A Unified Quadratic-Programming-Based Dynamical System Approach to Joint Torque Optimization of Physically Constrained Redundant Manipulators

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***Abstract*—In this paper, for joint torque optimization of redundant manipulators subject to physical constraints, we show that velocity-level and acceleration-level redundancy-resolution schemes both can be formulated as a quadratic programming (QP) problem subject to equality and inequality/bound constraints. To solve this QP problem online, a primal-dual dynamical system solver is further presented based on linear variational inequalities. Compared to previous researches, the presented QP-solver has simple piecewise-linear dynamics, does not entail real-time matrix inversion, and could also provide joint-acceleration information for manipulator torque control in the velocity-level redundancy-resolution schemes. The proposed QP-based dynamical system approach is simulated based on the PUMA560 robot arm with efficiency and effectiveness demonstrated.**

***Index Terms*—Linear variational inequalities (LVI), primal-dual solver, quadratic programming (QP), redundant manipulator, torque control.**

I. Introduction

A manipulator is said to be kinematically redundant when more degrees of freedom (DOF) than the minimum number required to execute a given task are available. Redundant manipulators have wider operational space and extra DOF to meet a number of functional constraints. One of the most fundamental tasks in operating redundant manipulators is the inverse kinematics problem, i.e., to find the joint trajectories, given the trajectories of the end-effector. This inverse kinematics problem could be handled to avoid joint physical limits, singularities, obstacles, and to optimize various performance criteria, while conducting the primary end-effector motion task.

One example of the performance criteria to be optimized is the joint torques, which is aimed at making a more effective utilization of input power from actuators by exploiting the extra DOF in redundant manipulators. Similar to the velocity-level redundancy resolution, most of the researchers use the pseudoinverse-type solution for the torque-minimizing redundancy resolution of manipulators. In particular, initial formulation for local torque minimization was proposed in the mid 1980s by Hollerback and Suh [1]. They designed several schemes at the joint accelerations level through the pseudoinverse and null-space method. However, the torque-minimizing redundancy resolution problem remains to be unsolved since the schemes exhibited instabilities/divergence, especially in long-range motions. Since then many researchers attempted to formulate other joint torque optimization schemes to eliminate the instability problem [2]–[7]. The recent thorough analysis [8], [9] showed that almost all the pseudoinverse-type acceleration-level redundancy resolution schemes, including the remedy methods, may still exhibit local instabilities in the form of abrupt increases in joint velocities, accelerations and torques.

Since the mid 1990's, the quadratic programming (QP) approaches have been developed for the redundancy resolution of robot manipulators, which include the torque-optimization schemes resolved at the acceleration-level, e.g., [10]–[20]. The QP formulations corresponding to specific pseudoinverse-type solutions have the potential to contain directly the physical/mechanical constraints in the form of inequalities, like joint limits, joint velocity/acceleration limits, and environmental obstacles [19], [20]. To solve the quadratic program, a compact QP solution method has also been established by using serial-processing techniques like workspace decomposition, Gaussian elimination with partial pivoting, and matrix inversion [11], [14]. Recently, as a parallel alternative to solve the QP-based redundancy-resolution problem in real time and to provide explicit solutions, a dynamical system approach in the form of recurrent neural networks has been developed, e.g., [16]–[20]. In view of the nature of parallel distributed computation and hardware implementation, the dynamical system approach is thought to have opened new avenue for online optimization.

In the past decade, various dynamical system solvers have been developed for solving online the constrained QP problems. These include among others the Lagrange neural networks [21], the gradient and projected network [22], the usual primal-dual network [17], and the dual neural network [18]. For a survey of the aforementioned dynamical solvers, see [23]. Here, two recent types of dynamical network systems for solving QP-based redundancy problems are reviewed. One is the usual primal-dual dynamical system in [17]. The primal-dual system was developed by minimizing the duality gap via gradient method, and thus the dynamic equations are usually complicated and with high-order nonlinear terms. Another recent result is the dual dynamical system [18]. Reducing system complexity, the dual network was proposed by only using dual decision variables and directly using KKT conditions with the projection operator. However, this dual network was for strictly convex optimization problems and entails the explicit matrix inversion.

As torque instabilities in local redundancy-resolution schemes gave birth to a multitude of different remedy methods, this paper unifies them into a general QP-based formulation and approach. The unified framework of these schemes may bring more insight into the wealth of existing solutions as well as a better understanding of future researches. Specifically, in this paper a general QP problem formulation is first established. This formulation could unify both acceleration-level and velocity-level redundancy-resolution schemes into a QP problem subject to equality and inequality/bound constraints. In addition, the torque instability appearing in pseudoinverse-type solutions due to large null-space joint velocity/acceleration is much eliminated, as this QP formulation takes into account joint physical limits to hinder the build-up of large velocity/acceleration. Second, a linear variational inequalities (LVI)-based primal-dual dynamical system solver is presented for the online computation of such QP-based redundancy-resolution schemes. Based on the LVI-reformulation of a QP problem, the presented dynamical solver has simple piecewise-linear dynamics and does not entail explicit matrix inversion. In addition, this QP solver could also provide accurate joint acceleration for the velocity-level redundancy-resolution schemes, which, in pseudoinverse-type or numerical QP solutions, is usually a problem that have to use numerical differentiation to approximate. Third, a number of comparison simulations are performed based on the PUMA560 robot arm to examine the characteristics and performance of the proposed QP-based primal-dual dynamical system approach. The efficiency and effectiveness are shown via a few illustrative examples.

II. Unified QP Formulation

The relationship between end-effector velocity $\dot{r} \in R^m$ and joint velocity $\dot{\theta} \in R^n$ for redundant manipulators can be represented as

$$\dot{r} = J(\theta)\dot{\theta} \tag{1}$$

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where Jacobian matrix $J(\theta) \in R^{m\times n}$. Differentiating (1) yields the acceleration-level relation

$$J(\theta)\ddot{\theta} = \ddot{r}_a \quad (2)$$

where $\ddot{r}_a := \ddot{r} - \dot{J}(\theta)\dot{\theta}$, and $\dot{J}(\theta)$ is the time derivative of J. Because the manipulator system is redundant, $m < n$. Equations (1) and (2) are both underdetermined, admitting an infinite number of solutions.

Conventionally, the general solutions to (1) and (2) are obtained as the pseudoinverse-type formulation, i.e., one minimum-norm particular solution plus a homogeneous solution: $\dot{\theta} = J^{+}\dot{r} + (I - J^{+}J)z_v$ or $\ddot{\theta} = J^{+}\ddot{r}_a + (I - J^{+}J)z_a$, where $J^{+} \in R^{n\times m}$ denotes the pseudoinverse of J, and z_v and z_a are arbitrary self-motion vectors determined as gradients of performance criteria. Evidently, there exist the following weakness in the pseudoinverse-type formulation: 1) except kinematic singularities (J being rank deficient), there also exist algorithmic singularities; 2) it is difficult to let z_v and z_a to incorporate inequality constraints directly; and 3) determining the magnitude of z_v and z_a is also a problem even for single performance index and is usually made based on a trial-and-error approach [10], [11], [19].

The manipulator's dynamic equation is [1], [24]:

$$H(\theta)\ddot{\theta} + c(\theta,\dot{\theta}) + g(\theta) = \tau \quad (3)$$

where $H(\theta)$ denotes the inertia matrix, $c(\theta,\dot{\theta})$ denotes the Coriolis/centrifugal force vector, $g(\theta)$ denotes the gravitational force vector, and $\tau \in R^n$ the joint torque vector. Considering (1)–(3) simultaneously, we can then incorporate dynamics into the redundancy resolution. The subject of this paper is to establish a unified QP approach to torque optimization and control of redundant manipulators. In the ensuing subsections, acceleration-level and velocity-level redundancy-resolution schemes are both formulated as a QP problem.

A. Acceleration-Level Redundancy Resolution

In this subsection, we formulate the acceleration-level redundancy resolution as a QP problem.

1) Minimum Torque Norm (MTN) Scheme: The MTN scheme subject to joint physical limits is originally written as

$$\min \quad \frac{\tau^T\tau}{2} \quad (4)$$

$$\text{s.t.} \quad J(\theta)\ddot{\theta} = \ddot{r}_a \quad (5)$$

$$\theta^- \leqslant \theta(t) \leqslant \theta^+ \quad (6)$$

$$\dot{\theta}^- \leqslant \dot{\theta}(t) \leqslant \dot{\theta}^+ \quad (7)$$

$$\ddot{\theta}^- \leqslant \ddot{\theta}(t) \leqslant \ddot{\theta}^+ \quad (8)$$

where (4) represents the two-norm of instantaneous joint torques to be minimized, and (6)–(8) are the bound constraints resulting from the joint, joint velocity, and joint acceleration limits.

As the redundancy is resolved at the acceleration level, the performance index (4) and the constraints (6)–(7) have to be converted into the expressions based on joint acceleration $\ddot{\theta}$. Substituting (3) into (4) yields the following local objective function at the acceleration level:

$$\frac{\tau^T\tau}{2} = \frac{\ddot{\theta}^T H^2\ddot{\theta}}{2} + (c+g)^T H\ddot{\theta} + \frac{(c+g)^T(c+g)}{2}.$$

The two-sided inequality constraint for joint limits avoidance in (6) can be converted as

$$\mu_p(\eta_p\theta^- - \theta(t)) \leqslant \ddot{\theta} \leqslant \mu_p(\eta_p\theta^+ - \theta(t)) \quad (9)$$

where, in light of the inertia movement, the critical coefficient $\eta_p \in (0,1)$ defines two critical areas (i.e., $[\theta^-, \eta_p\theta^-]$ and $[\eta_p\theta^+, \theta^+]$) for joint position variables such that there will appear a deceleration when the robot arm enters them, and the intensity coefficient $\mu_p > 0$ determines the magnitude of such a deceleration [18], [19]. Similarly, the avoidance of joint velocity limits $[\dot{\theta}^-, \dot{\theta}^+]$ in (7) can be converted as

$$\mu_v(\dot{\theta}^- - \dot{\theta}(t)) \leqslant \ddot{\theta} \leqslant \mu_v(\dot{\theta}^+ - \dot{\theta}(t)) \quad (10)$$

which guarantees that joint acceleration change its direction gradually as the joint velocity approaches its limit. The intensity coefficients μ_p and μ_v are selected such that the feasible region of $\ddot{\theta}$ made by the conversion of joint limits and joint velocity limit, i.e., (9) and (10), is normally not smaller than the original one made by joint acceleration limits, i.e., (8) [19].

Thus, with joint physical limits considered, the MTN redundancy-resolution scheme (4)–(8) can be formulated as the following quadratic program:

$$\min \quad \frac{\ddot{\theta}^T W\ddot{\theta}}{2} + b^T\ddot{\theta} \quad (11)$$

$$\text{s.t.} \quad J\ddot{\theta} = d \quad (12)$$

$$\xi^- \leqslant \ddot{\theta}(t) \leqslant \xi^+ \quad (13)$$

where, in this MTN scheme, $W := H^2$, $b := H(c+g)$, $d := \ddot{r}_a$, and $\xi^{\pm}$ is the resulting variable bounds by combining (8)–(10), i.e.,

$$\xi^- := \max\left(\mu_p(\eta_p\theta^- - \theta(t)),\ \mu_v(\dot{\theta}^- - \dot{\theta}(t)),\ \ddot{\theta}^-\right)$$
$$\xi^+ := \min\left(\mu_p(\eta_p\theta^+ - \theta(t)),\ \mu_v(\dot{\theta}^+ - \dot{\theta}(t)),\ \ddot{\theta}^+\right). \quad (14)$$

2) Inertia-Inverse Weighted Torque (IWT) Scheme: As analyzed via calculus of variations [4], the IWT scheme may result in resolutions with global characteristics. That is, the solution to the minimization of $\tau^T H^{-1}\tau/2$ subject to (5)–(8) also expresses the governing solution equation to the global optimization of kinetic energy

$$\min \quad \int_0^{t_f} \frac{\dot{\theta}^T H\dot{\theta}}{2}, \ \text{s.t.} \quad G_k(\theta,t) = 0,\ k = 1,\ldots,m \quad (15)$$

where t_f is the duration of the given manipulator task, and G_k are kinematic constraints. Similar to the MTN scheme, the IWT scheme can be reformulated as a QP in the same form as (11)–(13) where, in this IWT scheme, $W := H$, $b := c+g$, $d := \ddot{r}_a$, and $\xi^{\pm}$ is as in (14).

3) Minimum Acceleration Norm (MAN) Scheme: In the previous quadratic programs for the MTN and IWT schemes, the weighting coefficients W are respectively H^2 and H. As in our PUMA560-based simulations, the condition numbers are usually greater than 550 and 24. If ill-posed, the weighting coefficient may impose side effects like computational complexity and inaccuracy [18]. As extended directly from (and also an alternative of) the MTN and IWT schemes, we have the MAN redundancy-resolution scheme in the same QP formulation as (11)–(13) where, in this MAN scheme, $W := I$, $b := 0$, $d := \ddot{r}_a$, and $\xi^{\pm}$ is as in (14).

B. Velocity-Level Redundancy Resolution

In this subsection, we formulate the velocity-level redundancy resolution as a QP problem.

1) Minimum Kinetic Energy (MKE) Scheme: Being a local counterpart of global kinetic-energy minimization scheme (15), the following weighted criterion is defined and resolved at the velocity level:

$$\min \quad \frac{\dot{\theta}^T H\dot{\theta}}{2}$$

$$\text{s.t.} \quad J(\theta)\dot{\theta} = \dot{r} \quad (16)$$

$$\theta^- \leqslant \theta(t) \leqslant \theta^+ \quad (17)$$

$$\dot{\theta}^- \leqslant \dot{\theta}(t) \leqslant \dot{\theta}^+. \quad (18)$$

The above MKE scheme can be rewritten as the following QP, which is of the same form as in Section II-A but with different coefficients and decision variables, as follows:

$$\min \quad \frac{\dot{\theta}^T W \dot{\theta}}{2} + b^T \dot{\theta} \tag{19}$$

$$\text{s.t.} \quad J\dot{\theta} = d \tag{20}$$

$$\xi^- \leqslant \dot{\theta}(t) \leqslant \xi^+ \tag{21}$$

where, in this MKE scheme, $W := H$, $b := 0$, $d := \dot{r}$, and

$$\xi^- := \max\left(\mu_p(\theta^- - \theta(t)),\ \dot{\theta}^-\right)$$
$$\xi^+ := \min\left(\mu_p(\theta^+ - \theta(t)),\ \dot{\theta}^+\right). \tag{22}$$

2) Minimum Velocity Norm (MVN) Scheme: As shown in [8], the acceleration-level redundancy resolution (like the MTN, IWT and MAN schemes without considering joint limits) has the intrinsic instability problem. In addition, the velocity-level MKE method may also encounter instability problems for extremely long movements [10]. If the long end-effecter movement is a major concern, the MVN scheme could be applied by minimizing $\dot{\theta}^T\dot{\theta}/2$ subject to (16)–(18). This can be rewritten in the same QP formulation as (19)–(21) where, in this MVN scheme, $W := I$, $b := 0$, $d := \dot{r}$, and $\xi^{\pm}$ is as in (22).

Theorem 1. (QP Formulation): With joint physical constraints considered, the acceleration-level redundancy resolution and velocity-level redundancy resolution both can be unified as the following QP problem:

$$\min \quad \frac{x^T W x}{2} + b^T x \tag{23}$$

$$\text{s.t.} \quad Jx = d \tag{24}$$

$$\xi^- \leqslant x \leqslant \xi^+ \tag{25}$$

where decision vector x is defined respectively as $\ddot{\theta}$ in acceleration-level schemes and $\dot{\theta}$ in velocity-level schemes, and the coefficients (i.e., W, b, d, and $\xi^{\pm}$) are defined corresponding to a specific redundancy-resolution scheme.

Proof: Following the above analysis procedures. □

III. Primal-Dual Dynamical QP Solver

The previous section has developed a unified QP formulation for both acceleration-level and velocity-level redundancy-resolution schemes. However, the online efficient solution to the QP problem is still worth exploring. Numerical QP methods are not efficient enough for high-DOF sensor-based robotic systems: 1) they treat a two-sided inequality constraint as two one-sided inequality constraints, in addition to other serial-processing techniques and 2) in the velocity-level schemes, the joint acceleration required in torque control (3) has to be estimated through numerical differentiation [10], [25].

As a parallel alternative to continuous optimization and the remedy to the aforementioned weakness, in this section, a primal-dual dynamical QP solver is presented based on linear variational inequalities (LVI) [26]–[29]. By the duality theory [30], for the primal problem (23)–(25), its dual problem can be derived with the aid of dual decision variables. The dual decision variable is often defined as the Lagrangian multiplier for each constraint like (24) and (25). However, to reduce the QP-solver complexity, an elegant treatment is used to cancel the dual variable for bound constraint (25). That is, we only need to define the corresponding dual decision vector $y \in R^m$ for equality constraint (24). Thus, the primal-dual decision vector u and its bounds $u^{\pm}$ are constituted as

$$u := \begin{bmatrix} x \\ y \end{bmatrix},\ u^+ := \begin{bmatrix} \xi^+ \\ y^+ \end{bmatrix},\ u^- := \begin{bmatrix} \xi^- \\ -y^+ \end{bmatrix} \in R^{n+m} \tag{26}$$

where in hardware implementation/simulation, $\forall i$, elements y_i^+ are sufficiently large constants to represent $+\infty$. The convex set Ω made by u is then $\Omega = \{u \in R^{n+m} | u^- \leqslant u \leqslant u^+\}$. Defining coefficient matrix M and vector q as

$$M = \begin{bmatrix} W & -J^T \\ J & 0 \end{bmatrix},\ q = \begin{bmatrix} b \\ -d \end{bmatrix} \in R^{n+m} \tag{27}$$

we have the following equivalence result.

Theorem 2. (LVI Reformulation): Quadratic program (23)–(25) is equivalent to the following linear variational inequalities problem, i.e., to find a vector $u^* \in \Omega$, such that

$$(u - u^*)^T(Mu^* + q) \geqslant 0,\ \ \forall u \in \Omega. \tag{28}$$

Proof: See the Appendix. □

It is known that linear variational inequality (28) is equivalent to the following system of piecewise-linear equations [18], [19], [27]–[31]:

$$P_\Omega\left(u - (Mu + q)\right) - u = 0 \tag{29}$$

where $P_\Omega(\cdot)$ is the Ω-projection operator defined as $P_\Omega(u) = [P_\Omega(u_1), \ldots, P_\Omega(u_{n+m})]^T$ with

$$P_\Omega(u_i) = \begin{cases} u_i^-, & \text{if } u_i < u_i^- \\ u_i, & \text{if } u_i^- \leqslant u_i \leqslant u_i^+ \\ u_i^+, & \text{if } u_i > u_i^+ \end{cases},\ \forall i \in \{1, \ldots, n+m\}.$$

To solve (29), guided by dynamical-solver design methodologies [19], [28], [29], and [32], the following dynamical system, being the QP-solver for (23)–(25), is adopted:

$$\dot{u} = \gamma(I + M^T)\{P_\Omega\left(u - (Mu + q)\right) - u\}. \tag{30}$$

where γ is a positive design parameter used to scale the convergence rate of the system. Note that by the definition of u in (26), being the first n-dimensional component of $\dot{u}$, $\dot{x}$ is explicitly generated by (30). Thus, in the velocity-level MKE and MVN schemes, joint acceleration $\ddot{\theta}$ is obtained accurately and directly for torque control (3) in view of $x := \dot{\theta}$ and $\dot{x} = \ddot{\theta}$. That is, there is no numerical differentiation for $\ddot{\theta}$ in our approach.

Fig. 1 shows the block diagram of the QP-based dynamical system approach to redundancy resolution and torque control of redundant manipulators. Based on the current robot state, the online path planner first generates the end-effector command, i.e., r, $\dot{r}$, and $\ddot{r}$ if necessary. The redundancy resolution problem is then formulated as a quadratic program in the form of (23)–(25). As shown in Fig. 2, with simple matrix/vector augmentations and operations, the LVI-based dynamical system could generate optimal x (i.e., $\ddot{\theta}$ in acceleration-level schemes and $\dot{\theta}$ in velocity-level schemes). For velocity-level schemes, the signal $\dot{x}$ is also brought into use from the dynamical QP solver for joint torque control. Note that in real system, the modeling errors always exist and the feedback control should be applied [24], [33], [34].

It is also worth mentioning that an early model of (30) was proposed in [28] in the form of recurrent neural networks to solve linear projection equations only, while this dynamical system is elaborated here to solve general QP problems subject to hybrid constraints. As the primal and dual problems/variables are considered simultaneously, such a dynamical QP solver (30) is termed as an LVI-based primal-dual dynamical system or neural network. As bound constraint (25) is elegantly cast into Ω, primal-dual network (30) is composed of only $n+m$ neurons, of which the number is surprisingly the same as dual networks [18], [19]. The network architecture or computational complexity is thus simpler than other recurrent neural networks', also due to its piecewise-linear dynamics. In addition, there is no online inversion of W in our approach, which is prohibitively of $O(n^3)$ numerical operations existing in others' approaches. Furthermore, the next theorem can be generalized from [18], [19], [28], [35].

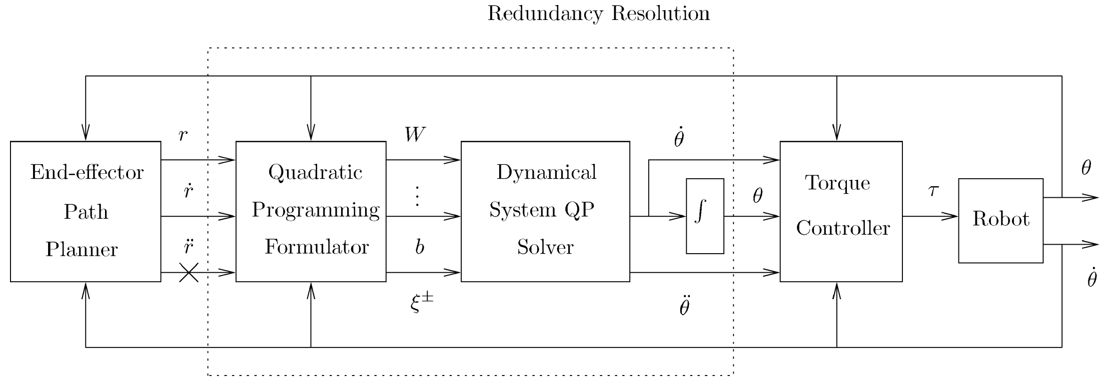

Fig. 1. QP-based dynamical system approach to redundancy resolution and torque control.

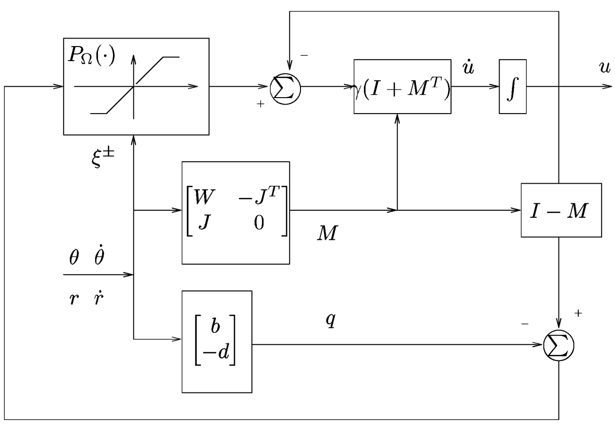
Fig. 2. Block diagram of LVI-based primal-dual QP solver.

Theorem 3. (Solver Convergence): Starting from any initial state, the state vector $u(t)$ of LVI-based primal-dual dynamical system (30) is convergent to an equilibrium point u^*, of which the first n elements constitute the optimal solution x^* to the QP problem in (23)–(25). Moreover, the exponential convergence can be achieved, provided that there exists a constant $\rho > 0$ such that $\|u - P_\Omega(u-(Mu+q))\|_2 \geqslant \rho\|u-u^*\|_2$.

Proof: Omitted due to space limitation. □

IV. Simulation Studies

A number of computer simulations of the aforementioned acceleration-level and velocity-level redundancy resolution schemes have been performed based on the Unimation PUMA560 robot arm [18], [36] through general QP formulation (23)–(25) and its dynamical solver (30). Due to space limitations, only a few illustrative simulation results are presented. With the end-effector positioning considered, $J(\theta) \in R^{3\times 6}$. Parameters η_p, μ_p and μ_v are respectively 0.9, 20, and 20. The desired motion of the end-effector is a straight-line Cartesian path with length $0.8\sqrt{2}$ m and task duration $T = 10$ seconds.

A. Remark 1

It is shown in Fig. 3 through the MTN examples that by taking into consideration joint physical limits, the build-up of very large null-space joint velocities/accelerations could be reduced, and provides a remedy for the torque instability problem, whereas the conventional MTN fails at time $t = 6.2$ s without considering joint physical limits.

B. Remark 2

In terms of torque profiles and motion trajectories as in Figs. 4 and 5, the velocity-level MVN, MKE and acceleration-level MAN schemes are superior to the MTN and IWT schemes for long movements. Specifically, MTN and IWT schemes may exhibit natural motion characteristics because of considering inertia matrix [18], but may also encounter intensive computational burden because of this sometimes ill-conditioned H-weighting matrix.

C. Remark 3

Joint physical variables have been kept within their mechanical limits by using the proposed QP-based redundancy-resolution schemes. As seen from simulation data, the end-effector positioning error is usually of (or less than) order 10^{-4} m.

V. Conclusion

This paper has established a general QP-based problem formulation for various acceleration-level and velocity-level redundancy-resolution schemes for manipulator torque optimization subject to joint physical constraints. To solve online this QP-based redundancy-resolution problem, an LVI-based primal-dual dynamical QP solver has been presented with simple piecewise-linear dynamics. Simulation results have demonstrated the effectiveness and efficiency of the proposed QP-based dynamical system approach to manipulator redundancy resolution. Future research directions may lie in the implementation of these algorithms on hardware and/or the development of a fast discrete-time QP solver for manipulator redundancy resolution.

Appendix

Proof of Theorem 2

It follows from [30] that the Lagrangian dual problem of (23)–(25) can be derived as

$$\max \quad -\frac{1}{2}x^TWx + d^Ty + \xi^{-T}\nu^- - \xi^{+T}\nu^+ \qquad (31)$$

$$\text{s.t.} \quad Wx + b - J^Ty - \nu^- + \nu^+ = 0 \qquad (32)$$

$$\text{with } y \text{ unrestricted}, \ \nu^- \geqslant 0, \ \nu^+ \geqslant 0. \qquad (33)$$

where y, ν^-, and ν^+ are dual-decision variables. Then, a necessary and sufficient condition for the optimum $(x^*, y^*, \nu^{-*}, \nu^{+*})$ of the primal (23)–(25) and its dual (31)–(33), is [30]

Primal feasibility:

$$Jx^* - d = 0, \qquad (34)$$
$$\xi^- \leqslant x^* \leqslant \xi^+;$$

Dual feasibility:

$$Wx + b - J^Ty - \nu^- + \nu^+ = 0, \qquad (35)$$
$$y \text{ unrestricted}, \ \nu^- \geqslant 0, \ \nu^+ \geqslant 0;$$

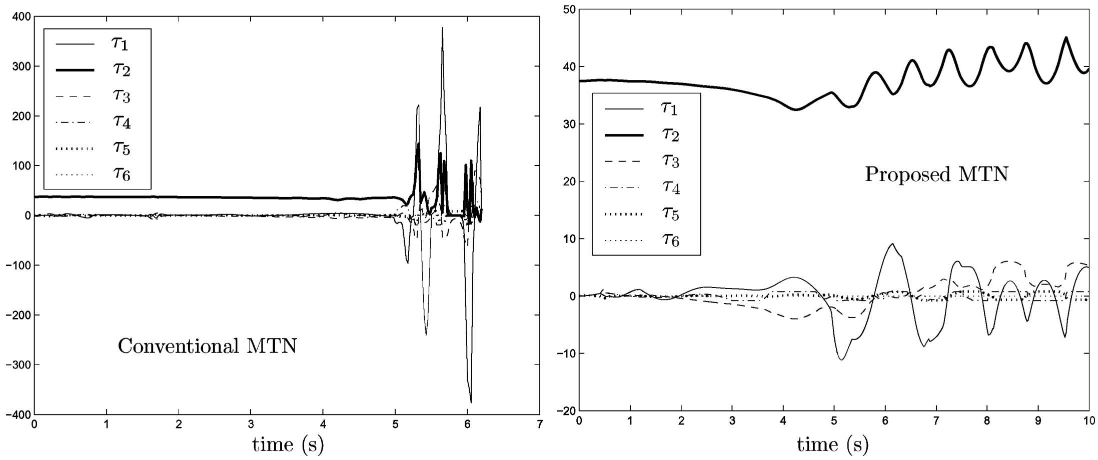

Fig. 3. Comparison of conventional and proposed MTN schemes.

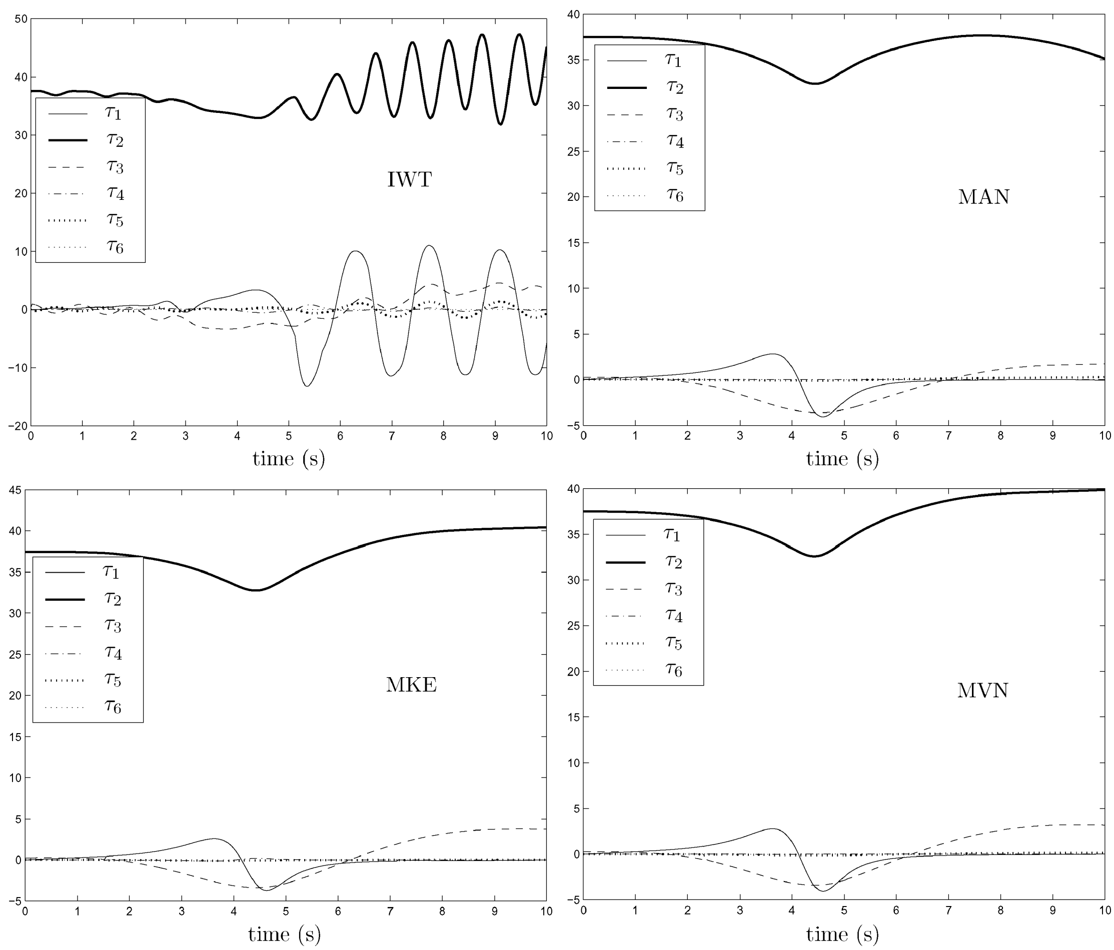

Fig. 4. Torques synthesized by QP-based resolution schemes.

Complementarity:

$$nu^{-*T}(-x^* + \xi^-) = 0, \quad (36)$$

$$nu^{+*T}(-\xi^+ + x^*) = 0. \quad (37)$$

To simplify the above necessary and sufficient condition, we further study dual variable vectors ν^{-*} and ν^{+*} in (35)–(37), which correspond to bound constraint (25). It follows from (36) and (37) that [18], [19], [31], [32]

$$\begin{cases} x_i^* = \xi_i^+ & \text{iff } \nu_i^{+*} > 0, \nu_i^{-*} = 0 \\ \xi_i^- < x_i^* < \xi_i^+ & \text{iff } \nu_i^{+*} = 0, \nu_i^{-*} = 0 \\ x_i^* = \xi_i^- & \text{iff } \nu_i^{+*} = 0, \nu_i^{-*} > 0. \end{cases}$$

By defining $\nu^* = \nu^{-*} - \nu^{+*}$, dual-feasibility constraint (35) becomes

$$Wx^* + b - J^T y^* = \nu^* \begin{cases} \leqslant 0, & x_i^* = \xi_i^+ \\ = 0, & x_i^* \in (\xi_i^-, \xi_i^+) \\ \geqslant 0, & x_i^* = \xi_i^- \end{cases}$$

which equals the following linear variational inequality [26]–[29]: to find an $x^* \in \Omega_1$ such that

$$(x - x^*)^T (Wx^* + b - J^T y^*) \geqslant 0, \ \forall x \in \Omega_1 \quad (38)$$

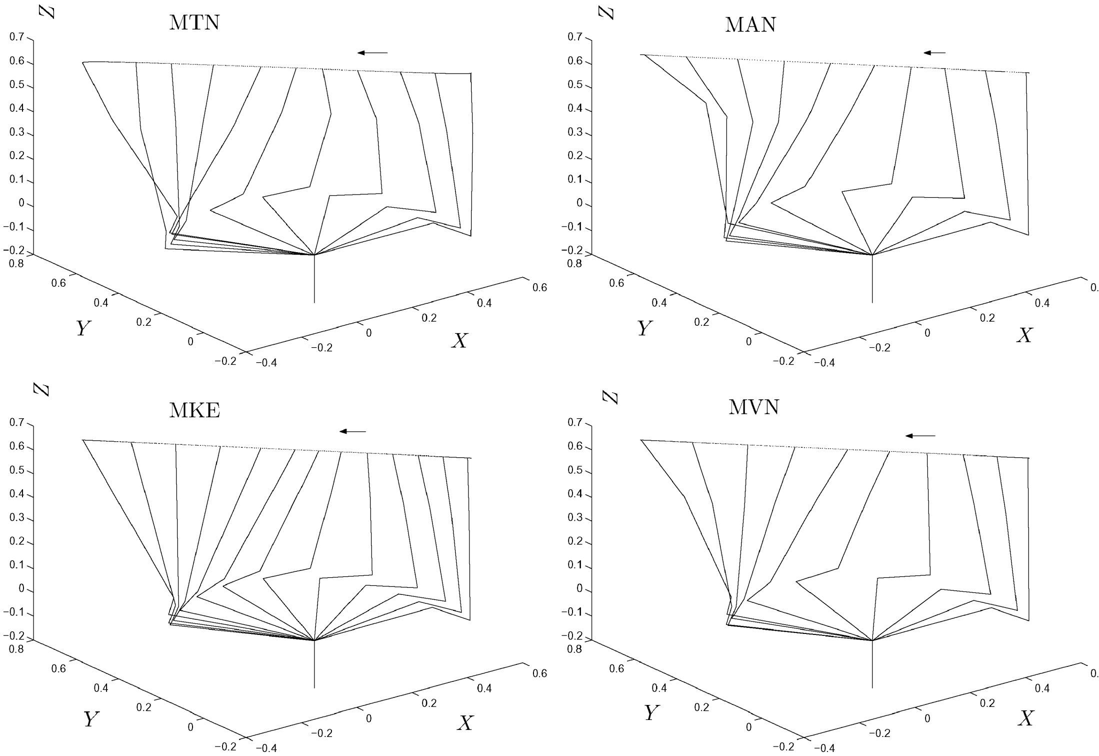

Fig. 5. Motion trajectories synthesized by QP-based resolution schemes.

where $\Omega_1 := \{x|\xi^- \leqslant x^* \leqslant \xi^+\}$. Similarly, defining $\Omega_2 := \{y|y \in R^m\}$, we have the following LVI for (34): to find a $y^* \in \Omega_2$ such that

$$(y - y^*)^T (Jx^* - d) \geqslant 0, \ \forall y \in \Omega_2. \tag{39}$$

Define $\Omega = \Omega_1 \times \Omega_2 = \{u = (x^T, y^T)^T \in R^{n+m} | \xi^- \leqslant x^* \leqslant \xi^+, \ y \, \mathrm{unrestricted}\}$ [26]–[29]. Linear variational inequalities (38) and (39) can be combined into one LVI problem; i.e., to find $u^* \in \Omega$ such that $\forall [x^T, y^T]^T \in \Omega$

$$\left(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x^* \\ y^* \end{bmatrix} \right)^T \left(\begin{bmatrix} W & -J^T \\ J & 0 \end{bmatrix} \begin{bmatrix} x^* \\ y^* \end{bmatrix} + \begin{bmatrix} b \\ -d \end{bmatrix} \right) \geqslant 0.$$

After defining $u^{\pm}$, M and q, respectively, in (26) and (27) for notation simplicity, the above LVI is exactly in the compact matrix form (28) as the equivalence of QP (23)–(25). □

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