Observer and observer-based H_∞ control of generalized Hamiltonian systems

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Abstract This paper deals with observer design for generalized Hamiltonian systems and its applications. First, by using the systems' structural properties, a new observer design method called Augment Plus Feedback is provided and two kinds of observers are obtained: non-adaptive and adaptive ones. Then, based on the obtained observer, H_∞ control design is investigated for generalized Hamiltonian systems, and an observer-based control design is proposed. Finally, as an application to power systems, an observer and an observer-based H_∞ control law are designed for single-machine infinite-bus systems. Simulations show that both the observer and controller obtained in this paper work very well.

Keywords: generalized Hamiltonian system, adaptive observer, zero-state detectable, observer-based H_∞ control, power system.

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1 Introduction

In state feedback control designs, if states of the system under consideration are not measurable, the need to design an observer is well understood by now. Since the appearance of Luenberger's observer[1], observer designs for dynamic systems have drawn considerable attention and seen many remarkable results[2−6]. The design of observers for linear systems has been well studied by now. But for general nonlinear systems, the design problem still remains challenging, though there are numerous observer results for certain classes of nonlinear systems.

In recent years, port-controlled Hamiltonian (PCH) systems[7] have been well investigated, see, e.g. refs. [8—15]. The Hamiltonian function in a PCH system is the total energy, i.e. sum of potential and kinetic energies in physical systems, and can play the role of Lyapunov function for the system. At present, there are two "hot" topics worth noticing about PCH systems: one is the application study of energy-based Lyapunov function method[9−13], and the other is how to express a nonlinear system as a dissipative Hamiltonian system, i.e. the generalized Hamiltonian realization problem[14,15]. In these topics, the key is the design of suitable state feedback control laws.

For observer design for Hamiltonian systems, Hebert et al. proposed a design method for a special class of generalized Hamiltonian systems, in which the output was assumed to be linear and the structure matrix was assumed to be constant, and then they used the method to design observers for synchronization of chaotic systems[16]. Lohmiller and Slotine used their earlier work on contraction analysis for nonlinear systems to design a globally convergent observer for a class of Hamiltonian systems[17]. However, to the authors' best knowledge, there are few results on observer design for general PCH systems.

This paper investigates observer design for PCH systems and its applications. Based on the systems' structural properties, a new observer design method called Augment plus Feedback is provided in this paper. The new method is different from traditional approaches, in which one usually gets a dynamic system of state error $e(:= x - \hat{x})$ and then shows that the error system converges to zero asymptotically. The new method has advantages in observer design for systems whose error dynamics are hard to obtain (e.g. PCH systems). By using the new method, two kinds of observers are obtained: one is a non-adaptive observer and the other is an adaptive one. Then, based on the obtained observer, H_∞ control design is investigated for generalized Hamiltonian systems, and an observer-based control design method is proposed. Finally, as an application to power systems, an observer and an observer-based H_∞ control law are designed for single-machine infinite-bus systems[18]. Simulations show that both the observer and controller obtained in this paper work very well.

2 Observer design

This section is to investigate observer design for PCH systems. We will provide a new design method called Augment Plus Feedback and design two observers for the systems.

Consider a PCH system with unknown structure parameter perturbations[10]:

$$\begin{cases} \dot{x} = [J(x,p) - R(x,p)]\dfrac{\partial H(x,p)}{\partial x} + g(x)u, \\ y = g^T(x)\dfrac{\partial H(x,0)}{\partial x}, \end{cases} \tag{1}$$

where $x \in R^n$, $u \in R^m$, p stands for the parametric perturbation, $J^T(x,p) = -J(x,p) \in R^{n\times n}$, $R(x,p) \in R^{n\times n}$ is positive semi-definite, and $H(x,p)$ is the Hamiltonian function, which has a minimum at x_0 when $p = 0$. Suppose that the states of system (1) are unmeasurable. We design observers for system (1) for the following two cases: $p = 0$ and $p \neq 0$.

2.1 Constant $p = 0$

In this case, the observer is designed under the following realistic assumptions:

S1: $\nabla H(x,0) \neq 0$ $(x \neq x_0)$, and system (1) is zero-state detectable[19] with respect to $y_1 := R^{\frac{1}{2}}(x,0)\nabla H(x,0)$ and u, that is, $y_1 \equiv 0$, $u \equiv 0 \Rightarrow x \to x_0$ $(t \to \infty)$, where $R^{\frac{1}{2}}(x,0)$ is defined as $R(x,0) = [R^{\frac{1}{2}}(x,0)]^2$ and $\nabla H := \frac{\partial H(x,0)}{\partial x}$.

S2: There are non-zero matrices $K(x)$, $K_1(x) \in R^{m\times n}$ such that

$$W(x) := R(x,0) + [g(x)K(x) + K^T(x)g^T(x)] \geqslant 0, \tag{2}$$

$K(x) = K_1(x)W(x)$ and

$$\dot{x} = [J(x,0) - W(x)]\nabla H(x,0) \tag{3}$$

is zero-state detectable with respect to $y_2 := W^{\frac{1}{2}}(x)\nabla H(x,0)$.

Remark 1. Since $R(x,0) \geqslant 0$, there must exist $K(x)$ such that (2) holds. After $K(x)$ is obtained, one can find $K_1(x)$ from $K(x) = K_1(x)W(x)$.

From Assumptions S1 and S2, we design an observer as follows:

$$\dot{\hat{x}} = [J(\hat{x},0) - R(\hat{x},0)]\frac{\partial H(\hat{x},0)}{\partial \hat{x}} + g(\hat{x})u + K^T(\hat{x})[y - g^T(\hat{x})\frac{\partial H(\hat{x},0)}{\partial \hat{x}}]. \tag{4}$$

Unlike the traditional approach, we will combine (1) and (4) to obtain an augmented system first, and then design a feedback law only based on the observer state $\hat{x}$ to make the augmented system be a PCH one with dissipation, from which we will show that system (4) can serve as an observer for system (1). The above method can be described as Augment Plus Feedback.

With (1) and (4), we have

$$\dot{X} = [\bar{J}_1(X) - \bar{R}_1(X)]\frac{\partial \bar{H}(X)}{\partial X} + \bar{g}(X)u, \tag{5}$$

where

$$\bar{J}_1(X) = \begin{pmatrix} J(x,0) & 0 \\ 0 & J(\hat{x},0) \end{pmatrix},$$

$$\bar{R}_1(X) = \begin{pmatrix} R(x,0) & 0 \\ -K^T(\hat{x})g^T(x) & R(\hat{x},0) + K^T(\hat{x})g^T(\hat{x}) \end{pmatrix},$$

$$X = (x^T, \hat{x}^T)^T, \quad \bar{H}(X) = H(x,0) + H(\hat{x},0),$$

$$\frac{\partial \bar{H}(X)}{\partial X} = \begin{pmatrix} \frac{\partial H(x,0)}{\partial x} \\ \frac{\partial H(\hat{x},0)}{\partial \hat{x}} \end{pmatrix}, \quad \bar{g}(X) = \begin{pmatrix} g(x) \\ g(\hat{x}) \end{pmatrix}.$$

Since $\bar{R}_1(X)$ is not symmetric, we design a control law only based on $\hat{x}$ as

$$u = -K(\hat{x})\frac{\partial H(\hat{x},0)}{\partial \hat{x}} + v. \tag{6}$$

Substituting (6) into (5) yields

$$\dot{X} = [\bar{J}(X) - \bar{R}(X)]\frac{\partial \bar{H}(X)}{\partial X} + \bar{g}(X)v, \tag{7}$$

where

$$\bar{J}(X) = \begin{pmatrix} J(x,0) & -g(x)K(\hat{x}) \\ K^T(\hat{x})g^T(x) & J(\hat{x},0) \end{pmatrix},$$

$$\bar{R}(X) = \begin{pmatrix} R(x,0) & 0 \\ 0 & R(\hat{x},0) + g(\hat{x})K(\hat{x}) + K^T(\hat{x})g^T(\hat{x}) \end{pmatrix}.$$

System (7) is a dissipative PCH system[7].

Theorem 1. Assume that S1 and S2 hold, and $p = 0$. Under the control (6), system (4) is a globally asymptotical observer for system (1).

Proof. Consider the energy flow of system (7). When $v = 0$, we obtain

$$\begin{aligned}\dot{\bar{H}}(X) &= -\frac{\partial \bar{H}^T}{\partial X}\bar{R}(X)\frac{\partial \bar{H}}{\partial X} \\ &= -\nabla^T H(x,0)R(x,0)\nabla H(x,0) - \nabla^T H(\hat{x},0)W(\hat{x})\nabla H(\hat{x},0) \leqslant 0, \end{aligned} \quad (8)$$

from which system (7) is stable. Furthermore, system (7) converges to the largest invariant set contained in

$$S = \left\{X : \frac{d\bar{H}}{dt} = 0\right\} = \left\{(x,\hat{x}) : R^{\frac{1}{2}}(x,0)\nabla H(x,0) = 0, \ \ W^{\frac{1}{2}}(\hat{x})\nabla H(\hat{x},0) = 0\right\}. \quad (9)$$

Since S1 and S2 hold, $W^{\frac{1}{2}}(\hat{x})\nabla H(\hat{x},0) = 0 \Rightarrow K(\hat{x})\nabla H(\hat{x},0) = 0$. Thus, when $v = 0$ and $W^{\frac{1}{2}}(\hat{x})\nabla H(\hat{x},0) = 0$, system (7) can be expressed as

$$\begin{cases} \dot{x} = [J(x,0) - R(x,0)]\nabla H(x,0), \\ \dot{\hat{x}} = [J(\hat{x},0) - W(\hat{x})]\nabla H(\hat{x},0) + K^T(\hat{x})g^T(x)\nabla H(x,0). \end{cases} \quad (10)$$

Because system (1) is zero-state detectable with respect to y_1, $y_1 = 0 \Rightarrow x \to x_0$ $(t \to \infty)$, and moreover, $\nabla H(x,0) \to 0$. Then, the second part of system (10) becomes

$$\dot{\hat{x}} = [J(\hat{x},0) - W(\hat{x})]\nabla H(\hat{x},0). \quad (11)$$

On the other hand, system (3) is zero-state detectable with respect to y_2. From $y_2 = W^{\frac{1}{2}}(\hat{x})\nabla H(\hat{x},0) \equiv 0$, we have $\hat{x} \to x_0$. Therefore, the above largest invariant set contains only one point, i.e. $(x_0^T, x_0^T)^T$. From the LaSalle invariant principle[20], system (7) is asymptotically stable and $||x-\hat{x}|| = ||x-x_0+x_0-\hat{x}|| \leqslant ||x-x_0||+||\hat{x}-x_0|| \longrightarrow 0$ $(t \to \infty)$.

Remark 2. As the control (6) is only a function of $\hat{x}$, the above observer can be realized in practice as shown in fig. 1.

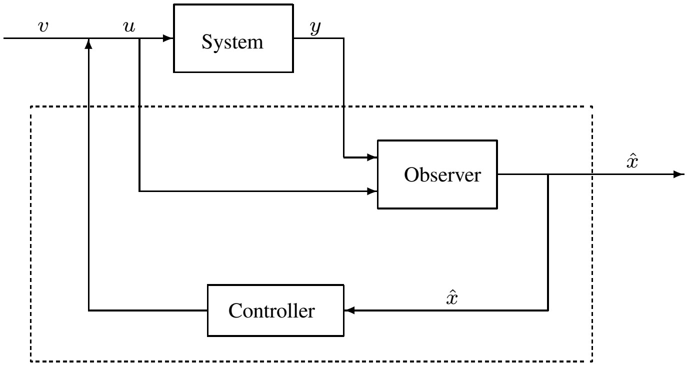

Fig. 1. Sketched drawing of realization.

Remark 3. Since $\hat{x}$ is obtained under the control law (6), $\hat{x}$ seems not the estimate of the true state. In fact, it is not the case. When $t \to \infty$, $\nabla H(\hat{x}, 0) \to 0$, and as a consequence, the controller (6) $\to 0$, as $t \to \infty$ ($v = 0$). Thus, when t is sufficiently large, the controller (6) does not affect the estimate of the true state. In practical control designs, one only needs to design $v = v(\hat{x})$ (the reference input) according to the need, and then add it to the control law (6) to obtain the complete controller. Obviously, the complete controller can ensure that: (i) $\hat{x} \to x$; (ii) it meets the designed control demand (see section 3).

In section 4, we will use Theorem 1 to solve a practical problem.

2.2 Constant $p \neq 0$

When $p \neq 0$, in order to obtain the estimate of the states of system (1), we have to design an adaptive observer for system (1), that is, to design an observer of the following form

$$\begin{cases} \dot{\hat{x}} = \alpha(\hat{x}, \hat{\theta}, y, u), \\ \dot{\hat{\theta}} = \beta(\hat{x}, \hat{\theta}, y, u), \end{cases} \tag{12}$$

such that $||x - \hat{x}|| \to 0$ ($t \to \infty$), where $\hat{\theta}$ is the estimate of θ, and $\theta = \theta(p)$ is an unknown vector.

In this case, the observer is designed under the following realistic assumpotions:

S1′: $\nabla H(x, 0) \neq 0$ ($x \neq x_0$), and the system is zero-state detectable with respect to $y_1 := R^{\frac{1}{2}}(x, p)\nabla H(x, 0)$ and u;

S2′: There exist non-zero matrices $L(x)$, $L_1(x) \in R^{n\times m}$ such that

$$W(x) := R(x, 0) + [g(x)L^T(x) + L(x)g^T(x)] \geqslant 0,$$

$L^T(x) = L_1^T(x)W(x)$ and the following system

$$\dot{x} = [J(x, 0) - W(x)]\nabla H(x, 0)$$

is zero-state detectable with respect to $y_2 := W^{\frac{1}{2}}(x)\nabla H(x, 0)$.

S3: There exists a constant matrix $\Phi \in R^{l\times m}$ such that

$$[J(x,p)-R(x,p)]\Delta_H(x,p)=g(x)\Phi^T\theta, \tag{13}$$

where $\Delta_H(x,p)=\frac{\partial H(x,p)}{\partial x}-\frac{\partial H(x,0)}{\partial x}$ and $\theta \in R^l$ is an unknown vector only about p.

From (13), system (1) can be rewritten as

$$\begin{cases} \dot{x}=[J(x,p)-R(x,p)]\dfrac{\partial H(x,0)}{\partial x}+g(x)\Phi^T\theta+g(x)u, \\ y=g^T(x)\dfrac{\partial H(x,0)}{\partial x}, \end{cases} \tag{14}$$

from which we design an observer of the form:

$$\begin{cases} \dot{\hat{x}}=[J(\hat{x},0)-R(\hat{x},0)]\dfrac{\partial H(\hat{x},0)}{\partial \hat{x}}+g(\hat{x})\Phi^T\hat{\theta}+g(\hat{x})u+L(\hat{x})\left[y-g^T(\hat{x})\dfrac{\partial H(\hat{x},0)}{\partial \hat{x}}\right], \\ \dot{\hat{\theta}}=Q\Phi y, \end{cases} \tag{15}$$

where $Q \in R^{l\times l}$ is a constant positive definite matrix, called the adaptation gain.

With (14) and (15), we obtain

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \\ \dot{\hat{\theta}} \end{pmatrix}=\begin{pmatrix} J(x,p)-R(x,p) & 0 \\ L(\hat{x})g^T(x) & J(\hat{x},0)-R(\hat{x},0)-L(\hat{x})g^T(\hat{x}) \\ Q\Phi g^T(x) & 0 \end{pmatrix}\begin{pmatrix} \nabla H(x,0) \\ \nabla H(\hat{x},0) \end{pmatrix}$$
$$+\begin{pmatrix} g(x)\Phi^T\theta \\ g(\hat{x})\Phi^T\hat{\theta} \\ 0 \end{pmatrix}+\begin{pmatrix} g(x) \\ g(\hat{x}) \\ 0 \end{pmatrix}u. \tag{16}$$

To make (16) a dissipative PCH system, we design a feedback law only based on $\hat{x}$ and $\hat{\theta}$ as follows:

$$u=-L^T(\hat{x})\nabla H(\hat{x},0)-\Phi^T\hat{\theta}+v. \tag{17}$$

Substituting it into (16) yields

$$\dot{X}=[\bar{J}(X,p)-\bar{R}(X,p)]\frac{\partial \bar{H}(X)}{\partial X}+\bar{g}(X)v, \tag{18}$$

where $X=(x^T,\hat{x}^T,\hat{\theta}^T)^T$, $\bar{H}(X)=H(x,0)+H(\hat{x},0)+\frac{1}{2}(\theta-\hat{\theta})^TQ^{-1}(\theta-\hat{\theta})$,

$$\bar{J}(X,p)=\begin{pmatrix} J(x,p) & -g(x)L^T(\hat{x}) & -g(x)\Phi^TQ \\ L(\hat{x})g^T(x) & J(\hat{x},0) & 0 \\ Q\Phi g^T(x) & 0 & 0 \end{pmatrix},$$

$$\bar{R}(X,p)=\begin{pmatrix} R(x,p) & 0 & 0 \\ 0 & W(\hat{x}) & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\frac{\partial \bar{H}(X)}{\partial X}=\begin{pmatrix} \nabla H(x,0) \\ \nabla H(\hat{x},0) \\ \frac{\partial \bar{H}(X)}{\partial \hat{\theta}} \end{pmatrix}, \qquad \bar{g}(X)=\begin{pmatrix} g(x) \\ g(\hat{x}) \\ 0 \end{pmatrix}.$$

Obviously, system (18) is a dissipative PCH one under Assumption S2′.

Theorem 2. Assume that S1′, S2′ and S3 hold. With the control (17), system (15) is an adaptive observer for system (1).

Proof. From the properties of dissipative Hamiltonian systems[14], system (18) is stable when $v = 0$. Assume that $X_0 = (x_0^T, \hat{x}_0^T, \hat{\theta}_0^T)^T$ is the system's equilibrium, then $\nabla H(x_0, 0) = \nabla H(\hat{x}_0, 0) = 0$. From S1′, $\hat{x}_0 = x_0$. Now consider the system's energy flow. When $v = 0$, we have

$$\begin{aligned}\dot{\bar{H}}(X) = &- \frac{\partial \bar{H}^T}{\partial X}\bar{R}(X,p)\frac{\partial \bar{H}}{\partial X} = -\nabla^T H(x,0)R(x,p)\nabla H(x,0)\\ &- \nabla^T H(\hat{x},0)W(\hat{x})\nabla H(\hat{x},0) \leqslant 0,\end{aligned} \tag{19}$$

from which we know that system (18) converges to the largest invariant set contained in

$$S = \left\{X :\ \frac{d\bar{H}}{dt} = 0\right\} = \left\{X :\ R^{\frac{1}{2}}(x,p)\nabla H(x,0) \equiv 0,\ \ W^{\frac{1}{2}}(\hat{x})\nabla H(\hat{x},0) \equiv 0\right\}. \tag{20}$$

Similar to the proof of Theorem 1, by using the zero-state detectability, we can show that: $R^{\frac{1}{2}}(x,p)\nabla H(x,0) \equiv 0$, $W^{\frac{1}{2}}(\hat{x})\nabla H(\hat{x},0) \equiv 0 \Rightarrow x \rightarrow x_0$, $\hat{x} \rightarrow \hat{x}_0$ $(t \rightarrow \infty)$. Therefore, $||x - \hat{x}|| = ||x - x_0 + x_0 - \hat{x}|| \leqslant ||x - x_0|| + ||\hat{x} - x_0|| \longrightarrow 0$ $(t \longrightarrow \infty)$.

Remark 4. Since the control (17) is a function of $\hat{x}$ and $\hat{\theta}$, the above adaptive observer can be realized.

Example 1. Consider system

$$\begin{cases} \dot{x} = [J(x,p) - R(x,p)]\dfrac{\partial H(x,p)}{\partial x} + g(x)u,\\ y = g^T(x)\nabla H(x,0), \end{cases} \tag{21}$$

where $H(x,p) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + \frac{1}{2}(1+p)x_3^2$,

$$J(x,p) = \begin{pmatrix} 0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 0 \end{pmatrix},\quad R(x,p) = \begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1+p \end{pmatrix},\quad g(x) = \begin{pmatrix} 0\\ 0\\ x_3 \end{pmatrix},$$

and p is a parametric perturbation satisfying $|p| < 1$.

Now we design an adaptive observer for system (21). Because $R(x,p) > 0$, $y_1 = R^{\frac{1}{2}}(x,p)\nabla H(x,0) \equiv 0 \Rightarrow \nabla H(x,0) \equiv 0$, from which we can obtain $x \rightarrow 0$ $(t \rightarrow \infty)$. Thus, S1′ holds. Choosing $L(x) = (0,\ 0,\ \frac{1}{2}x_3)^T$, $L_1 = (0,\ 0,\ \frac{x_3}{2(1+x_3^2)})^T$, we have

$$W(x) = R(x,0) + [g(x)L^T(x) + L(x)g^T(x)] = \begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1+x_3^2 \end{pmatrix} > 0,$$

and $L^T(x) = L_1^T(x)W(x)$. This means that S2′ holds, too. Next, we check S3. A straightforward computation shows that $\Delta_H(x,p) = \frac{\partial H(x,p)}{\partial x} - \frac{\partial H(x,0)}{\partial x} = (0,\ 0,\ px_3)^T$. Let $\theta = (1+p)p$, then

$$[J(x,p) - R(x,p)]\Delta_H(x,p) = g(x)\Phi^T\theta$$

holds, where $\Phi = 1$. Therefore, S3 holds. From Theorem 2,

$$\begin{cases} \dot{\hat{x}} = \left[\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right] \dfrac{\partial H(\hat{x},0)}{\partial \hat{x}} \\ \qquad + \begin{pmatrix} 0 \\ 0 \\ \hat{x}_3 \end{pmatrix} \hat{\theta} + \begin{pmatrix} 0 \\ 0 \\ \hat{x}_3 \end{pmatrix} u + \dfrac{1}{2} \begin{pmatrix} 0 \\ 0 \\ \hat{x}_3 \end{pmatrix} [y - \hat{x}_3^2], \\ \dot{\hat{\theta}} = ly \end{cases} \tag{22}$$

is an adaptive observer for system (21), where $u = -L^T(\hat{x})\nabla H(\hat{x},0) - \Phi^T\hat{\theta} + v = -\frac{1}{2}\hat{x}_3^2 - \hat{\theta} + v$, $\hat{\theta}$ is the estimate of θ, and $l > 0$ is the gain constant.

3 Observer-based H_∞ control

This section is to use the observer proposed in subsection 2.1 to study observer-based H_∞ control of PCH systems and propose a new control design method.

In this section, we only consider the unperturbed case, that is, $p = 0$ in system (1). For clarity, we will drop p in the following.

Consider a disturbed PCH system:

$$\begin{cases} \dot{x} = [J(x) - R(x)]\nabla H(x) + g_1(x)u + g_2(x)w, \\ y = g_1^T(x)\nabla H(x), \\ z = \rho g_2^T(x)\nabla H(x), \end{cases} \tag{23}$$

where $x \in R^n$, $u \in R^m$, $y \in R^m$ are the system's state, input and output, respectively; $J^T(x) = -J(x)$; $R(x) \geqslant 0$; $H(x)$ is the Hamiltonian function, which has a minimum at x_0; $z \in R^l$ is the penalty signal; $\rho > 0$ is a weighting coefficient; and $w \in R^l$ is the disturbance.

The observer-based H_∞ control problem of system (23) can be described as: Given a disturbance attenuation level $\gamma > 0$, based on the system's observer, design a control law $u = \alpha(\hat{x}, y)$ such that the closed-loop system's L_2 gain (from w to z) is bounded by γ, and meanwhile, the closed-loop system is asymptotically stable when $w = 0$.

Lemma 1[19]**.** Consider an affine system

$$\begin{cases} \dot{x} = f(x) + g(x)w, \quad f(x_0) = 0, \\ z = h(x), \end{cases} \tag{24}$$

where $x \in R^n$, $w \in R^s$ is the disturbance, and $z \in R^q$ is the penalty signal. If there exists a function $V(x) \geqslant 0$ ($V(x_0) = 0$) such that Hamilton-Jacobian inequality

$$\left(\frac{\partial V}{\partial x}\right)^T f(x) + \frac{1}{2\gamma^2}\left(\frac{\partial V}{\partial x}\right)^T gg^T \frac{\partial V}{\partial x} + \frac{1}{2}h^T h \leqslant 0 \tag{25}$$

holds, then the L_2 gain (from w to z) of system (24) is bounded by γ, i.e.

$$\int_0^T ||z(t)||^2 dt \leqslant \gamma^2 \int_0^T ||w(t)||^2 dt, \quad \forall w \in L_2[0,T]$$

holds, where γ is a positive number.

Assume that system (23) satisfies S1 and S2. From Theorem 1,

$$\dot{\hat{x}} = [J(\hat{x}) - R(\hat{x})]\nabla H(\hat{x}) + g_1(\hat{x})u + K^T(\hat{x})[y - g_1^T(\hat{x})\nabla H(\hat{x})], \quad (26)$$

is an observer for system (23), where

$$u = -K(\hat{x})\nabla H(\hat{x}) + v. \quad (27)$$

Set $\Lambda = \frac{\rho^2}{2} + \frac{1}{2\gamma^2}$, and still let $W(x) := R(x) + [g_1(x)K(x) + K^T(x)g_1^T(x)]$. Then, we have the following result.

Theorem 3. Assume that system (23) satisfies Assumptions S1 and S2. For the given $\gamma > 0$, if

$$R(x) + \Lambda[g_1(x)g_1^T(x) - g_2(x)g_2^T(x)] \geqslant 0, \quad W(x) - \Lambda g_1(x)g_1^T(x) \geqslant 0 \quad (28)$$

hold, then based on the observer (26) an H_∞ control law of system (23) can be designed as

$$v = -\Lambda\left[y - g_1^T(\hat{x})\nabla H(\hat{x})\right]. \quad (29)$$

Proof. Let $X = (x^T, \hat{x}^T)^T$, $\bar{H} = H(x) + H(\hat{x})$. With (23), (26) and (27), we obtain

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \left[\begin{pmatrix} J(x) & -g_1(x)K(\hat{x}) \\ K^T(\hat{x})g_1^T(x) & J(\hat{x}) \end{pmatrix} - \begin{pmatrix} R(x) & 0 \\ 0 & W(\hat{x}) \end{pmatrix}\right]\frac{\partial \bar{H}}{\partial X} + \begin{pmatrix} g_1(x) \\ g_1(\hat{x}) \end{pmatrix} v + \begin{pmatrix} g_2(x) \\ 0 \end{pmatrix} w.$$

Substituting (29) into the above yields

$$\dot{X} = [\bar{J}(X) - \bar{R}(X)]\nabla \bar{H}(X) + \bar{g}_2 w := f(X) + \bar{g}_2 w, \quad (30)$$

where $\bar{g}_2 = (g_2^T(x), 0)^T$,

$$\bar{J}(X) = \begin{pmatrix} J(x) & -g_1(x)K(\hat{x}) + \Lambda g_1(x)g_1^T(\hat{x}) \\ K^T(\hat{x})g_1^T(x) - \Lambda g_1(\hat{x})g_1^T(x) & J(\hat{x}) \end{pmatrix},$$

$$\bar{R}(X) = \begin{pmatrix} R(x) + \Lambda g_1(x)g_1^T(x) & 0 \\ 0 & W(\hat{x}) - \Lambda g_1(\hat{x})g_1^T(\hat{x}) \end{pmatrix}.$$

Set $V(X) = \bar{H}(X) + c$, where c is a constant such that $V(X) \geqslant 0$, $V(X_0) = 0$ ($X_0 = (x_0^T, x_0^T)^T$). From (28),

$$\begin{aligned} &\frac{\partial V^T}{\partial X} f(X) + \frac{1}{2\gamma^2}\frac{\partial V^T}{\partial X}\bar{g}_2\bar{g}_2^T\frac{\partial V}{\partial X} + \frac{1}{2}z^T z \\ &= -\nabla^T\bar{H}(X)\bar{R}(X)\nabla\bar{H}(X) + \frac{1}{2\gamma^2}\nabla^T H(x)g_2(x)g_2^T(x)\nabla H(x) \\ &\quad + \frac{\rho^2}{2}\nabla^T H(x)g_2(x)g_2^T(x)\nabla H(x) \\ &= -\nabla^T H(x)\{R(x) + \Lambda[g_1(x)g_1^T(x) - g_2(x)g_2^T(x)]\}\nabla H(x) \\ &\quad - \nabla^T H(\hat{x})[W(\hat{x}) - \Lambda g_1(\hat{x})g_1^T(\hat{x})]\nabla H(\hat{x}) \leqslant 0. \end{aligned}$$

From Lemma 1, the closed-loop system's L_2 gain is bounded by γ.

Next, we compute the energy flow of system (30). When $w = 0$, from (28) we have

$$\begin{aligned}\dot{\bar{H}} = & -\nabla^T H(x)[R(x) + \Lambda g_1(x) g_1^T(x)]\nabla H(x) - \nabla^T H(\hat{x})[W(\hat{x}) \\ & - \Lambda g_1(\hat{x}) g_1^T(\hat{x})]\nabla H(\hat{x}) \leqslant 0,\end{aligned}$$

which means that system (30) is stable. Since $\hat{x}$ is the asymptotical estimate of x, as $t \to \infty$ the above can be rewritten as

$$\dot{\bar{H}} = -\nabla^T H(x) R(x) \nabla H(x) - \nabla^T H(\hat{x}) W(\hat{x}) \nabla H(\hat{x}) \leqslant 0.$$

Therefore, as $t \to \infty$, system (30) converges to the largest invariant set contained in

$$S = \left\{X : \frac{d\bar{H}}{dt} = 0\right\} = \left\{(x, \hat{x}) : R^{\frac{1}{2}}(x)\nabla H(x) = 0, \;\; W^{\frac{1}{2}}(\hat{x})\nabla H(\hat{x}) = 0\right\}. \tag{31}$$

On the other hand, when $t \to \infty$ and $w = 0$, system (30) can be expressed as

$$\begin{cases} \dot{x} = [J(x) - R(x)]\nabla H(x) - g_1(x) K(\hat{x}) \nabla H(\hat{x}), \\ \dot{\hat{x}} = [J(\hat{x}) - W(\hat{x})]\nabla H(\hat{x}) + K^T(\hat{x}) g^T(x) \nabla H(x). \end{cases} \tag{32}$$

Since S2 holds, $W^{\frac{1}{2}}(\hat{x})\nabla H(\hat{x}) = 0 \Rightarrow K(\hat{x})\nabla H(\hat{x}) = 0$. Similar to the proof of Theorem 1, from the system's zero-state detectability, we can show that system is asymptotically stable.

Remark 5. The controller proposed in Theorem 3 is developed by using the structure properties of PCH systems, and not the one obtained from the existing state feedback controllers simply by replacing x with $\hat{x}$.

From (27) and (29), we obtain

$$u = -K(\hat{x})\nabla H(\hat{x}) - \Lambda[y - g_1^T(\hat{x})\nabla H(\hat{x})], \tag{33}$$

which is the complete controller. It can be shown that (33) can guarantee the following: (i) $||\hat{x} - x|| \to 0$ $(t \to \infty)$; (ii) meeting the demand of performances of the H_∞ control.

4 Observer-based H_∞ controller of single-machine power systems

As an application to power systems, this section designs an observer and an observer-based H_∞ controller for single-machine infinite-bus systems, respectively.

4.1 Observer for single-machine infinite-bus systems

Consider the 3rd-order model[18]

$$\begin{cases} \dot{\delta} &= \omega - \omega_0, \\ \dot{\omega} &= \dfrac{\omega_0}{M} P_m - \dfrac{D}{M}(\omega - \omega_0) - \dfrac{\omega_0 E'_q V_s}{M x'_{d\Sigma}} \sin\delta, \\ \dot{E}'_q &= -\dfrac{1}{T'_d} E'_q + \dfrac{1}{T_{do}} \dfrac{x_d - x'_d}{x'_{d\Sigma}} V_s \cos\delta + \dfrac{1}{T_{d0}} u_f, \end{cases} \tag{34}$$

where δ is the power angle, in radian; ω the rotor speed, in rad/s, $\omega_0 = 2\pi f_0$; E'_q the q-axis internal transient voltage, in per unit; x_d the d-axis reactance, in per unit; x'_d the

d-axis transient reactance, in per unit; u_f the excitation voltage, the control input, in per unit; M the inertia coefficient, in seconds; D the damping constant, in per unit; T_{d0}, T'_d are time constants, in seconds; P_m the mechanical power, assumed to be constant, in per unit; V_s infinite-bus voltage, in per unit; $x'_{d\Sigma} = x'_d + \frac{1}{2}x_L + x_T$.

Let $x_1 = \delta$, $x_2 = \omega - \omega_0$, $x_3 = E'_q$, $a = \frac{\omega_0}{M}P_m$, $b = \frac{D}{M}$, $c = \frac{\omega_0 V_s}{M x'_{d\Sigma}}$, $d = \frac{1}{T'_d}$, $e = \frac{1}{T_{do}}\frac{x_d - x'_d}{x'_{d\Sigma}}V_s$, $h = \frac{1}{T_{do}}$,

$$H(x) = -cx_3\cos x_1 - ax_1 + \frac{cd}{2e}x_3^2 + \frac{1}{2}x_2^2, \tag{35}$$

and set $y = -c\cos x_1 + \frac{cd}{e}x_3$, then system (34) can be expressed as

$$\begin{cases} \dot{x} = [J(x) - R(x)]\nabla H(x) + g_1(x)u, \\ y = g_1^T(x)\nabla H(x), \end{cases} \tag{36}$$

where $x = (x_1, x_2, x_3)^T$, $u = hu_f$,

$$J(x) = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ R(x) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & \frac{e}{c} \end{pmatrix}, \ g_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Now, we use Theorem 1 to design an observer for the system. First, we check Assumption S1 and S2 (Note: $H(x,0)$, $J(x,0)$ and $R(x,0)$ in both S1 and S2 should be replaced by $H(x)$, $J(x)$ and $R(x)$, respectively). Let $y_1 = R^{\frac{1}{2}}(x)\nabla H(x) \equiv 0$, $u \equiv 0$, then we obtain that $\frac{\partial H}{\partial x_2} = x_2 \equiv 0$, $\frac{\partial H}{\partial x_3} = -c\cos x_1 + \frac{cd}{e}x_3 \equiv 0$. From $x_2 \equiv 0$ and the form of system (36), it is easy to know that $a - cx_3\sin x_1 = 0$. Thus, we have

$$\begin{cases} x_2 = 0, \\ a - cx_3\sin x_1 = 0, \\ -c\cos x_1 + \dfrac{cd}{e}x_3 = 0, \end{cases} \tag{37}$$

which is just the condition that the equilibrium satisfies. Therefore, system (36) is zero-state detectable with respect to y_1 and u, which means that S1 holds.

Next, choose $K(x) = (0, 0, 1)$, $K_1(x) = (0,\ 0,\ \frac{c}{e+2c})$. It can be seen that $W(x) = R(x) + g_1(x)K(x) + K^T(x)g_1^T(x) = \mathrm{Diag}\{0,\ b,\ \frac{e}{c}+2\} \geqslant 0$ and $K(x) = K_1(x)W(x)$. Consider system

$$\dot{x} = \left[\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & \frac{e}{c}+2 \end{pmatrix}\right]\nabla H(x). \tag{38}$$

Similarly, we can show that system (38) is zero-state detectable with respect to $y_2 = W^{\frac{1}{2}}(x)\nabla H(x)$. Thus, S2 is satisfied, too.

From Theorem 1,

$$\dot{\hat{x}} = \left[\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & \frac{e}{c} \end{pmatrix}\right]\nabla H(\hat{x})$$

$$+\begin{pmatrix}0\\0\\1\end{pmatrix}u+\begin{pmatrix}0\\0\\1\end{pmatrix}\left[y+c\cos\hat{x}_1-\frac{cd}{e}\hat{x}_3\right] \tag{39}$$

is an asymptotical observer for system (34), where

$$u=-K(\hat{x})\nabla H(\hat{x})+v=c\cos\hat{x}_1-\frac{cd}{e}\hat{x}_3+v. \tag{40}$$

Digital simulations have been conducted to demonstrate the effectiveness of the observer (39). Choose a set of parameters: $\{\omega_0,\ M,P_m,D,V_s,x_d,x_d',x_{d\Sigma}',T_{do},T_d'\}=\{1,7.6,1,3,1.5,0.9,0.36,0.36,5,5\}$[12]. The simulation results are shown in figs. 2 and 3, from which it can be seen that the observer (39) is very effective.

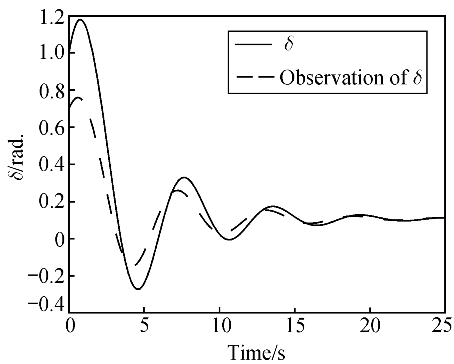

Fig. 2.

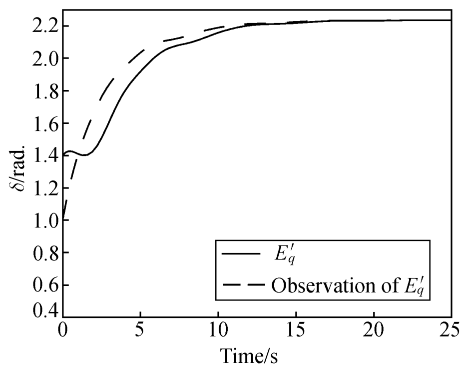

Fig. 3.

4.2 Observer-based H_∞ control law

Consider system (34) affected by disturbances:

$$\begin{cases}\dot{\delta} &= \omega-\omega_0,\\ \dot{\omega} &= \frac{\omega_0}{M}P_m-\frac{D}{M}(\omega-\omega_0)-\frac{\omega_0 E_q' V_s}{M x_{d\Sigma}'}\sin\delta+w_1,\\ \dot{E}_q' &= -\frac{1}{T_d'}E_q'+\frac{1}{T_{do}}\frac{x_d-x_d'}{x_{d\Sigma}'}V_s\cos\delta+\frac{1}{T_{d0}}u_f+w_2,\end{cases} \tag{41}$$

where w_1, w_2 are disturbances. In this case, system (41) can be expressed as

$$\begin{cases}\dot{x}=[J(x)-R(x)]\nabla H(x)+g_1(x)u+g_2 w,\\ y=g_1^T(x)\nabla H(x),\\ z=\rho g_2^T\nabla H(x),\end{cases} \tag{42}$$

where $g_2^T=\begin{pmatrix}0&1&0\\0&0&1\end{pmatrix}$, $w=(w_1,w_2)^T$, z is the chosen penalty signal, $\rho>0$ is the weighting constant, and others are the same as in subsection 4.1.

Now we use Theorem 3 to design an observer-based H_∞ controller for system (41). Choose

$$\rho<\rho^*:=\min\left\{\sqrt{\frac{2D}{M}},\ \sqrt{\frac{2M(x_d-x_d')}{\omega_0 T_{do}}+4}\right\},$$

let the disturbance attenuation level $\gamma>0$ be given. When $\gamma \geqslant \gamma^* := \max\left\{\sqrt{\frac{M}{2D-\rho^2 M}}, \sqrt{\frac{\omega_0 T_{do}}{2M(x_d-x'_d)+\omega_0 T_{do}(4-\rho^2)}}\right\}$, we can show that

$$R(x)+\Lambda[g_1(x)g_1^T(x)-g_2(x)g_2^T(x)] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & b-\Lambda & 0 \\ 0 & 0 & \frac{e}{c} \end{pmatrix} \geqslant 0,$$

$$W(x)-\Lambda g_1(x)g_1^T(x) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & \frac{e}{c}+2-\Lambda \end{pmatrix} \geqslant 0,$$

where $\Lambda = \frac{\rho^2}{2} + \frac{1}{2\gamma^2}$. On the other hand, the system satisfies S1 and S2 (see subsection 4.1). Thus, all the conditions of Theorem 3 are satisfied. From Theorem 3, based on (39), an observer-based H_∞ control law of system (42) can be given by

$$v = -\Lambda\left[y + c\cos\hat{x}_1 - \frac{cd}{e}\hat{x}_3\right]. \tag{43}$$

Substituting it into (40) yields

$$u_f = \frac{1}{h}u = \frac{1-\Lambda}{h}\left[c\cos\hat{x}_1 - \frac{cd}{e}\hat{x}_3\right] - \frac{\Lambda}{h}\,y. \tag{44}$$

Theorem 4. Given a disturbance attenuation level $\gamma > 0$, if $\gamma \geqslant \gamma^*$, then based on (39) an observer-based H_∞ control law of system (41) can be given by (44).

To test the effectiveness of the controller (44), some simulations have been conducted. Choose the same set of parameters as in subsection 4.1. A straightforward computation shows that $\rho^* = 0.8885$, $\gamma^* = 1.1551$. In simulating, we choose $\rho = 0.2$, $\gamma = 1.2$, and assume that a symmetrical three-phase short-circuit fault occurs near the bus during the time period $0.1s$—$0.25s$. The simulation results are shown in figs. 4 and 5.

Figs. 4 and 5 indicate that the closed-loop system is asymptotically stable, and the disturbance caused by the fault is attenuated out in a very short time. Simulation results show that the controller (44) works very well.

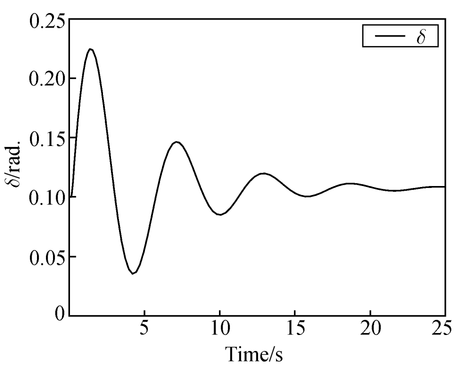

Fig. 4. Swing curve of z.

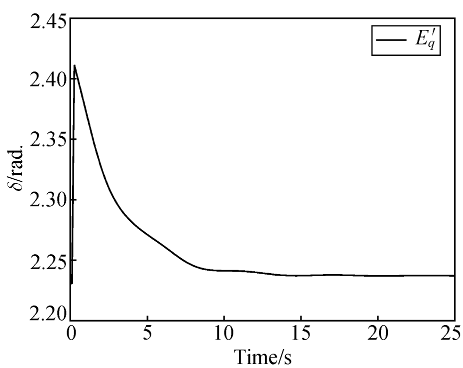

Fig. 5. Swing curve of E'_q.

5 Conclusion

This paper has investigated observer and observer-based H_∞ control design for PCH systems. Based on the systems' structural properties, this paper has provided a new observer design method called Augment Plus Feedback and obtained two kinds of observers: non-adaptive and adaptive ones. Using the obtained observer, this paper has dealt with the H_∞ control design for PCH systems, and proposed an observer-based control design method. As an application to power systems, we have designed an observer and an observer-based H_∞ control law for single-machine infinite-bus systems. Simulations show that both the observer and controller obtained in this paper are very effective.

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