Practical Adaptive Neural Control of Nonlinear Systems With Unknown Time Delays

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***Abstract*—Practical adaptive neural control is presented for a class of nonlinear systems with unknown time delays in strict-feedback form. Using appropriate Lyapunov–Krasovskii functionals, the unknown time delays are compensated for. Controller singularity problems are solved by practical neural network control. A novel differentiable control function is provided such that the practical design can be carried out in the decoupled backstepping design. It is proved that the proposed design method is able to guarantee semi-global uniform ultimate boundedness of all the signals in the closed-loop system, and the tracking error is proven to converge to a small neighborhood of the origin.**

***Index Terms*—Decoupled backstepping, differentiable control, nonlinear time-delay system, practical neural networks.**

I. Introduction

Robust control of systems with time delays has attracted much attention due to its mathematical challenge and application demand in real-time control. The existence of time delays may make the stabilization problem become much more difficult. Lyapunov–Krasovskii functionals [1], combined with the linear matrix inequality (LMI) technique [2], have been used to establish a framework for the stability and control of time-delay systems [3], [4]. There are basically two stability checking criteria—delay-dependent [5] and delay-independent [6]. Robust control of time-delay systems using the above-mentioned technique is also intensively investigated. However, for nonlinear systems with delay in the state, few results are reported. In [7] and [8], for a class of nonlinear time-delay systems in strict-feedback form, systematic and practical backstepping design has been presented. Under the mild assumption on the upper bound of the unknown time-delay, the proposed design based on the Lyapunov stability is delay-independent in the sense that the design is totally free from unknown delays except for their upper limits. The controller singularity problem is solved by introducing the practical design and using integral Lyapunov function. However, due to the integral operation, the controller is complicated in the practical implementation, and so is its derivation. Motivated by the results in [9] and [10], where the systems properties have been fully exploited such that a rather simple control scheme has been developed without using integral-Lyapunov functions and singularity problems have been avoided as well, we present a direct neural network controller for a class of time-delay systems in strict-feedback form. A continuous function is introduced to solve the differentiation problem at certain discontinuous points.

The main contribution of the paper lies in i) the novel introduction of continuous functions to provide smooth control functions that are differentiable to any required degree such that the practical control can be carried out in the backstepping design and ii) the employment of decoupled backstepping design, by which the stability analysis of the proposed practical control can be carried out in a nested matter to guarantee the closed-loop stability, and the residual set of each state in z_i coordinate can be iteratively individually determined.

Manuscript received February 17, 2004; revised September 12, 2004. This paper was recommended by Associate Editor Chi-Hsu Wang.

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Digital Object Identifier 10.1109/TSMCB.2005.846645

II. Problem Formulation

Consider a class of single-input-single-output (SISO) nonlinear time-delay systems, as in (1), shown at the bottom of the next page, where $\bar{x}_i = [x_1, \ldots, x_i]^T$, $x = [x_1, \ldots, x_n]^T \in R^n$, and $u \in R$ are the state variables and system input, respectively, $g_i(\cdot)$, $f_i(\cdot)$, and $h_i(\cdot)$ are unknown smooth functions, and τ_i are unknown time delays of the states $i = 1, \ldots, n$. The control objective is to design an adaptive controller for system (1) such that the state $x_1(t)$ follows a desired reference signal $y_d(t)$, whereas all signals in the closed-loop system are bounded. Define the desired trajectory $\bar{x}_{d(i+1)} = [y_d, \dot{y}_d, \ldots, y_d^{(i)}]^T$, $i = 1, \ldots, n-1$.

- A1) The system states $x(t)$ and part of their time derivatives $\dot{\bar{x}}_{n-1}(t)$ are all available for feedback.
- A2) The signs of $g_i(\cdot)$ are known, and there exist known constants $g_{\max} \geq g_{\min} > 0$ such that $g_{\min} \leq |g_i(\cdot)| \leq g_{\max}$, and $\partial g_n(x)/\partial x_n = 0$.
- A3) The desired trajectory vectors $\bar{x}_{di}$, $i = 2, \ldots, n$ are continuous and available, and $\bar{x}_{di} \in \Omega_{di} \subset R^i$ with Ω_{di} known compact sets.
- A4) The unknown smooth functions $h_i(\bar{x}_i(t))$ satisfy the inequality $|h_i(\bar{x}_i(t))| \leq \sum_{j=1}^{i} |x_j(t)| \varrho_{ij}(\bar{x}_i(t))$ with $\varrho_{ij}(\cdot)$ known smooth functions.
- A5) The size of the unknown time delays is bounded by a known constant, i.e., $\tau_i \leq \tau_{\max}$, $i = 1, \ldots, n$.

Without losing generality, we will only consider the case when $g_i(\cdot) > 0$. Note that the requirement for $\dot{\bar{x}}_{n-1}(t)$ is a constraint but realistic for many physical systems as we are not requiring $\dot{x}_n$, which is directly influenced by the control. In addition, $\partial g_n(x)/\partial x_n = 0$ means that

$$\dot{g}_n(x) = \left[\frac{\partial g_n(x)}{\partial x}\right]^T \dot{x}(t) = \sum_{i=1}^{n-1} \frac{\partial g_n(x)}{\partial x_i} \dot{x}_i(t) \tag{2}$$

which is only dependent on the state x. Obviously, $\dot{g}_i(\bar{x}_i)$, $i = 1, \ldots, n-1$ is also dependent on the state x only. As $g_i(\cdot)$ is a smooth function, we know that $\forall \bar{x}_i \in \Omega$ with Ω being a bounded compact set, there exists a known constant $g_{id} > 0$ such that $|\dot{g}_i(\cdot)| \leq g_{id}$. This nice property could be used to simplify the later controller design.

III. Preliminaries

A function approximator will be used to approximate the unknown nonlinear functions. In this paper, the radial basis function (RBF) neural network (NN) [11] is used to approximate the continuous function $h(Z) : R^q \to R$ as

$$h_{nn}(Z) = W^T S(Z) \tag{3}$$

where the input vector $Z \in \Omega_Z \subset R^q$, weight vector $W = [w_1, w_2, \ldots, w_l]^T \in R^l$, the NN node number $l > 1$, and $S(Z) = [s_1(Z), \ldots, s_l(Z)]^T$, with $s_i(Z)$ being chosen as the commonly used Gaussian functions, which have the form

$$s_i(Z) = e^{-(Z-\mu_i)^T(Z-\mu_i)/\eta_i^2}, \quad i = 1, \ldots l \tag{4}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{iq}]^T$ is the center of the receptive field, and η_i is the width of the Gaussian function. Universal approximation results in [12] and [13] indicate that, if l is chosen sufficiently large,

$W^T S(Z)$ can approximate any continuous function $h(Z)$ to any desired accuracy over a compact set $\Omega_Z \subset R^q$ to arbitrary accuracy in the form of

$$h(Z) = W^{*T} S(Z) + \epsilon(Z), \;\; \forall Z \in \Omega_Z \subset R^q \tag{5}$$

where W^* is the ideal constant weight vector, and $\epsilon(Z)$ is the approximation error that is bounded over the compact set, i.e., $|\epsilon(Z)| \leq \epsilon^*$, $\forall Z \in \Omega_Z$, where $\epsilon^* > 0$ is an unknown constant. The ideal weight vector W^* is an "artificial" quantity required for analytical purposes. W^* is defined as the value of W that minimizes $|\epsilon|$ for all $Z \in \Omega_Z \subset R^q$, i.e.,

$$W^* := \arg \min_{W \in R^l} \left\{ \sup_{Z \in \Omega_Z} |h(Z) - W^T S(Z)| \right\}. \tag{6}$$

Suppose that $x \in \Omega_Z$, where Ω_Z is a compact set. Define sets $\Omega_{c_z} \subset \Omega_Z$ and Ω_Z^0 as

$$\Omega_{c_z} := \{x \mid |x| < c_z\} \tag{7}$$
$$\Omega_Z^0 := \Omega_Z - \Omega_{c_z} \tag{8}$$

where constant $c_z > 0$ and "$-$" denotes the complement of set B in set A as $A - B := \{x | x \in A \text{ and } x \notin B\}$.

Lemma 1 [7], [8]: Set Ω_Z^0 is a compact set.

Lemma 2: Even function $q_i(x) : R \to R$, equation (9) shown at the bottom of the page, where $c_{qi} = [2(n-i)+1]!/\lambda_{bi}^{2(n-i)+1} [(n-i)!]^2$ [14], $\lambda_{ai}, \lambda_{bi} > 0$, and integer $i \in R^+$, is $(n-i)$th differentiable, i.e., $q_i(x) \in C^{n-i}$ and bounded by 1.

IV. Direct NN Control for First-Order System

We first consider the tracking problem of a first-order system by defining the tracking error $z_1 = x_1 - y_d$

$$\dot{z}_1(t) = g_1(x_1(t))u(t) + f_1(x_1(t)) + h_1(x_1(t-\tau_1)) - \dot{y}_d(t). \tag{10}$$

Certainty equivalent control based on feedback linearization takes the form $u(t) = (1/g_1(x_1))[-f_1(x_1) + v(t)]$. If $g_1(\cdot)$ and $f_1(\cdot)$ are unknown, their estimates $\hat{g}_1$ and $\hat{f}_1$ will be used instead to construct the controller and singularity problem may occur when $\hat{g}_1(x_1) = 0$. To avoid it, we will estimate the unknown term, e.g., $f_1(x_1)/g_1(x_1)$, as a whole rather than estimate the function $g_1(\cdot)$ and $f_1(\cdot)$ individually. Another design difficulty comes from the unknown time-delay τ_1, which can be compensated for by introducing the Lyapunov–Krasovskii functional in the form of

$$V_U(t) = \int_{t-\tau_1}^{t} U(x(\tau))d\tau \tag{11}$$

with $U(\cdot) \geq 0$ being a properly chosen function. The time derivative of $V_U(t)$ is $\dot{V}_U(t) = U(x(t)) - U(x(t-\tau_1))$, where the term $U(x(t-\tau_1))$ can be used to compensate for the unknown time-delay terms related to τ_1, whereas the remaining term $U(x(t))$ does not introduce any uncertainties to the system.

Consider scalar smooth function $V_{z_1} = (1/2)z_1^2(t)/g_1(x_1)$ and Lyapunov–Krasovskii functional $V_{U_1}(t) = (1/2g_{\min}) \int_{t-\tau_1}^{t} U_1(x_1(\tau))d\tau$ with $U_1(x_1(t)) = (1/2)x_1^2(t)\varrho_1(x_1(t)) \geq 0$. Noting Assumption A4), and using Young's inequality, we have

$$\begin{aligned}
&\dot{V}_{z_1}(t) + \dot{V}_{U_1}(t) \\
&\quad \leq z_1(t) \left\{ u(t) + \frac{1}{g_1(x_1)} \left[f_1(x_1(t)) - \dot{y}_d(t) \right.\right. \\
&\qquad \left.\left. + \frac{1}{2} z_1(t) \right] \right\} + \frac{1}{2g_{\min}} x_1^2(t) \varrho_1^2(x_1(t)) - \frac{\dot{g}_1(x_1)}{2g_1^2(x_1)} z_1^2(t) \\
&\qquad + \frac{1}{2g_1(x_1(t))} x_1^2(t-\tau_1) \varrho_1^2(x_1(t-\tau_1)) \\
&\qquad - \frac{1}{2g_{\min}} x_1^2(t-\tau_1) \varrho_1^2(x_1(t-\tau_1)).
\end{aligned} \tag{12}$$

As $g_1(x_1(t)) \geq g_{\min}$, it follows that

$$\frac{1}{2g_1} x_1^2(t-\tau_1) \varrho_1^2(x_1(t-\tau_1)) \leq \frac{1}{2g_{\min}} x_1^2(t-\tau_1) \varrho_1^2(x_1(t-\tau_1)).$$

In addition, from Assumption A2), we have $-\dot{g}_1(x_1) z_1^2 / 2g_1^2(x_1) \leq |\dot{g}_1(x_1)| z_1^2 / 2g_1^2(x_1) \leq (g_{1d}/2g_{\min}) z_1^2$. Thus, (12) becomes

$$\dot{V}_{z_1}(t) + \dot{V}_{U_1}(t) \leq z_1(t)[u(t) + Q_1(Z_1(t))] + \frac{g_{1d}}{2g_{\min}} z_1^2 \tag{13}$$

where

$$Q_1(Z_1) = \frac{1}{g_1(x_1)} \left[f_1(x_1) - \dot{y}_d + \frac{1}{2} z_1 \right] + \frac{1}{2g_{\min} z_1} x_1^2 \varrho_1^2(x_1)$$

with $Z_1 = [x_1, y_d, \dot{y}_d]^T \in \Omega_{Z_1} \subset R^3$ and Ω_{Z_1} being a compact set that will be specified later.

From (13), it is found that the controller design is free from unknown time-delay τ_1 at the present stage. Since $f_1(\cdot)$ and $g_1(\cdot)$ are unknown smooth functions, NNs will be used to approximate the function $Q_1(Z_1)$. According to the main result stated in [15], any real-valued continuous function can be arbitrarily closely approximated by a network of RBF type over a compact set. However, it is apparent that $Q_1(Z_1)$ is not continuous over the compact set Ω_{Z_1} at $z_1(t) = 0$. Therefore, we will reconstruct the compact set over which the NN approximation is feasible and valid. To this end, define

$$\Omega_{c_{z_1}} := \{z_1 \mid |z_1| < c_{z_1}\} \subset \Omega_{Z_1} \tag{14}$$
$$\Omega_{Z_1}^0 := \Omega_{Z_1} - \Omega_{c_{z_1}}. \tag{15}$$

$$\begin{cases} \dot{x}_i(t) = g_i(\bar{x}_i(t))x_{i+1}(t) + f_i(\bar{x}_i(t)) + h_i(\bar{x}_i(t-\tau_i)), \; 1 \leq i \leq n-1 \\ \dot{x}_n(t) = g_n(x)u + f_n(x(t)) + h_n(x(t-\tau_n)) \end{cases} \tag{1}$$

$$q_i(x) = \begin{cases} 1, & |x| \geq \lambda_{ai} + \lambda_{bi} \\ c_{qi} \int_{\lambda_{ai}}^{x} \left[\left(\frac{\lambda_{bi}}{2}\right)^2 - \left(\sigma - \lambda_{ai} - \frac{\lambda_{bi}}{2}\right)^2 \right]^{n-i} d\sigma, & \lambda_{ai} < x < \lambda_{ai} + \lambda_{bi} \\ c_{qi} \int_{x}^{-\lambda_{ai}} \left[\left(\frac{\lambda_{bi}}{2}\right)^2 - \left(\sigma + \lambda_{ai} + \frac{\lambda_{bi}}{2}\right)^2 \right]^{n-i} d\sigma, & -(\lambda_{ai} + \lambda_{bi}) < x < -\lambda_{ai} \\ 0, & |x| \leq \lambda_{ai} \end{cases} \tag{9}$$

From Lemma 1, we know that $\Omega_{Z_1}^0$ is a compact set, over which function $Q_1(Z_1)$ is continuous and

$$Q_1(Z_1) = W_1^{*T} S(Z_1) + \epsilon_1(Z_1), \quad Z_1 \in \Omega_{Z_1}^0 \tag{16}$$

where $\epsilon_1(Z_1)$ is the approximation error, satisfying $|\epsilon_1(Z_1)| \leq \epsilon_{z_1}^*$ with $\epsilon_{z_1}^* > 0$ being an unknown constant. As the ideal weight W_1^* is unknown, we will use its estimate $\hat{W}_1$ instead in the later controller design.

Remark 1: As the function approximation property (16) of NNs is only guaranteed in a compact set, $\Omega_{Z_1}^0$ in this case, the stability result obtained is in the sense of semi-global, i.e., it is only valid in this compact set, yet this compact set can be made as large as possible. Adaptive NN control can be easily constructed by choosing NN of sufficiently large size to guarantee that all the closed-loop signals stay within this compact set, provided that the system starts from a bounded initial compact set belonging to $\Omega_{Z_1}^0$. In addition, the size of the compact set is measurable by the closed-loop analysis, as shown later.

The control effort will be activated only in the compact set $\Omega_{Z_1}^0$ so that we would like to relax our control objective to boundedness of states around the origin rather than the asymptotic convergence to origin. Accordingly, consider the practical adaptive control

$$u(t) = \begin{cases} -k_1(t)z_1 - \hat{W}_1^T S(Z_1), & z_1 \in \Omega_{Z_1}^0 \\ 0, & z_1 \in \Omega_{c_{z_1}} \end{cases} \tag{17}$$

$$\dot{\hat{W}}_1 = \begin{cases} \Gamma_1[S(Z_1)z_1 - \sigma_1(\hat{W}_1 - W_1^0)], & z_1 \in \Omega_{Z_1}^0 \\ 0, & z_1 \in \Omega_{c_{z_1}} \end{cases} \tag{18}$$

where matrix $\Gamma_1 = \Gamma_1^T > 0$, W_1^0 is a constant vector, σ_1 is a small constant to introduce the σ-modification for the closed-loop system, and $k_1(t) > 0$ will be specified later.

Theorem 1: Consider the closed-loop systems consisting of the first-order plant (10) and the controller (17). If the gain is chosen as $k_1(t) = k_{10} + k_{11} + k_{12}(t)$ with constants $k_{10}^* \triangleq k_{10} - (g_{1d}/2g_{\min}) > 0$, $k_{11} > 0$, and

$$k_{12}(t) = \frac{\varepsilon_{10}}{z_1^2} \int_{t-\tau_{\max}}^{t} \frac{1}{2} x_1^2(\tau) \varrho_1^2(x_1(\tau)) d\tau \tag{19}$$

with constant $\varepsilon_{10} > 0$, and the NN weights are updated by (18), then for bounded initial conditions, all signals in the closed-loop systems are semi-global uniform ultimate bounded, and the vector Z_1 remains in a compact set Ω_{Z_1} specified by

$$\Omega_{Z_1} = \left\{ Z_1 \,\middle|\, |z_1| \leq \mu_1,\ \bar{x}_{d2} \in \Omega_{d2} \right\} \tag{20}$$

whose size $\mu_1 = \max\{\sqrt{2g_{\max}C_{01}}, c_{z_1}\}$, with C_{01} being defined later, can be adjusted by appropriately choosing the design parameters.

Proof: Consider the Lyapunov function candidate $V_1(t)$ as

$$V_1(t) = V_{z_1}(t) + V_{U_1}(t) + \frac{1}{2}\tilde{W}_1^T(t)\Gamma_1^{-1}\tilde{W}_1(t) \tag{21}$$

with $\tilde{(\cdot)} = \hat{(\cdot)} - (\cdot)^*$. Its time derivative along (13) is

$$\dot{V}_1 \leq z_1[u + Q_1(Z_1)] + \frac{g_{1d}}{2g_{\min}} z_1^2 + \tilde{W}_1^T \Gamma_1^{-1} \dot{\hat{W}}_1. \tag{22}$$

The stability analysis will be carried out in two regions: i) $z_1 \in \Omega_{Z_1}^0$ and ii) $z_1 \in \Omega_{c_{z_1}}$ by noting that $\Omega_{Z_1}^0 \cup \Omega_{c_{z_1}} = \Omega_{Z_1}$.

Region i) $z_1 \in \Omega_{Z_1}^0$: In this region, the control and the weights adaptation are invoked. Substituting (16)–(18) into (22) yields

$$\dot{V}_1 \leq -\left(k_{10} - \frac{g_{1d}}{2g_{\min}}\right) z_1^2 - k_{12}(t)z_1^2 - k_{11}z_1^2 + \epsilon(Z_1)z_1 - \sigma_1 \tilde{W}_1^T(\hat{W}_1 - W_1^0). \tag{23}$$

Noting the following inequalities:

$$\begin{aligned} -k_{11}z_1^2 + z_1\epsilon_1(Z_1) &\leq -k_{11}z_1^2 + |z_1|\epsilon_{z_1}^* \leq \frac{\epsilon_{z_1}^{*2}}{4k_{11}} - \sigma_1\tilde{W}_1^T(\hat{W}_1 - W_1^0) \\ &\leq -\frac{1}{2}\sigma_1\|\tilde{W}_1\|^2 + \frac{1}{2}\sigma_1\|W_1^* - W_1^0\|^2 \end{aligned}$$

and substituting (19) into (23), we have

$$\dot{V}_1(t) \leq -c_1 V_1(t) + \lambda_1 \tag{24}$$

where constants $\lambda_1, c_1 > 0$ with $\lambda_1 := (1/2)\sigma_1\|W_1^* - W_1^0\|^2 + (\epsilon_{z_1}^{*2}/4k_{11})$, and $c_1 := \min\{2k_{10}^* g_{\min}, \varepsilon_{10} g_{\min}, \sigma_1/\lambda_{\max}(\Gamma_1^{-1})\}$. Letting $\rho_1 := \lambda_1/c_1$, it follows that

$$0 \leq V_1(t) \leq \rho_1 + [V_1(0) - \rho_1]e^{-c_1 t} \leq \rho_1 + V_1(0). \tag{25}$$

Region ii) $z_1 \in \Omega_{c_{z_1}}$: In this region, $|z_1| < c_{z_1}$, i.e., z_1 is already bounded, and $\dot{\hat{W}}_1 = 0$. Since $z_1 = x_1 - y_d$ and y_d is bounded, x_1 is bounded. In addition, the adaptation for $\hat{W}_1$ has stopped, and $\|\hat{W}_1\|$ is kept unchanged in a bounded value.

From (21) and (25), we have

$$z_1^2 \leq 2g_{\max}V_1(t) \leq 2g_{\max}[\rho_1 + V_1(0)] \tag{26}$$

$$\|\hat{W}_1 - W_1^*\|^2 \leq \frac{2V_1(t)}{\lambda_{\min}(\Gamma_1^{-1})} \leq \frac{2[\rho_1 + V_1(0)]}{\lambda_{\min}(\Gamma_1^{-1})}. \tag{27}$$

Defining $C_{01} = \rho_1 + V_1(0)$, it has $|z_1| \leq \sqrt{2g_{\max}C_{01}}$. Noting that (26) holds for $|z_1| > c_{z_1}$, we readily have the compact set Ω_{Z_1} specified in (20), over which the NN approximation is carried out with its feasibility being guaranteed. ■

V. Direct NN Control for Nth-Order System

The n-step backstepping design is based on the change of coordinates: $z_1 = x_1 - y_d$, $z_i = x_i - \alpha_{i-1}$, $i = 2, \ldots, n$, where $\alpha_i(t)$ is an intermediate control, and $u(t)$ is designed in the last step. Let us define the compact sets as

$$\begin{aligned} \Omega_{z_i} &:= \{z_i \in \Omega_{Z_i} \mid |z_i| \leq c_{z_i}\} \\ \Omega_{z_i}^I &:= \{z_i \in \Omega_{Z_i} \mid c_{z_i} < |z_i| < c_{z_i} + c_{z_i}^\epsilon\} \\ \Omega_{z_i}^O &:= \{z_i \in \Omega_{Z_i} \mid |z_i| \geq c_{z_i} + c_{z_i}^\epsilon\} \end{aligned}$$

with Ω_{Z_i} being a compact set, $\Omega_{Z_i} = \Omega_{z_i} \cup \Omega_{z_i}^I \cup \Omega_{z_i}^O \cup \Omega_{di}$, and small constants $c_{z_i}, c_{z_i}^\epsilon > 0$. In each design step, the stability analysis is carried out in the three regions defined by these compact sets, respectively.

For clarity, let $\tilde{W}_i = \hat{W}_i - W_i^*$, constants $c_i := \min\{2k_{i0}^* g_{\min}, \varepsilon_{10} g_{\min}, \sigma_i/\lambda_{\max}(\Gamma_i^{-1})\}$, and $\lambda_i := (1/2)\sigma_i\|W_i^* - W_i^0\|^2 + (\epsilon_{z_i}^{*2}/4k_{i1})$, where $\hat{W}_i \in R^{l_i}$ is the estimate of ideal NN weight $W_i^* \in R^{l_i}$, $W_i^0 \in R^{l_i}$ is a constant vector, $\epsilon_{z_i}^*$ is the upper bound of the NN approximation error, i.e., $\|\epsilon_i(Z_i)\| \leq \epsilon_{z_i}^*$ with Z_i the corresponding inputs to be defined later, $k_{i0} > 0$ and $k_{i1} > 0$ are control gains satisfying $k_{i0}^* = k_{i0} - (g_{id}/2g_{\min}) - (1/2) > 0$, $i = 1, \ldots, n-1$ and $k_{i0}^* = k_{i0} - (g_{id}/2g_{\min}) > 0$, constant $\varepsilon_{i0} > 0$, matrix $\Gamma_i = \Gamma_i^T > 0$, and small constants $\sigma_i > 0$ introduces

σ-modification in adaptive control to be developed later. In addition, the quadratic functions $V_{z_i}(t)$, the Lyapunov–Krasovskii functionals $V_{U_i}(t)$, and the Lyapunov function candidates $V_i(t)$ are defined as

$$V_{z_i}(t) = \frac{1}{2g_i(\bar{x}_i)} z_i^2(t) \tag{28}$$

$$V_{U_i}(t) = \frac{1}{2g_{\min}} \int_{t-\tau_i}^{t} U_i(\bar{x}_i(\tau))d\tau \tag{29}$$

$$V_i(t) = V_{z_i}(t) + V_{U_i}(t) + \frac{1}{2}\tilde{W}_i^T(t)\Gamma_i^{-1}\tilde{W}_i(t) \tag{30}$$

where positive functions $U_i(\bar{x}_i(t)) = \sum_{j=1}^{i} x_j^2(t)\varrho_{ij}^2(\bar{x}_i(t))$. The unknown functions $Q_i(Z_i)$, $i = 1, \ldots, n$ will be approximated by NNs as

$$Q_i(Z_i) = W_i^{*T}S(Z_i) + \epsilon_i(Z_i), \forall Z_i \in \Omega_{Z_i}^0 \tag{31}$$

where

$$Q_i(Z_i) = \frac{1}{g_i(\bar{x}_i)}\left[f_i(\bar{x}_i) - \dot{\alpha}_{i-1} + \frac{1}{2}z_i\right] + \frac{1}{2g_{\min} z_i}\sum_{j=1}^{i} x_j^2(t)\varrho_{ij}^2(\bar{x}_i(t))$$

with

$$Z_i(t) = \left[\bar{x}_i, \dot{\bar{x}}_{i-1}, \alpha_{i-1}, \frac{\partial \alpha_{i-1}}{\partial x_1}, \frac{\partial \alpha_{i-1}}{\partial x_2}, \ldots, \frac{\partial \alpha_{i-1}}{\partial x_{i-1}}, \omega_{i-1}\right] \in \Omega_{Z_i}^0 \subset R^{3i}$$

$$\dot{\alpha}_{i-1} = \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\dot{x}_j + \omega_{i-1}$$

$$\omega_{i-1} = \frac{\partial \alpha_{i-1}}{\partial \bar{x}_{di}}\dot{\bar{x}}_{di} + \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial \hat{W}_j}\dot{\hat{W}}_j.$$

The practical adaptive control is proposed, for $i = 1, \ldots, n$

$$\alpha_i = q_i(z_i)[-k_i(t)z_i - \hat{W}_i^T S(Z_i)] \tag{32}$$

$$k_i(t) = k_{i0} + k_{i1} + \frac{\varepsilon_{i0}}{2z_i^2}\int_{t-\tau_{\max}}^{t}\sum_{j=1}^{i} x_j^2(\tau)\varrho_{ij}^2(\bar{x}_i(\tau))d\tau \tag{33}$$

$$\dot{\hat{W}}_i = q_i(z_i)\Gamma_i[S(Z_i)z_i - \sigma_i(\hat{W}_i - W_i^0)]. \tag{34}$$

Note that when $i = n$, α_n is actually the control input $u(t)$.

Step 1: Consider the z_1-subsystem

$$\dot{z}_1(t) = g_1(x_1(t))[z_2(t) + \alpha_1(t)] + f_1(x_1(t)) + h_1(x_1(t-\tau_1)) - \dot{y}_d(t). \tag{35}$$

Consider the Lyapunov function candidate in (30). Following the same procedure as in Section IV by applying Assumption A4) and Young's inequality, we obtain

$$\dot{V}_1 \le \left(\frac{g_{1d}}{2g_{\min}} + \frac{1}{2}\right)z_1^2 + \frac{1}{2}z_2^2 + z_1[\alpha_1 + Q_1(Z_1)] + \tilde{W}_1^T\Gamma_1^{-1}\dot{\hat{W}}_1. \tag{36}$$

Consider the practical adaptive neural control (32)–(34). The stability analysis is carried out in the following three regions, respectively.

a) Region 1—$z_1 \in \Omega_{z_1}^O$: As $q_1(z_1) = 1$ when $z_1 \in \Omega_{z_1}^O$, substituting (32)–(34) into (36) yields

$$\dot{V}_1(t) \le -c_1 V_1(t) + \lambda_1 + \frac{1}{2}z_2^2. \tag{37}$$

From (37), we know that if z_2 can be regulated as bounded, the boundedness of $V_1(t)$, z_1, x_1, and $\tilde{W}_1$ can be obtained, as can be seen from Theorem 1. The regulation of z_2 will be shown in the next steps.

b) Region 2—$z_1 \in \Omega_{z_1}^I$: As $c_{z_1} < |z_1| < c_{z_1} + c_{z_1}^\epsilon$, z_1 is bounded, and $x_1 = z_1 + y_d$ is also bounded. Considering $V_{z_1}(t)$ and $V_{U_1}(t)$, they are all bounded. Define positive function $V_{W1}(t) := (1/2)\tilde{W}_1^T(t)\Gamma_1^{-1}\tilde{W}_1(t)$. Its time derivation along (34) is

$$\dot{V}_{W1}(t) = q_1(z_1)\tilde{W}_1^T[S(Z_1)z_1 - \sigma_1(\hat{W}_1 - W_1^0)]. \tag{38}$$

Applying the inequalities

$$q_1(z_1)\tilde{W}_1^T S(Z_1)z_1 \le \frac{1}{2k_{W1}}q_1(z_1)\|\tilde{W}_1\|^2 + \frac{k_{W1}}{2}q_1(z_1)S^T(Z_1)S(Z_1)z_1^2$$

$$k_{W1} > 0$$

$$-q_1(z_1)\sigma_1\tilde{W}_1^T(\hat{W}_1 - W_1^0) \le -\frac{1}{2}q_1(z_1)\sigma_1\|\tilde{W}_1\|^2 + \frac{1}{2}q_1(z_1)\sigma_1\|W_1^* - W_1^0\|^2$$

then (38) becomes

$$\dot{V}_{W1}(t) \le -\frac{1}{2}q_1(z_1)\left(\sigma_1 - \frac{1}{k_{W1}}\right)\|\tilde{W}_1\|^2 + \frac{1}{2}q_1(z_1)[\sigma_1\|W_1^* - W_1^0\|^2 + k_{W1}S^T(Z_1)S(Z_1)z_1^2].$$

For $z_1 \in \Omega_{z_1}^I$, $q_1(z_1) \in (0,1)$, and $S(Z_1)$ is smooth and bounded. Choosing k_{W1} such that $\sigma_1^* := \sigma_1 - (1/k_{W1}) > 0$, and letting $\lambda_{W1} := \sup_{z_1 \in \Omega_{z_1}^I}\{\sigma_1\|W_1^* - W_1^0\|^2 + k_{W1}S^T S z_1^2\}$, we have

$$\dot{V}_{W1}(t) \le -\frac{1}{2}q_1(z_1)\sigma_1^*\|\tilde{W}_1\|^2 + \frac{1}{2}q_1(z_1)\lambda_{W1} \le -q_1(z_1)\frac{\sigma_1^*}{\lambda_{\max}(\Gamma_1^{-1})}V_{W1}(t) + \frac{1}{2}q_1(z_1)\lambda_{W1}. \tag{39}$$

Letting $c_{W1}^q := q_1(z_1)(\sigma_1^*/\lambda_{\max}(\Gamma_1^{-1}))$, $\lambda_{W1}^q := (1/2)q_1(z_1)\lambda_{W1}$, and $\rho_{W1}^q := \lambda_{W1}^q/c_{W1}^q$, it follows from (39) that

$$V_{W1}(t) \le [V_{W1}(0) - \rho_{W1}^q]e^{-c_{W1}^q t} + \rho_{W1}^q \le V_{W1}(0) + \rho_{W1}^q$$

from which we can conclude that $V_{W1}(t)$ is bounded, and hence, $\tilde{W}_1$ is bounded. Consider the Lyapunov function candidate $V_1(t)$ in (30). As it has been already shown that $V_{z_1}(t)$, $V_{U_1}(t)$, and $\tilde{W}_1$ are bounded, we can conclude that $V_1(t)$ is bounded for $z_1 \in \Omega_{z_1}^I$.

Remark 2: In this region, it is noted that although the function $q_1(z_1)$ is not of fixed value, the ultimate boundedness of the closed-loop signals is independent of $q_1(z_1)$, as can be seen from the definition of $V_{W1}(0)$ and ρ_{W1}^q.

c) Region 3—$z_1 \in \Omega_{z_1}$: In this region, $|z_1| \le c_{z_1}$ is already bounded, and $q_1(z_1) = 0$, $\dot{\hat{W}}_1 = 0$. Hence, $x_1 = z_1 + y_d$ is bounded, and $\|\hat{W}_1\|$ is kept unchanged in a bounded value. As $V_{z_1}(t)$ and $V_{U_1}(t)$ are smooth functions, we know that for bounded x_1 and z_1, $V_{z_1}(t)$ and $V_{U_1}(t)$ are bounded, and $V_1(t)$ is bounded.

Step i ($2 \le i \le n-1$): Similar procedures are taken as in Step 1. The dynamics of z_i-subsystem is given by

$$\dot{z}_i(t) = g_i(\bar{x}_i(t))[z_{i+1}(t) + \alpha_i(t)] + f_i(\bar{x}_i(t)) + h_i(\bar{x}_i(t-\tau_i)) - \dot{\alpha}_{i-1}(t).$$

Consider $V_i(t)$ in (30). Using Young's inequality and noting Assumption A4), the time derivative of $V_i(t)$ is

$$\dot{V}_i \leq -\left[\frac{\dot{g}_i(\bar{x}_i)}{2g_i^2(\bar{x}_i)} - \frac{1}{2}\right] z_i^2 + \frac{1}{2} z_{i+1}^2 + z_i[\alpha_i + Q_i(Z_i)] + \tilde{W}_i^T \Gamma_i^{-1} \dot{\hat{W}}_i. \quad (40)$$

Consider the control given by (32)–(34). Similarly as in Step 1, the stability analysis is carried out in the three regions, respectively: i) For $z_i \in \Omega_{z_i}^O$, we have $\dot{V}_i(t) \leq -c_i V_i(t) + \lambda_i + (1/2) z_{i+1}^2$, from which it can be seen that the stability of the z_i subsystem is dependent on z_{i+1}, which will be dealt with in the next steps; ii) for $z_i \in \Omega_{z_i}^I$, z_i is bounded, and it can be concluded backward that $z_{i-1}, \ldots, z_1$ are all bounded so that $x_i, x_{i-1}, \ldots, x_1$ can be guaranteed to be bounded as well. The boundedness of $\hat{W}_i$ can be obtained from the similar analysis carried out in Region 1 of *Step 1*; and iii) for $z_i \in \Omega_{z_i}$, it directly follows that z_i, x_i, and $\hat{W}_i$ are bounded.

Step n: In the final step, the actual control $u(t)$ appears in the dynamics of the z_n-subsystem given by

$$\dot{z}_n = g_n(\bar{x}_n(t))u + f_n(\bar{x}_n(t)) + h_n(\bar{x}_n(t - \tau_n)) - \dot{\alpha}_{n-1}(t).$$

Consider $V_n(t)$ in (30). The time derivative of $V_n(t)$ is

$$\dot{V}_n \leq -\frac{\dot{g}_n(x)}{2g_n^2(x)} z_n^2(t) + z_n[\alpha_n + Q_n(Z_n)] + \tilde{W}_n^T \Gamma_n^{-1} \dot{\hat{W}}_n. \quad (41)$$

Consider the control given by (32)–(34). Similarly, as in the previous steps, the stability analysis is carried out in the three regions, respectively: i) For $z_n \in \Omega_{z_n}^O$, $q_n(z_n) = 1$, the final control $u(t)$ is invoked, and the time derivative of $V_n(t)$ along (32)–(34) and (41) is $\dot{V}_n(t) \leq -c_n V_n(t) + \lambda_n$, from which we conclude that $V_n(t)$ is bounded; hence, $z_n, \hat{W}_n$ are bounded. ii) For $z_n \in \Omega_{z_n}^I$, z_n is already bounded. It can be concluded backward that all the previous z_ith subsystem, $i = 1, \ldots, n-1$, are stable, i.e., $z_i, \hat{W}_i, i = 1, \ldots, n-1$ are all bounded. As $x_i = z_i + \alpha_{i-1}, i = 2, \ldots, n, x_1 = z_1 + y_d$, and α_i, $i = 1, \ldots, n-1$ are smooth functions, we know that α_i are bounded, and hence, $x_i, i = 1, \ldots, n$ are bounded. The boundedness of $\hat{W}_n$ can be obtained from the similar analysis carried out in Region 1 of *Step 1*. iii) For $z_n \in \Omega_{z_n}$, the boundedness of z_n directly follows. Hence, z_i, x_i and $\hat{W}_i, i = 1, \ldots, n-1$ are bounded. As $q_n(z_n) = 0, \dot{\hat{W}}_n = 0$, $\|\hat{W}_n\|$ is kept unchanged in a finite value.

Theorem 2: Consider the closed-loop system consisting of the plant (1) under Assumptions A1)-A5) and the adaptive control laws (32)–(34). For bounded initial conditions within the compact set Ω_Z, the following properties hold.

i) All signals in the closed-loop system remain semi-global uniform ultimate bounded, and the vector $Z = [Z_1^T, \ldots, Z_n^T]^T$ remains in a compact set $\Omega_Z := \Omega_{Z_1} \cup \cdots \cup \Omega_{Z_n}$, which is specified as

$$\Omega_Z = \left\{ Z \left| \sum_{i=1}^n z_i^2 \leq 2g_{\max} C_0 \right. \sum_{i=1}^n \|\tilde{W}_i\|^2 \leq \frac{2C_0}{\lambda_{\min}(\Gamma_i^{-1})}, \bar{x}_{di} \in \Omega_{di}, i = 2, \ldots, n \right\} \quad (42)$$

where $C_0 > 0$ is a constant whose size depends on the initial conditions (as will be defined later in the proof);

ii) The closed-loop signal $z(t) = [z_1, \ldots, z_n]^T \in R^n$ will eventually converge to a compact set defined by

$$\Omega_S := \left\{ z \middle| \|z\|^2 \leq \mu \right\} \quad (43)$$

where $\mu > 0$ is a constant related to the design parameters and will be defined later in the proof, and Ω_S can be made as small as desired by an appropriate choice of the design parameters.

Proof: Consider the Lyapunov function candidate

$$V(t) = \sum_{i=1}^n \left[V_{z_i}(t) + V_{U_i}(t) + \frac{1}{2} \tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i \right] \quad (44)$$

where $V_{z_i}(t)$ and $V_{U_i}(t)$ are defined in (28) and (29), respectively. The following four cases are considered.

Case 1: All $z_i \in \Omega_{z_i}^O, i = 1, \ldots, n$. In this case, $q_i(z_i) = 1$, and as all the control effort including $\alpha_i(t), i = 1, \ldots, n-1$, and $u(t)$ are invoked, from the previous analysis, we have

$$\dot{V}_i(t) \leq -c_i V_i(t) + \lambda_i + \frac{1}{2} z_{i+1}^2, i = 1, \ldots, n-1 \quad (45)$$

$$\dot{V}_n(t) \leq -c_n V_n(t) + \lambda_n \quad (46)$$

where c_i and λ_i, $i = 1, \ldots, n$ are defined as before. Letting $\rho_n := \lambda_n / c_n$, it follows from (46) that

$$V_n(t) \leq [V_n(0) - \rho_n] e^{-c_n t} + \rho_n \leq V_n(0) + \rho_n \quad (47)$$

$$z_n^2 \leq 2g_{\max}[V_n(0) + \rho_n], \ \|\tilde{W}_n\|^2 \leq \frac{2[V_n(0) + \rho_n]}{\lambda_{\min}\{\Gamma_n^{-1}\}}. \quad (48)$$

It follows from (47) and (48) that $V_n(t)$ is bounded; hence, z_n and $\hat{W}_n$ are bounded. From (45) and (48), we have that

$$\dot{V}_i(t) \leq -c_i V_i(t) + \lambda_i + g_{\max}[V_{i+1}(0) + \rho_{i+1}]. \quad (49)$$

Letting $\rho_i = [\lambda_i + g_{\max}(V_{i+1}(0) + \rho_{i+1})]/c_i$, similarly as (47), we have

$$0 \leq V_i(t) \leq [V_i(0) - \rho_i] e^{-c_i t} + \rho_i \leq V_i(0) + \rho_i \quad (50)$$

$$z_i^2 \leq 2g_{\max}[V_i(0) + \rho_i], \ \|\tilde{W}_i\|^2 \leq \frac{2[V_i(0) + \rho_i]}{\lambda_{\min}\{\Gamma_i^{-1}\}} \quad (51)$$

from which we conclude that z_i and $\hat{W}_i$, $i = 1, \ldots, n-1$ are all uniformly bounded. From (47) and (50), we have

$$z_i^2 \leq 2g_{\max}[(V_i(0) - \rho_i) e^{-c_i t} + \rho_i] \quad (52)$$

i.e., $\lim_{t \to \infty} \|z\| \leq \sqrt{2g_{\max} \sum_{i=1}^n \rho_i}$. Since the above analysis is carried out for all $z_i \in \Omega_{z_i}^O$, i.e., $|z_i| \geq c_{z_i} + c_{z_i}^\epsilon, i = 1, \ldots, n$, we have that $z_{\min} \leq \lim_{t \to \infty} \|z\| \leq z_{\max}$ with $z_{\min} \triangleq \sqrt{\sum_{i=1}^n (c_{z_i} + c_{z_i}^\epsilon)^2}$ and $z_{\max} \triangleq \sqrt{2g_{\max} \sum_{i=1}^n \rho_i}$. Two possibilities should be considered. First, if $z_{\max} \geq z_{\min}$, this means that z, starting at $\Omega_{z_i}^O$, will finally converge to a boundary bigger than the inner boundary of $\Omega_{z_i}^O$. Second, if $z_{\max} \leq z_{\min}$, this means that z will finally converge to a boundary smaller than the inner boundary of $\Omega_{z_i}^O$. However, the fact is, when z crosses the inner boundary of $\Omega_{z_i}^O$, i.e., $z_{\min}$, it falls into another compact set where a different control is applied. The only properties that can be guaranteed for this possibility is $\lim_{t \to \infty} \|z\| \leq z_{\min}$. Thus, we have

$$\lim_{t \to \infty} \|z\| \leq \max \left\{ \sqrt{2g_{\max} \sum_{i=1}^n \rho_i}, \sqrt{\sum_{i=1}^n (c_{z_i} + c_{z_i}^\epsilon)^2} \right\}.$$

Case 2: All $z_i \in \Omega_{z_i}, i = 1, \ldots, n$. In this case, as $|z_i| < c_{z_i}$, all z_i's are bounded. As $q_i(z_i) = 0, \dot{\hat{W}}_i = 0, \alpha_i(t) = 0$, and $u(t) = 0$, it is known that $\|\hat{W}_i\|$ is kept unchanged in a bounded value, and $x_i = z_i + \alpha_{i-1}, i = 2, \ldots, n$, and $x_1 = z_1 + y_d$ are all bounded. Considering $V_{z_i}(t)$ and $V_{U_i}(t)$ and noting that $g_i(\cdot), \varrho_{ij}(\cdot)$ are smooth functions, we know that for bounded x_i, z_i and $\hat{W}_i$, $V_{z_i}(t)$ and $V_{U_i}(t)$ are bounded, i.e., there exists a finite C_B such that

$$V(t) \leq C_B. \quad (53)$$

Case 3: All $z_i \in \Omega^I_{z_i}$, $i = 1, \ldots, n$. In this case, z_i's are already bounded. Since z_1 and y_d are bounded, and $x_1 = z_1 + y_d$, x_1 is also bounded. From the analysis for Region 2 in *Step 1*, $\|\tilde{W}_1\|^2 \leq 2[V_{W1}(0) + \rho^q_{W1}]/\lambda_{\min}(\Gamma_1^{-1})$, from which we know that $\hat{W}_1$ is bounded. As $\alpha_1(t)$ is a smooth function of x_1, y_d, $\dot{y}_d$, and $\hat{W}_1$, α_1 is guaranteed bounded. From $x_2 = z_2 + \alpha_1$, x_2 is obviously bounded. Following the same analysis for Region 2 in *Step 1*, we can show that $\hat{W}_2$ is bounded. Similarly, the boundedness of all the other closed-loop signals $x_3, \ldots, x_n$ and $\hat{W}_3, \ldots, \hat{W}_n$ can be shown iteratively. Thus, there exists a finite C_C such that

$$V(t) \leq C_C. \tag{54}$$

Case 4: Some z_i's are satisfying $z_i \in \Omega^O_{z_i}$, while some z_j's are satisfying $z_j \in \Omega^I_{z_j}$ or $z_j \in \Omega_{z_j}$. For those $z_i \in \Omega^O_{z_i}$, $q_i(z_i) = 1$, the corresponding control effort $\alpha_i(t)$ or $u(t)$ and the parameter adaptation law for $\hat{W}_i$ are invoked, and from the previous analysis, we have that $\dot{V}_i(t) \leq -c_i V_i(t) + \lambda_i + (1/2)z^2_{i+1}$. From Case 1), we know that $z^2_{i+1} \leq 2g_{\max}(V_{i+1}(0) + \rho_{i+1})$. Defining $V_I(t) = \sum_i V_i(t)$ and positive constants, where $C^I_1 = \min_i\{c_i\}, C^I_2 = \sum_i[\lambda_i + g_{\max}(V_{i+1}(0) + \rho_{i+1})]$, and $\rho_I := C^I_2/C^I_1$, we have that $\dot{V}_I(t) \leq -C^I_1 V_I(t) + C^I_2$, i.e.,

$$V_I(t) \leq [V_I(0) - \rho_I]e^{-C^I_1 t} + \rho_I \leq V_I(0) + \rho_I. \tag{55}$$

For those $z_j \in \Omega^I_{z_j}$ or $z_j \in \Omega_{z_j}$, define $V_J(t) = \sum_j (V_{z_j} + V_{U_j} + (1/2)\tilde{W}^T_j \Gamma^{-1}_j \tilde{W}_j)$. As z_j is already bounded, it guarantees that the closed-loop signals in the previous steps, i.e., z_k, x_k, $\hat{W}_k$, $k = 1, \ldots, j-1$, are bounded, and the stability of the corresponding z_jth subsystems is independent of the signals in future steps. As α_{j-1} is a smooth function of $x_k, \hat{W}_k, k = 1, \ldots, j-1$, α_{j-1} is bounded, hence, $x_j = z_j + \alpha_{j-1}$ is bounded. The boundedness of $\hat{W}_j$ can be readily obtained following the similar analysis in Region 2 of *Step 1*. More optimally, for $z_j \in \Omega_{z_j}$, $\dot{\hat{W}}_j = 0$, $\hat{W}_j$ is kept unchanged in a bounded value. Thus, $V_J(t)$ is bounded, i.e., $V_J(t) \leq C_J$ with C_J being finite. Therefore, it can be obtained that

$$V(t) = V_I(t) + V_J(t) \leq V_I(0) + \rho_I + C_J. \tag{56}$$

Thus, from Cases 1)–4), we can conclude that

$$V(t) \leq C_0 \tag{57}$$

where $C_0 = \max\{C_B, C_C, \sum_{i=1}^n (V_i(0) + \rho_i), V_I(0) + \rho_I + C_J\}$. From (57), we know that $V_i(t)$, z_i, and $\hat{W}_i$, $i = 1, \ldots, n$ are bounded. Since $z_1 = x_1 - y_d$ and y_d is bounded, x_1 is bounded. For $x_2 = z_2 + \alpha_1$, since α_1 is function of bounded signals z_1, Z_1, $\hat{W}_1$, α_1 is thus bounded, which in turn leads to the boundedness of x_2. Following in the same way, we can prove one by one that all α_{i-1} and x_i, $i = 3, \ldots, n$ are bounded. Therefore, the systems' states $x_i, i = 1, \ldots, n$ are bounded.

Considering (44), we know that

$$\sum_{i=1}^n z_i^2 \leq 2g_{\max}V(t), \ \sum_{i=1}^n \|\tilde{W}_i\|^2 \leq \frac{2V(t)}{\lambda_{\min}(\Gamma_1^{-1}, \ldots, \Gamma_n^{-1})}. \tag{58}$$

From (57) and (58), we readily have the compact set Ω_Z defined in (42) over which the NN approximation is carried out with its feasibility being guaranteed.

In addition, for Cases 2)–4), we know that $\|z\|^2 = \sum_{i=1}^n z_i^2 \leq \sum_{i=1}^n (c_{z_i} + c^\epsilon_{z_i})^2$. Therefore, as $t \to \infty$, we can conclude that $\|z\|^2 \leq \mu$, where

$$\mu = \max\left\{\sqrt{2g_{\max}\sum_{i=1}^n \rho_i}, \sqrt{\sum_{i=1}^n (c_{z_i} + c^\epsilon_{z_i})^2}\right\} \tag{59}$$

i.e., the vector z will eventually converge to the compact set Ω_S defined in (43). ■

VI. Conclusion

Practical adaptive neural control has been addressed for a class of nonlinear systems with unknown time delays in strict-feedback form. The unknown time delays have been compensated for using appropriate Lyapunov–Krasovskii functionals. Controller singularity problems have been solved by employing practical NN control. Novel differentiable control functions have been applied in decoupled backstepping design. The proposed design has been proven to be able to guarantee semi-global uniform ultimate boundedness of all the signals in the closed-loop system, and the tracking error has been proven to converge to a small neighborhood of the origin.

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