Adaptive Robust Motion/Force Control of Holonomic-Constrained Nonholonomic Mobile Manipulators

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***Abstract*—In this paper, adaptive robust force/motion control strategies are presented for mobile manipulators under both holonomic and nonholonomic constraints in the presence of uncertainties and disturbances. The proposed control is robust not only to parameter uncertainties such as mass variations but also to external ones such as disturbances. The stability of the closed-loop system and the boundedness of tracking errors are proved using Lyapunov stability synthesis. The proposed control strategies guarantee that the system motion converges to the desired manifold with prescribed performance and the bounded constraint force. Simulation results validate that the motion of the system converges to the desired trajectory, and the constraint force converges to the desired force.**

***Index Terms*—Holonomic constraint, motion/force control, nonholonomic mobile manipulators.**

I. Introduction

MOBILE manipulators refer to robotic manipulators (or arms) mounted on mobile platforms (or vehicles). Such systems combine the advantages of mobile platforms and robotic arms and reduce their drawbacks. The mobile platform extends the arm's workspace, whereas the arm offers much operational functionality. Applications for such systems could be found in mining, construction, forestry, planetary exploration, and military [1]–[3].

Mobile manipulators possess strongly coupled dynamics of mobile platforms and manipulators. With the assumption of known dynamics, much research has been carried out. Input–output feedback linearization was investigated to control the mobile platform such that the manipulator is always positioned at the preferred configurations measured by its manipulability [4]. Similarly, through nonlinear-feedback linearization and decoupling dynamics in [5], force/position control of the end-effector along the same direction for mobile manipulators was proposed and applied to nonholonomic cart pushing. In [6], the effect of the dynamic interaction between the arm and the vehicle of a mobile manipulator was studied and nonlinear-feedback control for the mobile manipulator was developed to compensate for the dynamic interaction. In [7], coordination and control of mobile manipulators were presented with two basic task-oriented controls: end-effector task control and platform self-posture control. In [8], the concept of manipulability was generalized to the case of mobile manipulators and the optimization criteria in terms of manipulability were given to generate the controls of the system.

However, control of mobile manipulators with uncertainties is essential in many practical applications, especially for the case when the force of the end-effector should be considered. To handle unknown dynamics of mechanical systems, robust, and adaptive controls have been extensive investigated for robot manipulators and dynamic nonholonomic systems. Robust controls assume the known boundedness of unknown dynamics of the systems; nevertheless, adaptive controls could learn the unknown parameters of interest through adaptive tuning laws.

Under the assumption of a good understanding of dynamics of the systems understudy, model-based adaptive controls have been much investigated for dynamic nonholonomic systems. In [9], adaptive control was proposed for trajectory/force control of mobile manipulators subjected to holonomic and nonholonomic constraints with unknown inertia parameters, which ensures the motion of the system to asymptotically converge to the desired trajectory and force. In [10], adaptive state feedback and output feedback control strategies using state scaling and backstepping techniques were proposed for a class of nonholonomic systems in chained form with drift nonlinearity and parametric uncertainties. In [11], the nonholonomic kinematic subsystem was first transformed into a skew-symmetric form, and then a virtual adaptive control designed at the actuator level was proposed to compensate for the parametric uncertainties of the kinematic and dynamic subsystems.

In [12], robust adaptive control was proposed for dynamic nonholonomic systems with unknown inertia parameters and disturbances, in which adaptive control techniques were used to compensate for the parametric uncertainties and sliding mode control was used to suppress the bounded disturbances. In [13], adaptive robust force/motion control was presented systematically for holonomic mechanical systems and a large class of nonholonomic mechanical systems in the presence of uncertainties and disturbances.

Because of the difficulty in dynamic modeling, adaptive neural network control, a nonmodel-based approach, has been extensively studied for different classes of systems, such as

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robotic manipulators [15]–[17] and mobile robots [18]. In [19], adaptive neural network control for a robot manipulator in the task space was proposed, which neither requires the inverse dynamical model nor the time-consuming offline training process. In [20], the unidirectionality of the contact force of robot manipulators was explicitly included in modeling and the fuzzy control was developed. In [21], adaptive neural fuzzy control for function approximation had been investigated for uncertain nonholonomic mobile robots in the presence of unknown disturbances. In [22], adaptive neural network controls had been developed for the motion control of mobile manipulators subject to kinematic constraints.

In this paper, we shall consider a class of mechanical systems with both holonomic and nonholonomic constraints, such as nonholonomic mobile manipulators, and address the force/motion control for holonomic-constrained nonholonomic mobile-manipulator systems in the presence of parameter uncertainties and external disturbances. The main contributions of this paper are listed as follows.

1) The control design is developed in a systematic and unified manner for a class of mechanical systems with both holonomic and nonholonomic constraints.
2) Decoupling adaptive robust motion and force control strategies are presented for mobile manipulators with both parameter uncertainties and external disturbances.
3) Nonregressor-based control design is developed and carried out without imposing any restriction on the system dynamics.

The stability and the boundedness of tracking errors are proved using Lyapunov synthesis. The proposed control strategies guarantee that the motion of the system converges to the desired manifold and at the same time guarantee the boundedness of the constrained force. The simulation studies validate not only the motion of the system converging to the desired trajectory but also the constraint force converging to the desired force.

The rest of this paper is organized as follows. The mobile manipulator subject to simultaneous nonholonomic and holonomic constraints are briefly described in Section II. The main results of control design are presented in Section III. Simulation studies are presented by comparison between the proposed adaptive robust control and model-based control in Section IV. Concluding remarks are given in Section V.

II. System Description

Consider an n DOF mobile manipulator mounted on a nonholonomic mobile platform, as shown in Fig. 1. The constrained mechanical system can be described as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + d(t) = B(q)\tau + f \qquad (1)$$

where $q = [q_1, \ldots, q_n]^{\mathrm{T}} \in R^n$ denote the generalized coordinates, $M(q) \in R^{n\times n}$ is the symmetric bounded positive definite inertia matrix, $C(q,\dot{q})\dot{q} \in R^n$ denote the centripetal and Coriolis torques, $G(q) \in R^n$ is the gravitational torque vector, $d(t)$ denotes the external disturbances, $\tau \in R^m$ are the control inputs, $B(q) \in R^{n\times m}$ is a full-rank input transformation matrix and is assumed to be known because it is a function of fixed

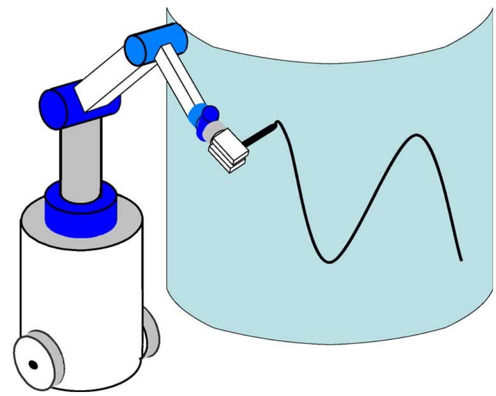

Fig. 1. Trajectory/force tracking of a mobile manipulator.

geometry of the system, and $f = [f_n, f_h]^{\mathrm{T}} = J^{\mathrm{T}}\lambda \in R^n$ with generalized constraint forces f_n and f_h for the nonholonomic and holonomic constraints, respectively, and $\lambda = [\lambda_n, \lambda_h]^{\mathrm{T}}$ denoting the Lagrangian multipliers with both the nonholonomic and holonomic constraints.

The generalized coordinates may be separated into two sets as $q = [q_v, q_a]^{\mathrm{T}}$ with $q_v \in R^{n_v}$ denoting the generalized coordinates for the vehicle and $q_a \in R^{n_a}$ denoting the coordinates of the arm. Therefore, we have

$$M(q) = \begin{bmatrix} M_v & M_{va} \\ M_{av} & M_a \end{bmatrix}$$
$$C(q,\dot{q}) = \begin{bmatrix} C_v & C_{va} \\ C_{av} & C_a \end{bmatrix}$$
$$G(q) = [G_v \quad G_a]^{\mathrm{T}}$$
$$d(t) = [d_v \quad d_a]^{\mathrm{T}}$$
$$B(q) = \begin{bmatrix} B_v & 0 \\ 0 & B_a \end{bmatrix}$$
$$\tau = [\tau_v \quad \tau_a]^{\mathrm{T}}$$
$$J = \begin{bmatrix} A & 0 \\ J_v & J_a \end{bmatrix}.$$

The vehicle is subjected to nonholonomic constraints, and the l nonintegrable and independent velocity constraints can be expressed as

$$A(q_v)\dot{q}_v = 0 \qquad (2)$$

where $A(q_v) = [A_1^{\mathrm{T}}(q_v), \ldots, A_l^{\mathrm{T}}(q_v)]^{\mathrm{T}} : R^{n_v} \to R^{l\times n_v}$ is the kinematic-constraint matrix, which is assumed to have full-rank l. In this paper, the vehicle is assumed to be completely nonholonomic. The effect of the constraints can be viewed as a restriction of the dynamics on the manifold Ω_n as

$$\Omega_n = \{(q_v, \dot{q}_v) | A(q_v)\dot{q}_v = 0\}. \qquad (3)$$

The generalized constraint forces for the nonholonomic constraints can be given by

$$f_n = A^{\mathrm{T}}(q_v)\lambda_n. \qquad (4)$$

Assume that the annihilator of the codistribution spanned by the covector fields $A_1(q_v), \ldots, A_l(q_v)$ is an $(n_v - l)$-dimensional smooth nonsingular distribution Δ on R^{n_v}. This distribution Δ is spanned by a set of $(n_v - l)$ smooth and linearly independent vector fields $H_1(q_v), \ldots, H_{n_v-l}(q_v)$, i.e., $\Delta = \text{span}\{H_1(q_v), \ldots, H_{n_v-l}(q)\}$, which satisfy, in local coordinates, the following relation [14]:

$$H^{\mathrm{T}}(q_v)A^{\mathrm{T}}(q_v) = 0 \tag{5}$$

where $H(q_v) = [H_1(q_v), \ldots, H_{n_v-l}(q_v)] \in R^{n_v \times (n_v - l)}$. Note that $H^{\mathrm{T}}H$ is of full rank. Constraints (2) implies the existence of vector $\dot{\eta} \in R^{n_v - l}$, such that

$$\dot{q}_v = H(q_v)\dot{\eta}. \tag{6}$$

Considering the nonholonomic constraints (2) and its derivative, the dynamics of mobile manipulator can be expressed as

$$\begin{aligned} &\begin{bmatrix} H^{\mathrm{T}}M_vH & H^{\mathrm{T}}M_{va} \\ M_{av}H & M_a \end{bmatrix} \begin{bmatrix} \ddot{\eta} \\ \ddot{q}_a \end{bmatrix} \\ &\quad + \begin{bmatrix} H^{\mathrm{T}}M_v\dot{H} + H^{\mathrm{T}}C_vH & H^{\mathrm{T}}C_{va} \\ M_{av}\dot{H} + C_{av}H & C_a \end{bmatrix} \begin{bmatrix} \dot{\eta} \\ \dot{q}_a \end{bmatrix} \\ &\quad + \begin{bmatrix} H^{\mathrm{T}}G_v \\ G_a \end{bmatrix} + \begin{bmatrix} H^{\mathrm{T}}d_v \\ d_a \end{bmatrix} \\ &\quad = \begin{bmatrix} H^{\mathrm{T}}B_v\tau_v \\ B_a\tau_a \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ J_v & J_a \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 0 \\ \lambda_h \end{bmatrix}. \end{aligned} \tag{7}$$

Let $\xi = [\eta \quad q_a]^{\mathrm{T}}$ and assume that the system (7) is subjected to k independent holonomic constraints

$$h(\xi) = 0, h(\xi) \in R^k \tag{8}$$

where $h(\xi)$ is of full rank. Define

$$J(\xi) = \partial h/\partial \xi \tag{9}$$

then the holonomic constraints could be further written as

$$J(\xi)\dot{\xi} = 0. \tag{10}$$

The holonomic constraint force can be converted to the joint space as

$$f_h = J^{\mathrm{T}}\lambda_h. \tag{11}$$

The holonomic constraint on the robot's end-effector can be viewed as restricting only the dynamics on the constraint manifold

$$\Omega_h = \left\{(\xi, \dot{\xi}) | h(\xi) = 0, J(\xi)\dot{\xi} = 0\right\}. \tag{12}$$

Assume that the arm is a series-chain multiple links with holonomic constraints (i.e., geometric constraints). Since the motion is subject to k-dimensional constraint, the configuration space of the holonomic system is left with $n_a - k$ DOF. From an appropriate manipulation of the constraint $h(\xi) = 0$, the vector q_a can be further rearranged and partitioned into $q_a = [q_a^1, q_a^2]^{\mathrm{T}}$, where $q_a^1 \in R^{n_a - k}$ describe the constrained motion of the arm and $q_a^2 \in R^k$ denote the remaining joint variables. Then

$$J(\xi) = \left[\frac{\partial h}{\partial \eta}, \frac{\partial h}{\partial q_a^1}, \frac{\partial h}{\partial q_a^2}\right]. \tag{13}$$

From [23], it could be concluded that q is the function of $\zeta = [\eta, q_a^1]^{\mathrm{T}}$, that is, $\xi = \varphi(\zeta)$, and we have

$$\dot{\xi} = L(\zeta)\dot{\zeta} \tag{14}$$

where $L(\zeta) = \partial\varphi/\partial\zeta$, $\ddot{\xi} = L(\zeta)\ddot{\zeta} + \dot{L}(\zeta)\dot{\zeta}$, and $L(\zeta)$, $J^1(\zeta) = J(\varphi(\zeta))$ satisfies the relationship

$$L^{\mathrm{T}}(\zeta)J^{1\mathrm{T}}(\zeta) = 0. \tag{15}$$

The dynamics (7), when it is restricted to the constraint surface, can be transformed into the reduced dynamics

$$M^1L(\zeta)\ddot{\zeta} + C^1\dot{\zeta} + G^1 + d^1(t) = u + J^{1\mathrm{T}}\lambda_h \tag{16}$$

where

$$\begin{aligned} M^1 &= \begin{bmatrix} H^{\mathrm{T}}M_vH & H^{\mathrm{T}}M_{va} \\ M_{av}H & M_a \end{bmatrix} \\ C^1 &= \begin{bmatrix} H^{\mathrm{T}}M_v\dot{H} & H^{\mathrm{T}}M_{va} \\ M_{av}H & M_a \end{bmatrix} \dot{L}(\zeta) \\ &\quad + \begin{bmatrix} H^{\mathrm{T}}M_v\dot{H} + H^{\mathrm{T}}C_vH & H^{\mathrm{T}}C_{va} \\ M_{av}\dot{H} + C_{av} & C_a \end{bmatrix} L(\zeta) \\ G^1 &= \begin{bmatrix} H^{\mathrm{T}}G_v \\ G_a \end{bmatrix} \\ d^1(t) &= \begin{bmatrix} H^{\mathrm{T}}d_v \\ d_a \end{bmatrix} \\ u &= B^1\tau, B^1 \\ &= \begin{bmatrix} H^{\mathrm{T}}B_v & 0 \\ 0 & B_a \end{bmatrix} \\ \zeta &= \begin{bmatrix} \eta \\ q_a^1 \end{bmatrix}. \end{aligned}$$

Property 2.1: Matrices M^1 and G^1 are uniformly bounded and uniformly continuous if ζ is uniformly bounded and continuous, respectively. Matrix C^1 is uniformly bounded and uniformly continuous if ζ and $\dot{\zeta}$ are uniformly bounded and continuous, respectively.

Multiplying L^{T} on both sides of (16), we can obtain

$$M_L\ddot{\zeta} + C_L\dot{\zeta} + G_L + d_L(t) = L^{\mathrm{T}}u \tag{17}$$

where

$$\begin{aligned} M_L &= L^T(\zeta)M^1L \\ C_L &= L^{\mathrm{T}}(\zeta)C^1 \\ G_L &= L^{\mathrm{T}}(\zeta)G^1 \\ d_L &= L^{\mathrm{T}}(\zeta)d^1(t). \end{aligned}$$

The force multiplier λ_h can be obtained by (16)

$$\lambda_h = Z\left(C^1\dot{\zeta} + G^1 + d^1(t) - u\right) \tag{18}$$

where

$$Z = \left(J^1(M^1)^{-1}J^{1\mathrm{T}}\right)^{-1} J^1(M^1)^{-1}.$$

Property 2.2: The matrix M_L is symmetric and positive definite, and we have the following inequalities:

$$\lambda_{\min}(M_L)\|x\|^2 \leq x^{\mathrm{T}}M_L x \leq \lambda_{\max}(M_L)\|x\|^2, \quad \forall x \in R^n \tag{19}$$

where $\lambda_{\min}$ and $\lambda_{\max}$ denote the minimum and maximum eigenvalues of M_L, respectively [15].

Property 2.3: The matrix $\dot{M}_L - 2C_L$ is skew symmetric.

Property 2.4: For holonomic systems, matrices $J^1(\zeta)$ and $L(\zeta)$ are uniformly bounded and uniformly continuous if ζ is uniformly bounded and continuous, respectively.

Remark 2.1: The matrix Z is bounded and continuous, since M^1 and $J^1(\zeta)$ are bounded and continuous from Property 2.1 and Property 2.4.

III. Control Design

Since the system is subjected to the nonholonomic constraint (2) and holonomic constraint (8), the states q_v, q_a^1, and q_a^2 are not independent. By a proper partition of q_a, q_a^2 is uniquely determined by $\zeta = [\eta, q_a^1]^{\mathrm{T}}$. Therefore, it is not necessary to consider the control of q_a^2.

Given a desired motion trajectory $\zeta_d(t) = [\eta_d, q_{ad}^1]^{\mathrm{T}}$ and a desired constraint force $f_d(t)$, or equivalently, a desired multiplier $\lambda_{hd}(t)$, we are to determine a control law such that for any $(\zeta(0), \dot{\zeta}(0)) \in \Omega$, $\zeta, \dot{\zeta}$, λ_h converge to a manifold Ω_d specified as Ω, where

$$\Omega_d = \left\{(\zeta, \dot{\zeta}, \lambda_h) | \zeta = \zeta_d, \dot{\zeta} = \dot{\zeta}_d, \lambda_h = \lambda_{hd}\right\}. \tag{20}$$

Assumption 3.1: The desired reference trajectory $\zeta_d(t)$ is assumed to be bounded and uniformly continuous and has bounded and uniformly continuous derivatives up to the second order. The desired Lagrangian multiplier $\lambda_{hd}(t)$ is also bounded and uniformly continuous.

Consider the following signals:

$$\begin{aligned} e_\zeta &= \zeta - \zeta_d \\ \dot{\zeta}_r &= \dot{\zeta}_d - K_\zeta e_\zeta \\ r &= \dot{e}_\zeta + K_\zeta e_\zeta \\ e_\lambda &= \lambda_h - \lambda_{hd} \end{aligned}$$

where $K_\zeta = \mathrm{diag}[K_{\zeta i}] > 0$.

Consider the control input u in the form

$$u = u_a - J^{1\mathrm{T}}u_b. \tag{21}$$

Then, (17) and (18) can be changed to

$$M_L\ddot{\zeta} + C_L\dot{\zeta} + G_L + d_L = L^{\mathrm{T}}u_a \tag{22}$$

$$\lambda_h = Z\left(C^1\dot{\zeta} + G^1 + d^1(t) - u_a\right) + u_b. \tag{23}$$

A. Model-Based Control

Under the assumption that the dynamics of mobile manipulators are known without considering external disturbances, consider the following control laws:

$$L^{\mathrm{T}}u_a = -K_p r - K_i\int_0^t r ds - \Phi_m \tag{24}$$

$$u_b = \chi_m\ddot{\zeta}_d + \lambda_{hd} - K_f e_\lambda \tag{25}$$

where

$$\begin{aligned} \Phi_m &= C_m\Psi_m \\ \chi_m &= ZL^{+\mathrm{T}}M_L \end{aligned} \tag{26}$$

with

$$\begin{aligned} C_m &= [M_L \quad C_L \quad G_L] \\ \Psi_m &= [\ddot{\zeta}_r \quad \dot{\zeta}_r \quad 1]^{\mathrm{T}} \\ L^+ &= (L^{\mathrm{T}}L)^{-1}L^{\mathrm{T}}. \end{aligned}$$

K_p, K_i, and K_f are positive definite.

Theorem 3.1: Consider the mechanical system without external disturbance described by (1), (2), and (8) with $d(t) = 0$. Using the control law (24) and (25), the following hold for any $(q(0), \dot{q}(0)) \in \Omega_n \cap \Omega_h$.

1) r converges to a set containing the origin as $t \to \infty$.
2) e_q and $\dot{e}_q$ converge to zero as $t \to \infty$.
3) e_λ and τ are bounded for all $t \geq 0$.

Proof (i): The closed-loop system dynamics can be rewritten as

$$M_L\dot{r} = L^{\mathrm{T}}u_a - \mu - C_L r \tag{27}$$

where $\mu = M_L\ddot{\zeta}_r + C_L\dot{\zeta}_r + G_L + d_L(t)$.

Substituting (24) into (27), the closed-loop dynamics are given by

$$M_L\dot{r} = -K_p r - K_i\int_0^t r ds - \Phi_m - \mu - C_L r. \tag{28}$$

Consider the Lyapunov function candidate

$$V = \frac{1}{2}r^{\mathrm{T}}M_L r + \frac{1}{2}\left(\int_0^t r ds\right)^{\mathrm{T}} K_i\int_0^t r ds \tag{29}$$

then

$$\dot{V} = r^{\mathrm{T}}\left(M_L\dot{r} + \frac{1}{2}\dot{M}_L r + K_i\int_0^t r ds\right). \tag{30}$$

Using Property 2.3, the time derivative of V along the trajectory of (28) is

$$\begin{aligned} \dot{V} &= -r^{\mathrm{T}}K_p r - r^{\mathrm{T}}\mu - r^{\mathrm{T}}\Phi_m \\ &\leq -r^{\mathrm{T}}K_p r \leq -\lambda_{\min}(K_p)\|r\|^2 \leq 0. \end{aligned}$$

Integrating both sides of the above equation gives

$$V(t) - V(0) \leq -\int_0^t r^{\mathrm{T}} K_p r ds \leq 0. \tag{31}$$

From above all, r converges to a small set containing the origin as $t \to \infty$. ■

Proof (ii): From (31), V is bounded, which implies that $r \in L_\infty^{n-k-l}$. We have $\int_0^t r^{\mathrm{T}} K_p r ds \leq V(0) - V(t)$, which leads to $r \in L_2^{n-k-l}$. From $r = \dot{e}_\zeta + K_\zeta e_\zeta$, it can be obtained that $e_\zeta, \dot{e}_\zeta \in L_\infty^{n-k-l}$. As we have established that $e_\zeta, \dot{e}_\zeta \in L_\infty$, from Assumption 3.1, we conclude that $\zeta(t), \dot{\zeta}(t), \dot{\zeta}_r(t), \ddot{\zeta}_r(t) \in L_\infty^{n-k-l}$, and $\dot{q} \in L_\infty^n$.

Therefore, all the signals on the right-hand side of (28) are bounded and we can conclude that $\dot{r}$ and $\ddot{\zeta}$ are bounded. Thus, $r \to 0$ as $t \to \infty$ can be obtained. Consequently, we have $e_\zeta \to 0$ and $\dot{e}_\zeta \to 0$ as $t \to \infty$. It follows that e_q and $\dot{e}_q \to 0$ as $t \to \infty$. ■

Proof (iii): Substituting the control (24) and (25) into the reduced order dynamics (23) yields

$$\begin{aligned}(I + K_f)e_\lambda &= Z(C^1\dot{\zeta} + G^1 + d^1(t) - u_a) + u_b \\ &= -ZL^{+^{\mathrm{T}}} M_L \ddot{\zeta} + \chi_m \ddot{\zeta}_d. \end{aligned} \tag{32}$$

Since $\ddot{\zeta}$ and Z are bounded, $\zeta \to \zeta_d$, $(-ZL^{+^{\mathrm{T}}} M_L \ddot{\zeta} + \chi_m \ddot{\zeta}_d)$ is also bounded, the size of e_λ can be adjusted by choosing the proper gain matrix K_f.

Since $r, \zeta, \dot{\zeta}, \zeta_r^1, \dot{\zeta}_r, \ddot{\zeta}_r$, and e_λ are all bounded, it is easy to conclude that τ is bounded from (24) and (25). ■

B. Robust Control

In practice, uncertainties and external disturbances do exist. The above control so designed may give degraded performance and may incur instability. Robust-control schemes can handle the uncertainties and external disturbances on the dynamics.

To facilitate the robust-control formulation, the following assumption is required.

Assumption 3.2: There exist some finite positive constants $c_{ri} > 0 (1 \leq i \leq 4)$, and finite nonnegative constant $c_{r5} \geq 0$ such that $\forall \zeta, \dot{\zeta} \in R^n$ [13]

$$\begin{aligned}\|M_L\| &\leq c_{r1} \\ \|C_L\| &\leq c_{r2} + c_{r3}\|\dot{\zeta}\| \\ \|G_L\| &\leq c_{r4} \\ \sup_{t\geq 0}\|d_L\| &\leq c_{r5}.\end{aligned}$$

Assumption 3.3: Time-varying positive function δ converges to zero as $t \to \infty$ and satisfies

$$\lim_{t\to\infty} \int_0^t \delta(\omega) d\omega = \rho < \infty$$

with finite constant ρ.

There are many choices for δ that satisfy the Assumption 3.3, e.g., $\delta = 1/(1+t)^2$.

Consider the following control law:

$$L^{\mathrm{T}} u_a = -K_p r - K_i \int_0^t r ds - \frac{r\Phi_r^2}{\|r\|\Phi_r + \delta} \tag{33}$$

$$u_b = \frac{\chi_r^2}{\chi_r + \delta}\ddot{\zeta}_d + \lambda_{hd} - K_f e_\lambda \tag{34}$$

where

$$\begin{aligned}\Phi_r &= C_r^{\mathrm{T}} \Psi_r \\ \chi_r &= c_1 \|Z^* L^{+^{\mathrm{T}}}\|\end{aligned}$$

with

$$\begin{aligned}C_r &= [c_{r1}\ c_{r2}\ c_{r3}\ c_{r4}\ c_{r5}]^{\mathrm{T}} \\ \Psi_r &= \left[\|\ddot{\zeta}_r\|\ \|\dot{\zeta}_r\|\ \|\dot{\zeta}\|\ \|\dot{\zeta}_r\|\ 1\ 1\right]^{\mathrm{T}} \\ Z^* &= \left(J^1 (M^*)^{-1} J^{1\mathrm{T}}\right)^{-1} J^1 (M^*)^{-1}.\end{aligned}$$

K_p, K_i, and K_f are positive definite and $\|L^{\mathrm{T}} M^* L\| = c_1$.

Theorem 3.2: Consider the mechanical system described by (1), (2), and (8). Using the control law (33) and (34), the following hold for any $(q(0), \dot{q}(0)) \in \Omega_n \cap \Omega_h$.

1) r converges to a set containing the origin as $t \to \infty$.
2) e_q and $\dot{e}_q$ converge to zero as $t \to \infty$.
3) e_λ and τ are bounded for all $t \geq 0$.

Proof (i): Substituting (33) into (27), the closed-loop dynamics is obtained

$$M_L \dot{r} = -K_p r - K_i \int_0^t r ds - \frac{r\Phi_r^2}{\|r\|\Phi_r + \delta} - \mu - C_L r. \tag{35}$$

Considering the Lyapunov function candidate (29), from Property 2.2, we have $(1/2)\lambda_{\min}(M_L) r^{\mathrm{T}} r \leq (1/2) r^{\mathrm{T}} M_L r \leq (1/2)\lambda_{\max}(M_L) r^{\mathrm{T}} r$. By using Property 2.3, the time derivative (30) of V along the trajectory of (35) is

$$\begin{aligned}\dot{V} &= -r^{\mathrm{T}} K_p r - r^{\mathrm{T}}\mu - \frac{\|r\|^2\Phi_r^2}{\|r\|\Phi_r + \delta} \\ &\leq -r^{\mathrm{T}} K_p r - \frac{\|r\|^2\Phi_r^2}{\|r\|\Phi_r + \delta} + \|r\|\Phi_r \\ &\leq -r^{\mathrm{T}} K_p r + \delta \\ &\leq -\lambda_{\min}(K_p)\|r\|^2 + \delta.\end{aligned}$$

Since δ is a time-varying function converging to zero as $t \to \infty$, and δ is bounded, there exists $t > t_1$ and $\delta \leq \rho_1$, when $\|r\| \geq \sqrt{\rho_1/\lambda_{\min}(K_p)}$, $\dot{V} \leq 0$. From above all, r converges to a small set containing the origin as $t \to \infty$.

Integrating both sides of the above equation gives

$$V(t) - V(0) \leq -\int_0^t r^{\mathrm{T}} K_p r ds + \rho < \infty. \tag{36}$$

■

Proof (ii): From (36), V is bounded, which implies that $r \in L_\infty^{n-k-l}$. We have $\int_0^t r^{\mathrm{T}} K_p r ds \leq V(0) - V(t) + \rho$, which leads to $r \in L_2^{n-k-l}$. From $r = \dot{e}_\zeta + K_\zeta e_\zeta$, it can be obtained that $e_\zeta, \dot{e}_\zeta \in L_\infty^{n-k-l}$. As we have established $e_\zeta, \dot{e}_\zeta \in L_\infty$, from Assumption 3.1, we conclude that $\zeta(t), \dot{\zeta}(t), \dot{\zeta}_r(t), \ddot{\zeta}_r(t) \in L_\infty^{n-k-l}$, and $\dot{q} \in L_\infty^n$.

Therefore, all the signals on the right-hand side of (35) are bounded, and we can conclude that $\dot{r}$ and $\ddot{\zeta}$ are bounded. Thus, $r \to 0$ as $t \to \infty$ can be obtained. Consequently, we have $e_\zeta \to 0$ and $\dot{e}_\zeta \to 0$ as $t \to \infty$. It follows that e_q and $\dot{e}_q \to 0$ as $t \to \infty$. ■

Proof (iii): Substituting the control (33) and (34) into the reduced order dynamics (23) yields

$$\begin{aligned}(I + K_f)e_\lambda &= Z\left(C^1\dot{\zeta} + G^1 + d^1(t) - u_a\right) + u_b \\ &= -ZL^{+^{\mathrm{T}}} M_L \ddot{\zeta} + \frac{\chi^2}{\chi + \delta}\ddot{\zeta}_d. \end{aligned} \quad (37)$$

Since $\ddot{\zeta}$ and Z are bounded, $\zeta \to \zeta_d$, $-ZL^{+^{\mathrm{T}}} M_L \ddot{\zeta} + (\chi^2/(\chi + \delta))\ddot{\zeta}_d$ is also bounded; the size of e_λ can be adjusted by choosing the proper gain matrix K_f.

Since r, ζ, $\dot{\zeta}$, ζ_r^1, $\dot{\zeta}_r$, $\ddot{\zeta}_r$, and e_λ are all bounded, it is easy to conclude that τ is bounded from (33) and (34). ■

C. Adaptive Robust Control

In developing robust-control law (33) and (34), the vector C_r is assumed to be known. However, in practice, it cannot be obtained easily. We can develop adaptive updating law to estimate the C_r [13].

Consider the adaptive robust-control law as

$$L^{\mathrm{T}} u_a = -K_p r - K_i \int_0^t r dt - \sum_{i=1}^5 \frac{r\hat{c}_{ri}\Psi_{ri}^2}{\|r\|\Psi_{ri} + \delta_i} \quad (38)$$

$$u_b = \frac{\hat{\chi}_r^2}{\hat{\chi}_r + \delta_1}\ddot{\zeta}_d + \lambda_{hd} - K_f e_\lambda \quad (39)$$

where

$$\dot{\hat{c}}_{ri} = -\omega_i \hat{c}_{ri} + \sum_{i=1}^5 \frac{\gamma_i \Psi_{ri}^2 \|r\|^2}{\|r\|\Psi_{ri} + \delta_i}, \qquad i = 1, \ldots, 5 \quad (40)$$

$$\hat{\chi}_r = \hat{c}_1 \left\| \hat{Z} L^{+^{\mathrm{T}}} \right\|. \quad (41)$$

$\hat{Z} = (J^1(\hat{M}^*)^{-1}J^{1\mathrm{T}})^{-1}J^1(\hat{M}^*)^{-1}$ with $\|L^{\mathrm{T}}\hat{M}^*L\| = \hat{c}_1$, K_p, K_i, K_f are positive definite; $\gamma_i > 0$, $\delta_i > 0$, and $\omega_i > 0$ satisfying Assumption 3.3

$$\int_0^\infty \delta_i(s)ds = \rho_{i\delta} < \infty$$

$$\int_0^\infty \omega_i(s)ds = \rho_{i\omega} < \infty$$

with the constants $\rho_{i\delta}$ and $\rho_{i\omega}$.

Theorem 3.3: Consider the mechanical system described by (1), (2), and (8). Using the control law (38) and (39), the following hold for any $(q(0), \dot{q}(0)) \in \Omega_n \cap \Omega_h$.

1) r converges to a set containing the origin as $t \to \infty$.
2) e_q and $\dot{e}_q$ converge to zero as $t \to \infty$.
3) e_λ and τ are bounded for all $t \geq 0$.

Proof (i): Substituting (38) into (27), the closed-loop dynamic equation is obtained

$$M_L\dot{r} = -K_p r - K_i \int_0^t r ds - \sum_{i=1}^5 \frac{r\hat{c}_{ri}\Psi_{ri}^2}{\|r\|\Psi_{ri} + \delta_i} - \mu - C_L r. \quad (42)$$

Consider the Lyapunov function candidate

$$V = \frac{1}{2}r^{\mathrm{T}} M_L r + \frac{1}{2}\left(\int_0^t r ds\right)^{\mathrm{T}} K_i \int_0^t r ds + \frac{1}{2}\tilde{C}_r \Gamma^{-1}\tilde{C}_r^{\mathrm{T}} \quad (43)$$

with $\tilde{C}_r = C_r - \hat{C}_r$.

Its derivative is

$$\dot{V} = r^{\mathrm{T}}\left(M_L\dot{r} + \frac{1}{2}\dot{M}_L r + K_i \int_0^t r dt\right) + \tilde{C}_r \Gamma^{-1}\dot{\tilde{C}}_r^{\mathrm{T}} \quad (44)$$

where $\Gamma = \mathrm{diag}[\gamma_i] > 0$, $i = 1, \ldots, 5$.

From Property 2.2, we have $(1/2)\lambda_{\min}(M_L)r^{\mathrm{T}}r \leq 1/2r^{\mathrm{T}}M_L r \leq (1/2)\lambda_{\max}(M_L)r^{\mathrm{T}}r$. By using Property 2.3, the time derivative of V along the trajectory of (42) is

$$\begin{aligned}\dot{V} =& -r^{\mathrm{T}}K_p r - r^{\mathrm{T}}\mu - r^{\mathrm{T}}\sum_{i=1}^5 \frac{r\hat{c}_{ri}\Psi_{ri}^2}{\|r\|\Psi_{ri} + \delta_i} \\ &+ \hat{C}_r^{\mathrm{T}}\Omega\Gamma^{-1}\tilde{C}_r - \sum_{i=1}^5 \frac{\|r\|^2\tilde{c}_{ri}\Psi_{ri}^2}{\|r\|\Psi_{ri} + \delta_i} \\ \leq& -r^{\mathrm{T}}K_p r + \|r\|\Phi_{ri} - \sum_{i=1}^5 \frac{\|r\|^2 c_{ri}\Psi_{ri}^2}{\|r\|\Psi_{ri} + \delta_i} + \hat{C}_r^{\mathrm{T}}\Omega\Gamma^{-1}\tilde{C}_r \\ \leq& -r^{\mathrm{T}}K_p r + C_r^{\mathrm{T}}\Delta + \hat{C}_r^{\mathrm{T}}\Omega\Gamma^{-1}\tilde{C}_r \\ =& -r^{\mathrm{T}}K_p r + C_r^{\mathrm{T}}\Delta + \hat{C}_r^{\mathrm{T}}\Omega\Gamma^{-1}(C_r - \hat{C}_r) \\ =& -r^{\mathrm{T}}K_p r + C_r^{\mathrm{T}}\Delta - \frac{1}{4}C_r^{\mathrm{T}}\Omega\Gamma^{-1}C_r + \frac{1}{4}C_r^{\mathrm{T}}\Omega\Gamma^{-1}C_r \\ &+ \hat{C}_r^{\mathrm{T}}\Omega\Gamma^{-1}C_r - \hat{C}_r^{\mathrm{T}}\Omega\Gamma^{-1}\hat{C}_r \\ =& -r^{\mathrm{T}}K_p r + C_r^{\mathrm{T}}\Delta - \left(\frac{1}{2}C_r^{\mathrm{T}} - \hat{C}_r^{\mathrm{T}}\right)\Omega\Gamma^{-1}\left(\frac{1}{2}C_r - \hat{C}_r\right) \\ &+ \frac{1}{4}C_r^{\mathrm{T}}\Omega\Gamma^{-1}C_r \\ \leq& -r^{\mathrm{T}}K_p r + C_r^{\mathrm{T}}\Delta + \frac{1}{4}C_r^{\mathrm{T}}\Omega\Gamma^{-1}C_r\end{aligned}$$

with $\Omega = \mathrm{diag}[\omega_i]$, $\Delta = [\delta_1, \delta_2, \ldots, \delta_5]^{\mathrm{T}}$, $i = 1, \ldots, 5$.

Therefore, $\dot{V} \leq -\lambda_{\min}(K_p)\|r\|^2 + C_r^{\mathrm{T}}\Delta + (1/4)C_r^{\mathrm{T}}\Omega\Gamma^{-1}C_r$. Since $C_r^{\mathrm{T}}\Delta + (1/4)C_r^{\mathrm{T}}\Omega\Gamma^{-1}C_r$ is bounded, there exists

$t > t_2$, $C_r^{\mathrm{T}}\Delta + (1/4)C_r^{\mathrm{T}}\Omega\Gamma^{-1}C_r \le \rho_2$, when $\|r\| \ge \sqrt{\rho_2/\lambda_{\min}(K_p)}$, $\dot{V} \le 0$, from above all, r converges to a small set containing the origin as $t \to \infty$. ■

Proof (ii): Integrating both sides of the above equation gives

$$V(t)-V(0) \le -\int_0^t r^{\mathrm{T}}K_p r ds + \int_0^t \left(C_r^{\mathrm{T}}\Delta + \frac{1}{4}C_r^{\mathrm{T}}\Omega\Gamma^{-1}C_r\right)ds. \tag{45}$$

Since C_r and Γ are constant, $\int_0^\infty \Delta ds = \rho_\delta = [\rho_{1\delta}, \ldots, \rho_{5\delta}]^{\mathrm{T}}$, $\int_0^\infty \Omega ds = \rho_\omega = [\rho_{1\omega}, \ldots, \rho_{5\omega}]^{\mathrm{T}}$, we can rewrite (45) as

$$\begin{aligned} V(t)-V(0) \le &-\int_0^t r^{\mathrm{T}}K_p r ds + C_r^{\mathrm{T}}\left(\int_0^t \Delta ds\right) \\ &+ \frac{1}{4}C_r^{\mathrm{T}}\left(\int_0^t \Omega ds\right)\Gamma^{-1}C_r ds \\ \le &-\int_0^t r^{\mathrm{T}}K_p r ds + C_r^{\mathrm{T}}\rho_\delta + C_r^{\mathrm{T}}\rho_\omega\Gamma^{-1}C_r \\ < &-\lambda_{\min}(K_p)\|r\|^2 + C_r^{\mathrm{T}}\rho_\delta + C_r^{\mathrm{T}}\rho_\omega\Gamma^{-1}C_r \\ < &\infty. \end{aligned} \tag{46}$$

Thus, V is bounded, which implies that $r \in L_\infty^{n-k-l}$. From (46), we have

$$\int_0^t r^{\mathrm{T}}K_p r ds \le V(0) - V(t) + C_r^{\mathrm{T}}\rho_\delta + C_r^{\mathrm{T}}\rho_\omega\Gamma^{-1}C_r \tag{47}$$

which leads to $r \in L_2^{n-k-l}$. From $r = \dot{e}_\zeta + K_\zeta e_\zeta$, it can be obtained that $e_\zeta, \dot{e}_\zeta \in L_\infty^{n-k-l}$. As we have established $e_\zeta, \dot{e}_\zeta \in L_\infty$, from Assumption 3.1, we conclude that $\zeta(t), \dot{\zeta}(t), \dot{\zeta}_r(t), \ddot{\zeta}_r(t) \in L_\infty^{n-k-l}$, and $\dot{q} \in L_\infty^n$.

Therefore, all the signals on the right-hand side of (42) are bounded, and we can conclude that $\dot{r}$ and $\ddot{\zeta}$ are bounded. Thus, $r \to 0$ as $t \to \infty$ can be obtained. Consequently, we have $e_\zeta \to 0$ and $\dot{e}_\zeta \to 0$ as $t \to \infty$. It follows that e_q and $\dot{e}_q \to 0$ as $t \to \infty$. ■

Proof (iii): Substituting the control (38) and (39) into the reduced order dynamics (23) yields

$$\begin{aligned} (I + K_f)e_\lambda &= Z\left(C^1\dot{\zeta} + G^1 + d^1(t) - u_a\right) + u_b \\ &= -ZL^{+^{\mathrm{T}}}M_L\ddot{\zeta} + \frac{\hat{\chi}_r^2}{\hat{\chi}_r + \delta_1}\ddot{\zeta}_d. \end{aligned} \tag{48}$$

Since $\ddot{\zeta}$ and Z are bounded, $\zeta \to \zeta_d$, $-ZL^{+^{\mathrm{T}}}M_L\ddot{\zeta} + (\hat{\chi}_r^2/(\hat{\chi}_r + \delta_1))\ddot{\zeta}_d$ is also bounded, the size of e_λ can be adjusted by choosing the proper gain matrix K_f.

Since r, ζ, $\dot{\zeta}$, ζ_r^1, $\dot{\zeta}_r$, $\ddot{\zeta}_r$, and e_λ are all bounded, it is easy to conclude that τ is bounded from (38) and (39). ■

IV. Simulations

To verify the effectiveness of the proposed adaptive robust control, let us consider the mobile-manipulator system shown in Fig. 2 [9]. The mobile manipulator is subjected to the following constraints:

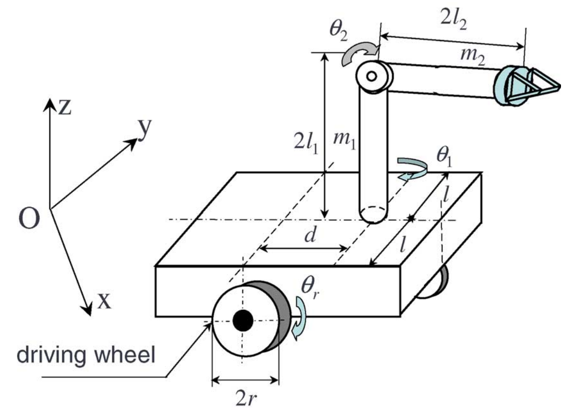

Fig. 2. Two-DOF manipulator mounted on a mobile platform.

$$\begin{aligned} -\dot{x}\sin\theta + \dot{y}\cos\theta + l\dot{\theta} + r\theta_L &= 0 \\ \dot{x}\sin\theta - \dot{y}\cos\theta + l\dot{\theta} + r\theta_R &= 0 \\ \dot{x}\cos\theta + \dot{y}\sin\theta &= 0. \end{aligned}$$

Using the Lagrangian approach, we can obtain the standard form (1) with

$$\begin{aligned} q_v &= [x, y, \theta]^{\mathrm{T}} \quad q_a = [\theta_1, \theta_2]^{\mathrm{T}} \quad q = [q_v, q_a]^{\mathrm{T}} \\ A &= [\cos\theta, \sin\theta, 0]^{\mathrm{T}} \\ M_{v11} &= \begin{bmatrix} m_{p12} + \frac{2I_w\sin^2\theta}{r^2} & -\frac{2I_w}{r^2}\sin\theta\cos\theta \\ -\frac{2I_w}{r^2}\sin\theta\cos\theta & m_{p12} + \frac{2I_w\cos^2\theta}{r^2} \end{bmatrix} \\ M_{v12} &= \begin{bmatrix} -m_{12}d\sin\theta \\ m_{12}d\cos\theta \end{bmatrix} \\ M_{v21} &= M_{v12}^{\mathrm{T}} \quad M_{v22} = M_{11}^1 \\ C_v &= \begin{bmatrix} \frac{2I_w}{r^2}\dot{\theta}\sin\theta\cos\theta & -\frac{2I_w}{r^2}\dot{\theta}\cos^2\theta & 0 \\ \frac{2I_w}{r^2}\dot{\theta}\sin^2\theta & -\frac{2I_w}{r^2}\dot{\theta}\sin\theta\cos\theta & 0 \\ -m_{12}d\dot{\theta}\cos\theta & -m_{12}d\dot{\theta}\cos\theta & 0 \end{bmatrix} \\ M_{11}^1 &= I_p + I_{12} + m_{12}d^2 + 2I_w l^2/r^2 \\ M_a &= \mathrm{diag}[I_{12}, I_2] \\ m_{p12} &= m_p + m_{12} \quad m_{12} = m_1 + m_2 \\ I_{12} &= I_1 + I_2 \\ M_{va} &= \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ I_{12} & 0.0 \end{bmatrix} \\ B &= \begin{bmatrix} \sin\theta/r & -\sin\theta/r & 0.0 & 0.0 \\ -\cos\theta/r & \cos\theta/r & 0.0 & 0.0 \\ -l/r & l/r & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \\ C_{va} &= 0.0, \quad C_{av} = C_{va}^{\mathrm{T}}, \quad C_a = 0.0 \\ G_v &= [0.0, 0.0, 0.0]^{\mathrm{T}} \quad G_a = [0.0, m_2 g l_2 \sin\theta_2]^{\mathrm{T}} \\ H &= \begin{bmatrix} -\tan\theta & 0.0 \\ 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} \\ \tau_v &= [\tau_L, \tau_r]^{\mathrm{T}} \quad \tau_a = [\tau_1, \tau_2]^{\mathrm{T}}. \end{aligned}$$

Given the desired trajectories

$$y_d = 1.5\sin(t)$$
$$\theta_d = 1.0\sin(t)$$
$$\theta_{1d} = \pi/4\,(1-\cos(t))$$

and the geometric constraints that the end-effector is subjected to as

$$\Phi = l_1 + l_2\sin(\theta_2) = 0$$
$$\lambda_{hd} = 10.0N$$

it is easy to have

$$J = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & l_2\cos\theta_2 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$J^1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & l_2\cos\theta \end{bmatrix}.$$

In the simulation, we assume that the parameters $m_p = m_1 = m_2 = 1.0$ kg, $I_w = I_p = 1.0$ kgm^2, $I_1 = I_2 = 1.0$ kgm^2, $I = 0.5$ kgm^2, $d = l = r = 1.0$ m, $2l_1 = 1.0$ m, $2l_2 = 0.6$ m, $q(0) = [0, 4, 0.785, 0.1]^{\mathrm{T}}$, $\dot{q}(0) = [0.0, 0.0, 0.0, 0.0]^{\mathrm{T}}$, and $\lambda_h(0) = 0$. According to Theorem 3.3, the control gains are selected as $K_p = \mathrm{diag}[1.0]$, $K_\zeta = \mathrm{diag}[1.0]$, $K_i = 0.0$, and $K_f = 0.5$. The adaptation gains in control law (33) are chosen as $\delta_i = \omega_i = 1/(1+t)^2$, $i = 1, \ldots, 5, C_r = [1.0, 1.0, 1.0, 1.0, 1.0]^T$, and $\Gamma = \mathrm{diag}[2.5]$. The disturbances on the mobile base are set as $0.1\sin(t)$ and $0.1\cos(t)$. Under the same initial conditions, control gains, and environment, we conduct the simulations by the adaptive robust control using (38) and (39) in Theorem 3.3 and the model-based control using (24) and (25) in Theorem 3.1, respectively. For the model-based control, we assume that the system model has 30% uncertainty. The trajectory-tracking performances of the adaptive robust control and the model-based control are illustrated in Figs. 3–5 and the velocities tracking and the input torques by the adaptive robust control are shown in Figs. 6 and 7, and Figs. 8 and 9 are the velocities tracking and the input torques by the model-based control, respectively. The force tracking of the adaptive robust control is shown in Fig. 10 and that of the model-based control is shown in Fig. 11. From the comparison of both controls, we can see the proposed adaptive robust control is more stable, smooth, and quickly converged. But, the tracking results of the model-based control are not satisfactory and the evolutions of velocities and torques fluctuate greatly in comparison with the adaptive robust control. Moreover, the constraint force using the model-based control changes more greatly, but this force using the adaptive robust control converges quickly to the desired force.

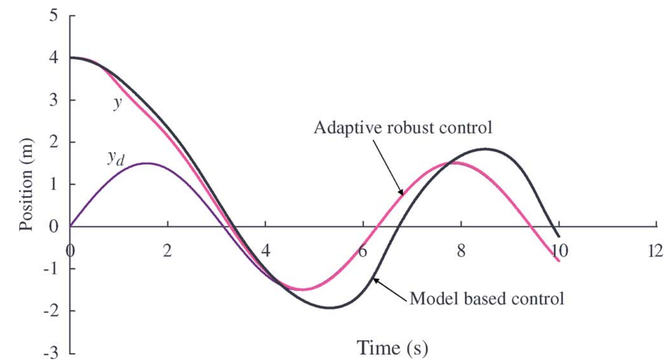

Fig. 3. Position of the joint.

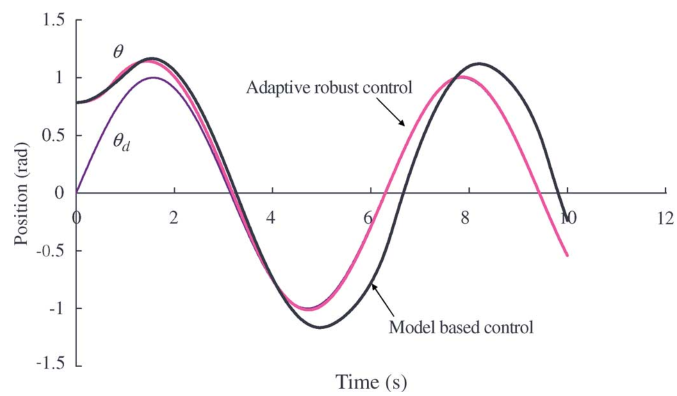

Fig. 4. Position of the joint.

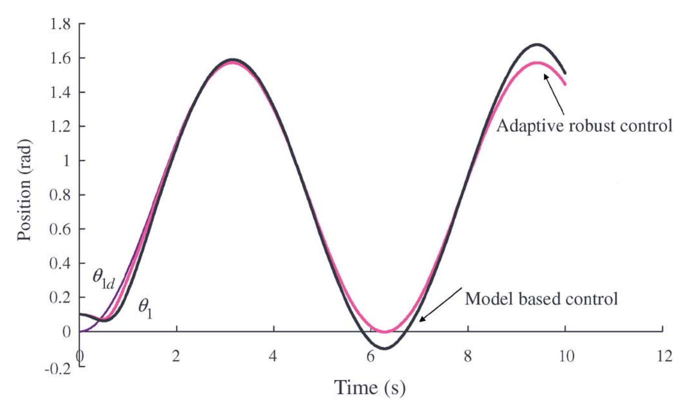

Fig. 5. Position of the joint.

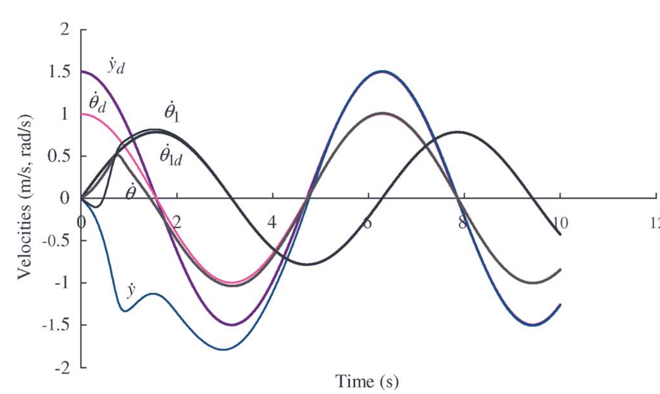

Fig. 6. Velocities of the joints by adaptive robust control.

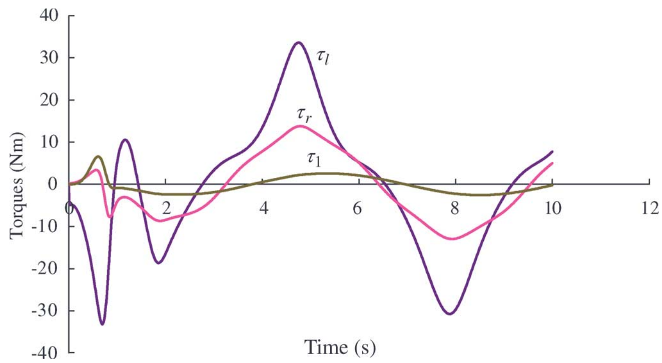

Fig. 7. Torques of the joints by adaptive robust control.

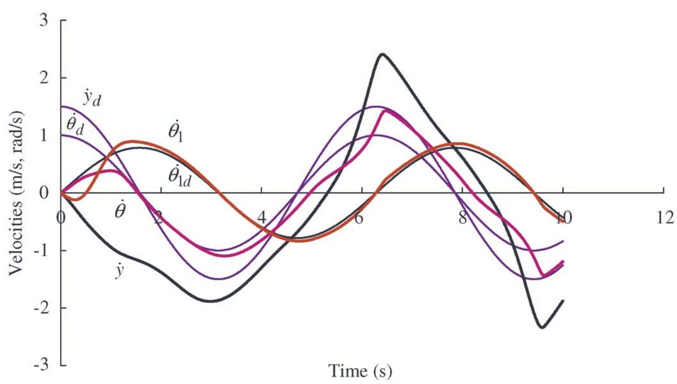

Fig. 8. Velocities of the joints by model-based control.

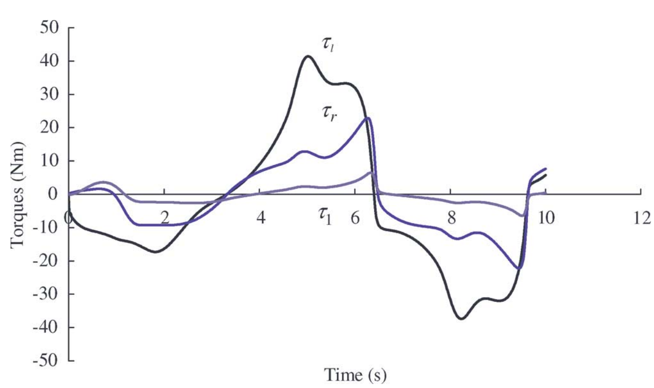

Fig. 9. Torques of the joints by model-based control.

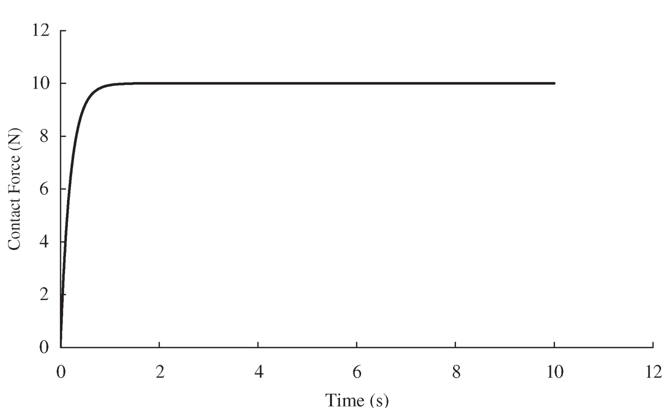

Fig. 10. Constraint force tracking of adaptive robust control.

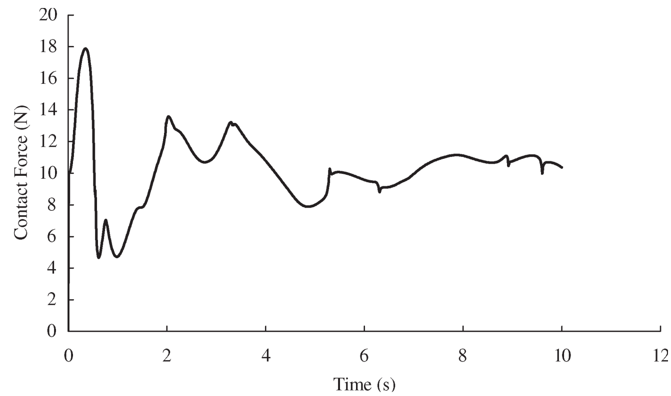

Fig. 11. Constraint force tracking of model-based control.

The simulation results show that the trajectory and force-tracking errors converge to zero, which validates the results of the controls (38) and (39) in Theorem 3.3.

V. Conclusion

In this paper, effective adaptive robust-control strategies have been presented systematically to control a class of holonomic-constrained noholonomic mobile manipulators in the presence of uncertainties and disturbances. The system stability and the boundedness of tracking errors are proved using Lyapunov synthesis. All control strategies have been designed to drive the system motion converge to the desired manifold and, at the same time, guarantee the boundedness of the constrained force. Simulation studies have verified that not only the states of the system converge to the desired trajectory but also the constraint force converges to the desired force.

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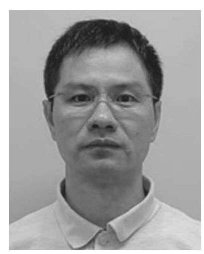

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