Adaptive Neural Control of Nonlinear Time-Delay Systems With Unknown Virtual Control Coefficients

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***Abstract*—In this paper, adaptive neural control is presented for a class of strict-feedback nonlinear systems with unknown time delays. The proposed design method does not require a priori knowledge of the signs of the unknown virtual control coefficients. The unknown time delays are compensated for using appropriate Lyapunov–Krasovskii functionals in the design. It is proved that the proposed backstepping design method is able to guarantee semi-global uniformly ultimately boundedness of all the signals in the closed-loop. In addition, the output of the system is proven to converge to a small neighborhood of the origin. Simulation results are provided to show the effectiveness of the proposed approach.**

***Index Terms*—Adaptive control, neural networks, nonlinear time-delay systems.**

I. Introduction

RECENT years have witnessed great progress in adaptive control of nonlinear systems due to great demands from industrial applications. In order to obtain global stability, some restrictions have to be made to nonlinearities such as matching conditions, extended matching conditions, or growth conditions [1]. To overcome these restrictions, a recursive design procedure called adaptive backstepping design was developed in [2]. Robust adaptive backstepping control has been studied for certain class of nonlinear systems [3]–[5] (to name just a few). While the earlier works such as [3], [4], [6] assumed the virtual control coefficients to be 1, adaptive control has been extended to parametric strict-feedback systems with unknown constant virtual control coefficients but with known signs (either positive or negative) [7] based on the cancellation backstepping design as stated in [8] by seeking for a cancellation of the coupling terms related to $z_i z_{i+1}$ in the next step of Lyapunov design. With the aid of neural network parametrization, adaptive control schemes have been further extended to certain classes of strict-feedback forms in which virtual control coefficients are unknown functions of states with known signs [9], [10]. For system $\dot{x} = f(x) + g(x)u$, the unknown virtual control function $g(x)$ causes great design difficulty in adaptive control. Based on feedback linearization, certainty equivalent control $u = [-\hat{f}(x) + v]/\hat{g}(x)$ is usually taken, where $\hat{f}(x)$ and $\hat{g}(x)$ are estimates of $f(x)$ and $g(x)$, and measures have to be taken to avoid controller singularity when $\hat{g}(x) = 0$. To avoid this problem, integral Lyapunov functions have been developed in [9], and semi-globally stable adaptive controllers are developed, which do not require the estimate of the unknown function $g(x)$. Although the system's virtual control coefficients are assumed to be unknown nonlinear functions of states, their signs are assumed to be known as strictly either positive or negative. Under this assumption, stable neural network controllers have been constructed in [10] by augmenting a robustifying portion, and in [11], [12] by estimating the derivation of the control Lyapunov function.

When there is no a priori knowledge about the signs of virtual control coefficients, adaptive control of such systems becomes much more difficult. The first solution was given in [13] for a class of first-order linear systems, where the Nussbaum-type gain was originally proposed. When the high-frequency control gains and their signs are unknown, gains of Nussbaum type [13] have been effectively used in controller design in solving the difficulty of unknown control directions [14], [15] in which the arguments of the constructed Nussbaum functions are required to be monotone increasing. This method was then generalized to higher-order linear systems in [16]. For nonlinear systems, some results have also been reported in the literature. Without the requirement for monotone increasing arguments for the Nussbaum functions, the same technique has been extended to higher order systems for constant virtual control coefficients [17], [18] using decoupled backstepping formally stated in [8] without seeking for the cancellation of the coupling terms related to $z_i z_{i+1}$ but to decouple z_i and z_{i+1} using Young's inequality and seek for the boundedness of z_{i+1} next. Under the assumption that the virtual control coefficients are time-varying, with unknown signs and bounded in finite intervals, it has also been used to construct robust adaptive control for a class of nonlinear systems with bounded disturbances by introducing exponentially decaying terms to handle the bounded disturbances [19]. The behavior of this class of control laws can be interpreted as the controller tries to sweep all possible control gains and stops when a stabilising gain is found.

Another challenging problem in control of nonlinear systems lies in robust control of nonlinear systems with time delays [20], [21]. The existence of time delays may degrade the control performance and make the stabilization problem become more difficult. By using appropriate *Lyapunov-Krasovskii functionals* [22], uncertainties from unknown time delays can be compensated for. A stabilizing controller design based on the above-mentioned functional was proposed in [23] for a class of nonlinear time-delay systems with a so-called "triangular structure". However, the uncertainties from unknown parameters or unknown nonlinear functions were not discussed. In [24], we studied a class of nonlinear time-delay systems, in

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which the virtual control coefficients are unknown constants with known sign and the system uncertainties are linearly parametrized with unknown constant parameters and known nonlinear functions. Practical stability was introduced to solve the singularity problem due to the appearance of $1/z_i$ or $1/z_i^2$ in the controller and the tracking error can be made to confine in a compact domain of attraction. When the virtual control coefficients are unknown nonlinear functions of states, the problem becomes even more complicated. Although the system's virtual control coefficients are assumed to be unknown nonlinear functions of states, their signs are assumed to be known as strictly either positive or negative. Under the same assumption, stable neural network controllers have also been constructed in [25] by compensating for the unknown time-delay terms completely under the assumption that signals $\dot{\bar{x}}_{n-1}$ are available for feedback and more strict assumption on the time delay terms.

Motivated by previous works on both unknown time-delay systems and unknown virtual control coefficient systems, two adaptive neural controllers without the requirements for $\dot{\bar{x}}_{n-1}$ are presented for a class of strict-feedback nonlinear systems with unknown time delays, and unknown nonlinear functions with unknown signs. For clarity, the first controller is developed based on distinct definitions of two separate compact sets $\Omega_{c_{z_i}} \subset \Omega_{Z_i}$ and $\Omega_{Z_i}^0 = \Omega_{Z_i} - \Omega_{c_{z_i}} \subset \Omega_{Z_i}$ where "$-$" denotes the complement operation. However, the controller has a "technical problem"—the intermediate controls are not differentiable at isolated points $|z_i| = c_{z_i}$. To solve this problem, one practical way is to simply set the differentiation at these points to be any finite value, say 0, and then every signal in the closed-loop system can be shown to be bounded. By modifying the first controller such that the intermediate controls are differentiable, we have the second controller for the class of systems in the paper—which is mathematically rigorous. To the best of our knowledge, there is little work dealing with such a kind of systems in the literature at present stage, except for some preliminary results presented in [25], [26]. The main contributions of the paper lie in:

i) the use of integral Lyapunov function in avoiding the controller singularity problem commonly encountered in adaptive feedback linearization control;
ii) the combination of Lyapunov-Krasovskii functional and the Young's inequality in eliminating the unknown time delay τ_i in the upper bounding function of the Lyapunov functional derivative, which makes neural network parametrization with known inputs possible;
iii) the use of the Nussbaum-type functions in solving the problem of the completely unknown control direction
iv) the novel introduction of smooth functions in making the intermediate control laws continuous and differentiable to certain desired order in solving the differentiability problems at some isolated points incurred in the first practical control.

The rest of the paper is organized as follows. The problem formulation and preliminaries are given in Section II. An adaptive neural controller design for first-order systems is presented in Section III. The scheme is extended to nth-order systems in Section IV. A simulation example is given in Section V, and followed by Section VI which concludes the work.

II. Problem Formulation and Preliminaries

Consider a class of single-input-single-output (SISO) nonlinear time-delay systems

$$\begin{aligned} \dot{x}_i(t) &= g_i(\bar{x}_i(t))\, x_{i+1}(t) + f_i(\bar{x}_i(t)) + h_i(\bar{x}_i(t-\tau_i)) \\ &\qquad i = 1, \cdots, n-1 \\ \dot{x}_n(t) &= g_n(x(t))\, u(t) + f_n(x(t)) + h_n(x(t-\tau_n)) \\ y(t) &= x_1(t) \end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T$, $x = [x_1, x_2, \cdots, x_n]^T \in R^n$, $u \in R, y \in R$ are the state variables, system input and output respectively, $g_i(\cdot)$ and $f_i(\cdot)$, $h_i(\cdot)$ are unknown smooth functions, and τ_i are unknown time delays of the states, $i = 1, \cdots, n$. The control objective is to design an adaptive controller for system (1) such that the output $y(t)$ follows a desired reference signal $y_d(t)$, while all signals in the closed-loop system are bounded. Define the desired trajectory $\bar{x}_{d(i+1)} = [y_d, \dot{y}_d, \cdots, y_d^{(i)}]^T$, $i = 1, \cdots, n-1$, which is a vector of y_d up to its ith time derivative $y_d^{(i)}$.

Assumption 1: Functions $g_i(\bar{x}_i)$ and their signs are unknown, and there exist constants g_{i0} and known smooth functions $\bar{g}_i(\bar{x}_i)$ such that $0 < g_{i0} \leq |g_i(\bar{x}_i)| \leq \bar{g}_i(\bar{x}_i)$, $\forall \bar{x}_i \in R^i$.

Assumption 2: Known smooth functions $\bar{g}_i(\bar{x}_i)$ take value in the unknown closed intervals $I_i := [l_i^-, l_i^+] \subset [g_{i0}, +\infty)$.

Assumption 3: The desired trajectory vectors $\bar{x}_{di}$, $i = 2, \cdots, n$ are continuous and available, and $\bar{x}_{di} \in \Omega_{di} \subset R^i$ with Ω_{di} known compact sets.

Remark 1: Assumption 1 implies that smooth functions $g_i(\bar{x}_i)$ are strictly either positive or negative, which is reasonable because $g_i(\bar{x}_i)$ being away from zero is the controllable condition of system (1), which is made in most control schemes [7], [27]. For a given practical system, the upper bounds of $g_i(\bar{x}_i)$ are not difficult to determine by choosing $\bar{g}_i(\bar{x}_i)$ large enough. It should be emphasized that the low bounds g_{i0}, the lower and upper bounds of the closed intervals l_i^- and l_i^+ are only required for analytical purposes, their true values are not necessarily known.

Accordingly, we define positive-definite functions $\beta_i(\bar{x}_i) = \bar{g}_i(\bar{x}_i)/|g_i(\bar{x}_i)|$, $i = 1, \cdots, n$. From Assumption 1, we know that $\beta_i(\bar{x}_i)$ are bounded by known functions as $1 < \beta_i(\bar{x}_i) \leq (\bar{g}_i(\bar{x}_i)/g_{i0})$.

Assumption 4: The unknown smooth functions $h_i(\bar{x}_i(t))$ satisfy the inequality $|h_i(\bar{x}_i(t))| \leq \varrho_i(\bar{x}_i(t))$ where $\varrho_i(\cdot)$ are known positive smooth functions.

This assumption is much more relaxed than $|h_i(\bar{x}_i(t))| \leq \sum_{j=1}^{i} |x_j(t)| \varrho_{ij}(\bar{x}_i(t))$ as has been made in [25].

Assumption 5: The unknown time delays are bounded by a known constant, i.e., $\tau_i \leq \tau_{\max}$, $i = 1, \cdots, n$.

Remark 2: There are many physical processes which are governed by nonlinear differential equations of the form (1). Examples are recycled reactors, recycled storage tanks and cold rolling mills [21]. In general, most of the recycling processes inherit delays in their state equations.

A. Nussbaum Type Gain

Any continuous function $N(s)\colon R \rightarrow R$ is a function of Nussbaum type if it has the following properties:

$$\lim_{s\rightarrow+\infty}\sup\frac{1}{s}\int_0^s N(\zeta)d\zeta = +\infty \tag{2}$$

$$\lim_{s\rightarrow+\infty}\inf\frac{1}{s}\int_0^s N(\zeta)d\zeta = -\infty. \tag{3}$$

For example, the continuous functions $\zeta^2\cos(\zeta)$, $\zeta^2\sin(\zeta)$, and $e^{\zeta^2}\cos((\pi/2)\zeta)$ suffice [28]. For clarity, the even Nussbaum function, $N(\zeta) = e^{\zeta^2}\cos((\pi/2)\zeta)$ is used throughout this paper.

Lemma 1: [17] Let $V(\cdot)$ and $\zeta(\cdot)$ be smooth functions defined on $[0,t_f)$ with $V(t) \ge 0$, $\forall t \in [0,t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds:

$$V(t) \le c_0 + \int_0^t (gN(\zeta)+1)\,\dot{\zeta}d\tau, \quad \forall t\in[0,t_f)$$

where g is a nonzero constant and c_0 represents some suitable constant, then $V(t)$, $\zeta(t)$ and $\int_0^t (gN(\zeta)+1)\dot{\zeta}d\tau$ must be bounded on $[0,t_f)$.

Lemma 2: Let $V(\cdot)$ and $\zeta(\cdot)$ be smooth functions defined on $[0,t_f)$ with $V(t)\ge 0, \forall t\in[0,t_f)$, and $N(\zeta)$ be an even smooth Nussbaum-type function. The following inequality holds:

$$0 \le V(t) \le c_0 + e^{-c_1 t}\int_0^t g(x(\tau))\,N(\zeta)\dot{\zeta}e^{c_1\tau}d\tau + e^{-c_1 t}\int_0^t \dot{\zeta}e^{c_1\tau}d\tau, \quad \forall t\in[0,t_f) \tag{4}$$

where constant $c_1 > 0$, $g(x(t))$ is a time-varying parameter which takes values in the unknown closed intervals $I := [l^-, l^+]$ with $0 \notin I$, and c_0 represents some suitable constant, then $V(t)$, $\zeta(t)$ and $\int_0^t g(x(\tau))N(\zeta)\dot{\zeta}d\tau$ must be bounded on $[0,t_f)$.

Proof: See Appendix I. □

B. Useful Continuous Functions

For the construction of differentiable control laws, two continuous functions are introduced as follows.

F1). Even function $q_i(x)\colon R \rightarrow R$ defined by (5) shown at the bottom of the page, where $c_{qi} = ([2(n-i)+1]!/(2\lambda)^{2(n-i)+1}[(n-i)!]^2)$, $\lambda > 0$ and $1\le i\le n$, is $(n-i)$th differentiable, i.e., $q_i(x)\in C^{n-i}$ and bounded by 1.

F2). Even function $\kappa(\cdot) : R \rightarrow R$

$$\kappa(x) = \frac{x^2\cosh(x)}{1+x^2}, \quad \forall x\in R \tag{6}$$

is continuous, and monotonic, i.e., for any $|x|\ge c$, where c is a positive constant, $\kappa(x)\ge\kappa(c)$.

C. Linearly Parametrized Neural Networks

A function approximator shall be used to approximate the unknown nonlinear functions. There are two basic types of artificial neural networks

1) linearly parametrized neural networks (LPNNs);
2) multilayer neural networks (MNNs).

In control engineering, the radial basis function (RBF) neural network (NN) as a kind of LPNNs is usually used as a tool for modeling nonlinear functions because of its nice approximation properties. The RBF NN can be considered as a two-layer network in which the hidden layer performs a fixed nonlinear transformation with no adjustable parameters, i.e., the input space is mapped into a new space. The output layer then combines the outputs in the latter space linearly. Therefore, it belongs to a class of linearly parameterized networks. In this paper, the following RBF NN [29] is used to approximate the continuous function $h(Z) : R^q \rightarrow R$

$$h_{nn}(Z) = W^T S(Z) \tag{7}$$

where the input vector $Z \in \Omega_Z \subset R^q$, weight vector $W = [w_1, w_2, \cdots, w_l]^T \in R^l$, the NN node number $l > 1$; and $S(Z) = [s_1(Z),\cdots,s_l(Z)]^T$, with $s_i(Z)$ being chosen as the commonly used Gaussian functions, i.e., $s_i(Z) = e^{-(Z-\mu_i)^T(Z-\mu_i)/\eta_i^2}$, $i = 1,\cdots,l$ where $\mu_i = [\mu_{i1},\mu_{i2},\cdots,\mu_{iq}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function. Universal approximation results in [30], [31] indicate that, if l is chosen sufficiently large, $W^T S(Z)$ can approximate any continuous function, $h(Z)$, to any desired accuracy over a compact set $\Omega_Z \subset R^q$ to arbitrary any accuracy in the form of $h(Z) = W^{*T}S(Z)+\epsilon(Z)$, $\forall Z\in\Omega_Z\subset R^q$ where W^* is the ideal constant weight vector, and $\epsilon(Z)$ is the approximation error which is bounded over the compact set, i.e., $|\epsilon(Z)|\le\epsilon^*$, $\forall Z\in\Omega_Z$ where $\epsilon^*>0$ is an unknown constant. The ideal weight vector W^* is an "artificial" quantity required for analytical purposes. W^* is defined as the value of W that minimizes $|\epsilon|$ for all $Z\in\Omega_Z\subset R^q$, i.e., $W^* := \arg\min_{W\in R^l}\{\sup_{Z\in\Omega_Z}|h(Z)-W^TS(Z)|\}$.

The stability results obtained in NN control literature are semi-global in the sense that, as long as the input variables Z of

$$q_i(x) = \begin{cases} 1, & x\in(2\lambda,+\infty)\cup(-\infty,-2\lambda) \\ c_{qi}\int_0^x \left[\lambda^2-(\sigma-\lambda)^2\right]^{n-i}d\sigma, & x\in[0,2\lambda] \\ c_{qi}\int_x^0 \left[\lambda^2-(\sigma+\lambda)^2\right]^{n-i}d\sigma, & x\in[-2\lambda,0] \end{cases} \tag{5}$$

the NNs remain within some pre-fixed compact set, $\Omega_Z \subset R^q$, where the compact set Ω_Z can be made as large as desired, there exists controller(s) with sufficiently large number of NN nodes such that all the signals in the closed-loop remain bounded.

It should be noted that RBF neural networks can be replaced by any linearly parameterized networks without any technical difficulty such as fuzzy systems, polynomial, splines and wavelet networks.

III. Adaptive Control for First-Order System

To illustrate the design methodology clearly, we first consider the tracking problem of a first-order system

$$\dot{x}_1(t) = g_1(x_1(t))\,u(t) + f_1(x_1(t)) + h_1(x_1(t-\tau_1)) \tag{8}$$

where $u(t)$ is the control input. Define the tracking error $z_1 = x_1 - y_d$, we have

$$\dot{z}_1(t) = g_1(x_1(t))\,u(t) + f_1(x_1(t)) + h_1(x_1(t-\tau_1)) - \dot{y}_d(t). \tag{9}$$

Define $\beta_1(x_1) = \bar{g}_1(x_1)/|g_1(x_1)|$, and a smooth scalar function

$$V_{z_1}(t) = \int_0^{z_1} \sigma\beta_1(\sigma + y_d)d\sigma.$$

By changing the variable $\sigma = \theta z_1$, we may rewrite V_{z_1} as $V_{z_1} = z_1^2 \int_0^1 \theta\beta_1(\theta z_1 + y_d)d\theta$. Noting that $1 \le \beta_1(\theta z_1 + y_d) \le \bar{g}_1(\theta z_1 + y_d)/g_{10}$, we have

$$\frac{z_1^2}{2} \le V_{z_1} \le \frac{z_1^2}{g_{10}} \int_0^1 \theta\bar{g}_1(\theta z_1 + y_d)d\theta. \tag{10}$$

Its time derivative is

$$\dot{V}_{z_1}(t) = z_1(t)\beta_1(x_1(t))\,\dot{z}_1(t) + \int_0^{z_1} \sigma\frac{\partial\beta_1(\sigma + y_d)}{\partial y_d}\dot{y}_d d\sigma.$$

Noting (9) and doing the integration by parts, we have

$$\begin{aligned}\dot{V}_{z_1}(t) &= z_1(t)\beta_1(x_1(t)) \\ &\quad\times [g_1(x_1(t))\,u(t) + f_1(x_1(t)) \\ &\qquad + h_1(x_1(t-\tau_1)) - \dot{y}_d(t)] \\ &\quad + \dot{y}_d(t)\left[\sigma\beta_1(\sigma + y_d)|_0^{z_1} - \int_0^{z_1}\beta_1(\sigma + y_d)d\sigma\right] \\ &= z_1(t)\left[\beta_1(x_1(t))\,g_1(x_1(t))\,u(t)\right. \\ &\qquad + \beta_1(x_1(t))\,f_1(x_1(t)) \\ &\qquad + \beta_1(x_1(t))\,h_1(x_1(t-\tau_1)) \\ &\qquad \left. -\dot{y}_d(t)\int_0^1 \beta_1(\theta z_1 + y_d)d\theta\right].\end{aligned}$$

Applying Assumption 4, we have

$$\begin{aligned}\dot{V}_{z_1}(t) &\le z_1(t)\left[\beta_1(x_1(t))\,g_1(x_1(t))\,u(t)\right. \\ &\qquad + \beta_1(x_1(t))\,f_1(x_1(t)) \\ &\qquad \left. -\dot{y}_d(t)\int_0^1 \beta_1(\theta z_1 + y_d)d\theta\right] \\ &\quad + |z_1(t)|\,\beta_1(x_1(t))\,\varrho_1(x_1(t-\tau_1)).\end{aligned} \tag{11}$$

Remark 3: It can be seen from (11) that the design difficulties are mainly from two uncertainties: unknown functions $f_1(\cdot)$, $\beta_1(\cdot)$ (due to unknown function $g_1(\cdot)$) and unknown time delay τ_1. Although $\varrho_1(\cdot)$ is known, state $x_1(t-\tau_1)$ should not appear in the designed controller as it is undetermined due to unknown τ_1. In addition, the unknown time delay τ_1 and the unknown function $\beta_1(x_1(t))$ are entangled together in a nonlinear fashion, which makes the problem even more complex to solve. Therefore, we have to convert these related terms into such a form that the uncertainties from τ_1 and $\beta_1(x_1(t))$ can be dealt with separately.

By using Young's Inequality, (11) becomes

$$\begin{aligned}\dot{V}_{z_1}(t) &\le z_1(t)\left[\beta_1(x_1(t))\,g_1(x_1(t))\,u(t)\right. \\ &\qquad + \beta_1(x_1(t))\,f_1(x_1(t)) \\ &\qquad \left. -\dot{y}_d(t)\int_0^1 \beta_1(\theta z_1 + y_d)d\theta\right] \\ &\quad + \frac{1}{2}z_1^2(t)\beta_1^2(x_1(t)) + \frac{1}{2}\varrho_1^2(x_1(t-\tau_1))\end{aligned} \tag{12}$$

where $\beta_1(x_1(t))$ and $\varrho_1(x_1(t-\tau_1))$ are separated and can be dealt with one by one as detailed later.

To overcome the design difficulties from the unknown time delay τ_1, the following Lyapunov-Krasovskii functional can be considered

$$V_{U_1}(t) = \frac{1}{2}\int_{t-\tau_1}^{t} U_1(x_1(\tau))\,d\tau, \quad U_1(x_1(t)) = \varrho_1^2(x_1(t)). \tag{13}$$

The time derivative of $V_{U_1}(t)$ is

$$\dot{V}_{U_1}(t) = \frac{1}{2}\varrho_1^2(x_1) - \frac{1}{2}\varrho_1^2(x_1(t-\tau_1))$$

which can be used to cancel the time-delay term on the right hand side of (12) and thus eliminate the design difficulty from the unknown time delay τ_1 without introducing any uncertainties to the system. For notation conciseness, we will omit the time variables t and $t-\tau_1$ after time-delay terms have been eliminated. Accordingly, we obtain

$$\dot{V}_{z_1} + \dot{V}_{U_1} \le z_1\beta_1(x_1)g_1(x_1)u + Q_1(Z_1)z_1 \tag{14}$$

where

$$\begin{aligned}Q_1(Z_1) &= \beta_1(x_1)f_1(x_1) - \dot{y}_d\int_0^1 \beta_1(\theta z_1 + y_d)d\theta \\ &\quad + \frac{1}{2}z_1\beta_1^2(x_1) + \frac{1}{2z_1}\varrho_1^2(x_1)\end{aligned} \tag{15}$$

with $Z_1 = [x_1, y_d, \dot{y}_d]^T \in \Omega_{Z_1} \subset R^3$, where Ω_{Z_1} is a compact set.

At present stage, suppose that the Lyapunov function candidate is chosen as $V_1(t) = V_{z_1}(t) + V_{U_1}(t)$. From (14), we know that we can design a stabilizing $u(t)$ which is free from unknown time delay τ_1 under the assumption of known system functions.

Note that if $Q_1(Z_1)$ is utilized to construct the controller, controller singularity may occur since $(1/2z_1)\varrho_1^2(x_1)$ is not well-defined at $z_1 = 0$. Therefore, care must be taken to guarantee the boundedness of the control. It is noted that the controller singularity takes place at the point $z_1 = 0$, where the control objective is supposed to be achieved. From a practical point of view, once the system reaches its origin, no control action should be taken for less power consumption. As $z_1 = 0$ is hard to detect owing to the existence of measurement noise, it is more practical to relax our control objective of convergence to a "ball" rather than the origin [24].

For ease of discussion, let us define sets $\Omega_{c_{z_1}} \subset \Omega_{Z_1}$ and $\Omega_{Z_1}^0$ as

$$\Omega_{c_{z_1}} := \{z_1 \| z_1| < c_{z_1}\} \tag{16}$$
$$\Omega_{Z_1}^0 := \Omega_{Z_1} - \Omega_{c_{z_1}} \tag{17}$$

where c_{z_1} is a constant that can be chosen arbitrarily small and "$-$" in (17) is used to denote the complement of set B in set A as

$$A - B := \{x | x \in A \text{ and } x \notin B\}.$$

Lemma 3: Set $\Omega_{Z_1}^0$ is a compact set.

Proof: See Appendix II. □

Under the assumption of known system functions, we have the practical robust control law to guarantee the closed-loop stability as detailed in Lemma 4.

Lemma 4: For the first-order system (8), if the practical robust control law is chosen as

$$u(t) = \begin{cases} N(\zeta_1)\left[k_1(t)z_1 + Q_1(Z_1)\right], & z_1 \in \Omega_{Z_1}^0 \\ 0, & z_1 \in \Omega_{c_{z_1}} \end{cases} \tag{18}$$
$$\dot{\zeta}_1 = k_1(t)z_1^2 + Q_1(Z_1)z_1 \tag{19}$$

where $k_1(t) \geq k^* > 0$ with k^* being any positive constant, then for bounded initial conditions, all the signals in the closed-loop system are globally uniformly ultimately bounded.

Proof: See Appendix III. It is shown that the compact set $z_1 \in \Omega_{c_{z_1}}$ is actually a domain of attraction. □

Remark 4: For the first-order system, the definition of the compact set $\Omega_{Z_1}^0$ in (17) and the corresponding practical control law $u(t)$ in (18) can guarantee the stability of the closed-loop system. To extend the above design methodology to higher-order systems, modification has to be made since $u(t)$ is not differentiable at $|z_1| = c_{z_1}$. We will discuss this issue at a later stage when the problem is clearly shown.

In the case that $f_1(\cdot)$ and $g_1(\cdot)$ are completely unknown, the proposed controller (18) in Lemma 4 is not feasible due to the unknown function $Q_1(Z_1)$. On the other hand, by employing the robust control in (18), control action is only activated when $z_1 \in \Omega_{Z_1}^0$. Apparently, $Q_1(Z_1)$ is continuous and well-defined over compact set $\Omega_{Z_1}^0$ and can be approximated by neural networks to arbitrary any accuracy as

$$Q_1(Z_1(t)) = W_1^{*T} S_1(Z_1) + \epsilon_1(Z_1) \tag{20}$$

where $|\epsilon_1(Z_1)| \leq \epsilon_{z_1}^*$ is the approximation error, $W_1^* \in R^{l_1}$ are unknown ideal constant weights, and $S_1(Z_1) \in R^{l_1}$ are the basis functions. Let us use its estimate $\hat{W}_1$ instead to form the adaptive control

$$u(t) = \begin{cases} N(\zeta_1)\left[k_1(t)z_1 + \hat{W}_1^T S_1(Z_1)\right], & z_1 \in \Omega_{Z_1}^0 \\ 0, & z_1 \in \Omega_{c_{z_1}} \end{cases} \tag{21}$$
$$\dot{\zeta}_1 = k_1(t)z_1^2 + \hat{W}_1^T S_1(Z_1)z_1 \tag{22}$$
$$\dot{\hat{W}}_1 = \Gamma_1\left[S_1(Z_1)z_1 - \sigma_1\hat{W}_1\right] \tag{23}$$

where matrix $\Gamma_1 = \Gamma_1^T > 0$, and small constant $\sigma_1 > 0$ is to introduce the σ-modification for the closed-loop system.

Theorem 1 summarizes the stability result for the proposed adaptive scheme, and shows that certain compact set is a domain of attraction.

Theorem 1: Consider the closed-loop systems consisting of the first-order plant (8) and controller (21), (22), if gain $k_1(t) = k_{10} + k_{11}(t)$ with $k_{10} > 0$ being a design constant, and $k_{11}(t)$ is chosen as

$$k_{11}(t) = \frac{1}{\varepsilon_1}\left[1 + \int_0^1 \theta\bar{g}_1(\theta z_1 + y_d)d\theta + \frac{1}{z_1^2}\int_{t-\tau_{\max}}^{t} \frac{1}{2}U_1(x_1(\tau))\,d\tau\right] \tag{24}$$

with constant $\varepsilon_1 > 0$, and the NN weights are updated by (23), then for bounded initial conditions $x_1(0)$ and $\hat{W}_1(0)$, all signals in the closed-loop system are semi-globally uniformly ultimately bounded, and the vector Z_1 remains in a compact set $\Omega_{Z_1}^0$ defined by

$$\Omega_{Z_1}^0 = \left\{Z_1 \middle| |z_1| \leq \mu_1, \frac{1}{2}\|\tilde{W}_1\|^2 \leq \frac{\mu_1}{\lambda_{\min}\left(\Gamma_1^{-1}\right)}, \bar{x}_{d2} \in \Omega_{d2}\right\}$$

whose size, $\mu_1 > 0$, can be adjusted by appropriately choosing the design parameters.

Proof: Let us consider the following Lyapunov function candidate

$$V_1(t) = V_{z_1}(t) + V_{U_1}(t) + \frac{1}{2}\tilde{W}_1^T(t)\Gamma_1^{-1}\tilde{W}_1(t) \tag{25}$$

where $\tilde{(\cdot)} = \hat{(\cdot)} - (\cdot)^*$. The time derivative of $V_1(t)$ along (14) is

$$\dot{V}_1 \leq z_1\beta_1(x_1)g_1(x_1)u + Q_1(Z_1)z_1 + \tilde{W}_1^T\Gamma_1^{-1}\dot{\hat{W}}_1. \tag{26}$$

For $z_1 \in \Omega_{Z_1}^0$, substituting (21) and (23) into (26), we have

$$\dot{V}_1 \leq \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + Q_1(Z_1)z_1 + \tilde{W}_1^T S_1(Z_1)z_1 - \sigma_1\tilde{W}_1^T\hat{W}_1. \tag{27}$$

Adding and subtracting $k_1(t)z_1^2 + \hat{W}_1^T S_1(Z_1)z_1$ on the right hand side of (27) and noting (20), we have

$$\begin{aligned}\dot{V}_1 \le& \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 - \dot{\zeta}_1 \\ &+ \hat{W}_1^T S_1(Z_1)z_1 + z_1\epsilon_{z_1} - \sigma_1\tilde{W}_1^T\hat{W}_1 \\ =& -k_1(t)z_1^2 + \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 \\ &+ z_1\epsilon_{z_1} - \sigma_1\tilde{W}_1^T\hat{W}_1 \end{aligned} \tag{28}$$

Noting $k_1(t) = k_{10} + k_{11}(t)$, (28) becomes

$$\begin{aligned}\dot{V}_1 \le& -k_{11}(t)z_1^2 + \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 \\ &-k_{10}z_1^2 + z_1\epsilon_{z_1} - \sigma_1\tilde{W}_1^T\hat{W}_1. \end{aligned} \tag{29}$$

Using the inequalities

$$-k_{10}z_1^2 + z_1\epsilon_{z_1} \le \frac{\epsilon_{z_1}^2}{4k_{10}} \le \frac{\epsilon_{z_1}^{*^2}}{4k_{10}}$$

$$-\sigma_1\tilde{W}_1^T\hat{W}_1 \le -\frac{1}{2}\sigma_1\|\tilde{W}_1\|^2 + \frac{1}{2}\sigma_1\|W_1^*\|^2$$

and substituting (24) into (29), we have

$$\begin{aligned}\dot{V}_1 \le& -\frac{z_1^2}{\varepsilon_1}\left[1 + \int_0^1 \theta\bar{g}_1(\theta z_1 + y_d)d\theta\right] \\ &- \frac{1}{\varepsilon_1}\int_{t-\tau_{\max}}^{t} \frac{1}{2}U_1(x_1(\tau))\,d\tau \\ &- \frac{1}{2}\sigma_1\|\tilde{W}_1\|^2 + \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 + c_{\epsilon 1}\end{aligned}$$

where

$$c_{\epsilon 1} = \frac{\epsilon_{z_1}^{*^2}}{4k_{10}} + \frac{1}{2}\sigma_1\|W_1^*\|^2. \tag{30}$$

Since $\tau_1 \le \tau_{\max}$ according to Assumption 5, inequality $\int_{t-\tau_1}^t U_1(x_1(\tau))d\tau \le \int_{t-\tau_{\max}}^t U_1(x_1(\tau))d\tau$ holds. From (10) and (13), we have

$$\begin{aligned}\dot{V}_1(t) \le& -\frac{g_{10}}{\varepsilon_1}V_{z_1} - \frac{1}{\varepsilon_1}V_{U_1} - \frac{1}{2}\sigma_1\|\tilde{W}_1\|^2 \\ &+ \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 + c_{\epsilon 1} \\ \le& -c_1V_1(t) + c_{\epsilon 1} + \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 \end{aligned} \tag{31}$$

where positive constant c_1 is defined by

$$c_1 := \min\left\{\frac{g_{10}}{\varepsilon_1}, \frac{1}{\varepsilon_1}, \frac{\sigma_1}{\lambda_{\min}(\Gamma_1^{-1})}\right\}. \tag{32}$$

Letting $\rho_1 := c_{\epsilon 1}/c_1$ and multiplying (31) by $e^{c_1 t}$, it becomes

$$\frac{d}{dt}\left(V_1(t)e^{c_1 t}\right) \le c_{\epsilon 1}e^{c_1 t} + \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 e^{c_1 t} + \dot{\zeta}_1 e^{c_1 t}. \tag{33}$$

Integrating (33) over $[0, t]$, we have

$$\begin{aligned}V_1(t) \le& \rho_1 + [V_1(0) - \rho_1]e^{-c_1 t} \\ &+ e^{-c_1 t}\int_0^t (\beta_1(x_1)g_1(x_1)N(\zeta_1) + 1)e^{c_1\tau}\dot{\zeta}_1 d\tau \\ \le& \rho_1 + V_1(0)e^{-c_1 t} \\ &+ e^{-c_1 t}\int_0^t (\beta_1(x_1)g_1(x_1)N(\zeta_1) + 1)e^{c_1\tau}\dot{\zeta}_1 d\tau. \end{aligned} \tag{34}$$

Applying Lemma 2, we can conclude that $V_1(t)$, $\int_0^t (\beta_1 g_1 N(\zeta_1) + 1)\dot{\zeta}_1 d\tau$, and $\zeta_1(t)$, hence $z_1(t)$, $\hat{W}_1$ are SGUUB on $[0, t_f)$. According to Proposition 2 in [15], if the solution of the closed-loop system is bounded, then $t_f = +\infty$. Let $c_{\beta 1}$ be the upper bound of $\int_0^t |(\beta_1(x_1)g_1(x_1)N(\zeta_1) + 1)\dot{\zeta}_1| d\tau$, then we have the following inequalities:

$$\begin{aligned}&e^{-c_1 t}\int_0^t (\beta_1(x_1)g_1(x_1)N(\zeta_1) + 1)e^{c_1\tau}\dot{\zeta}_1 d\tau \\ &\le \int_0^t \left|(\beta_1(x_1)g_1(x_1)N(\zeta_1) + 1)\dot{\zeta}_1\right| e^{-c_1(t-\tau)}d\tau \\ &\le \int_0^t \left|(\beta_1(x_1)g_1(x_1)N(\zeta_1) + 1)\dot{\zeta}_1\right| d\tau \le c_{\beta 1}.\end{aligned}$$

Thus, (34) becomes

$$V_1(t) \le (\rho_1 + c_{\beta 1}) + V_1(0)e^{-c_1 t} \tag{35}$$

where constant

$$V_1(0) = \int_0^{z_1(0)} \sigma\beta_1(\sigma + y_d(0))\,d\sigma + \frac{1}{2}\tilde{W}_1^T(0)\Gamma_1^{-1}\tilde{W}_1(0).$$

It follows from (10), (25) and (35) that

$$\frac{1}{2}z_1^2(t) \le V_{z_1}(t) \le V_1(t) \le (\rho_1 + \beta_1) + V_1(0)$$

$$\frac{1}{2}\|\tilde{W}_1\|^2 \le \frac{V_1(t)}{\lambda_{\min}(\Gamma_1^{-1})}.$$

By letting $\mu_1 = \sqrt{2(\rho_1 + c_{\beta 1}) + 2V_1(0)}$, we know that $|z_1| \le \mu_1$. We can readily conclude that there do exist a compact set $\Omega_{Z_1}^0$ such that $Z_1 \in \Omega_{Z_1}^0$, $\forall t \ge 0$. □

Remark 5: If system uncertainties are in the linear-in-the-parameter form as in [17], adaptive control can be used to solve the problem elegantly and the asymptotic stability can be guaranteed by applying Lemma 1. In this paper, the unknown functions are approximated by RBF NN, which has an intrinsic approximation error, therefore Lemma 1 is no longer applicable. To show the point clearly, the time derivative of $V_1(t)$ is re-written as

$$\dot{V}_1(t) \le -c_1V_1(t) + c_{\epsilon 1} + \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1. \tag{36}$$

Integrating (36) over $[0, t]$, we have

$$V_1(t) \le V_1(0) + c_{\epsilon 1}t + \int_0^t (\beta_1(x_1)g_1(x_1)N(\zeta_1) + 1)\dot{\zeta}_1 d\tau. \tag{37}$$

From (37), we cannot draw any conclusion for the boundedness of $V_1(t)$ or $\zeta_1(t)$ by applying Lemma 1 in [17] due to the extra term $c_{\epsilon 1}t$. From the definition of $c_{\epsilon 1}$ in (30), we know that $c_{\epsilon 1}$ is a function of NN approximation error $\epsilon_{z_1}^*$ and $(1/2)\sigma_1\|W_1^*\|^2$. Even though we can remove the latter by setting σ_1 as zero, the former effect from NN approximation error $\epsilon_{z_1}^*$ cannot be eliminated. The problem is successfully solved by multiplying the exponential term $e^{c_1 t}$ to both sides of (36) as did in the proof of Theorem 1. Consequently, the stability results can be drawn by invoking Lemma 2.

Remark 6: Although the system has been proven to converge into a compact set which is actually unknown due to unknown g_{10}, $\epsilon_{z_1}^*$, W_1^*, c_0, and $V_1(0)$, it is possible to adjust the size by appropriately choosing design parameters σ_1 and Γ_1.

Remark 7: The computation of the second integral of $k_{11}(t)$ in (24) should be conducted in the time interval $[t-\tau_{\max}, t]$. If the integration is conducted alternatively in $[0, t]$, the stability result may seem to hold. However, the integral result will progressively tend to a large value as the time increases, which may saturate the actuator and destroy the closed-loop stability. To avoid this, a rather conservative time interval $[t-\tau_{\max}, t]$ should be chosen for conducting the integration. The same conservative measure will be taken in the later recursive backstepping design.

Remark 8: Though it is known that the stability of time-delay systems depends on the size of the time delay, it is not necessarily true for general nonlinear systems as is illustrated by the following example. Consider the linear time-delay system

$$\dot{x}(t) = -bx(t-\tau)$$

with $b > 0$, $\tau > 0$. It has been proven that the linear time delay system is stable if $\tau < (1/b)$, and the system is unstable if τ is too large. However, for the forced linear time delay system given by

$$\dot{x}(t) = -bx(t-\tau) + u(t)$$

with $b > 0$, $\tau > 0$, subject to the sliding mode control

$$u(t) = -\mathrm{sgn}\,(x(t))\,[b_1\,|x(t-\tau)| + \epsilon]\,, \quad b_1 > b$$

we have the resulting nonlinear time delay closed-loop system

$$\dot{x}(t) + bx(t-\tau) + \mathrm{sgn}\,(x(t))\,[b_1\,|x(t-\tau)| + \epsilon] = 0. \quad (38)$$

For the nonlinear time delay system (38), consider the Lyapunov function candidate $V(t) = (1/2)x^2(t)$, we have

$$\begin{aligned}\dot{V}(t) =& -bx(t)x(t-\tau) - b_1\,|x(t)|\,|x(t-\tau)| - \epsilon\,|x(t)| \\ \leq& -\epsilon\,|x(t)| \leq 0.\end{aligned}$$

Apparently, the nonlinear time delay system (38) is stable for arbitrary τ. This also verifies the rich dynamic behaviors of nonlinear systems.

We have developed a practical adaptive neural control for first-order system (8). Now we are ready to extend the above design methodology to higher-order systems.

IV. Adaptive Backstepping Controller Design

In this section, the adaptive design will be extended to nth-order systems (1) and the stability results of the closed-loop system are presented.

Note that the extension is not straightforward as in the classical cases of backstepping design for nonlinear systems in strict feedback form without time delays. In the proposed recursive backstepping design, the computation of $\alpha_i(t)$ requires the computation of $\alpha_{i-1}(t)$. As a result, the unknown time-delay terms of all the previous subsystems will appear in Step i, which have to be compensated for one by one. Though the idea of Lyapunov-Krasovskii functional $V_{U_i}(t)$ shall be used to handle the unknown time delays terms as in Section III, different from the classical cases, the Lyapunov function candidate $V_i(t)$ is much more involved, in which the following terms $\int_{t-\tau_1}^t U_1(x_1(\tau))d\tau, \ldots, \int_{t-\tau_{i-1}}^t U_{i-1}(\bar{x}_{i-1}(\tau))d\tau$, and $\int_{t-\tau_i}^t U_i(\bar{x}_i(\tau))d\tau$ appeared i times, twice and once respectively rather than a simple summation of the previous ones. The derivations are very troublesome in order to see the choices of the above functionals clearly, and cannot be further simplified because of the nature of the problem.

A. Practical Controller Design

The backstepping design procedure contains n steps. At each step, an intermediate control function $\alpha_i(t)$ shall be developed using an appropriate Lyapunov function $V_i(t)$. The design of both the control laws and the adaptive laws are based on the following change of coordinates: $z_1 = x_1 - y_d$, $z_i = x_i - \alpha_{i-1}$, $i = 2, \cdots, n$. Note that the controller design based on such compact sets $\Omega_{Z_i}^0$ will render α_i not differentiable at points $|z_i| = c_{z_i}$. This appears to be a "technical problem" as the differentiation of α_i is not defined at these isolated points. To solve this problem, one practical way is to simply set the differentiation at these points to be any finite value, say 0, and then every signal in the closed-loop system can be shown to be bounded. Theoretically speaking, by doing so, there is no much loss either as these points are isolated and can be ignored. For ease and clarity of presentation, we assume that all the control functions are differentiable throughout this subsection.

For uniformity of notation, throughout this section, define estimation errors $\tilde{W}_i = \hat{W}_i - W_i^*$, compact sets $\Omega_{c_{z_i}}$ and $\Omega_{Z_i}^0$ as

$$\begin{aligned}\Omega_{c_{z_i}} &:= \{z_i \,|\, |z_i| < c_{z_i}\} \\ \Omega_{Z_i}^0 &:= \Omega_{Z_i} - \Omega_{c_{z_i}}\end{aligned}$$

with constants $c_{z_i} > 0$, and positive constants c_i, $c_{\epsilon i}$, ρ_i as

$$\begin{aligned}c_i &:= \min\left\{\frac{g_{i0}}{\varepsilon_i}, \frac{1}{\varepsilon_i}, \frac{\sigma_i}{\lambda_{\min}\left(\Gamma_i^{-1}\right)}\right\} \\ c_{\epsilon i} &:= \frac{\epsilon_{z_i}^{*^2}}{4k_{i0}} + \frac{1}{2}\sigma_i\,\|W_i^*\|^2 \\ \rho_i &:= \frac{c_{\epsilon i}}{c_i}\end{aligned}$$

where $\hat{W}_i \in R^{l_i}$ are the estimates of ideal NN weights $W_i^* \in R^{l_i}$, g_{i0} are the lower bounds of $|g_i(\bar{x}_i)|$, constants $0 < \varepsilon_i \leq 4$, small constants $\sigma_i > 0$, matrices $\Gamma_i = \Gamma_i^T > 0$, constants $k_{i0} > 0$, $\epsilon_{z_i}^*$ are the upper bounds of the NN approximation errors, i.e., $|\epsilon_i(Z_i)| \leq \epsilon_{z_i}^*$ with Z_i being the corresponding inputs to be defined later, and the following integral Lyapunov functions $V_{z_i}(t)$, the Lyapunov-Krasovskii functionals $V_{U_i}(t)$ with the positive scalar functions $U_i(\cdot)$, and the Lyapunov function candidates $V_i(t)$ as

$$V_{z_1}(t) = \int_0^{z_1} \sigma\beta_1(\sigma + y_d)d\sigma \quad (39)$$

$$V_{z_i}(t) = \int_0^{z_i} \sigma\beta_i(\bar{x}_{i-1}, \sigma + \alpha_{i-1})d\sigma, \quad i = 2, \cdots, n \quad (40)$$

$$V_{U_i}(t) = \frac{1}{2}\int_{t-\tau_i}^{t} U_i(\bar{x}_i(\tau))\,d\tau + \sum_{j=1}^{i-1}\int_{t-\tau_j}^{t} U_j(\bar{x}_j(\tau))\,d\tau, \quad i = 1,\cdots,n \quad (41)$$

$$V_i(t) = V_{z_i}(t) + V_{U_i}(t) + \frac{1}{2}\tilde{W}_i^T(t)\Gamma_i^{-1}\tilde{W}_i(t) \quad i = 1,\cdots,n \quad (42)$$

where positive functions $U_i(\bar{x}_i(t)) = \varrho_i^2(\bar{x}_i(t))$.

The adaptive neural control laws are as follows, for $i = 1,\cdots,n$

$$\alpha_i = \begin{cases} N(\zeta_i)\left[k_i(t)z_i + \hat{W}_i^T S_i(Z_i)\right], & z_i \in \Omega_{Z_i}^0 \\ 0, & z_i \in \Omega_{c_{z_i}} \end{cases} \quad (43)$$

$$\dot{\zeta}_i = k_i(t)z_i^2 + \hat{W}_i^T S_i(Z_i)z_i \quad (44)$$

$$\dot{\hat{W}}_i = \Gamma_i\left[S_i(Z_i)z_i - \sigma_i\hat{W}_i\right] \quad (45)$$

where $k_i(t) = k_{i0} + k_{i1}(t)$, $k_{i1}(t)$ is chosen as

$$k_{i1}(t) = \frac{1}{\varepsilon_i}\left[1 + \int_0^1 \theta\bar{g}_i(\bar{x}_{i-1}, \theta z_i + \alpha_{i-1})d\theta + \frac{1}{z_i^2}\int_{t-\tau_{\max}}^{t}\left(\frac{1}{2}U_i(\bar{x}_i(\tau)) + \sum_{j=1}^{i-1}U_j(\bar{x}_j(\tau))\right)d\tau\right] \quad (46)$$

and $S_i(Z_i) \in R^{l_i}$ are the basis functions with Z_i being the input vectors defined in (61) and (68) later.

Note that when $i = n$, α_n is actually the control input $u(t)$.

Step 1: Let us firstly consider the equation in (1) when $i = 1$, i.e.,

$$\dot{x}_1(t) = g_1(x_1(t))\,x_2(t) + f_1(x_1(t)) + h_1(x_1(t-\tau_1)). \quad (47)$$

From the definition for new states z_1 and z_2, i.e., $z_1 = x_1 - y_d$ and $z_2 = x_2 - \alpha_1$, we have

$$\dot{z}_1(t) = g_1(x_1(t))\left[z_2(t) + \alpha_1(t)\right] + f_1(x_1(t)) + h_1(x_1(t-\tau_1)) - \dot{y}_d(t). \quad (48)$$

Consider $V_{z_1}(t)$ in (39). Its time derivative along (48) is

$$\dot{V}_{z_1}(t) = z_1(t)\left[\beta_1(x_1(t))\,g_1(x_1(t))\,z_2(t) + \beta_1(x_1(t))\,g_1(x_1(t))\,\alpha_1(t) + \beta_1(x_1(t))\,f_1(x_1(t)) + \beta_1(x_1(t))\,h_1(x_1(t-\tau_1)) - \dot{y}_d(t)\int_0^1 \beta_1(\theta z_1 + y_d)d\theta\right]. \quad (49)$$

Following the same procedure as in Section III by choosing V_{U_1} in (41) and applying Assumption 4 and Young's inequality, we obtain

$$\dot{V}_{z_1} + \dot{V}_{U_1} \le z_1\beta_1(x_1)g_1(x_1)z_2 + z_1\beta_1(x_1)g_1(x_1)\alpha_1 + Q_1(Z_1)z_1 \quad (50)$$

where $Q_1(Z_1)$ is defined in (15).

As stated in Section III, the control objective now is to show that z_1 converges to certain domain of attraction rather than the origin. To this end, let us show the derivative of Lyapunov function candidate is nonpositive when $z_1 \in \Omega_{Z_1}^0$. Consider the Lyapunov function candidate $V_1(t)$ given in (42). Its time derivative along (50) is

$$\dot{V}_1(t) = z_1\beta_1(x_1)g_1(x_1)z_2 + z_1\beta_1(x_1)g_1(x_1)\alpha_1 + Q_1(Z_1)z_1 + \tilde{W}_1^T\Gamma_1^{-1}\dot{\tilde{W}}_1.$$

Choose the practical adaptive neural intermediate control law and NN weights updating law as given in (43)–(45) with $k_{11}(t)$ given in (46). Now, using the same procedure as in Section III, it can be shown that

$$\dot{V}_1(t) \le -\frac{z_1^2}{\varepsilon_1}\left[1 + \int_0^1 \theta\bar{g}_1(\theta z_1 + y_d)d\theta\right] - \frac{1}{\varepsilon_1}\int_{t-\tau_{\max}}^{t}\frac{1}{2}U_1(x_1(\tau))d\tau - \frac{1}{2}\sigma_1\|\tilde{W}_1\|^2 + \beta_1(x_1)g_1(x_1)z_1z_2 + \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 + c_{\epsilon 1}. \quad (51)$$

Noting that $\beta_1 g_1 z_1 z_2 \le (1/4)z_1^2 + \beta_1^2 g_1^2 z_2^2$, (51) becomes

$$\dot{V}_1(t) \le -\frac{z_1^2}{\varepsilon_1}\left[1 - \frac{\varepsilon_1}{4} + \int_0^1 \theta\bar{g}_1(\theta z_1 + y_d)d\theta\right] - \frac{1}{\varepsilon_1}\int_{t-\tau_{\max}}^{t}\frac{1}{2}U_1(x_1(\tau))d\tau - \frac{1}{2}\sigma_1\|\tilde{W}_1\|^2 + \beta_1^2(x_1)g_1^2(x_1)z_2^2 + \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 + c_{\epsilon 1}. \quad (52)$$

Remark 9: In the cancellation based backstepping design, the coupling term $\beta_1 g_1 z_1 z_2$ is left as it is and it will be cancelled in the next step by augmenting the Lyapunv candidate. In decoupled backstepping design, we will not seeking the cancellation of the coupling term $\beta_1 g_1 z_1 z_2$, but seeking the boundedness of z_2 in the next step. According to Lemma 2, if we could prove that z_2 is bounded, then the stability of z_1 is apparent and easy. This fundamental change makes control system design for this problem solvable [8].

Since $0 < \varepsilon_1 \le 4$, we have

$$\begin{aligned}\dot{V}_1(t) \le & -\frac{g_{10}}{\varepsilon_1}V_{z_1} - \frac{1}{\varepsilon_1}V_{U_1} - \frac{1}{2}\sigma_1\|\tilde{W}_1\|^2 + \beta_1^2(x_1)g_1^2(x_1)z_2^2 + \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 + c_{\epsilon 1} \\ \le & -c_1V_1(t) + c_{\epsilon 1} + \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 + \beta_1^2(x_1)g_1^2(x_1)z_2^2. \end{aligned} \quad (53)$$

Multiplying (53) by $e^{c_1 t}$, it becomes

$$\frac{d}{dt}\left(V_1(t)e^{c_1 t}\right) \le c_{\epsilon 1}e^{c_1 t} + \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 e^{c_1 t} + \dot{\zeta}_1 e^{C_{11} t} + \beta_1^2(x_1)g_1^2(x_1)z_2^2 e^{c_1 t}. \quad (54)$$

Integrating (54) over $[0, t]$, we have

$$\begin{aligned} V_1(t) \le\ & \rho_1 + [V_1(0) - \rho_1]\, e^{-c_1 t} \\ & + e^{-c_1 t}\int_0^t \left(\left(\beta_1(x_1)g_1(x_1)N(\zeta_1) + 1\right) e^{c_1\tau}\right)\dot{\zeta}_1 d\tau \\ & + e^{-c_1 t}\int_0^t \beta_1^2(x_1)g_1^2(x_1)z_2^2 e^{c_1\tau} d\tau \\ \le\ & \rho_1 + V_1(0)e^{-c_1 t} \\ & + e^{-c_1 t}\int_0^t \left(\left(\beta_1(x_1)g_1(x_1)N(\zeta_1) + 1\right) e^{c_1\tau}\right)\dot{\zeta}_1 d\tau \\ & + e^{-c_1 t}\int_0^t \beta_1^2(x_1)g_1^2(x_1)z_2^2 e^{c_1\tau} d\tau. \end{aligned} \quad (55)$$

Remark 10: In (55), if there is no extra term $e^{-c_1 t}\int_0^t \beta_1^2 g_1^2 z_2^2 e^{c_1\tau} d\tau$ within the inequality, we can conclude that $V_1(t)$, ζ_1, $\hat{W}_1$, are all bounded on $[0, t_f)$ according to Lemma 2. According to Proposition 2 in [15], $t_f = +\infty$ and we can claim that z_1, $\hat{W}_1$ are SGUUB. Remark 2.3 in [18] also explains the problem. Due to the presence of extra term $e^{-c_1 t}\int_0^t \beta_1^2 g_1^2 z_2^2 e^{c_1\tau} d\tau$ in (55), Lemma 2 cannot be applied directly. It was supposed in [17] that if z_2 can be regulated such that it is square integrable, the regulation of z_1 can be achieved. However, the situation is different in this paper. Owing to the introduction of exponential term in Lemma 2, the requirement for square integrability can be further relaxed to boundedness.

Noting Assumption 2, we have the following inequality [18]:

$$\begin{aligned} e^{-c_1 t}\int_0^t \beta_1^2 g_1^2 z_2^2 e^{c_1\tau} d\tau &= e^{-c_1 t}\int_0^t \bar{g}_1^2 z_2^2 e^{c_1\tau} d\tau \\ &\le e^{-c_1 t} l_1^{+2} \sup_{\tau\in[0,t]}\left[z_2^2(\tau)\right]\int_0^t e^{c_1\tau} d\tau \\ &\le \frac{1}{c_1} l_1^{+2} \sup_{\tau\in[0,t]}\left[z_2^2(\tau)\right]. \end{aligned} \quad (56)$$

Thus if z_2 can be regulated as bounded, then from (56) we can readily conclude the boundedness of the extra term $e^{-c_1 t}\int_0^t \beta_1^2 g_1^2 z_2^2 e^{c_1\tau} d\tau$.

The effect of $e^{-c_1 t}\int_0^t \beta_1^2 g_1^2 z_2^2 e^{c_1\tau} d\tau$ will be dealt with in the following steps.

Step i $(2 \le i \le n-1)$: Similar procedures are taken recursively for each step of $i = 2, \cdots, n-1$.

The time derivative of $z_i(t)$ is given by

$$\dot{z}_i(t) = g_i(\bar{x}_i(t))\left[z_{i+1}(t) + \alpha_i(t)\right] + f_i\left(\bar{x}_i(t)\right) + h_i\left(\bar{x}_i(t-\tau_i)\right) - \dot{\alpha}_{i-1}(t). \quad (57)$$

Consider $V_{z_i}(t)$ given in (40). Its time derivative is

$$\begin{aligned} \dot{V}_{z_i}(t) &= \frac{\partial V_{z_i}}{\partial z_i}\dot{z}_i + \frac{\partial V_{z_i}}{\partial \bar{x}_{i-1}}\dot{\bar{x}}_{i-1} + \frac{\partial V_{z_i}}{\partial \alpha_{i-1}}\dot{\alpha}_{i-1} \\ &= z_i\beta_i(\bar{x}_i)\dot{z}_i \\ &\quad + \int_0^{z_i} \sigma\left[\dot{\bar{x}}_{i-1}^T \frac{\partial \beta_i(\bar{x}_{i-1}, \sigma + \alpha_{i-1})}{\partial \bar{x}_{i-1}} + \dot{\alpha}_{i-1}\frac{\partial \beta_i(\bar{x}_{i-1}, \sigma + \alpha_{i-1})}{\partial \alpha_{i-1}}\right] d\sigma. \end{aligned} \quad (58)$$

Noting (57) and

$$\begin{aligned} &\int_0^{z_i} \sigma\dot{\bar{x}}_{i-1}^T \frac{\partial \beta_i(\bar{x}_{i-1}, \sigma + \alpha_{i-1})}{\partial \bar{x}_{i-1}} d\sigma \\ &\quad = z_i^2\dot{\bar{x}}_{i-1}^T \int_0^1 \theta\frac{\partial \beta_i(\bar{x}_{i-1}, \theta z_i + \alpha_{i-1})}{\partial \bar{x}_{i-1}} d\theta \\ &\quad \times \int_0^{z_i} \sigma\dot{\alpha}_{i-1}\frac{\partial \beta_i(\bar{x}_{i-1}, \sigma + \alpha_{i-1})}{\partial \alpha_{i-1}} d\sigma \\ &\quad = \dot{\alpha}_{i-1}\left[z_i\beta_i(\bar{x}_i) - z_i\int_0^1 \beta_i(\bar{x}_{i-1}, \theta z_i + \alpha_{i-1}) d\theta\right] \end{aligned}$$

(58) becomes

$$\begin{aligned} \dot{V}_{z_i}(t) = z_i(t)\Big[&\beta_i\left(\bar{x}_i(t)\right) g_i\left(\bar{x}_i(t)\right)\left(z_{i+1}(t) + \alpha_i(t)\right) \\ &+ \beta_i\left(\bar{x}_i(t)\right) f_i\left(\bar{x}_i(t)\right) \\ &+ \beta_i\left(\bar{x}_i(t)\right) h_i\left(\bar{x}_i(t-\tau_i)\right) \\ &+ z_i(t)\dot{\bar{x}}_{i-1}^T \int_0^1 \theta\frac{\partial \beta_i(\bar{x}_{i-1}, \theta z_i + \alpha_{i-1})}{\partial \bar{x}_{i-1}} d\theta \\ &- \dot{\alpha}_{i-1}\int_0^1 \beta_i(\bar{x}_{i-1}, \theta z_i + \alpha_{i-1}) d\theta\Big] \end{aligned}$$

where

$$\begin{aligned} \dot{\bar{x}}_{i-1} &= [\dot{x}_1, \dot{x}_2, \cdots, \dot{x}_{i-1}]^T \\ &= \big[g_1(x_1)x_2 + f_1(x_1) + h_1\left(x_1(t-\tau_1)\right), \\ &\qquad g_2(\bar{x}_2)x_3 + f_2(\bar{x}_2) + h_2\left(\bar{x}_2(t-\tau_2)\right), \cdots, \\ &\qquad g_{i-1}(\bar{x}_{i-1})x_i + f_{i-1}(\bar{x}_{i-1}) \\ &\qquad + h_{i-1}\left(\bar{x}_{i-1}(t-\tau_{i-1})\right)\big]^T. \end{aligned}$$

Since α_{i-1} is a function of $\bar{x}_{i-1}, \zeta_{i-1}, \bar{x}_{di}, \hat{W}_1, \ldots, \hat{W}_{i-1}, \dot{\alpha}_{i-1}$ can be expressed as

$$\begin{aligned} \dot{\alpha}_{i-1} &= \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\dot{x}_j + \omega_{i-1}(t) \\ &= \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\left[g_j(\bar{x}_j)x_{j+1} + f_j(\bar{x}_j) + h_j\left(\bar{x}_j(t-\tau_j)\right)\right] \\ &\quad + \omega_{i-1}(t) \end{aligned}$$

where

$$\omega_{i-1}(t) = \frac{\partial \alpha_{i-1}}{\partial \zeta_{i-1}}\dot{\zeta}_{i-1} + \frac{\partial \alpha_{i-1}}{\partial \bar{x}_{di}}\dot{\bar{x}}_{di} + \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial \hat{W}_j}\dot{\hat{W}}_j.$$

Note that the computation of $\dot{\alpha}_{i-1}$, which is required by the recursive backstepping design, and the appearance of $\dot{\bar{x}}_{i-1}$ make the unknown time delays of all the previous subsystems appear, which should all be compensated for in this step. In other words, Lyapunov–Krasovskii functionals (41) shall be utilized to compensate for not only the unknown time delay τ_i, but also $\tau_{i-1},\ldots,\tau_1$. This difficulty or complexity was avoided by assuming that $\dot{x}_{i-1}$ is available for feedback control in [25].

Applying Assumption 4 and using Young's Inequality, we have

$$\begin{aligned}
\dot{V}_{z_i}(t) =& z_i(t)\left[\beta_i(\bar{x}_i(t))\,g_i(\bar{x}_i(t))\,(z_{i+1}(t)+\alpha_i(t)) \right.\\
&+\beta_i(\bar{x}_i(t))\,f_i(\bar{x}_i(t))\\
&\left.+z_i(t)\bar{f}_{i-1}^T\int_0^1\theta\frac{\partial\beta_i(\bar{x}_{i-1},\theta z_i+\alpha_{i-1})}{\partial\bar{x}_{i-1}}d\theta\right]\\
&+\frac{1}{2}z_i^4(t)\left[\int_0^1\theta\frac{\partial\beta_i(\bar{x}_{i-1},\theta z_i+\alpha_{i-1})}{\partial\bar{x}_{i-1}}d\theta\right]^T\\
&\times\int_0^1\theta\frac{\partial\beta_i(\bar{x}_{i-1},\theta z_i+\alpha_{i-1})}{\partial\bar{x}_{i-1}}d\theta\\
&+\frac{1}{2}z_i^2(t)\beta_i^2(\bar{x}_i(t))+\frac{1}{2}\varrho_i^2(\bar{x}_i(t-\tau_i))\\
&-z_i\int_0^1\beta_i(\bar{x}_{i-1},\theta z_i+\alpha_{i-1})d\theta\\
&\times\left[\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(g_j(\bar{x}_j)x_{j+1}+f_j(\bar{x}_j))+\omega_{i-1}(t)\right]\\
&+\frac{1}{2}\left[\sum_{j=1}^{i-1}z_i^2\left(\int_0^1\beta_i(\bar{x}_{i-1},\theta z_i+\alpha_{i-1})d\theta\right)^2\right.\\
&\left.\times\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^2+2\varrho_j^2(\bar{x}_j(t-\tau_j))\right]
\end{aligned}\tag{59}$$

where $\bar{f}_{i-1} = [g_1(x_1)x_2+f_1(x_1),\cdots,g_{i-1}(\bar{x}_{i-1})x_i+f_{i-1}(\bar{x}_{i-1})]^T$.

Considering the Lyapunov-Krasovskii functional $V_{U_i}(t)$ as given in (41), we have

$$\begin{aligned}
\dot{V}_{z_i}+\dot{V}_{U_i}\le& z_i\beta_i(\bar{x}_i)g_i(\bar{x}_i)z_{i+1}\\
&+z_i\beta_i(\bar{x}_i)g_i(\bar{x}_i)\alpha_i+z_iQ_i(Z_i)
\end{aligned}\tag{60}$$

where

$$\begin{aligned}
Q_i(Z_i) =& \beta_i(\bar{x}_i)f_i(\bar{x}_i)+\frac{1}{2}z_i\beta_i^2(\bar{x}_i)+\frac{1}{2z_i}\varrho_i^2(\bar{x}_i)\\
&+z_i\bar{f}_{i-1}^T\int_0^1\theta\frac{\partial\beta_i(\bar{x}_{i-1},\theta z_i+\alpha_{i-1})}{\partial\bar{x}_{i-1}}d\theta\\
&+\frac{1}{2}z_i^3(t)\left[\int_0^1\theta\frac{\partial\beta_i(\bar{x}_{i-1},\theta z_i+\alpha_{i-1})}{\partial\bar{x}_{i-1}}d\theta\right]^T\\
&\times\int_0^1\theta\frac{\partial\beta_i(\bar{x}_{i-1},\theta z_i+\alpha_{i-1})}{\partial\bar{x}_{i-1}}d\theta\\
&+\sum_{j=1}^{i-1}\left[-\frac{\partial\alpha_{i-1}}{\partial x_j}(g_j(\bar{x}_j)x_{j+1}+f_j(\bar{x}_j))\right.\\
&\times\int_0^1\beta_i(\bar{x}_{i-1},\theta z_i+\alpha_{i-1})d\theta\\
&+\frac{1}{2}z_i\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^2\\
&\times\left(\int_0^1\beta_i(\bar{x}_{i-1},\theta z_i+\alpha_{i-1})d\theta\right)^2\\
&\left.+\frac{1}{z_i}\varrho_j^2(\bar{x}_j)\right]\\
&-\omega_{i-1}\int_0^1\beta_i(\bar{x}_{i-1},\theta z_i+\alpha_{i-1})d\theta\\
Z_i =& \left[\bar{x}_i,\alpha_{i-1},\frac{\partial\alpha_{i-1}}{\partial x_1},\frac{\partial\alpha_{i-1}}{\partial x_2},\cdots,\frac{\partial\alpha_{i-1}}{\partial x_{i-1}},\omega_{i-1}\right]\\
&\in\Omega_{z_i}\subset R^{2i+1}.
\end{aligned}\tag{61}$$

For the adaptive neural intermediate control law given in (43)–(45) with $k_{i1}(t)$ being given in (46), consider Lyapunov function candidate $V_i(t)$ given in (42). Its time derivative along (43)–(45) and (60) is

$$\begin{aligned}
\dot{V}_i(t)\le& -c_iV_i(t)+c_{\epsilon i}+\beta_i(\bar{x}_i)g_i(\bar{x}_i)N(\zeta_i)\dot{\zeta}_i+\dot{\zeta}_i\\
&+\beta_i^2(\bar{x}_i)g_i^2(\bar{x}_i)z_{i+1}^2.
\end{aligned}\tag{62}$$

Multiplying (62) by e^{c_it}, it becomes

$$\begin{aligned}
\frac{d}{dt}\left(V_i(t)e^{c_it}\right)\le& c_{\epsilon i}e^{c_it}+\beta_i(\bar{x}_i)g_i(\bar{x}_i)N(\zeta_i)\dot{\zeta}_ie^{c_it}\\
&+\dot{\zeta}_ie^{c_it}+\beta_i^2(\bar{x}_i)g_i^2(\bar{x}_i)z_{i+1}^2e^{c_it}.
\end{aligned}\tag{63}$$

Integrating (63) over $[0,t]$, we have

$$\begin{aligned}
V_i(t)\le& \rho_i+[V_i(0)-\rho_i]\,e^{-c_it}\\
&+e^{-c_it}\int_0^t(\beta_i(\bar{x}_i)g_i(\bar{x}_i)N(\zeta_i)+1)\,e^{c_i\tau}\dot{\zeta}_id\tau\\
&+e^{-c_it}\int_0^t\beta_i^2(\bar{x}_i)g_i^2(\bar{x}_i)z_{i+1}^2e^{c_i\tau}d\tau\\
\le& \rho_i+V_i(0)\\
&+e^{-c_it}\int_0^t(\beta_i(\bar{x}_i)g_i(\bar{x}_i)N(\zeta_i)+1)\,e^{c_i\tau}\dot{\zeta}_id\tau\\
&+e^{-c_it}\int_0^t\beta_i^2(\bar{x}_i)g_i^2(\bar{x}_i)z_{i+1}^2e^{c_i\tau}d\tau.
\end{aligned}\tag{64}$$

Remark 11: Similarly as discussed in Remark 10, if z_{i+1} can be regulated as bounded, we can readily guarantee the boundedness of the extra term $e^{-c_it}\int_0^t\beta_i^2g_i^2z_{i+1}^2e^{c_i\tau}d\tau$ in (64). Then applying Lemma 2, the boundedness of $V_i(t)$, $z_i(t)$, $\zeta_i(t)$ and $\hat{W}_i(t)$ can be readily obtained.

The effect of $e^{-c_it}\int_0^t\beta_i^2g_i^2z_{i+1}^2e^{c_i\tau}d\tau$ will be dealt with in the next step.

Step n. This is the final step, since the actual control $u(t)$ appears in the derivative of $z_n(t)$ as given in

$$\dot{z}_n = g_n(\bar{x}_n(t))\,u + f_n(\bar{x}_n(t)) + h_n(\bar{x}_n(t-\tau_n)) - \dot{\alpha}_{n-1}(t). \tag{65}$$

Consider the scalar function $V_{z_n}(t)$ given in (40). Its time derivative is

$$\begin{aligned}\dot{V}_{z_n}(t) = z_n(t)\Big[&\beta_n(x(t))\,g_n(x(t))\,u(t)\\ &+\beta_n(x(t))\,f_n(x(t))\\ &+\beta_n(x(t))\,h_n(x(t-\tau_n))\\ &+z_n(t)\dot{\bar{x}}_{n-1}^T\int_0^1\theta\frac{\partial\beta_n(\bar{x}_{n-1},\theta z_n+\alpha_{n-1})}{\partial\bar{x}_{n-1}}d\theta\\ &-\dot{\alpha}_{n-1}\int_0^1\beta_n(\bar{x}_{n-1},\theta z_n+\alpha_{n-1})d\theta\Big].\end{aligned}$$

Since α_{n-1} is a function of $\bar{x}_{n-1}$, ζ_{n-1}, $\bar{x}_{dn}$, $\hat{W}_1,\ldots,\hat{W}_{n-1}$, $\dot{\alpha}_{n-1}$ can be expressed as

$$\begin{aligned}\dot{\alpha}_{n-1} &= \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\dot{x}_j+\omega_{n-1}(t)\\ &=\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\left[g_j(\bar{x}_j)x_{j+1}+f_j(\bar{x}_j)\right.\\ &\qquad\left.+h_j(\bar{x}_j(t-\tau_j))\right]+\omega_{n-1}(t)\end{aligned}$$

where

$$\omega_{n-1}(t)=\frac{\partial\alpha_{n-1}}{\partial\zeta_{n-1}}\dot{\zeta}_{n-1}+\frac{\partial\alpha_{n-1}}{\partial\bar{x}_{dn}}\dot{\bar{x}}_{dn}+\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{W}_j}\dot{\hat{W}}_j.$$

Applying Assumption 4 and using Young's Inequality, we have

$$\begin{aligned}\dot{V}_{z_n}(t) = z_n(t)\Big[&\beta_n(x(t))\,g_n(x(t))\,u(t)+\beta_n(x(t))\,f_n(x(t))\\ &+z_n(t)\bar{f}_{n-1}^T\int_0^1\theta\frac{\partial\beta_n(\bar{x}_{n-1},\theta z_n+\alpha_{n-1})}{\partial\bar{x}_{n-1}}d\theta\Big]\\ &+\frac{1}{2}z_n^4(t)\left[\int_0^1\theta\frac{\partial\beta_n(\bar{x}_{n-1},\theta z_n+\alpha_{n-1})}{\partial\bar{x}_{n-1}}d\theta\right]^T\\ &\times\int_0^1\theta\frac{\partial\beta_n(\bar{x}_{n-1},\theta z_n+\alpha_{n-1})}{\partial\bar{x}_{n-1}}d\theta\\ &+\frac{1}{2}z_n^2(t)\beta_n^2(x(t))+\frac{1}{2}\varrho_n^2(x(t-\tau_n))\\ &-z_n\int_0^1\beta_n(\bar{x}_{n-1},\theta z_n+\alpha_{n-1})d\theta\\ &\times\left[\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\left(g_j(\bar{x}_j)x_{j+1}+f_j(\bar{x}_j)\right)+\omega_{n-1}(t)\right]\\ &+\frac{1}{2}\left[\sum_{j=1}^{n-1}z_n^2\left(\int_0^1\beta_n(\bar{x}_{n-1},\theta z_n+\alpha_{n-1})d\theta\right)^2\right.\\ &\left.\times\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2+2\varrho_j^2(\bar{x}_j(t-\tau_j))\right]\end{aligned}\tag{66}$$

where $\bar{f}_{n-1}=[g_1(x_1)x_2+f_1(x_1),\cdots,g_{n-1}(\bar{x}_{n-1})x_n+f_{n-1}(\bar{x}_{n-1})]^T$.

Considering the Lyapunov-Krasovskii functional $V_{U_n}(t)$ given in (41), we have

$$\dot{V}_{z_n}+\dot{V}_{U_n}\le z_n\beta_n(x)g_n(x)u+z_nQ_n(Z_n)\tag{67}$$

where

$$\begin{aligned}Q_n(Z_n)=&\beta_n(x)f_n(x)+\frac{1}{2}z_n\beta_n^2(x)+\frac{1}{2z_n}\varrho_n^2(x)\\ &+z_n\bar{f}_{n-1}^T\int_0^1\theta\frac{\partial\beta_n(\bar{x}_{n-1},\theta z_n+\alpha_{n-1})}{\partial\bar{x}_{n-1}}d\theta\\ &+\frac{1}{2}z_n^3(t)\left[\int_0^1\theta\frac{\partial\beta_n(\bar{x}_{n-1},\theta z_n+\alpha_{n-1})}{\partial\bar{x}_{n-1}}d\theta\right]^T\\ &\times\int_0^1\theta\frac{\partial\beta_n(\bar{x}_{n-1},\theta z_n+\alpha_{n-1})}{\partial\bar{x}_{n-1}}d\theta\\ &+\sum_{j=1}^{n-1}\left\{-\frac{\partial\alpha_{n-1}}{\partial x_j}\left(g_j(\bar{x}_j)x_{j+1}+f_j(\bar{x}_j)\right)\right.\\ &\qquad\times\int_0^1\beta_n(\bar{x}_{n-1},\theta z_n+\alpha_{n-1})d\theta\\ &\qquad+\frac{1}{2}z_n\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2\\ &\qquad\times\left[\int_0^1\beta_n(\bar{x}_{n-1},\theta z_n+\alpha_{n-1})d\theta\right]^2\\ &\qquad\left.+\frac{1}{z_n}\varrho_j^2(\bar{x}_j)\right\}\\ &-\omega_{n-1}\int_0^1\beta_n(\bar{x}_{n-1},\theta z_n+\alpha_{n-1})d\theta\end{aligned}$$

$$\begin{aligned}Z_n=&\left[x,\alpha_{n-1},\frac{\partial\alpha_{n-1}}{\partial x_1},\frac{\partial\alpha_{n-1}}{\partial x_2},\cdots,\frac{\partial\alpha_{n-1}}{\partial x_{n-1}},\omega_{n-1}\right]\\ &\in\Omega_{Z_n}\subset R^{2n+1}.\end{aligned}\tag{68}$$

For the adaptive neural control law given in (43)–(45) with $k_{n1}(t)$ being given in (46), consider the Lyapunov function candidate $V_n(t)$. Its time derivative along (43)–(45) and (67) is

$$\dot{V}_n(t)\le -c_nV_n(t)+c_{\epsilon n}+\beta_n(x)g_i(x)N(\zeta_n)\dot{\zeta}_n+\dot{\zeta}_n\tag{69}$$

Multiplying (69) by e^{c_nt}, it becomes

$$\frac{d}{dt}\left(V_n(t)e^{c_nt}\right)\le c_{\epsilon n}e^{c_nt}+\beta_n(x)g_n(x)N(\zeta_n)\dot{\zeta}_ne^{c_nt}+\dot{\zeta}_ne^{c_nt}.\tag{70}$$

Integrating (70) over $[0,t]$, we have

$$\begin{aligned}V_n(t)&\le\rho_n+[V_n(0)-\rho_n]\,e^{-c_nt}\\ &\quad+e^{-c_nt}\int_0^t\left(\beta_n(x)g_n(x)N(\zeta_n)+1\right)e^{c_n\tau}\dot{\zeta}_nd\tau\\ &\le\rho_n+V_n(0)\\ &\quad+e^{-c_nt}\int_0^t\left(\beta_n(x)g_n(x)N(\zeta_n)+1\right)e^{c_n\tau}\dot{\zeta}_nd\tau.\end{aligned}\tag{71}$$

Using Lemma 2, we can conclude that $V_n(t)$ and $\zeta_n(t)$, hence $z_n(t)$, $\hat{W}_n$ are SGUUB on $[0, t_f)$. From the boundedness of $z_n(t)$, the boundedness of the extra term $e^{-c_{n-1}t}\int_0^t \beta_{n-1}^2 g_{n-1}^2 z_n^2 e^{c_{n-1}\tau} d\tau$ at Step $(n-1)$ is readily obtained. Applying Lemma 2 for $(n-1)$ times backward, it can be seen from the above iterative design procedures that $V_i(t)$, $z_i(t)$, $\hat{W}_i(t)$ and hence $x_i(t)$ are SGUUB, $i = 1, \ldots, n-1$.

The following theorem shows the stability and control performance of the closed-loop adaptive system.

Theorem 2: Consider the closed-loop system consisting of the plant (1) under Assumptions 1–4, the adaptive neural control laws (43)–(46). We can guarantee the following properties under bounded initial conditions

i) all signals in the closed-loop system remain semi-globally uniformly ultimately bounded;
ii) the vectors Z_i remain in the compact set $\Omega_{Z_i}^0 \subset R^{2i+1}$, $i = 1, \ldots, n$;

specified as

$$\Omega_{Z_i}^0 := \left\{ Z_i \middle| |z_i| \le \mu_i, \|\tilde{W}_i\|^2 \le \frac{\mu_i^2}{\lambda_{\min}\left(\Gamma_i^{-1}\right)}, \bar{x}_{di} \in \Omega_{di} \right\}$$

whose sizes, $\mu_i > 0$, can be adjusted by appropriately choosing the design parameters.

Proof: Consider the Lyapunov function candidate $V_n(t)$ given in (42) with $V_{z_n}(t)$, $V_{U_n}(t)$ being defined in (40) and (41). From the previous derivation, we have

$$V_n(t) \le \rho_n + V_n(0) + e^{-c_n t}\int_0^t \left(\beta_n(x) g_n(x) N(\zeta_n) + 1\right) e^{c_n \tau}\dot{\zeta}_n d\tau.$$

From the above iterative design procedures from Step 1 to Step n, we can conclude $V_i(t)$, $\zeta_i(t)$, $z_i(t)$, $\hat{W}_i(t)$, $i = 1, \ldots, n$, and hence $x(t)$ are SGUUB.

Letting $c_{\beta n}$ be the upper bound of $e^{-c_n t}\int_0^t |\beta_n g_n N(\zeta_n) + 1| e^{c_n \tau}\dot{\zeta}_n d\tau$ and noting the definition of $V_n(t)$, we have

$$\frac{1}{2}z_n^2 \le V_n(t) \le (\rho_n + c_{\beta n}) + V_n(0)$$
$$\|\tilde{W}_n\|^2 \le \frac{2V_n(t)}{\lambda_{\min}\left(\Gamma_n^{-1}\right)}.$$

In the rest of the steps from $n-1$ to 1, we obtain

$$\begin{aligned} V_i(t) \le& \rho_i + V_i(0) \\ &+ e^{-c_i t}\int_0^t \left(\beta_i(\bar{x}_i) g_i(\bar{x}_i) N(\zeta_i) + 1\right) e^{c_i \tau}\dot{\zeta}_i d\tau \\ &+ e^{-c_i t}\int_0^t \beta_i^2(\bar{x}_i) g_i^2(\bar{x}_i) z_{i+1}^2 e^{c_i \tau} d\tau, \\ &i = 1, \ldots, n-1. \end{aligned}$$

Letting $c_{\beta i}$ be the upper bound of $e^{-c_i t}\int_0^t |\beta_i g_i N(\zeta_i)\dot{\zeta}_i + \dot{\zeta}_i + \beta_i^2 g_i^2 z_i^2| e^{c_i \tau} d\tau$ and noting the definition of $V_i(t)$, we have

$$\frac{1}{2}z_i^2 \le V_i(t) \le (\rho_i + c_{\beta i}) + V_i(0)$$
$$\|\tilde{W}_i\|^2 \le \frac{2V_i(t)}{\lambda_{\min}\left(\Gamma_i^{-1}\right)}$$

where constant

$$V_i(0) = \int_0^{z_i(0)} \sigma \beta_i\left(\bar{x}_{i-1}(0), \sigma + \alpha_{i-1}(0)\right) d\sigma + \frac{1}{2}\tilde{W}_i^T(0)\Gamma_i^{-1}\tilde{W}_i(0)$$

with $\beta_i(\bar{x}_{i-1}(0), \sigma + \alpha_{i-1}(0)) = \beta_1(\sigma + y_d(0))$ for $i = 1$.

By letting $\mu_i = \sqrt{2(\rho_i + c_{\beta i} + V_i(0))}$, we know that $|z_i| \le \mu_i$. We can conclude that there do exist compact sets $\Omega_{Z_i}^0$ such that $Z_i \in \Omega_{Z_i}^0$, $\forall t \ge 0$. □

Remark 12: For the choice of $k_{i1}(t)$ in (46), it is found that if c_{z_i} is chosen to be very small, $k_{i1}(t)$ will take a very large value, which may saturate the control actuator. To solve this problem, we would like to find an alternative for $k_{i1}(t)$ such that it provides smooth control input, and at the same time guarantees the stability result. One such choice is

$$k_{i1}(t) = \frac{1}{\varepsilon_i}\left[1 + \int_0^1 \theta \bar{g}_i(\bar{x}_{i-1}, \theta z_i + \alpha_{i-1}) d\theta + \frac{\cosh(z_i)}{1 + z_i^2}\int_{t-\tau_{\max}}^t \left(\frac{1}{2}U_i\left(\bar{x}_i(\tau)\right) + \sum_{j=1}^{i-1} U_j\left(\bar{x}_j(\tau)\right)\right) d\tau\right].$$

Following the same derivation procedure and using the property of function $\kappa(\cdot)$ in (6), we can readily obtain (62) with c_i being modified/changed to

$$c_i := \min\left\{\frac{g_{i0}}{\varepsilon_i}, \frac{\kappa(c_{z_i})}{\varepsilon_i}, \frac{\sigma_i}{\lambda_{\min}\left(\Gamma_i^{-1}\right)}.\right\}$$

Although the bounded region may be enlarged by introducing the function $\kappa(\cdot)$, there are still design flexibility from ε_i, Γ_i and σ_i, which can help reduce the bounded region. Note that such modifications together with the choice of function $\kappa(\cdot)$ are also not unique and worth further investigation.

Remark 13: Note that the choices of $\beta_i(\bar{x}_i)$ are not unique [9]. As an alternative, we can choose $\beta_i(\bar{x}_i) = 1/|g_i(\bar{x}_i)|$. In this case, the upper bound function of $|g_i(\bar{x}_i)|$, i.e., $\bar{g}_i(\bar{x}_i)$ are not necessarily known. The smooth integral scalar function becomes

$$V_{z_i} = \int_0^{z_i} \frac{\sigma}{|g_i(\bar{x}_{i-1}, \sigma + \alpha_{i-1})|} d\sigma, \quad i = 1, \cdots, n.$$

By Mean Value Theorem, V_{z_i} can be rewritten as

$$V_{z_i} = \frac{\lambda_s z_i^2}{|g_i(\bar{x}_{i-1}, \lambda_s z_i + \alpha_{i-1})|}, \quad \lambda_s \in (0,1).$$

From Assumption 1, $0 \le g_{i0} \le |g_i(\bar{x}_i)|$, we know that $V_{z_i}(t)$ is a positive definite function and $V_{z_i}(t) \le (\lambda_s/g_{10})z_i^2$. For conciseness of presentation, we give the control and adaptive laws directly without proof, as well as the stability results.

Theorem 3: For system (1), we choose the adaptive neural control laws (43)–(45), where $k_i(t) = k_{i0} + k_{i1}(t)$ with constant $k_{i0} > 0$ and $k_{i1}(t)$ is chosen as

$$k_{i1}(t) = \frac{1}{\varepsilon_i}\left[1 + \lambda_s + \frac{1}{z_i^2} \times \int_{t-\tau_{\max}}^t \frac{1}{2}\left(\frac{1}{2}U_i\left(\bar{x}_i(\tau)\right) + \sum_{j=1}^{i-1} U_j\left(\bar{x}_j(\tau)\right)\right) d\tau\right] \tag{72}$$

with $0 < \varepsilon_{i0} \leq 4$, $\lambda_s \in (0,1)$. Then, under the bounded initial conditions, all signals in the closed-loop system remain bounded and the tracking error converges to a small neighborhood around zero by appropriately choosing design parameters.

Similar as the modification made to k_{i1} in Remark 12, we can modify (72) to

$$k_{i1}(t) = \frac{1}{\varepsilon_i}\left[1 + \lambda_s + \frac{\cosh(z_i)}{1+z_i^2} \times \int_{t-\tau_{\max}}^{t} \frac{1}{2}\left(\frac{1}{2}U_i\left(\bar{x}_i(\tau)\right) + \sum_{j=1}^{i-1} U_j\left(\bar{x}_j(\tau)\right)\right) d\tau\right] \quad (73)$$

for a relatively gentle control gain.

B. Differentiable Controller Design

Though the nondifferentiability of the intermediate controls can be solved in a very practical way as discussed in the previous subsection. In fact, this problem can also be solved theoretically by modifying the control laws such that they are differentiable to certain desired order as will be discussed below. It should be pointed out that the solution is not unique. For clarity, only one such a solution is presented.

It can be seen that the computation of $\alpha_i(t)$ requires that of $\dot{\alpha}_{i-1}(t)$. This is also the case for the computation of $\alpha_{i-1}, \cdots$, and α_2, which requires to compute $\dot{\alpha}_{i-2}, \cdots$, and $\dot{\alpha}_1$ respectively. Therefore, we know that the computation of α_i shall include that of $\alpha_1^{(i-1)}, \alpha_2^{(i-2)}, \cdots$, and $\dot{\alpha}_{i-1}$. This rule applies to the rest of the steps till the last step n. We can conclude that α_i need to be at least $(n-i)$th differentiable. By using the property of (n-i)th order differentiable function $q_i(z_i)$ in (5), the intermediate control, α_i in (43) can be easily modified to satisfy the required $(n-i)$th order differentiability as

$$\alpha_i^q = q_i(z_i)N(\zeta_i)\left[k_i(t)z_i + \hat{W}_i^T S_i(Z_i)\right], \quad i = 1, \cdots, n-1 \quad (74)$$

where $q_i(z_i)$ is defined in (5). It can be easily verified by actual differentiation.

The above modification not only guarantees the differentiability of the intermediate controls, but also preserves the closed-loop stability of the practical control design by noticing that $\alpha_i^q = \alpha_i \; \forall z_i \in \Omega_{Z_i}^0$. In fact, the stability analysis remains the same as before for $z_i \in \Omega_{Z_i}^0$.

V. Simulation

To illustrate the proposed adaptive neural control algorithms, we consider the following second-order time-delay system

$$\begin{aligned}
\dot{x}_1(t) &= g_1(x_1)x_2(t) + f_1(x_1) + h_1\left(x_1(t-\tau_1)\right)\\
\dot{x}_2(t) &= g_2(x)u(t) + f_2(x) + h_2\left(x(t-\tau_2)\right)\\
y_1(t) &= x_1(t)
\end{aligned}$$

where $g_1(x_1) = 1 + x_1^2$, $g_2(x) = 3 + \cos(x_1x_2)$, $f_1(x_1) = x_1(t)e^{-0.5x_1(t)}$, $f_2(x) = x_1(t)x_2^2(t)$, $h_1(x_1) = 2x_1^2$, and $h_2(x) = 0.2x_2\sin(x_2)$. Apparently, by choosing $\varrho_1(x_1) = 2x_1^2$ and $\varrho_2(x) = 0.2|x_2|$, Assumption 4 satisfies.

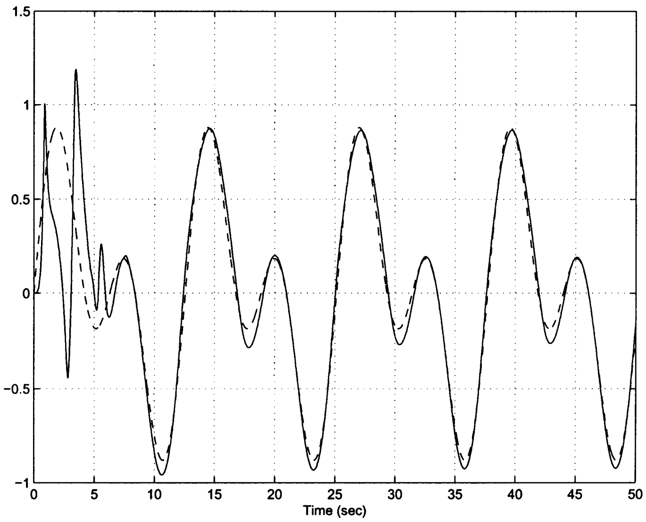

Fig. 1. Output $y(t)$("—") and reference y_d("- -").

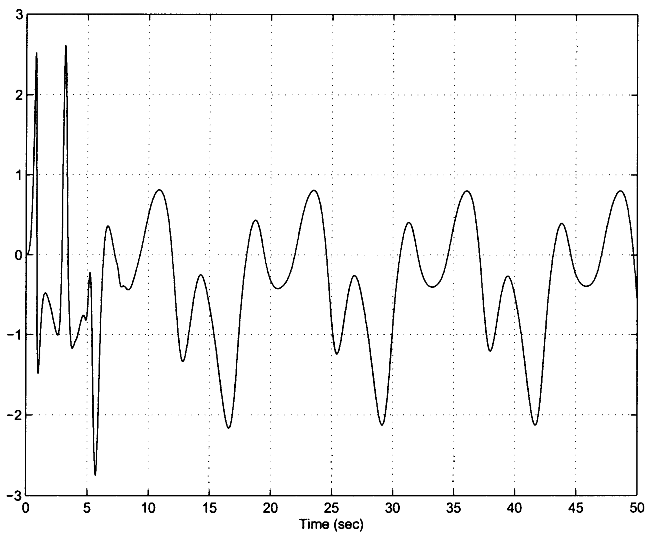

Fig. 2. Trajectory of state $x_2(t)$.

Choose the initial condition $[x_1(0), x_2(0)]^T = [0,0]^T$, the time delay $\tau_1 = \tau_2 = 2$ sec., and the desired reference signal $y_d = 0.5[\sin(t) + \sin(0.5t)]$. For the design of neural adaptive controller, let $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$. For simplicity, simulation is carried out based on Theorem 3 for the case $\beta_i(\bar{x}_i) = 1/|g_i(\bar{x}_i)|$. The intermediate control α_i and control $u(t)$ are given by (74) and (43) respectively with $k_{i1}(t)$ being chosen in (73) as

$$\begin{aligned}
\alpha_1(t) &= s_1(z_1)N(\zeta_1)\left[k_1(t)z_1 + \hat{W}_1^T S_1(Z_1)\right]\\
u(t) &= \begin{cases} N(\zeta_2)\left[k_2(t)z_2 + \hat{W}_2^T S_2(Z_2)\right], & |z_2| \geq c_{z_2}\\ 0, & \text{otherwise}\end{cases}\\
\dot{\zeta}_i &= k_i(t)z_i^2 + \hat{W}_i^T S_i(Z_i)z_i, \quad i = 1,2\\
\dot{\hat{W}}_i &= \Gamma_i\left[S_i(Z_i)z_i - \sigma_i\left(\hat{W}_i - W_i^0\right)\right], \quad i = 1,2
\end{aligned}$$

where $N(\zeta_i) = e^{\zeta_i^2}\cos((\pi/2)\zeta_i)$, $i = 1, 2$ are the Nussbaum functions, $Z_1 = [x_1, y_d, \dot{y}_d]^T$, $Z_2 =$

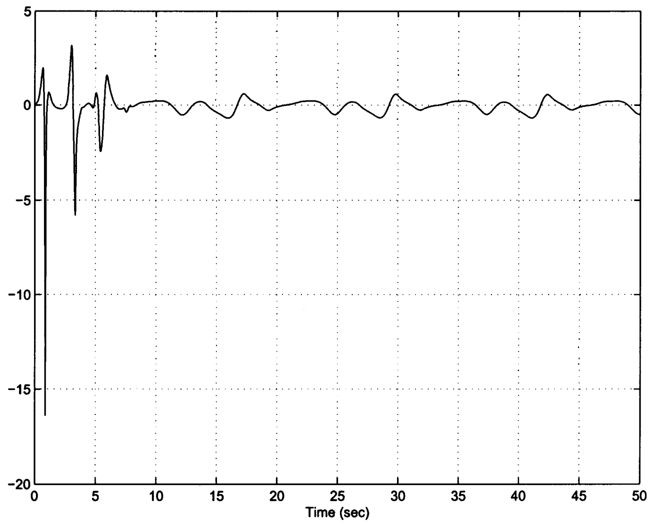

Fig. 3. Control input $u(t)$.

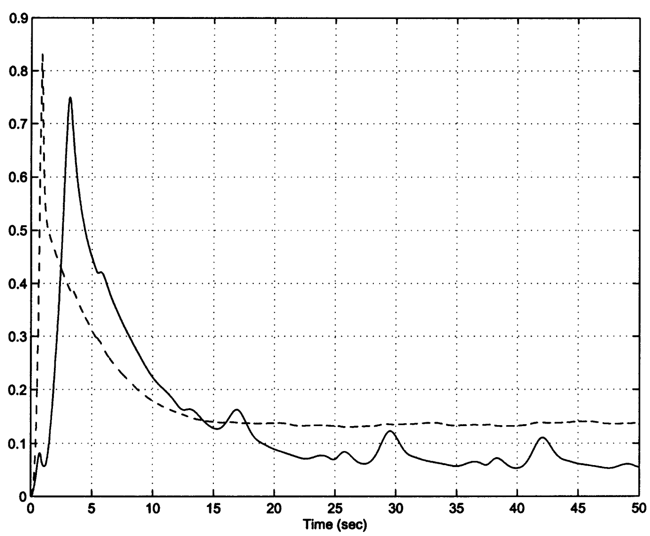

Fig. 4. Norms of NN weights $\|\hat{W}_1\|$("—") and $\|\hat{W}_2\|$("- -").

$[x_1, x_2, \alpha_1, (\partial\alpha_1/\partial x_1), \omega_1]^T$, and $k_i(t) = k_{i0} + k_{i1}(t)$ with constant $k_{i0} > 0$ and $k_{i1}(t)$ being chosen as

$$k_{i1}(t) = \frac{1}{\varepsilon_i}\left[1 + \lambda_s + \frac{\cosh(z_i)}{1+z_i^2} \times \int_{t-\tau_{\max}}^{t} \frac{1}{2}\left(\frac{1}{2}U_i\left(\bar{x}_i(\tau)\right) + \sum_{j=1}^{i-1} U_j\left(\bar{x}_j(\tau)\right)\right) d\tau\right].$$

The following design parameters are adopted in the simulation: $\Gamma_1 = \text{diag}[0.2]$, $\Gamma_2 = \text{diag}[0.4]$, $\sigma_1 = \sigma_2 = 0.5$, $W_1^0 = W_2^0 = 0.01$, $\varepsilon_1 = 4$, $\varepsilon_2 = 4$, $\lambda_s = 0.5$, and $c_{z_1} = c_{z_2} = 1.0e^{-7}$.

In practice, the selection of the centers and widths of RBF has a great influence on the performance of the designed controller. According to [30], Gaussian RBF NNs arranged on a regular lattice on R^n can uniformly approximate sufficiently smooth functions on closed, bounded subsets. Accordingly, in the following simulation studies, the centers and widths are chosen on a regular lattice in the respective compact sets. Specifically, neural networks $\hat{W}_1^T S_1(Z_1)$ contains 27 nodes (i.e., $l_1 = 27$) with centers $\mu_l(l = 1, \cdots, l_1)$ evenly spaced in $[-1, +1] \times [-1, +1] \times [-1, +1]$, and widths $\eta_l^2 = 1(l = 1, \cdots, l_1)$. Neural networks $\hat{W}_2^T S_1(Z_2)$ contains 243 nodes (i.e., $l_2 = 243$) with centers $\mu_l(l = 1, \cdots, l_2)$ evenly spaced in $[-1, +1] \times [-1.5, +1] \times [-1.5, +1] \times [-5, +5] \times [-5, +5]$, and widths $\eta_2^2 = 8(l = 1, \cdots, l_2)$. The initial weight estimates are assumed to me 0, i.e., $\hat{W}_1(0) = 0.0$ and $\hat{W}_2(0) = 0.0$.

Fig. 1 shows that good tracking performance is achieved after 10 seconds learning periods. Fig. 2 shows that the state x_2 in the closed-loop is also bounded. Figs. 3 and 4 show the boundedness of the control input and the NN weights in the control loop.

VI. Conclusion

An adaptive neural-based control has been addressed for a class of parametric-strict-feedback nonlinear systems with unknown time delays. The proposed design method does not require a priori knowledge of the signs of the unknown virtual control coefficients. The unknown time delays have been compensated for by using appropriate Lyapunov-Krasovskii functionals. The proposed systematic backstepping design method has been proved to be able to guarantee semi-global uniformly ultimately boundedness of all the signals. In addition, the output of the system has been proven to converge to a small neighborhood of the origin. Simulation has been conducted to show the effectiveness of the proposed approach.

Appendix A
Proof of Lemma 2

Proof: For easy reference, re-write (4) as

$$V(t) \leq c_0 + e^{-c_1 t}\int_0^t g\left(x(\tau)\right) N(\zeta)\dot{\zeta} e^{c_1\tau} d\tau + e^{-c_1 t}\int_0^t \dot{\zeta} e^{c_1\tau} d\tau, \quad \forall t \in [0, t_f) \tag{75}$$

Since $g(x(t)) \in [l^-, l^+]$, let us define $g_{\max} = \max\{|l^-|, |l^+|\}$ and $g_{\min} = \min\{|l^-|, |l^+|\}$ for convenience. We first show that $\zeta(t)$ is bounded on $[0, t_f)$ by seeking a contradiction. Suppose that $\zeta(t)$ is unbounded and two cases should be considered:

1) $\zeta(t)$ has no upper bound;
2) $\zeta(t)$ has no lower bound.

Case (i): $\zeta(t)$ has no upper bound on $[0, t_f)$. In this case, there must exist a monotone increasing variable $\{\omega_i = \omega(t_i) = \zeta(t_i)\}$ with $\omega_0 = |\zeta(t_0)| > 0$, $\lim_{i\to+\infty} t_i = t_f$, and $\lim_{i\to+\infty} \omega_i = +\infty$.

For clarity, define

$$N_g(\omega_i, \omega_j) = \int_{\omega_i}^{\omega_j} g\left(x(\tau)\right) N\left(\omega(\tau)\right) e^{-c_1(t_j-\tau)} d\omega(\tau)$$

with an understanding that $N_g(\omega_i, \omega_j) = N_g(\omega(t_i), \omega(t_j)) = N_g(t_i, t_j)$ for notation convenience, and $\omega_i \leq \omega_j, \tau \in [t_i, t_j]$.

Using integral inequality $(b-a)m_{f1} \le \int_a^b f(x)dx \le (b-a)m_{f2}$ with $m_{f1} = \inf_{a\le x\le b} f(x)$ and $m_{f2} = \sup_{a\le x\le b} f(x)$, and noting that $g(x(t)) \le g_{\max}$, $0 < e^{-c_1(t-\tau)} \le 1$ for $\tau \in [0,t]$, we have

$$\begin{aligned} |N_g(\omega_i,\omega_j)| &\le g_{\max}(\omega_j-\omega_i)\sup_{\omega\in[\omega_i,\omega_j]}|N(\omega)| \\ &= g_{\max}(\omega_j-\omega_i)e^{\omega_j^2} \end{aligned} \tag{76}$$

for the Nussbaum function $N(\omega) = e^{\omega^2}\cos((\pi/2)\omega)$, which is positive for $\omega \in (4m-1, 4m+1)$ and negative for $\omega \in (4m+1, 4m+3)$ with m an integer.

Let us first consider the case $g(x) > 0$. First, let us consider the interval $[\omega_0, \omega_{m_1}] = [\omega_0, 4m-1]$, and the following expression:

$$N_g(\omega_0,\omega_{m_1}) = \int_{\omega_0}^{\omega_{m_1}} g(x(\tau))\, e^{-c_1(t_{m_1}-\tau)}N(\omega)d\omega(\tau).$$

Applying (76), we have

$$|N_g(\omega_0,\omega_{m_1})| \le g_{\max}(4m-1-\omega_0)e^{(4m-1)^2}. \tag{77}$$

Next, let us observe variation in the interval $[\omega_{m_1},\omega_{m_2}] = [4m-1, 4m+1]$. Noting that $N(\omega) \ge 0$, $\forall\omega \in [\omega_{m_1},\omega_{m_2}]$, we have the following inequality

$$N_g(\omega_{m_1},\omega_{m_2}) \ge \int_{4m-\epsilon_1}^{4m+\epsilon_1} g(x(\tau))\, e^{-c_1(t_{m_2}-\tau)}N(\omega(\tau))\,d\omega(\tau)$$

with $\epsilon_1 \in (0,1)$. Similarly using the integral inequality by noting that $g(x(t)) \ge g_{\min}$, $e^{-c_1(t-\tau)} \ge e^{-c_1(t_{m_2}-t_{m_1})}$ for $\tau \in [t_{m_1}, t_{m_2}]$, we have

$$\begin{aligned} N_g(\omega_{m_1},\omega_{m_2}) &\ge 2\epsilon_1 g_{\min}e^{-c_1(t_{m_2}-t_{m_1})}\inf_{\omega\in[\omega_{m_1},\omega_{m_2}]}N(\omega) \\ &= c_{b_1}e^{(4m-\epsilon_1)^2} \end{aligned} \tag{78}$$

where $c_{b_1} = 2\epsilon_1 g_{\min}\cos((\pi/2)\epsilon_1)e^{-c_1(t_{m_2}-t_{m_1})}$.

It is known that if $|f_1(x)| \le a_1$ and $f_2(x) \ge a_2$, then $f_1(x)+f_2(x) \ge a_2 - a_1$, and inequalities (77) and (78) yield

$$\begin{aligned} N_g(\omega_0,\omega_{m_2}) \ge e^{(4m-1)^2}\Big\{&c_{b_1}e^{\left[2(4m-1)(1-\epsilon_1)+(1-\epsilon_1)^2\right]} \\ &-g_{\max}(4m-1-\omega_0)\Big\} \end{aligned}$$

which can be further written as

$$\begin{aligned} \frac{1}{\omega_{m_2}}N_g(\omega_0,\omega_{m_2}) \ge \frac{e^{(4m-1)^2}}{4m+1}\Big\{&c_{b1}e^{\left[2(4m-1)(1-\epsilon_1)+(1-\epsilon_1)^2\right]} \\ &-g_{\max}(4m-1-\omega_0)\Big\}. \end{aligned} \tag{79}$$

The following property is useful for our derivation:

$$\begin{aligned} \lim_{x\to+\infty}\frac{b_0e^{x^2}(e^{ax}-b_1x+b_2)}{x+b_3} &= +\infty, \\ x+b_3 \ne 0, \quad b_0,b_1,a &> 0 \end{aligned} \tag{80}$$

which can be easily proven by applying the L'Hopital's Rule as

$$\begin{aligned} &\lim_{x\to+\infty}\frac{b_0e^{x^2}(e^{ax}-b_1x+b_2)}{x+b_3} \\ &\quad= \lim_{x\to+\infty}\frac{\frac{\partial}{\partial x}\left[b_0e^{x^2}(e^{ax}-b_1x+b_2)\right]}{\frac{\partial}{\partial x}(x+b_3)} = +\infty. \end{aligned}$$

Using property (80) and noting $(1-\epsilon_1) \in (0,1)$, from (79), we have

$$\lim_{m\to+\infty}\frac{1}{\omega_{m_2}}N_g(\omega_0,\omega_{m_2}) = +\infty. \tag{81}$$

We have shown that $\lim_{m\to+\infty}(1/(4m+1))N_g(\omega_0, 4m+1) = +\infty$, now we would like to show that $\lim_{m\to+\infty}(1/(4m+3))N_g(\omega_0, 4m+3) = -\infty$.

To this end, let us first observe the interval $[\omega_0,\omega_{m_2}] = [\omega_0, 4m+1]$. Similarly, applying (76), we can obtain

$$|N_g(\omega_0,\omega_{m_2})| \le g_{\max}(4m+1-\omega_0)e^{(4m+1)^2} \tag{82}$$

Then, let us consider the next immediate interval $[\omega_{m_2},\omega_{m_3}] = [4m+1, 4m+3]$. Noting that $N(\omega) \le 0$, $\forall\omega \in [\omega_{m_2},\omega_{m_3}]$, as for $\omega \in [\omega_{m_1},\omega_{m_2}]$, we have the following inequality:

$$\begin{aligned} &N_g(\omega_{m_2},\omega_{m_3}) \\ &\quad\le \int_{4m+2-\epsilon_2}^{4m+2+\epsilon_2} g(x(\tau))\, e^{-c_1(t_{m_2}-\tau)}N(\omega(\tau))\,d\omega(\tau) \\ &\quad\le -c_{b_2}e^{(4m+2-\epsilon_2)^2} \end{aligned} \tag{83}$$

where $c_{b_2} = 2\epsilon_2 g_{\min}\cos((\pi/2)\epsilon_2)e^{-c_1(t_{m_3}-t_{m_2})}$ and $\epsilon_2 \in (0,1)$.

It is also known that if $|f_1(x)| \le a_1$ and $f_2(x) \le a_2$, then $f_1(x)+f_2(x) \le a_2 + a_1$. Accordingly, inequalities (82) and (83) lead to

$$\begin{aligned} N_g(\omega_0,\omega_{m_3}) \le -e^{(4m+1)^2}\Big\{&c_{b_2}e^{\left[2(4m+1)(1-\epsilon_2)+(1-\epsilon_2)^2\right]} \\ &-g_{\max}(4m+1-\omega_0)\Big\} \end{aligned}$$

which can be further written as

$$\begin{aligned} \frac{1}{\omega_{m_3}}N_g(\omega_0,\omega_{m_3}) \le -\frac{e^{(4m+1)^2}}{4m+3}\Big\{&c_{b_2}e^{\left[2(4m+1)(1-\epsilon_2)(1-\epsilon_2)^2\right]} \\ &-g_{\max}(4m+1-\omega_0)\Big\}. \end{aligned} \tag{84}$$

Using property (80) and noting $(1-\epsilon_2) \in (0,1)$, from (84), we have

$$\lim_{m\to+\infty}\frac{1}{\omega_{m_3}}N_g(\omega_0,\omega_{m_3}) = -\infty. \tag{85}$$

Therefore, from (81) and (85), we can conclude that, $g(x) > 0$

$$\lim_{\omega_j\to+\infty}\sup\frac{1}{\omega_j}N_g(\omega_0,\omega_j) = +\infty \tag{86}$$

$$\lim_{\omega_j\to+\infty}\inf\frac{1}{\omega_j}N_g(\omega_0,\omega_j) = -\infty. \tag{87}$$

In what follows, we would like to show that (86) and (87) also hold for $g(x) < 0$. Let us observe the following intervals $[\omega_0, 4m-1]$, $[4m-1, 4m+1]$ and, $[\omega_0, 4m+1]$ and $[4m+1, 4m+3]$, respectively for $g(x) < 0$. In the intervals $[\omega_0, 4m-1]$ and $[\omega_0, 4m+1]$, inequalities (77) and (82) remain. In the interval $[4m-1, 4m+1]$, noting that $g(x) < 0$ and $N(\omega) \geq 0$, we can similarly obtain

$$\begin{aligned} N_g(\omega_{m_1}, \omega_{m_2}) &\leq \int_{4m-\epsilon_1}^{4m+\epsilon_1} g(x(\tau)) e^{-c_1(t_{m_2}-\tau)} N(\omega(\tau)) d\omega(\tau) \\ &\leq -c_{b_1} e^{(4m-\epsilon_1)^2}. \end{aligned} \quad (88)$$

Combining (77) and (88) yields

$$\frac{1}{\omega_{m_2}} N_g(\omega_0, \omega_{m_2}) \leq -\frac{e^{(4m-1)^2}}{4m+1} \times \left\{ c_{b_1} e^{\left[2(4m-1)(1-\epsilon_1)+(1-\epsilon_1)^2\right]} - g_{\max}(4m-1-\omega_0) \right\}. \quad (89)$$

Using the property (80) and noting $(1-\epsilon_1) \in (0,1)$, from (89), we have

$$\lim_{m \to +\infty} \frac{1}{\omega_{m_2}} N_g(\omega_0, \omega_{m_2}) = -\infty. \quad (90)$$

In the interval $[4m+1, 4m+3]$, noting that $g(x) < 0$ and $N(\omega) \leq 0$, we have

$$\begin{aligned} N_g(\omega_{m_2}, \omega_{m_3}) &\geq \int_{4m+2-\epsilon_2}^{4m+2+\epsilon_2} g(x(\tau)) e^{-c_1(t_{m_2}-\tau)} N(\omega(\tau)) d\omega(\tau) \\ &\geq c_{b_2} e^{(4m+2-\epsilon_2)^2}. \end{aligned} \quad (91)$$

Combining the inequalities (82) and (91) on the intervals $[\omega_0, 4m+1]$ and $[4m+1, 4m+3]$ respectively, we have

$$\frac{1}{\omega_{m_3}} N_g(\omega_0, \omega_{m_3}) \geq \frac{e^{(4m+1)^2}}{4m+3} \left\{ c_{b_2} e^{\left[2(4m+1)(1-\epsilon_2)+(1-\epsilon_2)^2\right]} - g_{\max}(4m+1-\omega_0) \right\}. \quad (92)$$

Similarly using the property (80) and noting $(1-\epsilon_2) \in (0,1)$, from (92), we have

$$\lim_{m \to +\infty} \frac{1}{\omega_{m_3}} N_g(\omega_0, \omega_{m_3}) = +\infty. \quad (93)$$

From (90) and (93), we can also obtain (86) and (87). Therefore, we can conclude that (86) and (87) hold no matter $g(x(t)) > 0$ or $g(x(t)) < 0$.

Dividing (75) by $\omega_i = \zeta(t_i) > 0$ yields

$$\begin{aligned} 0 \leq \frac{V(t_i)}{\omega(t_i)} &\leq \frac{c_0}{\omega(t_i)} + \frac{\omega(t_i)-\omega(t_0)}{\omega(t_i)} \sup_{\omega \in [\omega(t_0), \omega(t_i)]} e^{-c_1(t_i-\tau)} \\ &\quad + \frac{1}{\omega(t_i)} \int_{\omega(t_0)}^{\omega(t_i)} g(x(\tau)) N(\omega(\tau)) e^{-c_1(t_i-\tau)} d\omega(\tau) \\ &= \frac{c_0}{\omega(t_i)} + \left(1 - \frac{\omega(t_0)}{\omega(t_i)}\right) \\ &\quad + \frac{1}{\omega(t_i)} \int_{\omega(t_0)}^{\omega(t_i)} g(x(\tau)) N(\omega(\tau)) e^{-c_1(t_i-\tau)} d\omega(\tau). \end{aligned} \quad (94)$$

On taking the limit as $i \to +\infty$, hence $t_i \to t_f$, $\omega(t_i) \to +\infty$, (94) becomes

$$0 \leq \lim_{i \to +\infty} \frac{V(t_i)}{\omega(t_i)} \leq 1 + \lim_{i \to +\infty} \frac{1}{\omega(t_i)} N_g(\omega(t_0), \omega(t_i))$$

which takes a contradiction as can be seen from (87). Therefore, $\zeta(t)$ is upper bounded on $[0, t_f)$.

Case (ii): $\zeta(t)$ has no lower bound on $[0, t_f)$. There must exist a monotone increasing variable $\{\omega_i = \omega(\underline{t}_i) = -\zeta(\underline{t}_i)\}$ with $\omega_0 = |\zeta(\underline{t}_0)| > 0$, $\lim_{i \to +\infty} \underline{t}_i = t_f$, and $\lim_{i \to +\infty} \omega_i = +\infty$.

Dividing (75) by $\omega_i = -\zeta(\underline{t}_i) > 0$ yields

$$\begin{aligned} 0 \leq \frac{V(\underline{t}_i)}{\omega(\underline{t}_i)} &\leq \frac{c_0}{\omega(\underline{t}_i)} - \frac{1}{\omega(\underline{t}_i)} \int_{\omega(\underline{t}_0)}^{\omega(\underline{t}_i)} e^{-c_1(\underline{t}_i-\tau)} d(\omega(\tau)) \\ &\quad - \frac{1}{\omega(\underline{t}_i)} \int_{\omega(\underline{t}_0)}^{\omega(\underline{t}_i)} g(x(\tau)) N(-\omega(\tau)) e^{-c_1(\underline{t}_i-\tau)} d(\omega(\tau)) \end{aligned} \quad (95)$$

Noting that $N(\cdot)$ is an even function, i.e., $N(\omega) = N(-\omega)$, (95) becomes

$$\begin{aligned} 0 \leq \frac{V(\underline{t}_i)}{\omega(\underline{t}_i)} &\leq \frac{c_0}{\omega(\underline{t}_i)} - \frac{1}{\omega(\underline{t}_i)} \int_{\omega(\underline{t}_0)}^{\omega(\underline{t}_i)} e^{-c_1(\underline{t}_i-\tau)} d\omega(\tau) \\ &\quad - \frac{1}{\omega(\underline{t}_i)} \int_{\omega(\underline{t}_0)}^{\omega(\underline{t}_i)} g(x(\tau)) N(\omega(\tau)) e^{-c_1(\underline{t}_i-\tau)} d\omega(\tau) \\ &\leq \frac{c_0}{\omega(\underline{t}_i)} - \frac{\omega(\underline{t}_i)-\omega(\underline{t}_0)}{\omega(\underline{t}_i)} \inf_{\omega \in [\omega(\underline{t}_0), \omega(\underline{t}_i)]} e^{-c_1(\underline{t}_i-\tau)} \\ &\quad - \frac{1}{\omega(\underline{t}_i)} \int_{\omega(\underline{t}_0)}^{\omega(\underline{t}_i)} g(x(\tau)) N(\omega(\tau)) e^{-c_1(\underline{t}_i-\tau)} d\omega(\tau) \\ &= \frac{c_0}{\omega(\underline{t}_i)} - \left(1 - \frac{\omega(\underline{t}_0)}{\omega(\underline{t}_i)}\right) e^{-c_1 \underline{t}_i} \\ &\quad - \frac{1}{\omega(\underline{t}_i)} \int_{\omega(\underline{t}_0)}^{\omega(\underline{t}_i)} g(x(\tau)) N(\omega(\tau)) e^{-c_1(\underline{t}_i-\tau)} d\omega(\tau). \end{aligned}$$

Taking the limit as $i \to +\infty$, hence $\underline{t}_i \to t_f, \omega \underline{t}_i) \to +\infty$, we have

$$0 \le \lim_{i\to+\infty} \frac{V(\underline{t}_i)}{\omega(\underline{t}_i)} \le -e^{-c_1 t_f} - \lim_{i\to+\infty} \frac{1}{\omega(\underline{t}_i)} N_g\left(\omega(\underline{t}_0), \omega(\underline{t}_i)\right)$$

which takes a contradiction as can be seen from (86). Therefore, $\zeta(t)$ is lower bounded on $[0, t_f)$.

Therefore, $\zeta(t)$ must be bounded on $[0, t_f)$. In addition, $V(t)$ and $\int_0^t g(x(\tau))N(\zeta)\dot{\zeta}d\tau$ are bounded on $[0, t_f)$. □

APPENDIX B
PROOF OF LEMMA 3

Proof: First, we show that $\Omega_{Z_1}^0$ is a *closed* set. From (17) and applying De Morgan's laws, we have

$$\Omega_{Z_1}^{0c} = \Omega_{Z_1}^c \cup \Omega_{c_{z_1}} \tag{96}$$

where $\Omega_{Z_1}^{0c}$ and $\Omega_{Z_1}^c$ are the complements of $\Omega_{Z_1}^0$ and Ω_{Z_1} respectively. Since Ω_{Z_1} is a *compact* set, i.e., it is *closed* and *bounded*, $\Omega_{Z_1}^c$ is an *open* set. In addition, $\Omega_{c_{z_1}}$ is also an open set from its definition. From (96), we know that $\Omega_{Z_1}^{0c}$ is an open set, which means that its complement $\Omega_{Z_1}^0$ is a closed set. Second, from (17), we know that $\Omega_{Z_1}^0 \subset \Omega_{Z_1}$. Since a closed subset of a compact set is compact, we can conclude that $\Omega_{Z_1}^0$ is a compact set. □

APPENDIX C
PROOF OF LEMMA 4

Proof: We first show that all the closed-loop signals are GUUB for $z_1 \in \Omega_{Z_1}^0$. Consider the following Lyapunov function candidate

$$V_1(t) = V_{z_1}(t) + V_{U_1}(t)$$

Its time derivative along (14) is

$$\dot{V}_1(t) \le z_1\beta_1(x_1)g_1(x_1)u + Q_1(Z_1)z_1 \tag{97}$$

For $z_1 \in \Omega_{Z_1}^0$, substituting (18) into (97) yields

$$\dot{V}_1(t) \le \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + Q_1(Z_1)z_1 \tag{98}$$

Adding and subtracting $k_1(t)z_1^2 + Q_1(Z_1)z_1$ on the right hand side of (98), we have

$$\begin{aligned}\dot{V}_1(t) \le& \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 - \dot{\zeta}_1 + Q_1(Z_1)z_1\\ \le& -k_1^* z_1^2 + \beta_1(x_1)g_1(x_1)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1\end{aligned} \tag{99}$$

Integrating (99) over $[0, t]$, $\forall t \in [0, t_f)$, we have the following inequality

$$\begin{aligned}V_1(t) + \int_0^t k_1^* z_1^2(\tau)d\tau \le V_1(0)\\ + \int_0^t \left[\beta_1\left(x_1(\tau)\right)g_1\left(x_1(\tau)\right)N\left(\zeta_1(\tau)\right) + 1\right]\dot{\zeta}_1(\tau)d\tau\end{aligned} \tag{100}$$

Since $\int_0^t k_1^* z_1^2(\tau)d\tau \ge 0$, we further have

$$\begin{aligned}V_1(t) \le& V_1(0)\\ &+ \int_0^t \left[\beta_1\left(x_1(\tau)\right)g_1\left(x_1(\tau)\right)N\left(\zeta_1(\tau)\right) + 1\right]\dot{\zeta}_1(\tau)d\tau\end{aligned}$$

Applying Lemma 1, we can conclude that $V_1(t)$, $\int_0^t(\beta_1 g_1 N(\zeta_1) + 1)\dot{\zeta}_1 d\tau$, and $\zeta_1(t)$ are bounded. Since $(1/2)z_1^2(t) \le V_{z_1}(t) \le V_1(t)$, we know that $z_1(t)$ are bounded on $[0, t_f)$. According to Proposition 2 in [15], if the solution of the closed-loop is bounded, then $t_f = +\infty$. From (100), $z_1(t)$ is square integrable and as an immediate result, x_1, u and $\dot{z}_1$ are also bounded on $[0, +\infty]$. Since $\dot{z}_1 \in L^\infty$, and $z_1 \in L^2 \cap L^\infty$, by Barbalat's lemma, $\lim_{t\to+\infty} z_1 = 0$. Note that the above results are obtained for $z_1 \in \Omega_{Z_1}^0$, therefore we can guarantee that $\Omega_{c_{z_1}}$ is domain of attraction. □

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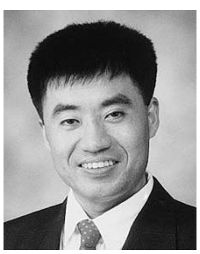

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