left hand side of Eqn (4) can be written as

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = D(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + G(q) - D(q)\dot{r} - C(q, \dot{q})r$$
$$= \Psi P - [D(q)\dot{r} + C(q, \dot{q})r]. \qquad (16)$$

Let the control law take the following form:

$$\tau = \bar{\Lambda}^{-1}[\hat{D}(q)\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{G}(q) + K(t)r + K_i\int_0^t r(\tau)\,d\tau] \qquad (17)$$

$$= \bar{\Lambda}^{-1}[\Psi\hat{P} + K(t)r + K_i\int_0^t r(\tau)\,d\tau], \qquad (18)$$

where $\bar{\Lambda} = \mathrm{diag}[\bar{\lambda}_i] > 0$ is an estimate of the input scaling matrix Λ, which is not to be updated on-line. For $\bar{\Lambda} = I$, the above controller coincides with the controller given in [1] for the usual robotic dynamics, when $\Lambda = I$ in Eqn (4). This gives a PID type controller because of the special definition of r, with K_i introduced to eliminate static tracking errors. It is interesting that K_i does not appear in the derivative of the Lyapunov function used in the stability proof; see Appendix A. From Eqns (4), (16) and (18), we have an error equation of the form

$$D(q)\dot{r} + C(q, \dot{q})r + \Lambda\bar{\Lambda}^{-1}K(t)r + \Lambda\bar{\Lambda}^{-1}K_i\int_0^t r(\tau)\,d\tau = \Psi\eta \qquad (19)$$

where

$$\eta = P - \hat{P}_2, \qquad \hat{P}_2 = M(\Lambda\bar{\Lambda}^{-1})\hat{P}, \qquad (20)$$

where $M(\cdot)$ is a diagonal matrix defined by Eqn (10). Since P, Λ and $\bar{\Lambda}$ are constant

$$\dot{\eta} = -\dot{\hat{P}}_2 = -M(\Lambda\bar{\Lambda}^{-1})\dot{\hat{P}}, \qquad (21)$$

i.e.

$$\dot{\eta}_i = -\alpha_i\dot{\hat{p}}_i, \qquad \alpha_i = \lambda_i/\bar{\lambda}_i, \qquad \text{for } \eta_i, \qquad \hat{p}_i \in R^{n_i}. \qquad (22)$$

The closed-loop stability of Eqn (19) is given by the following theorem.

Theorem 3.1. Consider the mapping (19) under the following assumptions:

(i) $\Lambda\bar{\Lambda}^{-1}K = K^T\Lambda\bar{\Lambda}^{-1} > 0$, $\Lambda\bar{\Lambda}^{-1}K_i = K_i^T\Lambda\bar{\Lambda}^{-1} \geq 0$

(ii) the gradient parameter adaptation for $\hat{P}$, i.e.

$$\dot{\hat{P}} = \Gamma\Psi^T r \qquad (23)$$

with Γ satisfying $M(\Lambda\bar{\Lambda}^{-1})\Gamma = \Gamma^T M(\Lambda\bar{\Lambda}^{-1})$,

then

(1) $e \in L_n^2 \cap L_n^\infty$, is continuous and $\to 0$ as $t \to \infty$, and $\dot{e} \in L_n^2$;

(2) $\hat{P}$ is bounded;

(3) $r \to 0$ as $t \to \infty$, consequently, $\dot{e} \to 0$ as $t \to 0$.

Proof. See Appendix A.

Remarks

(1) The estimate of input scaling $\bar{\Lambda}$ is a fixed constant. In actual implementation, $\bar{\Lambda}$ may be chosen as identity matrix I.

(2) In the theorem, K and K_i should be chosen such that $\Lambda\bar{\Lambda}^{-1}K$ and $\Lambda\bar{\Lambda}^{-1}K_i$ are symmetric positive definite. Usually, K and K_i are chosen diagonal matrices, therefore the condition is satisfied automatically.

(3) It is clear that the parameter adaptation algorithm (23) is the same as that for unit input scaling robots [1] apart from the restriction on Γ. When Γ is chosen to be a diagonal matrix (a common choice), the symmetry condition of $M(\Lambda\bar{\Lambda}^{-1})\Gamma$ is automatically satisfied.

(4) From the above remarks, it can be seen that the adaptive control laws for unit input scaling robots may be directly applicable to unknown input scaling robots when K, K_i and Γ are diagonal positive definite matrices (common choices). This explains why adaptive control laws for unit input scaling robot work in reality even though the scaling may not be known exactly.

(5) Since $\tilde{P} = P - M(\Lambda\bar{\Lambda}^{-1})\hat{P}$, $\hat{P}$ will never converge to its true value P. Suppose $P(t \to \infty) = 0$ under persistent excitation condition

$$\hat{P} = M^{-1}(\Lambda\bar{\Lambda}^{-1})P = M(\Lambda^{-1}\bar{\Lambda})P. \tag{24}$$

However, whether $\hat{P}$ converges to P or $M(\Lambda\bar{\Lambda}^{-1})P$ is not a big issue here, as long as $\tilde{P}$ is bounded, $e(t)$ and $\dot{e}(t) \to 0$ as $t \to \infty$, which is true from **Theorem 3.1.**

(6) In the presence of unmodelled dynamics, or in the presence of external disturbance, the parameters can drift along an equilibrium manifold until an instability results [9]. To solve this problem, σ-modification to the parameter update law is one available method to get boundedness of all signals but nonzero tracking errors; for detail see [10].

4. SIMULATION TESTS

Let us consider a planar two-link manipulator, whose dynamics are described by

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \Lambda\tau, \tag{25}$$

where

$$D(q) = \begin{bmatrix} P_1 + 2P_3\cos q_2 & P_2 + P_3\cos q_2 \\ P_6 + P_7\cos q_2 & P_6 \end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix} -P_3\dot{q}_2\sin q_2 & -P_3(\dot{q}_1 + \dot{q}_2)\sin q_2 \\ P_7\dot{q}_1\sin q_2 & 0.0 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} P_4\cos q_1 + P_5\cos(q_1 + q_2) \\ P_8\cos(q_1 + q_2) \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix},$$

where $P_2 = P_6$ and $P_3 = P_7$ since $D(q) = D(q)^T$. Therefore, for

$$D(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + G(q) = \Psi(q, \dot{q}, \dot{q}_r, \ddot{q}_r)P \tag{26}$$

we have

$$P = [P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8]^T$$

$$\Psi = \begin{bmatrix} \ddot{q}_{r1} & \ddot{q}_{r2} & \beta & \cos q_1 & \cos(q_1 + q_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddot{q}_{r1} + \ddot{q}_{r2} & \gamma & \cos(q_1 + q_2) \end{bmatrix}$$

with

$$\beta = (2\ddot{q}_{r1} + \ddot{q}_{r2})\cos q_2 - (\dot{q}_2\dot{q}_{r1} + (\dot{q}_1 + \dot{q}_2)\dot{q}_{r2})\sin q_2$$

$$\gamma = \ddot{q}_{r1}\cos q_2 + \dot{q}_1\dot{q}_{r1}\sin q_2$$

and the parameters are as shown in Table 1.

4.1. Trajectory planning

The desired trajectory for each axis is expressed as a Hermite polynomial of the third degree in t with continuous bounded position, velocity and bounded acceleration. The general expression for the desired position trajectory is

$$q_d(t, t_d) = q_0 + \left(-2\frac{t^3}{t_d^3} + 3\frac{t^2}{t_d^2}\right)(q_f - q_0), \tag{27}$$

where q_0 and q_f are the arm initial and final positions, and t_d represents the time at which the desired arm trajectory reaches the desired final position.

In the simulation tests, the following values were chosen:

$$\begin{gathered} t_d = 1.0\,\text{sec}, \qquad q_0 = [0, 0, 0, 0]^T\,\text{rad}, \qquad \dot{q}_0 = [0, 0, 0, 0]^T\,\text{rad} \\ q_d(0) = [0, 0, 0, 0]^T\,\text{rad}, \qquad q_d(t_d) = [1.0, 2.0]^T\,\text{rad} \\ \hat{P}(0) = [2.0, 1.0, 0.5, 3.0, 1.6, 1.0, 0.5, 1.6]^T\text{kgm}^2 \\ \bar{\Lambda} = \text{diag}[1.0]. \end{gathered} \tag{28}$$

It is clear that $\hat{P}(0) \neq P$ and $\bar{\Lambda} \neq \Lambda$.

4.2. Non-adaptive control

When the parameter adaptation algorithm (23) is not activated, the position and velocity tracking of the robot are as shown in Figs 1 and 2, respectively, under the controls which are shown in Fig. 3. It can be seen that this control scheme has static errors as well as big tracking errors.

4.3. Adaptive control

When the parameter adaptation algorithm (23) is activated with $\Gamma = \text{diag}[0.5]$, the position and velocity tracking of the robot are as shown in Figs 4 and 5, respectively.

Table 1. Parameters used for simulation

Parameter	Value
$[P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8]$	$[1.66, 0.42, 0.63, 3.75, 1.25, 0.42, 0.63, 1.25]$ kgm^2
$[\lambda_1, \lambda_2]$	$[0.6, 0.6]$

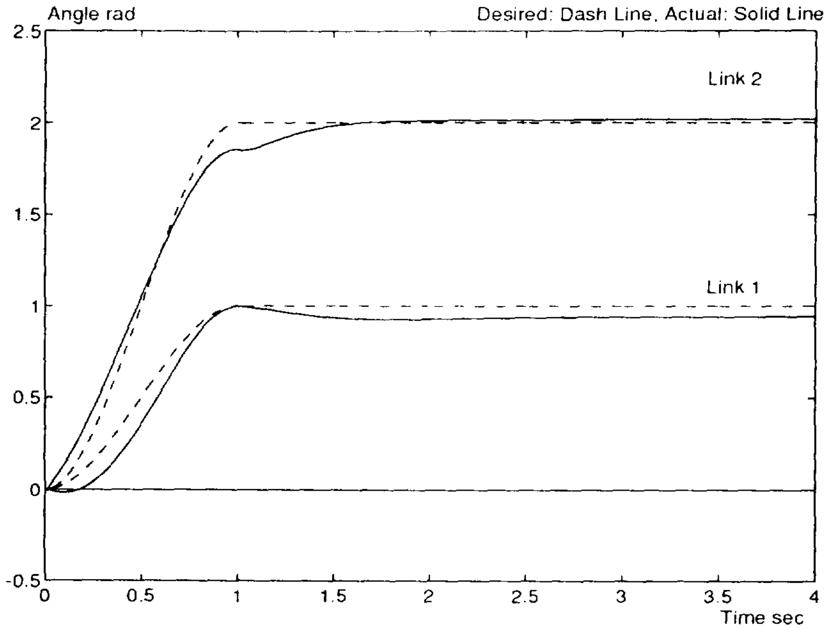

Fig. 1. Position tracking without adaptation.

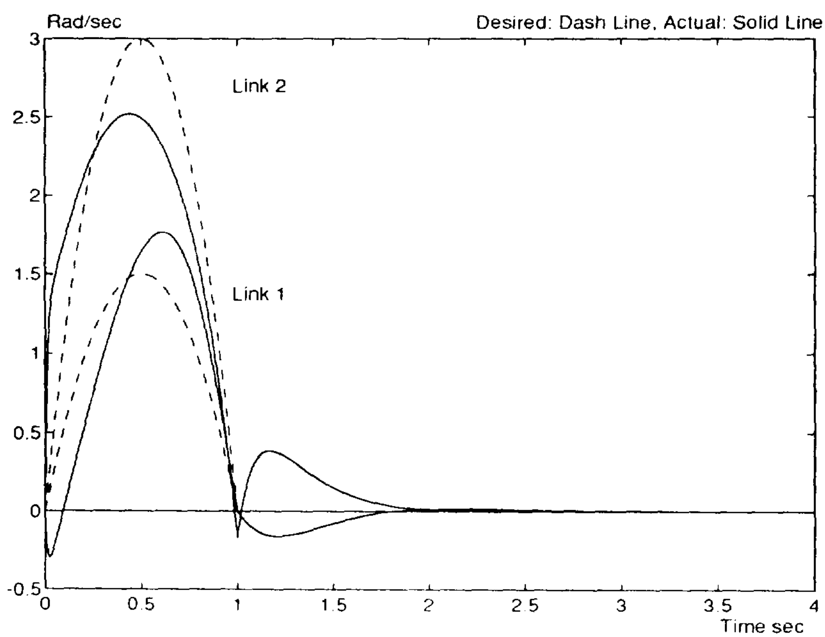

Fig. 2. Velocity tracking without adaptation.

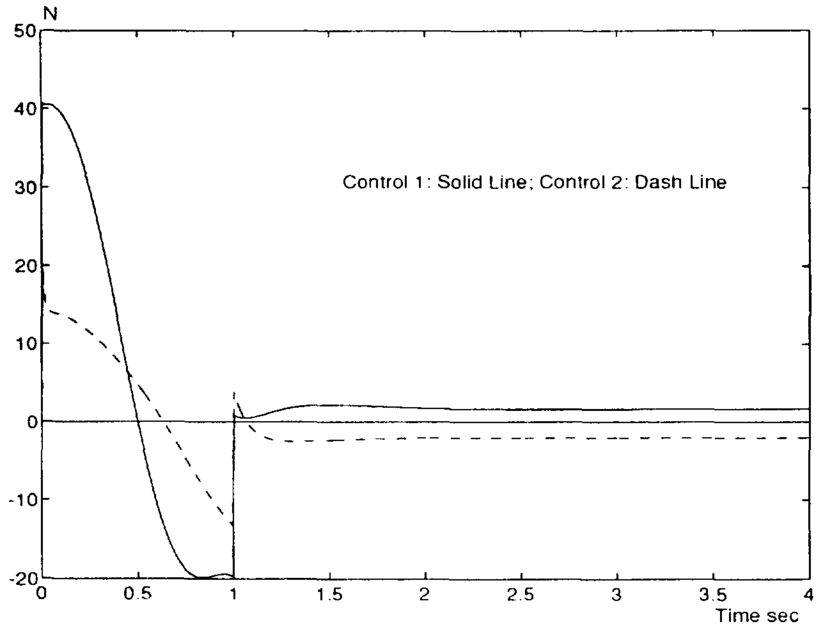

Fig. 3. Control signal variations without adaptation.

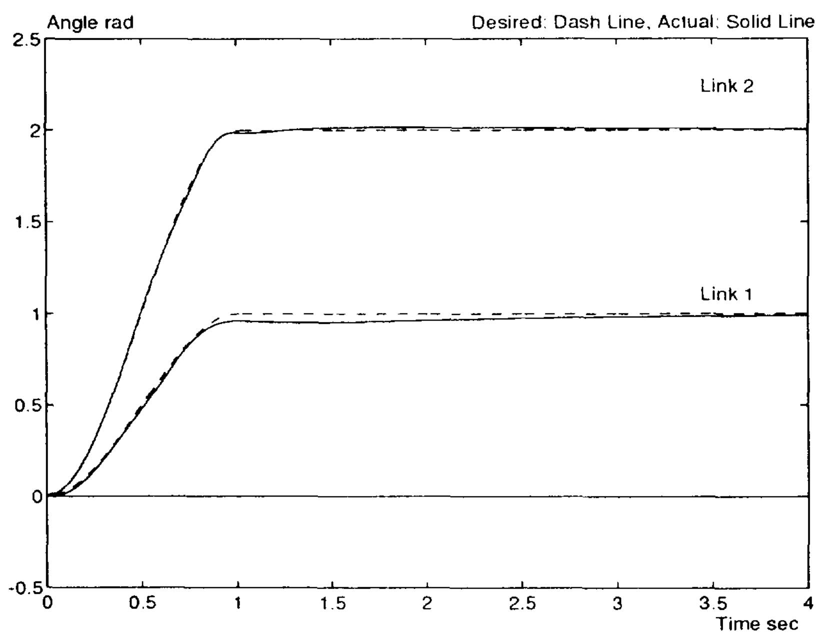

Fig. 4. Position tracking with adaptation.

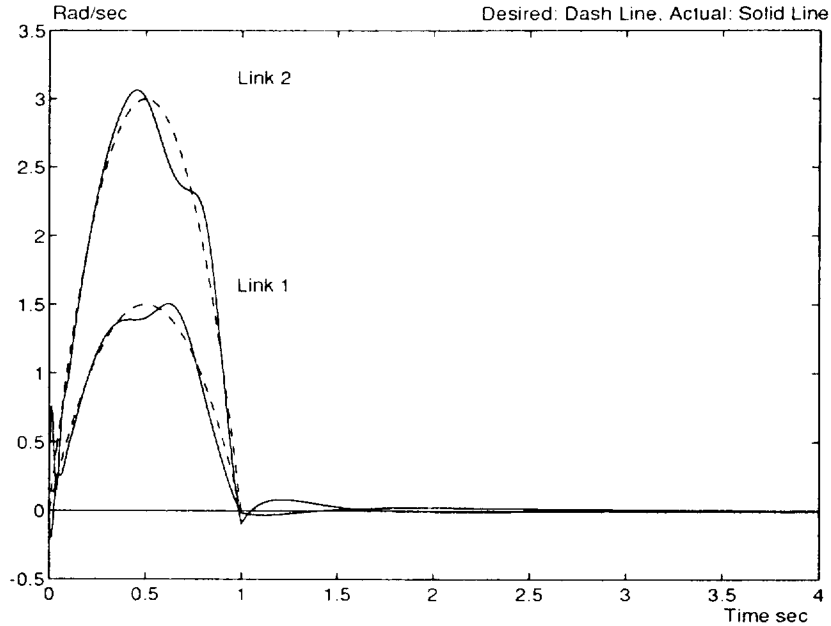

Fig. 5. Velocity tracking with adaptation.

The adaptive controls and the parameters are shown in Figs 6 and 7, respectively. It can be seen that both tracking and steady state errors are much smaller than for the non-adaptive case because of the "learning" mechanism. It can be seen that the estimated parameters are also bounded.

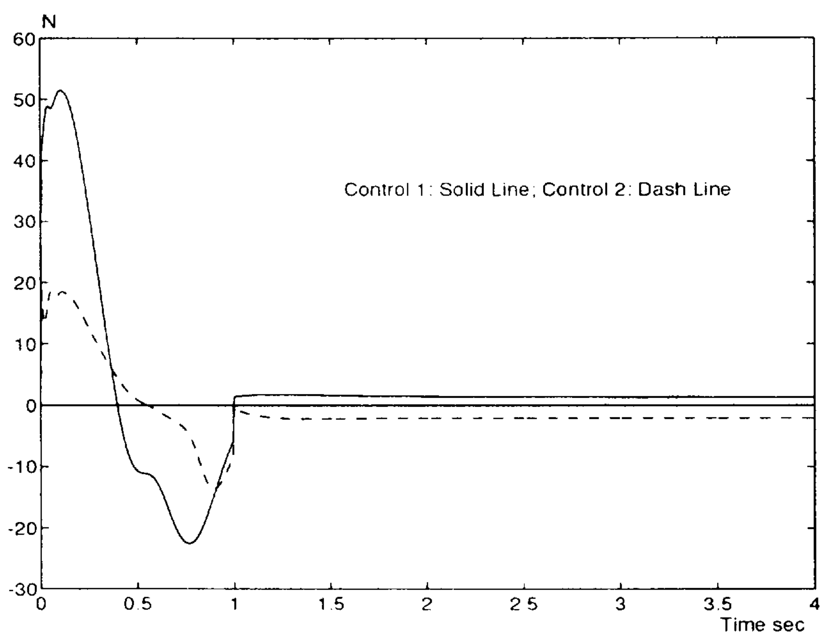

Fig. 6. Control signal variations with adaptation.

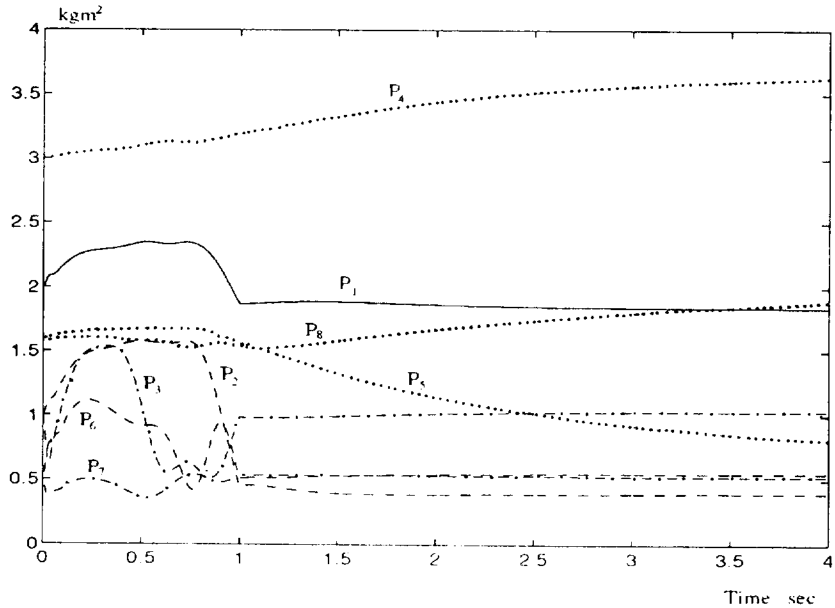

Fig. 7. Parameter variations with adaptation.

5. CONCLUSIONS

In this paper, an adaptive control law for robots having unknown input scalings has been presented. It has been shown that the adaptive control law is very effective in solving both parameter uncertainties of linear-in-the-parameters dynamics and the unknown input scalings. It has been shown that all the signals in the closed-loop are bounded, and tracking errors converge to zero. By applying the same procedure as shown in this paper, some of the adaptive control laws in the literature for unit scaling robots may be extended to unknown input scaling cases as well. Because of the introduction of the input scalings, uncertainties in the transformation mechanisms of robots are allowed.

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APPENDIX A: PROOF OF THEOREM 3.1

Consider the non-negative function V defined by

$$V(t) = \tfrac{1}{2}r^TD(q)r + \tfrac{1}{2}\left(\int_0^t r(\tau)\,\mathrm{d}\tau\right)^T \Lambda\bar{\Lambda}^{-1}K_i\left(\int_0^t r(\tau)\,\mathrm{d}\tau\right) + \tfrac{1}{2}\eta^TQ^{-1}\eta, \tag{A1}$$

where $Q = Q^T > 0$.

Differentiating V along the system's trajectory (19), and recalling the skew-symmetric property of $\dot{D}(q) - 2C(q, \dot{q})$, the symmetric property of $D(q)$ and the assumptions, we have

$$\begin{aligned}\dot{V} &= r^TD(q)\dot{r} + \tfrac{1}{2}r^T\dot{D}(q)r + r^T\Lambda\bar{\Lambda}^{-1}K_i\left(\int_0^t r(\tau)\,\mathrm{d}\tau\right) + \eta^TQ^{-1}\dot{\eta}\\ &= r^TD(q)\dot{r} + r^TC(q, \dot{q})r + r^T\Lambda\bar{\Lambda}^{-1}K_i\left(\int_0^t r(\tau)\,\mathrm{d}\tau\right) + \eta^TQ^{-1}\dot{\eta}\\ &= r^T\left[D(q)\dot{r} + C(q, \dot{q})r + \Lambda\bar{\Lambda}^{-1}K_i\left(\int_0^t r(\tau)\,\mathrm{d}\tau\right)\right] + \eta^TQ^{-1}\dot{\eta}\\ &= -r^T\Lambda\bar{\Lambda}^{-1}K_r + r^T\Psi\eta + \eta^TQ^{-1}\dot{\eta}.\end{aligned} \tag{A2}$$

By letting

$$\dot{\eta} = -Q\Psi^Tr, \tag{A3}$$

we have

$$\dot{V} = -r^T\Lambda\bar{\Lambda}^{-1}Kr.$$

Then

$$\int_0^t r^T\Lambda\bar{\Lambda}^{-1}Kr\,\mathrm{d}\tau = -V(t) + V(0), \tag{A4}$$

i.e.

$$\lambda_{\min}(\Lambda\bar{\Lambda}^{-1}K)\int_0^t r^Tr\,\mathrm{d}\tau \leqslant \int_0^t r^T\Lambda\bar{\Lambda}^{-1}Kr\,\mathrm{d}\tau \leqslant V(0).$$

(1) Since $V(0)$ and $\lambda_{\min}(\Lambda\bar{\Lambda}^{-1}K)$ are positive constants, it follows that $r \in L_2^n$. Consequently, from **Lemma 1.1**, $e \in L_2^n \cap L_\infty^n$ is continuous and $\to 0$ as $t \to \infty$, $\dot{e} \in L_2^n$.

(2) Since

$$\dot{V} = -r^T \Lambda\bar{\Lambda}^{-1} K r \leqslant 0$$

it follows that $0 \leqslant V(t) \leqslant V(0)$, $\forall t \geqslant 0$. Hence $V(t) \in L_\infty$, $\Rightarrow$ η and $\int_0^t r(\tau)\,\mathrm{d}\tau \in L_\infty$. From Eqn (20), we obtain that $\hat{P}$ is bounded. By letting $Q = M(\Lambda\bar{\Lambda}^{-1})\Gamma$, from Eqn (A3), we obtain

$$\dot{\hat{P}} = \Gamma \Psi^T r, \tag{A5}$$

which is one of the assumptions of **Theorem 3.1.** Note that Γ should be chosen such that $M(\Lambda\bar{\Lambda}^{-1})\Gamma = \Gamma^T M(\Lambda\bar{\Lambda}^{-1})$.

(3) For $r \in L_2^n$, $\int_0^t r(\tau)\,\mathrm{d}\tau$ and $\Psi\eta \in L_\infty^n$, we have $\dot{r} \in L_\infty^n$, from Eqn (19). Since $\dot{r} \in L_\infty^n$, r is uniformly continuous. The proof is completed using the implication

r uniformly continuous, and $r \in L_2^n$, $\Rightarrow$
$r \to 0$ as $t \to \infty$, $\Rightarrow$
$\dot{e} \to 0$.

QED.