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Discussion

Comments on "Adaptive global stabilization of nonholonomic systems with strong nonlinear drifts" [Systems & Control Letters 46 (2002) 195–205]

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Abstract

In the paper to be commented, there is a technical problem inherent in the input based switching, and it is highlighted here to avoid possible failure when the scheme is used in practice.

This is to confirm that the controller presented in [1] is interesting and inspiring, and tries to offer another different scheme to drive $x_0(t_0)=0$ away from zero in a finite time without escaping for the first equation

$$\dot{x}_0 = u_0 + \phi_0^{\mathrm{T}}(x_0)\theta \tag{1}$$

of the perturbed canonical nonholonomic system. In Eq. (1), x_0 and $u_0 \in R$ are the state and control, respectively, $\phi_0(x_0) \in \mathbb{R}^p$ are known smooth nonlinear functions of x_0, and $\theta \in \mathbb{R}^p$ are unknown constant parameters.

In this comment, however, it is shown that there is a possibility that the switching condition, $u_0(t) < u_0^*$, will never be met if $\phi_0^{\mathrm{T}}(x_0)\theta < 0$, subsequently no switching happens and the control system fails in practical applications.

To show the problem of the scheme, let us follow the argument of [1], and do further analysis.

When $x_0(t_0)=0$, the control u_0 in [1] is given by

$$u_0(t) = x_0 p_0(x_0, \hat{\theta}_0) + \eta_0, \quad \text{if } x_0 p_0(x_0, \hat{\theta}_0) + \eta_0 \geqslant u_0^* \tag{2}$$

where $0 < u_0^* < \eta_0$, $p_0(x_0, \hat{\theta}_0) = -\phi_0^{\mathrm{T}}\hat{\theta}_0 - \sqrt{k_0^2 + (\phi_0^{\mathrm{T}}\hat{\theta}_0)^2}$ with constant $k_0 > 0$, and parameter estimate $\hat{\theta}_0$ is updated by $\dot{\hat{\theta}}_0 = \Gamma_0 x_0 \phi(x_0)$ with $\Gamma_0 = \Gamma^{\mathrm{T}} > 0$.

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The strategy of the control was interpreted as follows: At the initial time t_0, $x_0(t_0) = 0$, the positive constant η_0 in u_0 drives the state $x_0(t)$ away from zero in the positive direction. "*Since* $p_0(x_0, \hat{\theta}_0) < 0$, $\forall (x_0(t), \hat{\theta}_0(t)) \in \mathbb{R}^{p+1}$ *and noticing the structure of the* x_0 *subsystem,* u_0 *starts to decreasing and approaches zero*".

After detailed analysis, it is found that there is a possibility that u_0 never decreases to zero if $\phi_0^{\mathrm{T}}(x_0)\theta < 0$, thus the switching condition $u_0(t) < u_0^*$ will never be met if u_0^* is chosen too large.

To show the above statement clearly, substitute (2) into (1) to obtain the closed-loop dynamics of x_0-subsystem as

$$\dot{x}_0 = -x_0\left(\sqrt{k_0^2 + (\phi_0^{\mathrm{T}}\hat{\theta}_0)^2} - \phi_0^{\mathrm{T}}\tilde{\theta}_0\right) + \eta_0, \quad u_0(t) \geqslant u_0^*, \tag{3}$$

where $\tilde{\theta}_0 = \theta - \hat{\theta}_0$. Since the system has been proven to be stable, as time increases, x_0 should approach its equilibrium point x_0^{e} defined by

$$- x_0^{\mathrm{e}}\left(\sqrt{k_0^2 + (\phi_0^{\mathrm{T}}\hat{\theta}_0^{\mathrm{e}})^2} - \phi_0^{\mathrm{T}}\tilde{\theta}_0^{\mathrm{e}}\right) + \eta_0 = 0, \tag{4}$$

where $\hat{\theta}_0^{\mathrm{e}}$ and $\tilde{\theta}_0^{\mathrm{e}}$ denote the steady state values of $\hat{\theta}_0$ and $\tilde{\theta}_0$, respectively.

For analysis purpose, Eq. (4) can be further simplified, under the assumption that $\tilde{\theta}_0 = 0$, i.e. $\hat{\theta}_0 = \theta$, as

$$x_0^{\mathrm{e}} = \frac{\eta_0}{\sqrt{k_0^2 + (\phi_0^{\mathrm{T}}\theta)^2}} > 0. \tag{5}$$

Substituting it into (2), we have the control signal at the equilibrium as

$$u_0^{\mathrm{e}} = \frac{-\eta_0\phi_0^{\mathrm{T}}\theta}{\sqrt{k_0^2 + (\phi_0^{\mathrm{T}}\theta)^2}}. \tag{6}$$

From (6), it can be seen that

$$u_0^{\mathrm{e}} > 0, \quad \text{if } \phi_0^{\mathrm{T}}\theta < 0. \tag{7}$$

Apparently, if u_0^* is chosen smaller than u_0^{e}, i.e. $u_0^* < u_0^{\mathrm{e}}$, then $u_0(t)$ will stop at u_0^{e} and never decreases to u_0^*, and the switching condition $u_0(t) < u_0^*$ will never be satisfied even if $t \to \infty$, thus switching never occurs. In this sense, the whole control fails. Since θ is unknown, u_0^{e} is unknown as well. It is indeed difficult to choose such a $u_0^* > u_0^{\mathrm{e}}$.

Of course, by assuming that $\phi_0^{\mathrm{T}}\theta \geqslant 0$, we have $u_0^* > 0 \geqslant u_0^{\mathrm{e}}$, and then the switching condition $u_0(t) < u_0^*$ will be satisfied as time increases under the ideal condition $\hat{\theta}_0 = \theta$. In practice, the dynamic response is more complicated as the convergence of parameter estimation error is not guaranteed. In fact, the corresponding equilibrium point is given by

$$(x_0^{\mathrm{e}}, u_0^{\mathrm{e}}) = \left(\frac{\eta_0}{\sqrt{k_0^2 + (\phi_0^{\mathrm{T}}\hat{\theta}_0^{\mathrm{e}})^2} - \phi_0^{\mathrm{T}}\tilde{\theta}_0^{\mathrm{e}}}, \frac{-\eta_0\phi_0^{\mathrm{T}}\theta}{\sqrt{k_0^2 + (\phi_0^{\mathrm{T}}\hat{\theta}_0^{\mathrm{e}})^2} - \phi_0^{\mathrm{T}}\tilde{\theta}_0^{\mathrm{e}}}\right). \tag{8}$$

For technical correctness, we may assume that $\phi_0^{\mathrm{T}}\theta \geqslant 0$, and measures are taken such that $\sqrt{k_0^2 + (\phi_0^{\mathrm{T}}\hat{\theta}_0^{\mathrm{e}})^2} > \phi_0^{\mathrm{T}}\tilde{\theta}_0^{\mathrm{e}}$, to guarantee the switching condition $u_0(t) < u_0^*$ be met. However, this assumption severely restricts the class of systems that the controller can be applied to because it is hard or unrealistic to know the sign of the unknown drift nonlinear term in practice. Apart from pointing out this technical problem, no quick fix could be offered at this point of time.

References

[1] K.D. Do, J. Pan, Adaptive global stabilization of nonholonomic systems with strong nonlinear drifts, Systems Control Lett. 46 (2002) 195–205.