Adaptive Neural Network Control of Nonlinear Systems by State and Output Feedback

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***Abstract*— This paper presents a novel control method for a general class of nonlinear systems using neural networks (NN's). Firstly, under the conditions of the system output and its time derivatives being available for feedback, an adaptive state feedback NN controller is developed. When only the output is measurable, by using a high-gain observer to estimate the derivatives of the system output, an adaptive output feedback NN controller is proposed. The closed-loop system is proven to be semi-globally uniformly ultimately bounded (SGUUB). In addition, if the approximation accuracy of the neural networks is high enough and the observer gain is chosen sufficiently large, an arbitrarily small tracking error can be achieved. Simulation results verify the effectiveness of the newly designed scheme and the theoretical discussions.**

***Index Terms*— Adaptive control, high-gain observer, neural networks, nonlinear system, output feedback control.**

I. Introduction

In recent years, controller design for systems having complex nonlinear dynamics becomes an important and challenging topic. Many remarkable results in this area have been obtained owing to the advances in geometric nonlinear control theory, and in particular, feedback linearization techniques [1]–[3]. Both state feedback and output feedback linearization methods were studied in the literature. Under certain assumptions, these output feedback controllers can guarantee the global stability of the closed-loop systems based on state observers [4]–[7]. Applications of these approaches are quite limited because they relay on the exact knowledge of the plant nonlinearities. In order to relax some of the exact model-matching restrictions, several adaptive schemes have recently been introduced to solve the problem of parametric uncertainties [8]–[14]. At the present stage they are only applicable for a kind of affine systems which can be linearly parametrized. A general control structure for adaptive feedback linearization was given by $u = \hat{N}(x)/\hat{D}(x)$ where $\hat{D}(x)$ must be bounded away from zero for all time, which is called a *well-defined* controller [16]. However, it is not easy to design an adaptive law to satisfy such a condition. The existing controllers are usually given locally and/or require additional prior knowledge about the systems. Other problems of current adaptive control techniques such as nonlinear control laws which are difficult to derive, geometrically increasing complexity with the number of unknown parameters, and the general difficulty for real-time applications have compelled researchers to look for more applicable methods.

In the past several years, active research has been carried out in neural network control [16]–[30]. The massive parallelism, natural fault tolerance and implicit programming of neural network computing architectures suggest that they may be good candidates for implementing real-time adaptive control for nonlinear dynamical systems. It has been proven that artificial neural networks can approximate a wide range of nonlinear functions to any desired degree of accuracy under certain conditions. The feasibility of applying neural networks for modeling unknown functions in dynamic systems has been demonstrated in several studies [19]–[21]. From these works, it was shown that for stable and efficient on-line control using the backpropagation (BP) learning algorithm, the identification must be sufficiently accurate before control action is initiated. In practical control applications, it is desirable to have systematic method of ensuring the stability, robustness, and performance properties of the overall system. Recently, several good NN control approaches have been proposed based on Lyapunov's stability theory [16]–[18], [22]. One main advantage of these schemes is that the adaptive laws were derived based on the Lyapunov synthesis method and therefore guarantee the stability of systems. A limitation lies that they can only be applied to relatively simple classes of nonlinear plants such as affine systems.

A novel direct adaptive NN controller using Lyapunov stability theory is developed in this paper for a general class of nonlinear systems. Both state feedback and output feedback control are studied. The overall system is proved to be semi-globally uniformly ultimately bounded and the tracking error converges to a small neighborhood of the origin.

The paper is organized as follows. Section II describes the class of nonlinear systems to be controlled and the control problem. Section III gives the structure and approximation properties of the neural networks. An adaptive NN controller based on state feedback is discussed in Section IV. In Section V, we study the output feedback control problem using a high-gain observer. The effectiveness of the proposed controllers is illustrated through an example in Section VI.

II. Problem Statement

Consider a single-input single-output (SISO) nonlinear system

$$y^{(n)} = f\big(y, y^{(1)}, y^{(2)}, \ldots, y^{(n-1)}, u\big) \qquad (1)$$

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where

$y \in R$	measured output;
$u \in R$	control input;
$y^{(i)}\ (i = 1, 2, \ldots, n)$	ith time derivatives of the output y;
$f(\cdot) : R^{n+1} \to R$	unknown nonlinear function.

It is should be noted that, unlike most recent results, the nonlinearity $f(\cdot)$ is an implicit function with respect to u. The control objective can be described as: given a desired output, $y_d(t)$, find a control, u, such that the output of the system tracks the desired trajectory with an acceptable accuracy, while all the states and the control remain bounded.

Let $x = [x_1, x_2, \ldots, x_n]^T = [y, y^{(1)}, \ldots, y^{(n-1)}]^T \in R^n$ be the state vector, we may represent system (1) in a state space model

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \quad \vdots \\ \dot{x}_n = f(x, u) \\ y = x_1 \end{cases} \tag{2}$$

Definition 1: The solution of (2) is semi-globally uniformly ultimately bounded (SGUUB), if for any Σ, a compact subset of R^n and all $x(t_0) = x_0 \in \Sigma$, there exist a $\mu > 0$ and a number $T(\mu, x_0)$ such that $\|x(t)\| < \mu$ for all $t > t_0 + T$.

Definition 2: Let U be an open subset of R^{n+1}. A mapping $f(w) : U \to R$ is said to be Lipschitz on U if there exists a positive constant L such that

$$|f(w_a) - f(w_b)| \leq L|w_a - w_b|$$

for all $(w_a, w_b) \in U$. We say L a Lipschitz constant for $f(w)$. We say $f(w)$ is Locally Lipschitz if each point of U has a neighborhood Ω_0 in U such that the restriction $f \mid \Omega_0$ is Lipschitz.

Lemma 1: Let a mapping $f : U \to R$ be C^1. Then, f is locally Lipschitz. Moreover, if $\Omega \subset U$ is compact, then, the restriction $f \mid \Omega$ is Lipschitz. (The proof can be found in [31].)

The following assumptions are made for system (2).

Assumption 1: $f(x, u)$ is C^1 for $(x, u) \in R^{n+1}$ and $f(x, u)$ is a smooth function with respect to input u.

Assumption 2: $\partial f(x, u)/\partial u \neq 0$ for all $(x, u) \in R^{n+1}$, and the sign of $\partial f(x, u)/\partial u$ is known.

Remark 2.1: Without losing generality, we shall assume that the sign of $\partial f(x, u)/\partial u$ is positive. Under Assumptions 1–2, system (2) includes the class of affine systems discussed in [14], [16], [23]. In the literature, intensive research has been done for systems in which $f(x, u)$ can be described by an affine form

$$f(x, u) = b(x) + a(x)u$$

with $b(x)$ and $a(x)$ being linearly parametrized, and $|a(x)| \neq 0$ for $x \in R^n$. However, their results cannot be applied to nonaffine systems, e.g.,

$$f(x, u) = x_1^2 + 0.15u^3 + 0.1\left(1 + x_2^2\right)u + \sin(0.1u) \tag{3}$$

$$f(x, u) = \frac{x_1}{1 + x_2^2} + u^5 + u^3 + ue^{u^2}. \tag{4}$$

Even if the description of system nonlinearities (3) and (4) are known exactly, it is not easy to design an explicit feedback control for achieving feedback linearization. When the structure of $f(x, u)$ in (2) is unknown it is even more difficult to construct the controller. Many results of feedback linearization methods [1]–[17] cannot be applied to such kinds of nonaffine nonlinear systems.

Remark 2.2: Assumption 2 is usually required in adaptive control design [2], [10], [11], [16], [32]. It implies that the sign of the high frequency gain is known.

Assumption 3: The reference signal $y_d(t)$, $y_d^{(1)}(t)$, $y_d^{(2)}(t), \ldots, y_d^{(n)}(t)$ are smooth and bounded.

Define vector x_d and ζ as

$$\begin{aligned} x_d &= \left[y_d, \dot{y}_d, \ldots, y_d^{(n-1)}\right]^T, \quad x_d \in R^n \\ \zeta &= x - x_d \end{aligned} \tag{5}$$

and a filtered tracking error as

$$e = [\Lambda \quad 1]^T \zeta \tag{6}$$

where $\Lambda = [\lambda_1, \lambda_2, \ldots, \lambda_{n-1}]^T$ is an appropriately chosen coefficient vector so that $\zeta(t) \to 0$ as $e(t) \to 0$, (i.e. $s^{n-1} + \lambda_{n-1}s^{n-2} + \cdots + \lambda_1$ is Hurwitz). Then, the time derivative of the filtered tracking error can be written as

$$\dot{e} = f(x, u) - y_d^{(n)}(t) + [0 \quad \Lambda^T]\zeta. \tag{7}$$

Define a continuous function

$$\text{sat}(e) = \begin{cases} 1 - \exp(-e/\gamma), & e > 0 \\ -1 + \exp(e/\gamma), & e \leq 0 \end{cases} \tag{8}$$

with γ being any small positive constant. As $\gamma \to 0$, $\text{sat}(e)$ approaches a step transition from -1 at $e = 0^-$ to 1 at $e = 0^+$ continuously.

We have the following lemma to establish the existence of an ideal control, u^*, that brings the output of the system to the desired trajectory.

Lemma 2: Consider system (2) satisfying Assumptions 1–3, $x(0) \in \Omega_x \subset R^n$ and $x_d \in \Omega_d \subset R^n$, where Ω_x and Ω_d are two compact sets. There exists an ideal control input, u^*, such that

$$\dot{e} = -k_v e - k_v \text{sat}(e) \tag{9}$$

where k_v is a positive constant. Subsequently, (9) leads to $\lim_{t\to\infty} |y(t) - y_d(t)| = 0$.

Proof: Plus and minus $k_v e + k_v \text{sat}(e)$ to the right-hand side of the error equation (7), we obtain

$$\dot{e} = f(x, u) + \nu - k_v e - k_v \text{sat}(e) \tag{10}$$

where ν is defined as

$$\nu = k_v e + k_v \text{sat}(e) - y_d^{(n)}(t) + [0 \quad \Lambda^T]\zeta.$$

From Assumption 2, we know that $\partial f(x, u)/\partial u \neq 0$ for all $(x, u) \in R^{n+1}$. Considering the fact that $\partial \nu/\partial u = 0$, we obtain

$$\frac{\partial[f(x, u) + \nu]}{\partial u} \neq 0.$$

Using the implicit function theorem [34], for every $x(0) \in \Omega_x$, $x_d \in \Omega_d$ and every value of x, there exists a continuous ideal control input $u^*(z)$ with $z = [x^T, \nu]^T \in R^{n+1}$ such that

$$f(x, u^*) + \nu = 0. \tag{11}$$

Under the action of u^*, (10) and (11) imply that (9) holds.

Define a Lyapunov function candidate

$$V = \frac{e^2}{2}.$$

Differentiating V along system (9) yields $\dot{V} = -k_v e^2 - k_v e * \mathrm{sat}(e)$. Considering $e * \mathrm{sat}(e) \geq 0$, we have $\dot{V} \leq -k_v e^2 \leq 0$. Since $V \geq 0$ and $\dot{V} \leq 0$, this shows the closed-loop stability in the sense of Lyapunov, thus, e is bounded. From (9), $\dot{e}$ is also bounded. For $x(0) \in \Omega_x$ and $x_d \in \Omega_d$, where Ω_x and Ω_d are two compact sets, therefore

$$\int_0^\infty -\dot{V}\, dt < \infty. \tag{12}$$

Considering (8) and (9), we obtain that $\ddot{V}$ is bounded. Using Barbalat's Lemma [3] in connection with (12), we conclude that $\dot{V}$ goes to zero with $t \to \infty$ and hence $\lim_{t\to\infty} e = 0$. This implies that $\lim_{t\to\infty} |y(t) - y_d(t)| = 0$. □

III. Function Approximation Using RBF Neural Networks

In control engineering, a NN is usually taken as a function approximator which emulates a given nonlinear function up to a small error tolerance. It has been proven [27]–[30] that any continuous functions can be uniformly approximated by a linear combination of Gaussians. The radial basis function (RBF) network can be considered as a two-layer network in which the hidden layer performs a fixed nonlinear transformation with no adjustable parameters to map the input space into an intermediate space, then the output layer combines the outputs of the intermediate layer linearly as the outputs of the whole network. Therefore, they belong to a class of linearly parameterized networks, and can be described as

$$u_{\mathrm{nn}}(W, z) = W^T S(z) \tag{13}$$

with the input vector $z \in \Omega_z \subset R^{n+1}$, weight vector $W \in R^l$, and basis function vector

$$S(z) = [s_1(z), s_2(z), \ldots, s_l(z)]^T \in R^l. \tag{14}$$

Commonly used RBF's are the Gaussian functions, which have the form

$$s_i(z) = \exp\left[\frac{-(z-\mu_i)^T(z-\mu_i)}{\sigma_i^2}\right], \quad i = 1, 2, \ldots, l \tag{15}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{i,n+1}]^T$ is the center of the receptive field and σ_i is the width of the Gaussian function.

In this paper, we shall consider $\hat{u}_{\mathrm{nn}}(\hat{W}, z)$ be the neural network controller under construction with $\hat{W}$ being the estimates of the NN weight W. Since $u^*(z)$ is a continuous function, according to [27], [28], on a compact set $\Omega_z \subset R^{n+1}$, there exist ideal weights W^* so that the ideal input $u^*(z)$ can be approximated by an ideal RBF neural network $u^*_{\mathrm{nn}}(z) = W^{*T} S(z)$. Thus, we have

$$u^*(z) = u^*_{\mathrm{nn}}(z) + \varepsilon_u(z) \tag{16}$$

where $\varepsilon_u(z)$ is called the NN approximation error. The NN approximation error is a critical quantity, representing the minimum possible deviation of the ideal approximator u^*_{nn} from the unknown ideal control $u^*(z)$. The NN approximation error can be reduced by increasing the number of the adjustable weights. Universal approximation results for neural networks [30] indicate that, if NN node number l is sufficiently large, then $|\varepsilon_u(z)|$ can be made arbitrarily small on a compact region.

The ideal weight vector W^* is an "artificial" quantity required for analytical purposes. W^* is defined as the value of W that minimizes $|\varepsilon_u(z)|$ for all $z \in \Omega_z \subset R^{n+1}$ in a compact region, i.e.,

$$W^* := \arg\min_{W\in R^l}\left\{\sup_{z\in\Omega_z} |u^*(z) - W^T S(z)|\right\}, \quad \Omega_z \subset R^{n+1}. \tag{17}$$

Assumption 4: On a compact set $\Omega_z \subset R^{n+1}$, the ideal neural network weights W^* satisfies

$$\|W^*\| \leq w_m \tag{18}$$

where w_m is a positive constant.

One property of the ideal NN $u^*_{\mathrm{nn}}(z)$ is given as follows.

Lemma 3: Suppose system (2) satisfies Assumptions 1–4 with (x, u) in a compact set Ω. Then the following inequality holds:

$$|f(x, u^*_{\mathrm{nn}}) + \nu| \leq C_1 \varepsilon |e| + C_2 \varepsilon \tag{19}$$

where C_1 and C_2 are positive constants and $\varepsilon = \sup_{z\in\Omega_z}\{|\varepsilon_u(z)|\}$.

Proof: See Appendix A. □

IV. State Feedback NN Control

In this section, under the condition that the full state x of system (2) is available for feedback, we proceed to design an adaptive controller using RBF neural networks.

A. Controller Structure and Error Dynamic

Let the NN controller take the form

$$u = \hat{u}_{\mathrm{nn}}(z) = \hat{W}^T S(z) \tag{20}$$

with $\hat{W}$ being the estimates of the NN weight W^* and $S(z)$ being the known basis function vector. Define the weight estimation error as

$$\tilde{W} = \hat{W} - W^*.$$

In order to establish the error system, we make the Taylor series expansion of $f(x, u^*_{\mathrm{nn}})$ at $\hat{u}_{\mathrm{nn}}(z) = \hat{W}^T S(z)$

$$f(x, u^*_{\mathrm{nn}}) = f(x, \hat{u}_{\mathrm{nn}}) - \Delta f^{[1]}_{\hat{u}_{\mathrm{nn}}} \tilde{W}^T S(z) + O(*) \tag{21}$$

where

$$\Delta f^{[1]}_{\hat{u}_{\mathrm{nn}}} = \left.\frac{\partial f(x,u)}{\partial u}\right|_{u=\hat{u}_{\mathrm{nn}}}$$

$$O(*) = \Delta f^{[2]}_{\hat{u}_{\mathrm{nn}}}[\tilde{W}^T S(z)]^2 - \cdots + \Delta f^{[i]}_{\hat{u}_{\mathrm{nn}}}[-\tilde{W}^T S(z)]^i + \cdots$$

with $\Delta f^{[k]}_{\hat{u}_{\rm nn}}$ being defined as

$$\Delta f^{[k]}_{\hat{u}_{\rm nn}} = \left.\frac{\partial^{(k)} f(x,u)}{k!\partial u^{(k)}}\right|_{u=\hat{u}_{\rm nn}}, \quad k = 2, 3 \ldots.$$

From (10) and (21), we obtain the error system for $u = \hat{u}_{\rm nn}$

$$\begin{aligned}\dot{e} &= f(x,\hat{u}_{\rm nn}) + \nu - k_v e - k_v \mathrm{sat}(e) \\ &= f(x,u^*_{\rm nn}) + \nu + \Delta f^{[1]}_{\hat{u}_{\rm nn}} \tilde{W}^T S(z) - O(*) \\ &\quad - k_v e - k_v \mathrm{sat}(e). \end{aligned} \tag{22}$$

Further, if $\Omega \subset R^{n+1}$ is a compact set, considering Assumption 1 and Lemma 1, we know that $f(x,u)$ and $\Delta f^{[k]}_{\hat{u}_{\rm nn}}$ $(k = 1, 2, \ldots, n)$ are Lipschitz for $(x,u) \in \Omega$. Similar to the proof of Lemma 3, we can derive $\|x\| \le d_1 + d_2|e|$ with d_1 and d_2 being positive constants, and there exist positive constants L_0 to L_5 such that

$$|O(*)| \le (L_0\|\hat{W}\| + L_1|e| + L_2)|\tilde{W}^T S(z)| \tag{23}$$

$$\left|\Delta f^{[1]}_{\hat{u}_{\rm nn}}\right| \le L_3\|\hat{W}\| + L_4|e| + L_5. \tag{24}$$

B. Weight Update Law and Stability Analysis

We here present the NN weight tuning algorithm that can guarantee the system stability and the tracking error $y(t) - y_d(t)$ to be suitably small. The weight update law is chosen as

$$\begin{aligned}\dot{\hat{W}} &= -(\kappa_0\|\hat{W}\| + \kappa_1|e| + \kappa_2)S(z)e \\ &\quad - \delta(\|\hat{W}\| + |e| + 1)\|S(z)\||e|\hat{W}\end{aligned} \tag{25}$$

where κ_0, κ_1, κ_2, and δ are positive constants. The first term on the right-hand side of (25) is a modified backpropagation algorithm and the second term corresponds to the e-modification [33] usually used in robust adaptive control, which is applied for improving the robustness of the controller in the presence of the NN approximation error. The following theorem shows the tracking ability of the proposed NN controller and the stability of the closed-loop system.

Theorem 1: For system (2), if the controller is given by (20) and the neural network weights are updated by (25), with

1) assumptions 1–4 being satisfied;
2) existence of two compact sets D_w and D_e such that $\hat{W}(0) \in D_w$ and $e(0) \in D_e$;

then, for a suitably chosen design parameter k_v, the filtered tracking error e, neural network weight $\hat{W}$ and all system states are SGUUB. In addition, the tracking error can be made arbitrarily small by increasing the controller gains and neural network node number.

Proof: Consider a Lyapunov function candidate as

$$V = \frac{e^2}{2} + \frac{1}{2}\hat{W}^T\hat{W}. \tag{26}$$

Differentiating (26) along (22) and (25), we have

$$\begin{aligned}\dot{V} &= -\hat{W}^T(\kappa_0\|\hat{W}\| + \kappa_1|e| + \kappa_2)S(z)e \\ &\quad - \delta(\|\hat{W}\| + |e| + 1)\|S(z)\||e|\|\hat{W}\|^2 \\ &\quad + e\big[-k_v e - k_v \mathrm{sat}(e) + f(x,u^*_{\rm nn}) + \nu \\ &\quad + \Delta f^{[1]}_{\hat{u}_{\rm nn}}(\hat{W} - W^*)^T S(z) - O(*)\big].\end{aligned}$$

Using (19), (23), and (24), we obtain the following inequality:

$$\begin{aligned}\dot{V} &\le -(\kappa_0\|\hat{W}\| + \kappa_1|e| + \kappa_2)\hat{W}^T S(z)e \\ &\quad - \delta(\|\hat{W}\| + |e| + 1)\|S(z)\||e|\|\hat{W}\|^2 \\ &\quad - k_v e^2 - k_v e \cdot \mathrm{sat}(e) + (\varepsilon C_1|e| + \varepsilon C_2)|e| \\ &\quad + \Delta f^{[1]}_{\hat{u}_{\rm nn}}\hat{W}^T S(z)e + (L_3\|\hat{W}\| \\ &\quad + L_4|e| + L_5)|W^{*T}S(z)e| \\ &\quad + (L_0\|\hat{W}\| + L_1|e| + L_2)\|\hat{W}\||e|\|S(z)\|. \end{aligned} \tag{27}$$

From Assumption 2 and (24), we have

$$L_3\|\hat{W}\| + L_4|e| + L_5 \ge \Delta f^{[1]}_{\hat{u}_{\rm nn}} > 0.$$

By using $\|W^*\| \le w_m$ and $\|\tilde{W}\| \le \|\hat{W}\| + w_m$, (27) can be written as

$$\begin{aligned}\dot{V} &\le [|\kappa_0 - L_3|\|\hat{W}\| + |\kappa_1 - L_4||e| + |\kappa_2 - L_5|]|\hat{W}^T S(z)e| \\ &\quad - \delta(\|\hat{W}\| + |e| + 1)\|S(z)\||e|\|\hat{W}\|^2 \\ &\quad - k_v e^2 - k_v|e| + k_v e I_e + (\varepsilon C_1|e| + \varepsilon C_2)|e| \\ &\quad + (L_3\|\hat{W}\| + L_4|e| + L_5)w_m\|S(z)\||e| \\ &\quad + (L_0\|\hat{W}\| + L_1|e| + L_2)(\|\hat{W}\| + w_m)|e|\|S(z)\| \end{aligned} \tag{28}$$

where I_e is defined as

$$I_e = \begin{cases} \exp(-e/\gamma), & e > 0 \\ -\exp(e/\gamma), & e \le 0 \end{cases} \tag{29}$$

Define the following positive constants:

$$\alpha_1 = |\kappa_0 - L_3| + L_0 \tag{30}$$

$$\alpha_2 = |\kappa_1 - L_4| + L_1 \tag{31}$$

$$\alpha_3 = |\kappa_2 - L_5| + L_2 + (L_0 + L_3)w_m \tag{32}$$

$$\alpha_4 = (L_2 + L_5)w_m$$

$$\alpha_5 = (L_1 + L_4)w_m$$

$$\alpha_s = \sup_{z \in \Omega_z}\{\|S(z)\|\}.$$

Equation (28) can be further written as

$$\begin{aligned}\dot{V} &\le -(k_v - \varepsilon C_1 - \alpha_5\alpha_s)e^2 - (k_v - \varepsilon C_2 - \alpha_4\alpha_s)|e| \\ &\quad + k_v e I_e - \|S(z)\|[\delta e^2\|\hat{W}\|^2 + \delta|e|\|\hat{W}\|^2 + \delta|e|\|\hat{W}\|^3] \\ &\quad + \|S(z)\|[\alpha_1|e|\|\hat{W}\|^2 + \alpha_2 e^2\|\hat{W}\| + \alpha_3|e|\|\hat{W}\|] \\ &\le -\left(\frac{k_v}{2} - \varepsilon C_1 - \alpha_5\alpha_s\right)e^2 - (k_v - \alpha_4\alpha_s)|e| + k_v e I_e \\ &\quad - \frac{k_v}{2}\left[\left(|e| - \frac{\varepsilon C_2}{k_v}\right)^2 - \left(\frac{\varepsilon C_2}{k_v}\right)^2\right] \\ &\quad - \delta|e|\|\hat{W}\|\|S(z)\|\left(\|\hat{W}\| - \frac{\alpha_1}{2\delta}\right)^2 \\ &\quad - \delta e^2\|S(z)\|\left[\left(\|\hat{W}\| - \frac{\alpha_2}{2\delta}\right)^2 - \left(\frac{\alpha_2}{\delta}\right)^2\right] \\ &\quad - \frac{\delta}{2}|e|\|S(z)\|\left[\left(\|\hat{W}\| - \frac{\alpha_3}{\delta}\right)^2 - \left(\frac{\alpha_3}{\delta}\right)^2\right] \\ &\quad - \frac{\delta}{2}|e|\|S(z)\|\left[\left(\|\hat{W}\| - \frac{\alpha_1^2}{4\delta^2}\right)^2 - \left(\frac{\alpha_1^2}{4\delta^2}\right)^2\right]. \end{aligned} \tag{33}$$

Thus, (33) can be expressed as

$$\dot{V} \le -\beta_1 e^2 - \beta_2 |e| + k_v e I_e + \frac{(\varepsilon C_2)^2}{2k_v}$$

where

$$\beta_1 = \frac{k_v}{2} - \varepsilon C_1 - \left(\alpha_5 + \frac{\alpha_2^2}{4\delta}\right)\alpha_s \tag{34}$$

$$\beta_2 = k_v - \left(\alpha_4 + \frac{\alpha_1^4}{32\delta^3} + \frac{\alpha_3^2}{2\delta}\right)\alpha_s. \tag{35}$$

Define

$$k_0 = \max\left\{2\varepsilon_0 C_1 + \left(2\alpha_5 + \frac{\alpha_2^2}{2\delta}\right)\alpha_s + \beta_0, \left(\alpha_4 + \frac{\alpha_1^4}{32\delta^3} + \frac{\alpha_3^2}{2\delta}\right)\alpha_s + \beta_0\right\}$$

where ε_0 and β_0 are positive constants. Since α_1 to α_5, δ, α_s, and C_1 are positive constants, we know that k_0 is a positive constant. If choosing $k_v > k_0$, we can guarantee that $\beta_1 > \beta_0$ and $\beta_2 > \beta_0$.

Since, for $e \in R$, $e * \exp(-e/\gamma)$ has a maximum value of γ/e_0 at $e = \gamma$, with e_0 being the natural exponential ($e_0 = 2.7183$). From (29), we have $k_v e I_e \le k_v \gamma / e_0$. Define a set

$$\Theta_e := \left\{e(t) \mid |e| \le \max\left[\sqrt{\frac{k_v}{\beta_1 e_0}\gamma + \frac{(\varepsilon C_2)^2}{2\beta_1 k_v}}, \left(\frac{k_v}{\beta_1 e_0}\gamma + \frac{(\varepsilon C_2)^2}{2\beta_2 k_v}\right)\right]\right\}. \tag{36}$$

Since C_2, γ and e_0 are positive constants, $k_v > k_0$, $\beta_1 > \beta_0$ and $\beta_2 > \beta_0$, we conclude that Θ_e is a compact set. $\dot{V}$ is negative as long as $e(t)$ is outside the compact set Θ_e. According to a standard Lyapunov theorem [32], we conclude that the filtered error $e(t)$ is bounded and will converge to Θ_e. Next we prove the boundedness of weight vector $\hat{W}$. Considering the Lyapunov function candidate

$$V_1 = \frac{1}{2}\hat{W}^T \hat{W}$$

and taking the derivative of V_1 along (25) with respect to time, we have

$$\begin{aligned}\dot{V}_1 &= -\hat{W}^T(\kappa_0\|\hat{W}\| + \kappa_1|e| + \kappa_2)S(z)e \\ &\quad - \delta(\|\hat{W}\| + |e| + 1)\|S(z)\||e|\|\hat{W}\|^2 \\ &\le (\kappa_0\|\hat{W}\| + \kappa_1|e| + \kappa_2)\|S(z)\|\|\hat{W}\||e| \\ &\quad - \delta(\|\hat{W}\| + |e| + 1)\|S(z)\||e|\|\hat{W}\|^2 \\ &\le -\|S(z)\|\|\hat{W}\||e|\left\{\delta\left[\|\hat{W}\| + \frac{1}{2}\left(1 + |e| - \frac{\kappa_0}{\delta}\right)\right]^2 \right. \\ &\quad \left. - \frac{\delta}{4}\left(1 + |e| - \frac{\kappa_0}{\delta}\right)^2 - k_1|e| - \kappa_2\right\}.\end{aligned}$$

Define

$$\Theta_w := \left\{\hat{W}(t) \mid \|\hat{W}\| \le \sup_{e \in \Theta_e}\left[\frac{1}{2}\left(1 + |e| - \frac{\kappa_0}{\delta}\right) + \sqrt{\frac{1}{4}\left(1 + |e| - \frac{\kappa_0}{\delta}\right)^2 + \frac{1}{\delta}(\kappa_1|e| + \kappa_2)}\right]\right\}.$$

Since the filtered tracking error e is bounded, we conclude that Θ_w is a compact set, and $\dot{V}_1 \le 0$ as long as $\hat{W}(t)$ is outside Θ_w. Now define

$$\Theta := \{(e, \hat{W}) \mid e \in \Theta_e, \hat{W} \in \Theta_w\}.$$

If we initialize $\hat{W}(0)$ inside D_w and $e(0)$ inside D_e, there exists a constant T such that all trajectories will converge to Θ and remain in Θ for all $t > T$. This implies that the closed-loop system is SGUUB. The filtered tracking error will converge to the small compact set Θ_e which is a (ε, γ)-neighborhood of the origin. Since $s^{n-1} + \lambda_{n-1}s^{n-2} + \cdots + \lambda_1$ is Hurwitz, $y(t) - y_d(t) \to \Theta_e$ as $e(t) \to \Theta_e$. Because γ can be chosen as any small positive constant, and ε can be as small as desired by increasing the number of neural nodes l, we conclude that arbitrarily small tracking error can be achieved. □

Remark 4.1: If a high tracking accuracy is required, a large number of NN nodes should be chosen such that ε is small enough to achieve the desired tracking performance. The parameters κ_0, κ_1, κ_2 and δ in adaptive law (25) can also be designed to make α_1, α_2 and α_3 in (30)–(32) small. Equation (36) shows that, the larger β_1 and β_2 are, the smaller the tracking error will be. Therefore the control performance are adjustable through the choices of k_v, γ, κ_0, κ_1, κ_2 and δ.

Remark 4.2: Compared with the traditional linearization techniques, the proposed adaptive NN controller clearly has an advantage, i.e., there is no need to exactly cancel the nonlinearities of the systems. Even if the nonlinear part $f(x, u)$ can be written as an affine form, $f(x, u) = b(x) + a(x)u$, when $b(x)$ and $a(x)$ are unknown, it is still difficult to design a controller $u = [-\hat{b}(x) + \nu]/\hat{a}(x)$ to cancel the nonlinear parts $b(x)$ and $a(x)$ while guaranteeing $\hat{a}(x) \ne 0$. If $f(x, u)$ cannot be written as an explicit function with respect to u, the traditional geometric methods are not applicable to such a control problem.

Remark 4.3: The weight update law (25) is derived from the Lyapunov method and the e-modification [33] term is introduced to achieve the robustness in the presence of the NN approximation error. There is no requirement for persistent excitation condition for tracking convergence. In addition, the NN controller needs not to be trained off-line.

V. Output Feedback Control

When only plant output $y = x_1$ is measurable and the rest of the system states are not available for feedback, we need to estimate $x_2, x_3, \ldots, x_n$ to implement the feedback control. We present a control structure of output feedback control in Fig. 1, and generate the estimates of the time derivatives by a high-gain observer presented in the lemma below.

Lemma 4: Suppose the function $y(t)$ and its first n derivatives are bounded. Consider the following linear system

$$\begin{aligned}\epsilon\dot{\xi}_1 &= \xi_2 \\ \epsilon\dot{\xi}_2 &= \xi_3 \\ &\vdots \\ \epsilon\dot{\xi}_{n-1} &= \xi_n \\ \epsilon\dot{\xi}_n &= -b_1\xi_n - b_2\xi_{n-1} - \cdots - b_{n-1}\xi_2 - \xi_1 + y(t)\end{aligned} \tag{37}$$

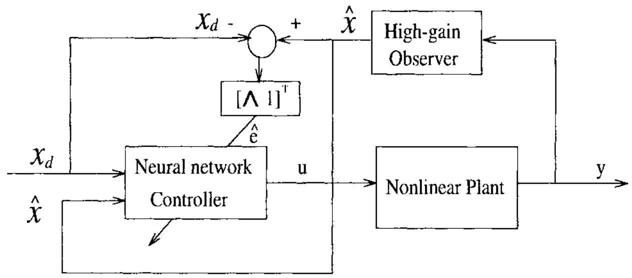

Fig. 1. Adaptive output feedback NN control using high-gain observer.

where the parameters b_1 to b_{n-1} are chosen so that the polynomial $s^n + b_1 s^{n-1} + \cdots + b_{n-1}s + 1$ is Hurwitz. Then, there exist positive constants h_k, $k = 2, 3, \ldots, n$, and t^* such that for all $t > t^*$ we have

$$\frac{\xi_{k+1}}{\epsilon^k} - y^{(k)} = -\epsilon\psi^{(k+1)} \quad k = 1, \ldots, n-1 \tag{38}$$

$$\left|\frac{\xi_{k+1}}{\epsilon^k} - y^{(k)}\right| \le \epsilon h_{k+1} \quad k = 1, \ldots, n-1 \tag{39}$$

where ϵ is any small positive constant, $\psi = \xi_n + b_1\xi_{n-1} +, \ldots, +b_{n-1}\xi_1$ and $|\psi^{(k)}| \le h_k$. $\psi^{(k)}$ denotes the kth derivative of ψ. (The proof of Lemma 4 can be found in [4]).

Having observer (37), we define the following variables:

$$\begin{aligned}
\xi &= [\xi_1, \xi_2, \ldots, \xi_n]^T \\
\hat{x} &= \left[x_1, \frac{\xi_2}{\epsilon}, \frac{\xi_3}{\epsilon^2}, \ldots, \frac{\xi_n}{\epsilon^{n-1}}\right]^T \\
\hat{\zeta} &= \hat{x} - x_d \\
&= \left[x_1 - y_d, \frac{\xi_2}{\epsilon} - \dot{y}_d, \frac{\xi_3}{\epsilon^2} - \ddot{y}_d, \ldots, \frac{\xi_n}{\epsilon^{n-1}} - y_d^{(n-1)}\right]^T \\
\hat{e} &= [\Lambda \quad 1]^T\hat{\zeta} = e + \epsilon\Lambda^T\Psi \qquad (40) \\
\hat{\nu} &= k_v\hat{e} + k_v \text{sat}(\hat{e}) - y_d^{(n)}(t) + [0 \quad \Lambda^I]\hat{\zeta} \\
\hat{z} &= [\hat{x}^T, \hat{\nu}]^T \qquad (41)
\end{aligned}$$

where $\Psi^T = [0, \ddot{\psi}, \ldots, \psi^{(n)}]$. The NN controller based on observer (37) is given by

$$u = \hat{u}_{\text{nn}}^o(z) = \hat{W}^T S(\hat{z}). \tag{42}$$

The weight update law is chosen as

$$\begin{aligned}
\dot{\hat{W}} = &-(\kappa_0\|\hat{W}\| + \kappa_1|\hat{e}| + \kappa_2)S(\hat{z})\hat{e} \\
&- [\delta(\|\hat{W}\| + |\hat{e}| + 1)\|S(\hat{z})\| + \epsilon|\hat{e}|]|\hat{e}|\hat{W}
\end{aligned} \tag{43}$$

where ϵ, κ_0, κ_1, κ_2, and δ are positive constants. The closed-loop error equation (10) becomes

$$\dot{e} = f(x, \hat{u}_{\text{nn}}^o) + \nu - k_v e - k_v \text{sat}(e) \tag{44}$$

In order to make the proof of the main theorem easier to follow, two lemmas are first provided.

Lemma 5: Consider the basis functions of Gaussian RBF NN (15) with $\hat{z}$ being the input vector

$$s_i(\hat{z}) = \exp\left[\frac{-(\hat{z} - \mu_i)^T(\hat{z} - \mu_i)}{\sigma_i^2}\right], \quad i = 1, 2, \ldots, l \tag{45}$$

we have

$$S(\hat{z}) = S(z) + \epsilon S_t \tag{46}$$

where S_t is a bounded function vector.

Proof: See Appendix B. □

Lemma 6: For the ideal neural network control $u_{\text{nn}}^* = W^{*T}S(z)$, the nonlinear function $f(x, u_{\text{nn}}^*)$ can be expressed as

$$\begin{aligned}
f(x, u_{\text{nn}}^*) = f(x, \hat{u}_{\text{nn}}^o) + \left(\Delta f_{\hat{u}_{\text{nn}}^o}^{[1]} + O_f\right)[W^{*T}S(z) \\
- \hat{W}^T S(\hat{z})]
\end{aligned} \tag{47}$$

where $O_f = \Delta f_{\hat{u}_{\text{nn}}^o}^{[2]}[u_{\text{nn}}^* - \hat{W}^T S(\hat{z})] + \cdots + \Delta f_{\hat{u}_{\text{nn}}^o}^{[i]} [u_{\text{nn}}^* \hat{W}^T S(\hat{z})]^{i-1} + \cdots$. In addition, if z is in a compact set $\Omega_z \subset R^{n+1}$, there exist positive constants L_0 to L_5 that

$$|O_f| \le L_0\|\hat{W}\| + L_1|e| + L_2 \tag{48}$$

$$\left|\Delta f_{\hat{u}_{\text{nn}}^o}^{[1]}\right| \le L_3\|\hat{W}\| + L_4|e| + L_5. \tag{49}$$

Proof: See Appendix C. □

In the following theorem, we discuss the convergence of the tracking error and the stability of the closed-loop system in combination with the high-gain observer (37).

Theorem 2: Consider the closed-loop system consisting of system (2), observer (37), controller (42) and adaptive law (43). Under the conditions that

(1) assumptions 1–4 being satisfied;
(2) existence of three compact sets D_w, D_e and D_ξ such that $\hat{W}(0) \in D_w$, $e(0) \in D_e$ and $\xi(0) \in D_\xi$

for a suitably chosen design parameter k_v, the closed-loop system is SGUUB. The tracking error can be made arbitrarily small by increasing the approximation accuracy of the neural networks and the high-gain $1/\epsilon$ of the state observer.

Proof: See Appendix D. □

Remark 5.1: The high-gain observer (37) may exhibit a peaking phenomenon in the transient behavior. The input saturation method introduced in [5], [14] may be used to overcome such a problem. Thus during the short transient period when the state estimates exhibit peaking, the controller saturates to prevent peaking from being transmitted to the plant.

Remark 5.2: The adaptive output feedback NN controller proposed here is easy to implement because it is simply a state feedback design with a linear high-gain observer without a priori knowledge of the nonlinear systems. Unlike exact linearization approach [1]–[3], it is not necessary to search for a nonlinear transformation and an explicit control function.

Remark 5.3: The neural networks used in this paper are two-layer linearly parametrized NN's. If nonlinearly parametrized NN's (such as Sigmoidal multilayer neural networks) are used, the approximation accuracy might be improved. Similar results of this paper can still be achieved by suitably modifying

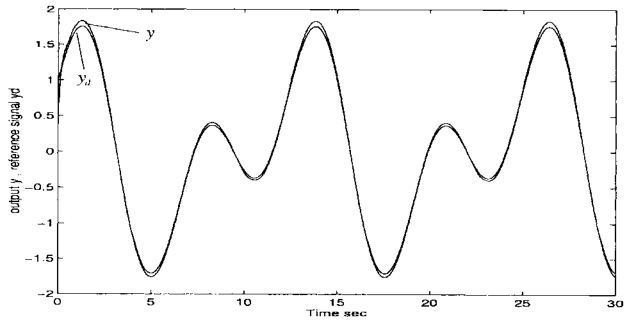

Fig. 2. Tracking performance of state feedback control.

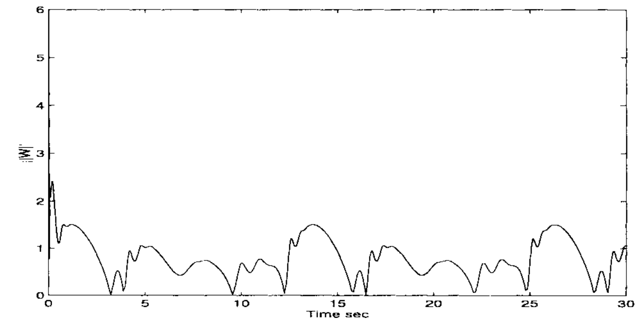

Fig. 4. Norm of estimated weights $||\hat{W}||$.

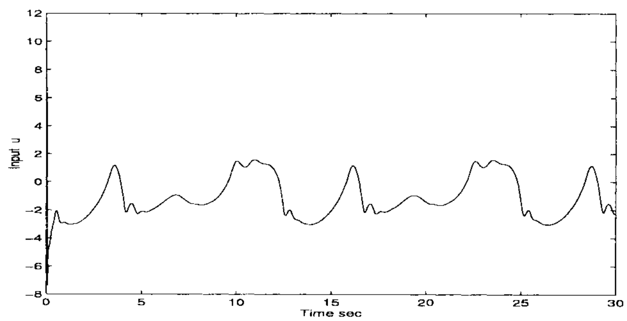

Fig. 3. Control input of state feedback control.

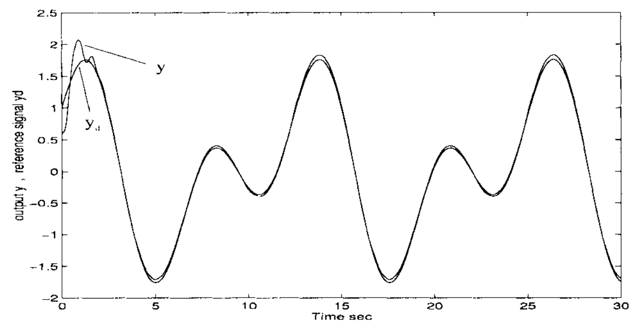

Fig. 5. Tracking performance of output feedback control.

the weight update laws using the techniques given in [16] and [18].

VI. Simulation Study

An example is used to illustrate the effectiveness of the proposed adaptive controller for unknown nonaffine nonlinear systems. Consider a nonlinear plant

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1^2 + 0.15u^3 + 0.1(1 + x_2^2)u + \sin(0.1u) \\ y &= x_1. \end{aligned} \tag{50}$$

Since the nonlinearities in the plant is an implicit function with respect to u, it is impossible to get an explicit controller for system feedback linearization. In this example, we suppose that there is no a priori knowledge of the system nonlinearities. As $\partial f(x,u)/\partial u = 0.45u^2 + 0.1 + 0.1x_2^2 + 0.1\cos(0.1u) > 0$ for all $(x,u) \in R^{n+1}$, Assumption 2 is satisfied. The tracking objective is to make the output $y(t)$ follow a desired reference $y_d(t) = \sin(t) + \cos(0.5t)$. The initial conditions are $x(0) = [0.6, 0.5]^T$.

The neural network controller $u(t) = \hat{W}^T S(z)$ has been chosen with $l = 8$, $\mu_i = 0.0$ and $\sigma_i^2 = 0.1$ for $i = 1, 2, \ldots, l$. Other controller parameters are chosen as $\Lambda = 10.0$, $k_v = 2.0$, $\gamma = 0.03$. The initial conditions for neural networks are $\hat{W}(0) = 0.0$.

1) State Feedback Result: When x_1 and x_2 are measurable, we choose the adaptive NN controller $u(t) = \hat{W}^T S(z)$ with the input vector $z = [x_1, x_2, \nu]^T$. The parameters in the weight update law (25) are chosen as $\kappa_0 = \kappa_1 = \kappa_2 = 10.0$ and $\delta = 5.0$. Fig. 2 shows that the output y tracks the reference y_d effectively, and Fig. 3 indicates the history of the control input u. The norm of the weight estimates is also given in Fig. 4 to illustrate the boundedness of the NN weight estimates. The results of the simulation show good transient performance and the tracking error is small with all the signals in the closed-loop system being bounded.

2) Output Feedback Result: When x_2 is not measurable, a high-gain observer is designed as follows:

$$\begin{aligned} \epsilon\dot{\xi}_1 &= \xi_2 \\ \epsilon\dot{\xi}_2 &= \xi_3 \\ \epsilon\dot{\xi}_3 &= -b_1\xi_3 - b_2\xi_2 - \xi_1 + y(t) \end{aligned} \tag{51}$$

with the parameters $\epsilon = 0.01$, $b_1 = 1.0$, $b_2 = 3.0$ and the initial condition $\xi(0) = [0.0, 0.0, 0.0]^T$. The estimate of vector z is $\hat{z} = [x_1, \xi_2/\epsilon, \hat{\nu}]^T$. We use the output feedback adaptive NN controller proposed in Section V to control the system. In order to avoid the peaking phenomenon, the saturation of the control input $u(t)$ is ± 4.0.

Figs. 5–8 illustrate the simulation result of the adaptive output feedback controller. It can be seen that, after a short period of peaking shown in Fig. 6, the tracking error and the state estimate error becomes small and the saturation mechanism in Fig. 7 becomes idle. The plots indicate the satisfied tracking performance with bounded closed-loop system signals.

VII. Conclusion

The main contribution of this paper is the development of two novel adaptive NN controllers for a general class of nonlinear systems by state feedback and output feedback. Compared with previous adaptive controllers, the proposed controllers are applicable to a larger class of nonlinear systems and does not require an off-line training phase for neural networks. The overall system is proved to be SGUUB and the

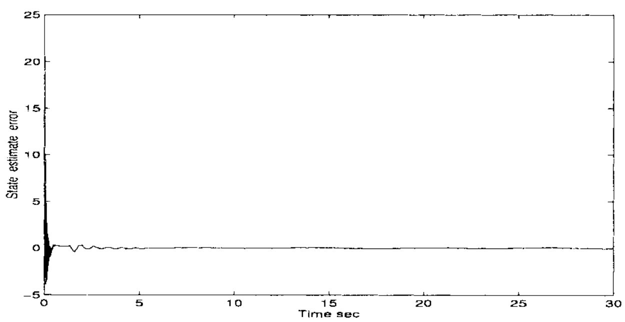

Fig. 6. The estimate error $\hat{x}_2 x_2$ in high-gain observer.

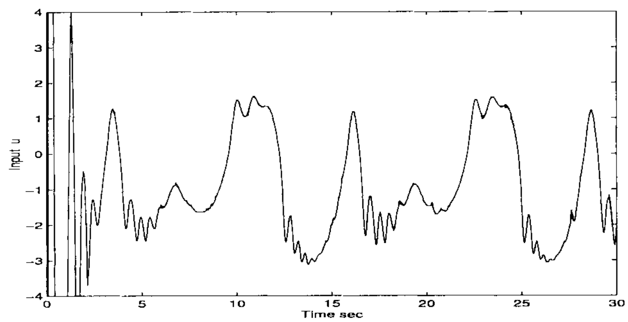

Fig. 7. Control input of output feedback control.

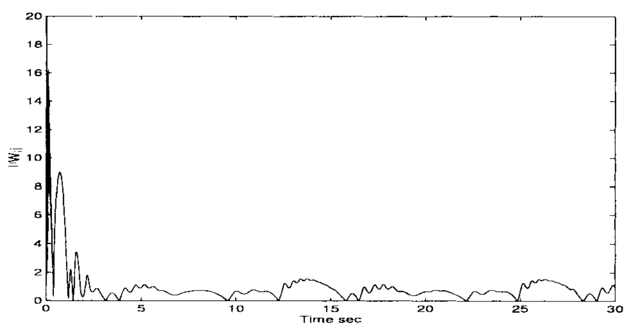

Fig. 8. Norm of estimated weights $||\hat{W}||$.

tracking error converges to an adjustable set. The theoretical analysis and the simulation results show that the proposed scheme is effective in controlling nonlinear dynamic systems.

APPENDIX/PROOFS

A. Proof of Lemma 3

Since $f(x,u)$ is a smooth function with respect to u (Assumption 2), we can write the Taylor series expansion of $f(x,u)$ at a given point $u_{\rm nn}^* = W^{*T}S(z)$ as

$$f(x,u) = f(x,u_{\rm nn}^*) + \Delta f_{u_{\rm nn}^*}^{[1]}(u - u_{\rm nn}^*) + O(\cdot)(u - u_{\rm nn}^*)$$

where

$$\Delta f_{u_{\rm nn}^*}^{[k]} = \left.\frac{\partial^{(k)} f(x,u)}{k!\partial u^{(k)}}\right|_{u=u_{\rm nn}^*}, \quad k = 1,2,\ldots$$
$$O(\cdot) = \Delta f_{u_{\rm nn}^*}^{[2]}(u - u_{\rm nn}^*) + \Delta f_{u_{\rm nn}^*}^{[3]}(u - u_{\rm nn}^*)^2 + \cdots + \Delta f_{u_{\rm nn}^*}^{[i]}(u - u_{\rm nn}^*)^{i-1} + \cdots.$$

Let $u = u^*$, according to (16), we have that $u - u_{\rm nn}^* = \varepsilon_u(z)$. Thus

$$f(x,u^*) = f(x,u_{\rm nn}^*) + \Delta f_{u_{\rm nn}^*}^{[1]}\varepsilon_u(z) + O(\cdot)\varepsilon_u(z)$$

As $\Delta f_{u_{\rm nn}^*}^{[k]}$ are continuous functions, from Lemma 1 we know that $\Delta f_{u_{\rm nn}^*}^{[1]}$ and $O(\cdot)$ are Lipschitz on the compact set Ω. Since $||W^*||$ is bounded and u^* is a function of x and x_d, by using $\varepsilon = \sup_{z\in\Omega_z}\{|\varepsilon_u(z)|\}$, there exist three constants a_1, a_2, and a_3 such that

$$|f(x,u^*) - f(x,u_{\rm nn}^*)| = \left|\Delta f_{u_{\rm nn}^*}^{[1]}\varepsilon_u(z) + O(\cdot)\varepsilon_u(z)\right| \leq a_1\varepsilon||x|| + a_2\varepsilon||\nu|| + a_3\varepsilon.$$

From (5) and (6) and Assumption 3, we can derive $||x|| \leq d_1 + d_2|e|$ and $||\nu|| \leq d_3 + d_4|e|$ with positive constants d_1 to d_4. Hence there exist positive constants C_1 and C_2 such that

$$|f(x,u^*) - f(x,u_{\rm nn}^*)| \leq C_1\varepsilon|e| + C_2\varepsilon \tag{52}$$

with $C_1 = a_1(d_2 + d_4)$ and $C_2 = a_1 d_1 + a_2 d_3 + a_3$. From (11) and (52), we have

$$|f(x,u^*) - f(x,u_{\rm nn}^*)| = |f(x,u_{\rm nn}^*) + \nu| \leq C_1\varepsilon|e| + C_2\varepsilon.$$

□

B. Proof of Lemma 5

According to (38)–(41), $\hat{z}$ can be rewritten as

$$\hat{z} = z - \epsilon\bar{\psi} \tag{53}$$

with $\bar{\psi}$ being suitably chosen bounded vector. Substituting (53) into (45), we have

$$s_i(\hat{z}) = s_i(z)\cdot\exp\left[\frac{2\epsilon(z-\mu_i)^T\bar{\psi}}{\sigma_i^2}\right]\cdot\exp\left[\frac{-(\epsilon||\bar{\psi}||)^2}{\sigma_i^2}\right], \quad i = 1,2,\ldots,l. \tag{54}$$

The Taylor series expansions of $\exp[\frac{2\epsilon(z-\mu_i)^T\bar{\psi}}{\sigma_i^2}]$ and $\exp[\frac{-(\epsilon||\bar{\psi}||)^2}{\sigma_i^2}]$ at zero are

$$\exp\left[\frac{2\epsilon(z-\mu_i)^T\bar{\psi}}{\sigma_i^2}\right] = 1 + \epsilon O_{1i}$$
$$\exp\left[\frac{-(\epsilon||\bar{\psi}||)^2}{\sigma_i^2}\right] = 1 + \epsilon O_{2i}$$

where O_{1i} and O_{2i} denote the remaining factors of the higher-order terms of the Taylor series expansions. Thus, (54) can be written as

$$s_i(\hat{z}) = s_i(z)(1+\epsilon O_{1i})(1+\epsilon O_{2i}) = s_i(z) + \epsilon s_{ti}$$

where $s_{ti} = s_i(z)\epsilon O_{1i}(1+\epsilon O_{2i}) + s_i(z)O_{2i}$. Since $s_i(\hat{z})$ and $s_i(z)$ are bounded basis functions, s_{ti} is also bounded. It follows from (14) that

$$S(\hat{z}) = S(z) + \epsilon S_t$$

where $S_t = [s_{t1}, s_{t2}, \ldots, s_{tl}]^T$ is a bounded vector function.

□

C. Proof of Lemma 6

The Taylor series expansion of $f(x,u)$ at $\hat{u}_{\rm nn}^o = \hat{W}^T S(\hat{z})$ is

$$f(x,u) = f\big(x, \hat{u}_{\rm nn}^o\big) + \big(\Delta f^{[1]}_{\hat{u}_{\rm nn}^o} + O_f\big)[u - \hat{W}^T S(\hat{z})]$$

where

$$O_f = \Delta f^{[2]}_{\hat{u}_{\rm nn}^o}[u - \hat{W}^T S(\hat{z})] + \cdots + \Delta f^{[i]}_{\hat{u}_{\rm nn}^o}[u - \hat{W}^T S(\hat{z})]^{i-1} + \cdots.$$

Let $u = u^*_{\rm nn} = W^{*T} S(z)$, we know that (47) holds.

Since $\Delta f^{[k]}_{\hat{u}_{\rm nn}^o}$, $(k = 2, 3, \ldots,)$, are continuous functions, we know from Lemma 1 that $\partial f(x,u)/\partial u$ and O_f are Lipschitz on the compact set Ω_z. Since $\hat{u}_{\rm nn}^o$ is a function of $\hat{W}$, x and x_d, there exist positive constants a_4 to a_9 such that

$$\big|\Delta f^{[1]}_{\hat{u}_{\rm nn}^o}\big| \le a_4\|\hat{W}\| + a_5\|x\| + a_6$$
$$|O_f| \le a_7\|\hat{W}\| + a_8\|x\| + a_9.$$

From (5) and (6), we can derive $\|x\| \le d_1 + d_2|e|$ with d_1 and d_2 being positive constants, which means that positive constants L_0 to L_5 exist such that (48) and (49) hold. □

D. Proof of Theorem 2

The proof is similar to that of [4]. We first assume that the system signals remain in a compact set, then, we show that, by properly choosing controller parameters, the system trajectories in fact remain in the compact set. Choose a Lyapunov function candidate

$$V = \frac{e^2}{2} + \frac{1}{2}\hat{W}^T\hat{W}. \tag{55}$$

Differentiating (55) along (43) and (44) yields

$$\begin{aligned}\dot{V} &= -\hat{W}^T(\kappa_0\|\hat{W}\| + \kappa_1|\hat{e}| + \kappa_2)S(\hat{z})\hat{e}\\ &\quad - \delta(\|\hat{W}\| + |\hat{e}| + 1)\|S(\hat{z})\||\hat{e}|\|\hat{W}\|^2\\ &\quad - \epsilon|\hat{e}|^2\|\hat{W}\|^2 + e\big[-k_v e - k_v \text{sat}(e) + f\big(x, \hat{u}^o_{\rm nn}\big) + \nu\big].\end{aligned}$$

From (40), (46), and (47), the following equation follows

$$\begin{aligned}\dot{V} &= -(\kappa_0\|\hat{W}\| + \kappa_1|\hat{e}| + \kappa_2)\hat{W}^T S(\hat{z})\hat{e}\\ &\quad - \delta(\|\hat{W}\| + |\hat{e}| + 1)\|S(\hat{z})\||\hat{e}|\|\hat{W}\|^2\\ &\quad - \epsilon|\hat{e}|^2\|\hat{W}\|^2 + k_v(\hat{e} - \epsilon\Lambda^T\Psi)^2\\ &\quad - k_v(\hat{e} - \epsilon\Lambda^T\Psi) * \text{sat}(\hat{e} - \epsilon\Lambda^T\Psi)\\ &\quad + [f(x, \hat{u}^*_{\rm nn}) + \nu]e - \big(\Delta f^{[1]}_{\hat{u}^o_{\rm nn}} + O_f\big)\\ &\quad \times [W^{*T}S(z) - \hat{W}^T S(\hat{z})]e.\end{aligned}$$

By using (8), we obtain

$$(\hat{e} - \epsilon\Lambda^T\Psi)\text{sat}(\hat{e} - \epsilon\Lambda^T\Psi) \ge |\hat{e}| - |\epsilon\Lambda^T\Psi| - \gamma/e_0$$

where e_0 is the natural exponential (i.e., $e_0 = 2.7183$). Considering (18), (19), (48) and $L_3\|\hat{W}\| + L_4|e| + L_5 \ge \Delta f^{[1]}_{\hat{u}^o_{\rm nn}} > 0$, we have

$$\begin{aligned}\dot{V} &\le (|\kappa_0 - L_3|\|\hat{W}\| + |\kappa_1 - L_4||\hat{e}| + |\kappa_2 - L_5|)|\hat{W}^T S(\hat{z})\hat{e}|\\ &\quad - \delta(\|\hat{W}\| + |\hat{e}| + 1)\|S(\hat{z})\||\hat{e}|\|\hat{W}\|^2 + (\epsilon C_1|e| + \epsilon C_2)|e|\\ &\quad - \epsilon|\hat{e}|^2\|\hat{W}\|^2 - k_v(\hat{e} - \epsilon\Lambda^T\Psi)^2 - k_v(|\hat{e}| - |\epsilon\Lambda^T\Psi|\\ &\quad - \gamma/e_0) + [(L_0 + L_3)\|\hat{W}\| + (L_1 + L_4)|e| + L_2\\ &\quad + L_5]w_m\|S(z)\||e| + (L_0\|\hat{W}\| + L_1|e|\\ &\quad + L_2)\|\hat{W}\|\|S(\hat{z})\||e|.\end{aligned} \tag{56}$$

From (40) and (46)

$$\|S(z)\| \le \|S(\hat{z})\| + \epsilon\|S_t\|$$
$$|e| = |\hat{e} - \epsilon\Lambda^T\Psi| \le |\hat{e}| + |\epsilon\Lambda^T\Psi|.$$

Define

$$\alpha_1 = |\kappa_0 - L_3| + L_0 \tag{57}$$

$$\alpha_2 = |\kappa_1 - L_4| + L_1 \tag{58}$$

$$\alpha_3 = |\kappa_2 - L_5| + L_2 + (L_0 + L_3)w_m + 2L_1|\epsilon\Lambda^T\Psi| \tag{59}$$

$$\begin{aligned}\alpha_4 &= k_v(\epsilon\Lambda^T\Psi)^2 + [(L_1 + L_4)|\epsilon\Lambda^T\Psi| + L_2 + L_5]\\ &\quad \times w_m\|S(z)\||\epsilon\Lambda^T\Psi| + \varepsilon(C_1|\epsilon\Lambda^T\Psi| + C_2) + k_v\gamma/e_0\\ &= \varepsilon\alpha_{4a} + \epsilon\alpha_{4b} + k_v\gamma/e_0\end{aligned} \tag{60}$$

$$\begin{aligned}\alpha_5 &= k_v\epsilon\Lambda^T\Psi + [2(L_1 + L_4)|\epsilon\Lambda^T\Psi| + L_2 + L_5]\\ &\quad \times w_m\|S(z)\| + \varepsilon(C_1|\epsilon\Lambda^T\Psi| + C_2)\\ &= \varepsilon\alpha_{5a} + \epsilon\alpha_{5b} + \alpha_{5c}\end{aligned} \tag{61}$$

$$\alpha_6 = (L_1 + L_4)\|S(z)\|w_m + \varepsilon C_1 = \varepsilon C_1 + \alpha_{6a} \tag{62}$$

$$\alpha_7 = (L_0 + L_3)w_m\|S_t\|\epsilon = \epsilon\alpha_{7a} \tag{63}$$

where α_1 to α_2 are positive constants and α_3 to α_7 are positive definite and bounded functions. α_{4a}, α_{4b}, α_{4c}, α_{5a}, α_{5b}, α_{5c}, α_{6a}, and α_{7a} are properly chosen bounded functions. Thus, (56) can be rewritten as

$$\begin{aligned}\dot{V} &\le -(k_v - \alpha_6)\hat{e}^2 - (k_v - \alpha_5)|\hat{e}| + \alpha_4 + \alpha_7\|\hat{W}\||\hat{e}|\\ &\quad - \epsilon(\|\hat{W}\||\hat{e}|)^2 - \|S(\hat{z})\|[\delta\hat{e}^2\|\hat{W}\|^2 + \delta|\hat{e}|\|\hat{W}\|^2\\ &\quad + \delta|\hat{e}|\|\hat{W}\|^3] + \|S(\hat{z})\|[\alpha_1|\hat{e}|\|\hat{W}\|^2 + \alpha_2\hat{e}^2\|\hat{W}\|\\ &\quad + \alpha_3|\hat{e}|\|\hat{W}\|]\\ &\le -(k_v - \alpha_6)\hat{e}^2 - (k_v - \alpha_5)|\hat{e}| + \alpha_4 - \frac{\delta}{2}|\hat{e}|\|S(\hat{z})\|\|\hat{W}\|^2\\ &\quad - \delta|\hat{e}|\|\hat{W}\|\|S(\hat{z})\|\left[\left(\|\hat{W}\| - \frac{\alpha_1}{2\delta}\right)^2 - \left(\frac{\alpha_1}{2\delta}\right)^2\right]\\ &\quad - \delta\hat{e}^2\|S(\hat{z})\|\left[\left(\|\hat{W}\| - \frac{\alpha_2}{2\delta}\right)^2 - \left(\frac{\alpha_2}{2\delta}\right)^2\right]\\ &\quad - \frac{\delta}{2}|\hat{e}|\|S(\hat{z})\|\left[\left(\|\hat{W}\| - \frac{\alpha_3}{\delta}\right)^2 - \left(\frac{\alpha_3}{\delta}\right)^2\right]\\ &\quad - \epsilon(\|\hat{W}\||\hat{e}|)^2 + \alpha_7\|\hat{W}\||\hat{e}|\end{aligned} \tag{64}$$

$$\begin{aligned}&\le -\left(\frac{k_v}{2} - \alpha_6 - \frac{\alpha_2^2}{4\delta}\|S(\hat{z})\|\right)\hat{e}^2\\ &\quad - \left(k_v - \alpha_{5c} - \frac{\alpha_3^2}{2\delta}\|S(\hat{z})\|\right)|\hat{e}| + \alpha_4\\ &\quad - \frac{k_v}{2}\left[\left(|\hat{e}| - \frac{\varepsilon\alpha_{5a} + \epsilon\alpha_{5b}}{k_v}\right)^2 - \left(\frac{\varepsilon\alpha_{5a} + \epsilon\alpha_{5b}}{k_v}\right)^2\right]\\ &\quad - \frac{\delta}{2}|\hat{e}|\|S(\hat{z})\|\left[\left(\|\hat{W}\| - \frac{\alpha_1^2}{4\delta}\right)^2 - \frac{\alpha_1^4}{16\delta^2}\right]\\ &\quad - \epsilon\left[\left(\|\hat{W}\||e| - \frac{\alpha_{7a}}{2}\right)^2 - \left(\frac{\alpha_{7a}}{2}\right)^2\right].\end{aligned} \tag{65}$$

Consider (59) to (63), (65) becomes

$$\begin{aligned}\dot{V} \le& -\left[\frac{k_v}{2} - \varepsilon C_1 - \alpha_{6a} - \frac{\alpha_2^2}{4\delta}\|S(\hat{z})\|\right]\hat{e}^2\\ &- \left[k_v - \alpha_{5c} - \left(\frac{\alpha_1^4}{32\delta} + \frac{\alpha_3^2}{2\delta}\right)\|S(\hat{z})\|\right]|\hat{e}|\\ &+ \frac{(\varepsilon\alpha_{5a} + \epsilon\alpha_{5b})^2}{2k_v} + \varepsilon\alpha_{4a} + \epsilon\alpha_{4b} + k_v\gamma/e_0 + \epsilon\frac{\alpha_{7a}^2}{4}\\ \le& -\beta_1\hat{e}^2 - \beta_2|\hat{e}| + \varepsilon\alpha_9 + \epsilon\alpha_8 + k_v\gamma/e_0\end{aligned}$$

where

$$\beta_1 = \frac{k_v}{2} - \varepsilon C_1 - \alpha_{6a} - \frac{\alpha_2^2}{4\delta}\alpha_s \tag{66}$$

$$\beta_2 = k_v - \alpha_{5c} - \left(\frac{\alpha_1^4}{32\delta} + \frac{\alpha_3^2}{2\delta}\right)\alpha_s \tag{67}$$

$$\begin{aligned}\alpha_s &= \sup\{\|S(\hat{z})\|\}\\ \alpha_8 &= \sup\left\{\alpha_{4b} + \frac{\alpha_{7a}^2}{4} + \frac{\epsilon(\alpha_{5b})^2}{2k_v}\right\}\\ \alpha_9 &= \sup\left\{\alpha_{4a} + \frac{2\epsilon\alpha_{5a}\alpha_{5b} + \varepsilon(\alpha_{5b})^2}{2k_v}\right\}.\end{aligned}$$

Define

$$\begin{aligned}k_0 = \max\Bigg\{&2\varepsilon_0 C_1 + 2\alpha_{6a} + \frac{\alpha_2^2}{2\delta}\alpha_s + \beta_0,\\ &\alpha_{5c} + \left(\frac{\alpha_1^4}{32\delta} + \frac{\alpha_3^2}{2\delta}\right)\alpha_s + \beta_0\Bigg\}\end{aligned}$$

where ε_0 and β_0 are positive constants. Since α_1 to α_3, α_{5c} and α_{6a} are bounded, α_s, δ and C_1 are positive constants, we know that k_0 is a positive constant. Therefore, if choosing $k_v > k_0$, (66) and (67) show that $\beta_1 > \beta_0$ and $\beta_2 > \beta_0$.

Define

$$\begin{aligned}\Theta_e := \Bigg\{e(t) : |e| \le \max\Bigg[&\sqrt{\frac{1}{\beta_1}\left(\frac{k_v\gamma}{e_0} + \varepsilon\alpha_9 + \epsilon\alpha_8\right) + \epsilon\Lambda^T\Psi},\\ &\frac{1}{\beta_2}\left(\frac{k_v\gamma}{e_0} + \varepsilon\alpha_9 + \epsilon\alpha_8\right) + \epsilon\Lambda^T\Psi\Bigg]\Bigg\}.\end{aligned} \tag{68}$$

Since α_8, α_9, γ, ϵ and e_0 are positive constants, $k_v > k_0$, $\beta_1 > \beta_0$ and $\beta_2 > \beta_0$, the set Θ_e is a compact set. $\dot{V}$ is negative as long as $e(t)$ is outside the compact set Θ_e. Hence, e and $\hat{e}$ are bounded.

Next we prove the boundedness of the NN weight vector $\hat{W}$. Considering the Lyapunov function candidate

$$V_1 = \frac{1}{2}\hat{W}^T\hat{W}$$

and taking the derivative of V_1 along (43) with respect to time, we obtain

$$\begin{aligned}\dot{V}_1 =& -\hat{W}^T(\kappa_0\|\hat{W}\| + \kappa_1|\hat{e}| + \kappa_2)S(\hat{z})\hat{e}\\ &- \delta(\|\hat{W}\| + |\hat{e}| + 1)\|S(\hat{z})\||\hat{e}|\|\hat{W}\|^2 - \epsilon(\|\hat{W}\||\hat{e}|)^2\\ \le& (\kappa_0\|\hat{W}\| + \kappa_1|\hat{e}| + \kappa_2)\|S(\hat{z})\|\|\hat{W}\||\hat{e}|\\ &- \delta(\|\hat{W}\| + |\hat{e}| + 1)\|S(\hat{z})\||\hat{e}|\|\hat{W}\|^2\\ \le& -\|S(\hat{z})\|\|\hat{W}\||\hat{e}|\Bigg\{\delta\left[\|\hat{W}\| + \frac{1}{2}\left(1 + |\hat{e}| - \frac{\kappa_0}{\delta}\right)\right]^2\\ &- \frac{\delta}{4}\left(1 + |\hat{e}| - \frac{\kappa_0}{\delta}\right)^2 - \kappa_1|\hat{e}| - \kappa_2\Bigg\}.\end{aligned}$$

Define

$$\begin{aligned}\Theta_w := \Bigg\{\hat{W}(t) : \|\hat{W}\| \le \sup_{e\in\Theta_e}\Bigg[&\frac{1}{2}\left(1 + |\hat{e}| - \frac{\kappa_0}{\delta}\right)\\ &+ \sqrt{\frac{1}{4}\left(1 + |\hat{e}| - \frac{\kappa_0}{\delta}\right)^2 + \frac{1}{\delta}(\kappa_1|\hat{e}| + \kappa_2)}\Bigg]\Bigg\}.\end{aligned}$$

Since $\hat{e}$ is bounded, the set Θ_w is also a compact set. $\dot{V}_1 \le 0$ as long as $\hat{W}(t)$ is outside Θ_w. Now define

$$\Theta := \{(e, \hat{W}, \xi) \mid e \in \Theta_e, \hat{W} \in \Theta_w, \xi(t) \in \Theta_\xi\}.$$

If we initialize $\hat{W}(0)$ inside D_w, $e(0)$ inside D_e, $\xi(0)$ inside D_ξ, and choose a large enough k_v guaranteeing $\beta_1 > \beta_0$ and $\beta_2 > \beta_0$, then there exists a constant T such that all trajectories will converge to Θ and remain in Θ for all $t > T$. This implies that the closed-loop system is SGUUB. The filtered tracking error will converge to the small compact set Θ_e which is a $(\varepsilon, \epsilon, \gamma)$-neighborhood of the origin.

Since $s^{n-1} + \lambda_{n-1}s^{n-2} +, \ldots, +\lambda_1$ is Hurwitz, $y(t) - y_d(t) \to \Theta_e$ as $e(t) \to \Theta_e$. In addition, because ε and γ can be made arbitrarily small by increasing the number of neural nodes l and the state observer gain $1/\epsilon$ can be designed arbitrarily large, we conclude that arbitrarily small tracking error is achievable. □

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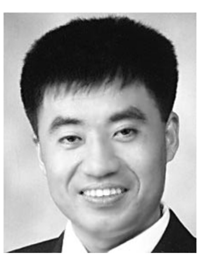

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