2) Quadratic Stabilization: The linear upper bound of w can be described as

$$\left|\begin{matrix} \sin(\theta_l)-\sin(\hat{\theta}_1) \\ c\theta_m \sin(\theta_m) \end{matrix}\right| \leq \left|\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ c & 0 & c & 0 & 0 & 0 & 0 \end{bmatrix} z\right| \triangleq |C_z z|.$$

When c is given up to 0.6 for simulation, there exists a strictly feasible solution for the problem (19) to find the quadratic Lyapunov function through LMI tool in Matlab [23].

Fig. 2 and 3 show simulation results in terms of the observer error and DSC error responses without considering the model uncertainty (dash-dot line in the figures) and in the presence of $\Delta f_1 = 0.6\theta_m \sin(\theta_m)$ (solid line in the figures). For the initial condition given as $x(0) = [1.2, 0, 2, 0]$, it is shown that all the observer errors converge to zero and so does the controller error. As mentioned in Section III-B, the model uncertainty enters directly the observer error dynamics, so it results in a larger convergence time of estimation error (see Fig. 2). Furthermore, as we expect the quadratic stability of the closed-loop system with knowledge of the precalculated quadratic Lyapunov function shown previously, the tracking error converges to zero with the larger convergence time for the given uncertainty [see Fig. 3(c)]. As a result, in order to find a set of controller and observer gains such that there exists a numerical solution to satisfy the sufficient condition LMI (19), this design procedure can be simplified, based on the separation principle discussed in Section III-B. As long as DSC is designed appropriately to guarantees the quadratic stability for the certain nonlinear system with full state feedback assumption, the tracking error depends on the observer error. In other words, under the same hypothesis, the tracking error will stay within arbitrarily small bound depending on DSC gains if the observer is robust enough for its error to converges to sufficiently small bound under the model uncertainty.

V. Conclusion

This note developed a systematic analysis method, which guarantees quadratic stability via an observer-based DSC for a class of nonlinear systems. Based upon a separation principle for the augmented error dynamics, eigenvalues of the observer and the DSC can be assigned independently. Furthermore, it was shown that model uncertainties in the nonlinear system are related to only the observer error dynamics. This enables us to use DSC design methodology without considering the model uncertainty as long as the observer is robust enough to compensate for the uncertainty. However, the application of this proposed method is limited to the single-input nonlinear systems. Also, more extensive study to design the robust observer to compensate for a larger class of model uncertainties is quite necessary for more various applications. The authors hope that these problems will be studied in future work.

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Direct Adaptive Control for a Class of MIMO Nonlinear Systems Using Neural Networks

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***Abstract*—In this note, direct adaptive neural network (NN) control is studied for a class of multiple-input–multiple-output nonlinear systems based on input–output discrete-time model with unknown interconnections between subsystems. By finding an orthogonal matrix to tune the NN weights, the closed-loop system is proven to be semiglobally uniformly ultimately bounded. The control performance of the closed-loop system is guaranteed by suitably choosing the design parameters.**

***Index Terms*—Adaptive control, discrete-time systems, multiple-input–multiple-output (MIMO) systems, neural networks (NNs).**

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I. Introduction

For ease of controller design, it is convenient to model processes in discrete time because the process data are typically available only at discrete time instants. The inevitable nonlinearity of the dynamics of many processes motivates us to explore nonlinear models. The wide range of applications of linear difference equations naturally lead to the search for nonlinear difference equations that can be used to represent general nonlinear systems. However, it is almost impossible to obtain a discrete-time model directly from discretizing a continuous-time nonlinear model. To make the situation even worse, the model structure may be unclear either. Input–output models are usually used to avoid the embarrassment of discretizing a continuous-time model. Furthermore, there are fewer analytical mathematical tools in discrete-time than that in continuous-time. These may be the reasons why research on control of nonlinear discrete-time systems is much behind than that for continuous-time counter parts.

Applications of neural networks (NNs) in system identification and control have been extensively studied in the past ten years. It has been shown that successful identification and control may be possible using NNs for complex nonlinear dynamic systems whose mathematical models are not available from first principles. In [1] and [2], it was shown that for stable and efficient online control using the backpropagation (BP) learning algorithm, the identification must be sufficiently accurate before the control action is initiated. In practical applications, it is desirable to have a systematic method of ensuring stability, robustness, and performance properties of the overall system. Recently, several good NN control approaches have been proposed based on Lyapunov's stability theory [3]–[7]. One main advantage of these schemes is that the adaptive laws were derived based on Lyapunov synthesis and, therefore, guarantee the stability of continuous-time systems without the requirement for offline training.

However, most of these good results are restricted in continuous-time domain. Due to the much difference between continuous-time domain and discrete-time domain, a stable control system designed in continuous-time may become unstable in discrete-time, thus we may run into troubles when we implement these NN controllers in a digital control system in which the data are typically available only at discrete time instants. Therefore, discrete-time NN control is necessary and significant. In [1], input–output-based NN control is studied for a class of nonlinear dynamical discrete-time systems. Further theoretical foundations and insights, which are essential for the design of NN control based on inverse controller, are provided in [8], in which the relative degree of discrete-time systems is well explained. In [9], multilayer NN is used in control of a class of discrete-time nonlinear systems with general relative degree. As mentioned previously, offline training is needed to provide a good starting point for the online adaptive control which uses the backpropagation learning algorithm. In [10], direct adaptive NN control is presented for a class of discrete-time unknown nonlinear systems with general relative degree in the presence of bounded disturbances. The NN control scheme can be applied to the system without off-line training. Unfortunately, all these good NN controllers are designed for single-input–single-output (SISO) discrete-time nonlinear systems. For multiple-input–multiple-output (MIMO) nonlinear systems, how to tune the NN weights is still an open problem, especially when there exists unknown strong interconnections between subsystems. In [11], the NN control is studied for a very special class of discrete-time MIMO nonlinear systems with relative degree of one and without any interconnections between subsystems. For a class of MIMO sampled-data nonlinear systems under the assumption that all the states are available, NN-based adaptive control is studied in [12] for a class of discrete-time nonlinear systems, which is derived by discretizing the original continuous-time nonlinear model using second-order approximation. However, it still needs further investigation to show that whether this discrete-time model structure can represent the original system. Some other elegant works in discrete-time domain can be found in [13]–[16].

In this note, using high-order NNs (HONNs), adaptive controller design is investigated for a class of unknown discrete-time MIMO nonlinear systems with unknown interconnections between subsystems. The controller can be applied directly to the system without the requirement of offline training if the node number of the NNs is sufficient large. By finding an orthogonal matrix to tune the NN weight matrix, the overall system is proved to be SGUUB, and the tracking error converges to a small neighborhood of the origin.

The main contributions of this note are as follows.

i) An effective control scheme is proposed for a class of MIMO discrete-time systems with complex subsystem interconnections.
ii) SGUUB stability is guaranteed in the presence of unknown bounded disturbances.
iii) Different from previous one step parameter update law, new τ-step update laws are essential to solve the problem of τ-step ahead predictor
iv) By finding an orthogonal matrix $Q(k)$ to tune the NN weights, meaningful stability results is elegantly provided without much technical difficulty.

II. System Dynamics and Stability Notions

In discrete-time formulations, one of the most popular nonlinear representations is the nonlinear auto regressive moving average with exogenous inputs (NARMAX) model [17]. Many $p \times p$ MIMO processes can be represented by a NARMAX model known as τ-step ahead observer equation as follows:

$$\begin{aligned} y(k+\tau) = {} & F_\tau(Y(k), U_{k-1}(k), D_{k-1}(k), \bar{d}(k)) \\ & + G_\tau(Y(k), U_{k-1}(k))u(k) + d(k+\tau-1) \end{aligned} \tag{1}$$

where τ is the system delay, or the relative degree of the system, $y(k) = [y_1(k), y_2(k), \ldots, y_p(k)]^T \in R^p, u(k) = [u_1(k), u_2(k), \ldots, u_p(k)]^T \in R^p$ are system outputs and inputs, respectively, $d(k) = [d_1(k), d_2(k), \ldots, d_p(k)]^T$ denotes the external unmeasured disturbance vector bounded by a known constant $d_0 > 0$, i.e., $\|d(k)\| \leq d_0, Y(k)$ is a vector containing current and past outputs, $U_{k-1}(k)$ is a vector containing only past inputs, and $D_{k-1}(k)$ is a vector containing the past disturbances. In particular, they are defined as

$$\bar{d}(k) = [d(k+\tau-2), \ldots, d(k)]^T, \qquad \text{if } \tau \geq 2$$

$$\begin{aligned} U_{k-1}(k) = {} & [u_1(k-1), \ldots, u_1(k-m_1) \\ & u_2(k-1), \ldots, u_2(k-m_2) \\ & \cdots \\ & u_p(k-1), \ldots, u_p(k-m_p)]^T \end{aligned} \tag{2}$$

$$\begin{aligned} Y(k) = {} & [y_1(k), \ldots, y_1(k-n_1+1) \\ & y_2(k), \ldots, y_2(k-n_2+1) \\ & \cdots \\ & y_p(k), \ldots, y_p(k-n_p+1)]^T \end{aligned} \tag{3}$$

$$\begin{aligned} D_{k-1}(k) = {} & [d_1(k-1), \ldots, d_1(k-t_1+1) \\ & d_2(k-1), \ldots, d_2(k-t_2+1) \\ & \cdots \\ & d_p(k-1), \ldots, d_p(k-t_p+1)]^T \end{aligned}$$

nonlinear system function $F_\tau(*)$, and matrix, $G_\tau(*)$, are defined as

$$F_\tau(k) = [f_{\tau_i}(Y(k), U_{k-1}(k), D_{k-1}(k), \bar{d}(k))] \in R^p$$
$$G_\tau(k) = [g_{\tau_{ij}}(Y(k), U_{k-1}(k))] \in R^{p \times p}$$

n_i denotes the length of the ith subsystem's outputs, and m_i is the length of the ith subsystem's inputs, which satisfies $m_i < n_i, i = 1, \ldots, p; t_i$ is the length of the ith disturbance, $i = 1, \ldots, p; f_{\tau_i}(Y(k), U_{k-1}(k), D_{k-1}(k), \bar{d}(k))$ and $g_{\tau_{ij}}(Y(k), U_{k-1}(k)), i, j = 1, \ldots, p$ are smooth nonlinear functions.

Assumption 1: The function vector $F_\tau(Y(k), U_{k-1}(k), D_{k-1}(k), \bar{d}(k))$ is locally Lipschitz in $D_{k-1}(k)$ and $\bar{d}(k)$ at $(0, 0)$, i.e., there exist Lipschitz constants L_1 and L_2 such that

$$\begin{aligned}&\|F_\tau(Y(k), U_{k-1}(k), D_{k-1}(k), \bar{d}(k))\\&\quad - F_\tau(Y(k), U_{k-1}(k), 0, 0)\|\\&\quad \le L_1\|D_{k-1}(k)\| + L_2\|\bar{d}(k)\|\end{aligned}$$

with L_1 and L_2 being positive constants.

Suppose that the objective is to design control $u(k)$ to drive the system output $y(k)$ follow a known and bounded trajectory $y_d(k) = [y_{d_1}(k), y_{d_2}(k), \ldots, y_{d_p}(k)]^T$.

Definition 1: The future outputs, $y(k+i), i > 0$, of discrete-time system (1) are said to be quasi-determined future outputs, if the future outputs are independent of the current control $u(k)$.

From Definition 1, it is clear that future outputs $y(k+1), \ldots, y(k+\tau-1)$ in (1) are all *quasi-determined future outputs* as they are independent of the current control $u(k)$ though they are influenced by the unknown external disturbances of the past and the future. As the external disturbances are unknown, their effects could not be cancelled through control action. Thus, we are interested in designing robust control for (1) using the results for the ideal case when the unknown disturbances are isolated.

Assumption 2: The desired trajectory $y_d(k) \in \Omega_{yd} \subset R^p, \forall k > 0$ is smooth and known, where Ω_{yd} is a small subset of Ω_y and $\Omega_y \triangleq \{\chi(k) | \chi(k) = y(k)\} \subset R^p$.

Define error vector $e(k) = y(k) - y_d(k) = [e_1(k), e_2(k), \ldots, e_p(k)]^T$. Noting (1), the error equation of $e(k)$ can then be written as

$$\begin{aligned}e(k+\tau) = F_\tau(k) - y_d(k+\tau) + G_\tau(k)u(k)\\ + d(k+\tau-1).\end{aligned} \tag{4}$$

Definition 2: The solution of (4) is semiglobally uniformly ultimately bounded (SGUUB), if for any Ω_y and Ω_u, compact subsets of R^p and all $y(k_0 - i) \in \Omega_y, i = 0, \ldots, \max\{n_1, \ldots, n_p\} - 1, u(k_0 - j) \in \Omega_u, j = 1, \ldots, \tau + \max\{m_1, \ldots, m_p\}$, and all *quasi-determined future outputs* are in Ω_y, there exist an $\epsilon > 0$, and a number N such that $\|e(k)\| < \epsilon$ for all $k \ge k_0 + N$.

Lemma 1: Consider the linear time-varying discrete-time system

$$x(k+1) = A(k)x(k) + Bu(k) \quad y(k) = Cx(k) \tag{5}$$

where $A(k), B$, and C are appropriately dimensional matrices with B and C are constant matrices. Let $\Phi(k_1, k_0)$ be the state transition matrix corresponding to $A(k)$ for system (5), i.e., $\Phi(k_1, k_0) = \prod_{k=k_0}^{k_1-1} A(k)$. If $\|\Phi(k_1, k_0)\| < 1, \forall k_1 > k_0 \ge 0$, then (5) is: i) globally exponentially stable for the unforced system (i.e., $u(k) = 0$); and ii) bounded-input–bounded-output (BIBO) stable.

III. Desired Control and HONN

A. Desired Control

In this section, the existence of an ideal control is investigated. Error dynamics (4) can be written as

$$\begin{aligned}e(k+\tau) = F(Y(k), U_{k-1}(k)) - y_d(k+\tau)\\ + G_\tau(Y(k), U_{k-1}(k))u(k)\\ + \Delta F(k) + d(k+\tau-1)\end{aligned} \tag{6}$$

where

$$\begin{aligned}F(Y(k), U_{k-1}(k)) &= F_\tau(Y(k), U_{k-1}(k), 0, 0)\\ \Delta F(k) &= F_\tau(Y(k), U_{k-1}(k), D_{k-1}, \bar{d}(k))\\ &\quad - F(Y(k), U_{k-1}(k)).\end{aligned}$$

We can see that $\Delta F(k)$ is generated by the external disturbances. By Assumption 1 and the boundedness of disturbances, we can conclude that $\Delta F(k) \le L_1\|D_{k-1}(k)\| + L_2\|\bar{d}(k)\|$ is bounded.

If $F(Y(k), U_{k-1}(k))$ and $G_\tau(Y(k), U_{k-1}(k))$ are known and $G_\tau^{-1}(Y(k), U_{k-1}(k))$ exists, then we can choose the ideal control input as

$$u^*(k) = G_\tau^{-1}(Y(k), U_{k-1}(k))\Big[y_d(k+\tau) - F(Y(k), U_{k-1}(k))\Big]. \tag{7}$$

Thus, we have the closed-loop error equation

$$\begin{aligned}\|e(k+\tau)\| &= \|\Delta F(k) + d(k+\tau-1)\|\\ &\le L_1\|D_{k-1}(k)\| + L_2\|\bar{d}(k)\| + \|d(k+\tau-1)\|.\end{aligned}$$

If there are no disturbances, i.e., $D_{k-1}(k) = 0$ and $d(k+\tau-1) = 0$, we have $e(k+\tau) = 0$, which is achieved in τ steps. Under this condition, the desired control, $u^*(k)$, is the so-called τ-step deadbeat control, or *exact tracking* control, which is well defined and has been proven to be unique in [8]. In practice, $u^*(k)$ is not realizable as $F(k)$ and $G_\tau(k)$ are unknown. In the following, adaptive NNs shall be used to approximate the unknown desired control $u^*(k)$, which is introduced for analytical purpose only. Note that saturation control is out of the scope of the technical notes, and further research will be carried out for other types of ideal controls rather than deadbeat control.

Remark 1: It is obvious that if there is no disturbances in the system, i.e., $D_{k-1}(k) = 0$ and $d(k) = 0$, the tracking error $e(k+\tau) = 0$. If $D_{k-1}(k) \ne 0$ and $d(k) \ne 0$, the error equation is $e(k+\tau) = \Delta F(k) + d(k+\tau-1)$, thus, exact tracking cannot be obtained though bounded due to Assumption 1. Instead, in this note, we propose SGUUB stability of the system in the presence of the unknown bounded disturbances.

Assumption 3: The desired control $u^*(k)$ is within the compact set $\Omega_{u^*} \subset \Omega_u \quad \forall y(k-i) \in \Omega_y \subset R^p, \quad i = 0, \ldots, \max\{n_1, \ldots, n_p\} - 1$ and $\forall u(k-j) \in \Omega_u \subset R^p, j = 1, \ldots, \tau + \max\{m_1, \ldots, m_p\}$.

The desired trajectory is assumed to be chosen such that the system can achieve since it is meaningless to ask the system to track an unrealistic trajectory. Assumption 3 is only introduced for mathematical rigor (stating that the desired control u^* is within the capability of the control system) as the boundedness of the actual control $u(k)$ is established via Lyapunov analysis later.

B. NN Approximation

In control engineering, NN is usually used as a function approximator to emulate unknown nonlinear ideal control $u^*(k)$. For convenience, let us consider the high-order NNs [18]

$$\begin{aligned}\phi(W, z) = W^T S(z), \qquad W \in R^{l\times p} \quad \text{and} \quad S(z) \in R^l\\ S(z) = [s_1(z), s_2(z), \ldots, s_l(z)]^T\end{aligned} \tag{8}$$

$$s_i(z) = \prod_{j\in I_i} [s(z_j)]^{d_j(i)}, \qquad i = 1, 2, \ldots, l \tag{9}$$

where $z = [z_1, z_2, \ldots, z_q]^T \in \Omega_z \subset R^q$, positive integer l denotes the NN node number, and p is the dimension of function vector, $\{I_1, I_2, \ldots, I_l\}$ is a collection of l not-ordered subsets of $\{1, 2, \ldots, q\}$ and $d_j(i)$ are nonnegative integers, W is an adjustable synaptic weight matrix, $s(z_j)$ is chosen as hyperbolic tangent function

$$s(z_j) = \frac{e^{z_j} - e^{-z_j}}{e^{z_j} + e^{-z_j}}.$$

By examining (7), the desired control input $u^*(k)$ is a function of $Y(k)$, $U_{k-1}(k)$ and $y_d(k+\tau)$. Thus, there exist ideal weights W^* such that the smooth function vector $u^*(k)$ can be approximated by an ideal NN on a compact set $\Omega_z \subset R^q$

$$u^*(k) = W^{*T} S(\bar{z}(k)) + \epsilon_z \tag{10}$$

where

$$\bar{z}(k) = \begin{bmatrix} Y(k) \\ U_{k-1}(k) \\ y_d(k+\tau) \end{bmatrix} \in \Omega_z \subset R^q$$
$$\epsilon_z = [\epsilon_{z_1}, \ldots, \epsilon_{z_p}]^T$$

with $q = \sum_{i=1}^{p}(n_i + m_i + 1)$ and ϵ_z is the bounded NN approximation error vector satisfying $\|\epsilon_z\| \leq \epsilon_0$ on the compact set, which can be reduced by increasing the number of the adjustable weights. The ideal weight matrix W^* is an "artificial" quantity required for analytical purpose, and is defined as that minimizes $\|\epsilon_z\|$ for all $\bar{z} \in \Omega_z \subset R^q$ in a compact region, i.e.,

$$W^* \triangleq \arg\min_{W \in \Omega_w} \left\{ \sup_{z \in \Omega_z} |u^*(k) - W^T S(\bar{z}(k))| \right\}$$
$$\Omega_z \subset R^q \text{ and compact set } \Omega_w \subset R^{l \times p} \quad (11)$$

In general, the ideal NN weight matrix, W^*, is unknown though constant, its estimate, $\hat{W}$, should be used for controller design which will be discussed in Section IV.

Though HONN is used for analysis, other linear-in-parameters function approximators such as polynomials, splines, fuzzy systems and wavelet networks, among others, can also be used to construct the controller presented in this note without any difficulty as stated in [7].

IV. Controller Design and Stability Analysis

In this section, we present the robust adaptive NN controller for (1) under some mild conditions.

Assumption 4: For (1), assume $G_\tau(k)$ is a full-rank matrix, and there exists an orthogonal matrix $Q(k) \in R^{p \times p}$, such that the eigenvalues of $Q(k)G_\tau^{-1}(k)$ are upper and lower bounded by $0 < (b/(1-\sigma\gamma)) \leq \lambda\{Q(k)G_\tau^{-1}(k)\} \leq a$, where a and b are constant numbers, $\sigma > 0, \gamma > 0$, and $0 < \sigma\gamma < 1$ (γ is the adaptation gain and σ is a positive constant indicates the leakage term of σ-modification used in weight update and $\lambda\{M\}$ denotes the eigenvalue of M).

Remark 2: If $G_\tau(k)$ is totally unknown, there is no valid method to construct such a $Q(k)$. However, if we know some properties of $G_\tau(k)$, then we may select such a $Q(k)$ that satisfies the requirement. For example, if all the eigenvalues of $G_\tau(k)$ are larger than zero, then we can select identity matrix $Q(k) = I$; if all the eigenvalues of $G_\tau(k)$ are less than zero, then we can choose $Q(k) = -I$. In practice, there are some physical systems possessing such a nice property, which include rigid robotic arms, and flexible joint robots, where the input matrix $G_\tau = M^{-1}(q), 0 < \alpha_1 I \leq M(q) \leq \alpha_2 I$ with q denotes the coordinates, M denotes the inertia matrix and constants α_1 and $\alpha_2 > 0$.

Once we find such an orthogonal matrix $Q(k)$, we are ready to present the direct adaptive controller and the weights updating law as

$$u(k) = \hat{W}^T(k)S(\bar{z}(k)) \quad (12)$$
$$\hat{W}(k+1) = \hat{W}(k-\tau+1) - \Gamma\Big[S(\bar{z}(k-\tau+1))e^T(k+1)Q(k-\tau+1) + \sigma\hat{W}(k-\tau+1)\Big] \quad (13)$$

where $\Gamma = \gamma I$ is a diagonal matrix with $\gamma > 0, \sigma$ is a positive constant number, $\hat{W}(k) \in R^{l \times p}$ and $S(\bar{z}(k)) \in R^l$. The σ-modification is used here to eliminate the need of persistent exciting (PE) condition for parameter convergence. In comparison with the standard parameter adaptation algorithms, it should be noted that parameter adaptation algorithm (13) is of τ steps ahead in order to solve the control problem of general τth-order nonlinear systems. In fact, the current estimate $\hat{W}(k)$ is deviated from the estimate, $\hat{W}(k-\tau)$, of τ steps earlier rather than that of the previous step. Without the introduction of the τ step update law, the problem is not solvable.

Substituting controller (12) into (6), the error (6) can be rewritten as

$$e(k+\tau) = F(Y(k), U_{k-1}) - y_d(k+\tau) + G_\tau(k)\hat{W}^T(k)S(\bar{z}(k)) + \Delta F(k) + d(k+\tau-1). \quad (14)$$

Adding and subtracting $G_\tau(k)u^*(\bar{z}(k))$ on the right-hand side of (14) and noting (10), we have

$$e(k+\tau) = F(Y(k), U_{k-1}) - y_d(k+\tau) + G_\tau(k)u^*(k) + G_\tau(k)[\hat{W}^T(k)S(\bar{z}(k)) - W^{*T}S(\bar{z}(k)) - \epsilon_z] + \Delta F(k) + d(k+\tau-1). \quad (15)$$

Substituting (7) into (15) leads to

$$e(k+\tau) = G_\tau(k)[\tilde{W}^T(k)S(\bar{z}(k)) - \epsilon_z] + D(k) \quad (16)$$

where $\tilde{W}(k) = \hat{W}(k) - W^*$ and $D(k) = \Delta F(k) + d(k+\tau-1)$. Since $\Delta F(k), d_{k+\tau-1}$ are due to the existence of external disturbances and they are bounded, we can consider that $D(k)$ is bounded by a positive constant D_0, i.e., $\|D(k)\| < D_0$.

Control input (12) can be rewritten as

$$u(k) = (\tilde{W}(k) + W^*)^T S(\bar{z}(k)) = u^*(k) + \tilde{W}^T(k)S(\bar{z}(k)) - \epsilon_z.$$

Because $u^*(k) \in \Omega_{u^*}$ and Ω_{u^*} is a subset of Ω_u under Assumption 3, there must exist a nonzero compact set $\Omega_w \subset R^{l \times p}$ such that any $\tilde{W}(k) \in \Omega_w$ guarantees $u(k) \in \Omega_u$. Since Ω_{yd} is a small subset of Ω_y under Assumption 2, there must exist a large enough compact set $\Omega_e \subset R^p$, such that for any $e(k) \in \Omega_e$ guarantees that $y(k) \in \Omega_y$.

Theorem 1: Consider the closed-loop system consisting of system (4), controller (12), and adaptation law (13). There exist compact sets $\Omega_{y_0} \subset \Omega_y, \Omega_{w_0} \subset \Omega_w$, and positive constants l^*, γ^*, and σ^* such that if

i) Assumptions 1)–4) being satisfied, the condition at time instant k_0 is initialized as

$$y(k_0 - j) \in \Omega_{y_0}, \qquad j = 0, \ldots, \max\{n_1, \ldots, n_p\} - 1$$
$$u(k_0 - j) \in \Omega_u, \qquad j = 1, \ldots, \tau + \max\{m_1, \ldots, m_p\}$$
$$\tilde{W}(k_0 - j) \in \Omega_{w_0}, \qquad j = 0, \ldots, \tau - 1;$$

ii) the *quasi-determined future outputs* at time instant $k_0, y(k_0+1), \ldots, y(k_0+\tau-1)$ are all in compact set Ω_y;

iii) the design parameters are suitably chosen such that $l > l^*, \sigma < \sigma^*$ and $\gamma < \gamma^*$ with γ being the eigenvalue of Γ;

then, the closed-loop system is SGUUB.

Proof: We have illustrated that there exists an ideal control $u^*(k)$ which guarantees that $e(k+\tau) = 0$ if there is no unknown disturbance. Since all the assumptions are only valid in compact set Ω_y and Ω_u, we must prove that the system outputs and inputs will remain in these compact sets all the time indeed. At time instant k, suppose that all past inputs are in Ω_u, current output and all past outputs are in Ω_y, the *quasi-determined future outputs*, $y(k+1), \ldots, y(k+\tau-1)$, are all in Ω_y, all past NN weight errors are in Ω_w, we will prove that all these conditions still hold after time instant k and the tracking error converges into a small neighborhood of zero.

Choose the Lyapunov function candidate as

$$J(k) = b\sum_{j=0}^{\tau-1}\text{tr}\{e(k+j)e^T(k+j)\} + \sum_{j=0}^{\tau-1}\text{tr}\{\tilde{W}^T(k+j)\Gamma^{-1}\tilde{W}(k+j)\} \quad (17)$$

where b is a positive constant, defined in Assumption 4. Apparently, the Lyapunov function candidate $J(k)$ contains the states of the error dynamics of the systems (15), and the parameter adaptation (13). Note that the future variables $e(k+1), \ldots, e(k+\tau-1)$ and $\tilde{W}(k+1), \ldots \tilde{W}(k+$

$\tau-1)$, are all quasi-determined at time instant k as they are independent of current control $u(k)$. We have shown that $y(k+\tau-1),\ldots,y(k+1)$ are all independent of $u(k)$, so are $e(k+\tau-1),\ldots,e(k+1)$. For the same reason, it can be shown that $\tilde{W}(k+\tau-1),\ldots,\tilde{W}(k+1)$ are all determined at time instant k. For example

$$\begin{aligned}\tilde{W}(k+\tau-1) = \tilde{W}(k-1) &- \Gamma\Big[S(\bar{z}(k-1))e^T(k+\tau-1)Q(k-1)\\ &+ \sigma\hat{W}(k-1)\Big]\end{aligned}$$

is uniquely determined since: i) $e^T(k+\tau-1)$ is quasi-determined, and ii) all other signals are well defined at time instant k.

The first difference of (17) along (12), (13), and (16) is given by

$$\begin{aligned}\Delta J(k) &= be^T(k+\tau)e(k+\tau) - be^T(k)e(k)\\ &\quad+ \mathrm{tr}\{\tilde{W}^T(k+\tau)\Gamma^{-1}\tilde{W}(k+\tau)\}\\ &\quad- \mathrm{tr}\{\tilde{W}^T(k)\Gamma^{-1}\tilde{W}(k)\}\\ &= be^T(k+\tau)e(k+\tau) - be^T(k)e(k)\\ &\quad+ \mathrm{tr}\{-\sigma\tilde{W}^T(k)\hat{W}(k) - \sigma\hat{W}^T(k)\tilde{W}(k)\}\\ &\quad+ \mathrm{tr}\{-\tilde{W}^T(k)S(\bar{z}(k))e^T(k+\tau)Q(k)\}\\ &\quad+ \mathrm{tr}\{-Q^T(k)e(k+\tau)S^T(\bar{z}(k))\tilde{W}(k)\}\\ &\quad+ \mathrm{tr}\{\sigma\hat{W}^T(k)\Gamma S(\bar{z}(k))e^T(k+\tau)Q(k)\}\\ &\quad+ \mathrm{tr}\{\sigma Q^T(k)e(k+\tau)S^T(\bar{z}(k))\Gamma\hat{W}(k)\}\\ &\quad+ \mathrm{tr}\{\sigma^2\hat{W}^T(k)\Gamma\hat{W}(k)\} + \mathrm{tr}\{Q^T(k)e(k+\tau)\\ &\quad\times S^T(\bar{z}(k))\Gamma S(\bar{z}(k))e^T(k+\tau)Q(k)\}\\ &= be^T(k+\tau)e(k+\tau) - be^T(k)e(k)\\ &\quad- 2\sigma\,\mathrm{tr}\{\tilde{W}^T(k)\hat{W}(k)\} + \sigma^2\mathrm{tr}\{\hat{W}^T(k)\Gamma\hat{W}(k)\}\\ &\quad- 2\mathrm{tr}\{\tilde{W}^T(k)S(\bar{z}(k))e^T(k+\tau)Q(k)\}\\ &\quad+ 2\sigma\,\mathrm{tr}\{\hat{W}^T(k)\Gamma S(\bar{z}(k))e^T(k+\tau)Q(k)\}\\ &\quad+ \mathrm{tr}\{Q^T(k)e(k+\tau)S^T(\bar{z}(k))\Gamma S(\bar{z}(k))\\ &\quad\times e^T(k+\tau)Q(k)\}.\end{aligned}$$

Noting that

$$\begin{aligned}&-2\sigma\,\mathrm{tr}\{\tilde{W}^T(k)\hat{W}(k)\}\\ &\quad= -\sigma\|\tilde{W}\|_F^2 - \sigma\|\hat{W}\|_F^2 + \sigma\|W^*\|_F^2\\ &\sigma^2\mathrm{tr}\{\hat{W}^T(k)\Gamma\hat{W}(k)\} = \sigma^2\gamma\|\hat{W}\|_F^2\\ &-2\mathrm{tr}\{\tilde{W}^T(k)S(\bar{z}(k))e^T(k+\tau)Q(k)\}\\ &\quad= -2e^T(k+\tau)Q(k)\tilde{W}^T(k)S(\bar{z}(k))\\ &2\sigma\,\mathrm{tr}\{\hat{W}^T(k)\Gamma S(\bar{z}(k))e^T(k+\tau)Q(k)\}\\ &\quad= 2\sigma\gamma e^T(k+\tau)Q(k)\hat{W}^T(k)S(\bar{z}(k))\\ &\mathrm{tr}\{Q^T(k)e(k+\tau)S^T(\bar{z}(k))\Gamma S(\bar{z}(k))e^T(k+\tau)Q(k)\}\\ &\quad= S^T(\bar{z}(k))\Gamma S(\bar{z}(k))e^T(k+\tau)e(k+\tau)\\ &Q(k)Q^T(k) = Q^T(k)Q(k) = I\end{aligned}$$

we have

$$\begin{aligned}\Delta J(k) &= be^T(k+\tau)e(k+\tau) - be^T(k)e(k) - \sigma\|\tilde{W}\|_F^2\\ &\quad- \sigma(1-\sigma\gamma)\|\hat{W}\|_F^2 + \sigma\|W^*\|_F^2\\ &\quad- 2e^T(k+\tau)Q(k)\tilde{W}^T(k)S(\bar{z}(k))\\ &\quad+ 2\sigma\gamma e^T(k+\tau)Q(k)\hat{W}^T(k)S(\bar{z}(k))\\ &\quad+ S^T(\bar{z}(k))\Gamma S(\bar{z}(k))e^T(k+\tau)e(k+\tau).\end{aligned}$$

Noticing (16), we can obtain

$$\begin{aligned}\tilde{W}^TS(\bar{z}(k)) &= G_\tau^{-1}(k)[e(k+\tau) - D(k)] + \epsilon_z\\ \hat{W}^TS(\bar{z}(k)) &= G_\tau^{-1}(k)[e(k+\tau) - D(k)]\\ &\quad+ \epsilon_z + W^{*T}(\bar{z}(k)).\end{aligned}$$

Thus

$$\begin{aligned}\Delta J(k) &= be^T(k+\tau)e(k+\tau) - be^T(k)e(k) - \sigma\|\tilde{W}\|_F^2\\ &\quad- \sigma(1-\sigma\gamma)\|\hat{W}\|_F^2 + \sigma\|W^*\|_F^2\\ &\quad- 2e^T(k+\tau)Q(k)G_\tau^{-1}(k)e(k+\tau)\\ &\quad+ 2e^T(k+\tau)\beta(k)\\ &\quad+ 2\sigma\gamma e^T(k+\tau)Q(k)G_\tau^{-1}(k)e(k+\tau)\\ &\quad+ 2\sigma\gamma e^T(k+\tau)\alpha(k)\\ &\quad+ S^T(\bar{z}(k))\Gamma S(\bar{z}(k))e^T(k+\tau)e(k+\tau)\end{aligned}$$

where $\alpha(k) = Q(k)[-G_\tau^{-1}(k)D(k) + \epsilon_z + W^{*T}S(\bar{z}(k))]$ and $\beta(k) = Q(k)[-G_\tau^{-1}(k)D(k)+\epsilon_z]$. Since ϵ_z, $D(k)$ and $W^{*T}S(\bar{z}(k))$ are all bounded, it is reasonable to assume that both $\alpha(k)$ and $\beta(k)$ are bounded. For convenience of analysis, let $\alpha_i(k) \le \alpha_{0i}$ and $\beta_i(k) \le \beta_{0i}$, where α_{0i} and β_{0i} denote the ith elements of constant vectors, α_0 and β_0, respectively, which are only introduced to establish the stability results rather than for controller design.

Remark 3: From Assumption 4, we know that $Q(k)G_\tau^{-1}(k)$ has p linearly independent eigenvectors, and can be written in the form $Q(k)G_\tau^{-1}(k) = T(k)\Lambda(k)T^{-1}(k)$, where $\Lambda(k)$ is a diagonal matrix with the eigenvalues of $Q(k)G_\tau^{-1}(k)$ as its entries and $T(k)$ is the corresponding invertible matrix consists of the eigenvectors. The technical benefit due to the existence of matrix $Q(k)$, subsequently, the existence of matrix $T(k)$, is apparent in merging the three items, $(2(1-\sigma\gamma)/(b))Q(k)G_\tau^{-1}(k), I$ and $(\gamma(1+\sigma+l)/b)I$ in (18), to continue the meaningful stability deduction of the note as shown here.

Combining with the following facts:

$$\begin{aligned}&S^T(\bar{z}(k))\Gamma S(\bar{z}(k)) = \gamma S^T(\bar{z}(k))S(\bar{z}(k))\\ &S^T(\bar{z}(k))S(\bar{z}(k)) < l\\ &2e^T(k+\tau)\beta(k) \le \gamma e^T(k+\tau)e(k+\tau) + \frac{1}{\gamma}\beta_0^T\beta_0\\ &2\sigma\gamma e^T(k+\tau)\alpha(k) \le \sigma\gamma e^T(k+\tau)e(k+\tau) + \sigma\gamma\alpha_0^T\alpha_0\end{aligned}$$

we further obtain

$$\begin{aligned}\Delta J(k) &\le -be^T(k+\tau)\left\{\frac{2(1-\sigma\gamma)}{b}Q(k)G_\tau^{-1}(k) - I\right.\\ &\quad\left.- \gamma\frac{1+\sigma+l}{b}I\right\}e(k+\tau) - be^T(k)e(k)\\ &\quad-\sigma\|\tilde{W}\|_F^2 - \sigma(1-\sigma\gamma)\|\hat{W}\|_F^2 + C_0\\ &\le -be^T(k+\tau)T(k)\left\{\frac{2(1-\sigma\gamma)}{b}\Lambda(k) - I\right.\\ &\quad\left.- \gamma\,\frac{1+\sigma+l}{b}I\right\}T^{-1}(k)e(k+\tau)\\ &\quad-be^T(k)e(k) + C_0\end{aligned} \tag{18}$$

with $C_0 = \sigma\|W^*\|_F^2 + \sigma\gamma\alpha_0^T\alpha_0 + (1/\gamma)\beta_0^T\beta_0$ being a positive constant. From Assumption 4, we know that

$$\frac{1-\sigma\gamma}{b}\Lambda(k) > I \text{ and } 0 < \sigma\gamma < 1\ (\sigma > 0 \text{ and } \gamma > 0).$$

Accordingly, we have

$$\begin{aligned}\Delta J(k) &\le -be^T(k+\tau)T(k)\left\{I - \gamma\frac{1+\sigma+l}{b}I\right\}\\ &\quad\times T^{-1}(k)e(k+\tau) - be^T(k)e(k) + C_0\\ &\le -\{b - \gamma(1+\sigma+l)\}e^T(k+\tau)e(k+\tau)\\ &\quad-be^T(k)e(k) + C_0.\end{aligned}$$

If we choose the design parameters as follows:

$$\gamma < \frac{b}{1+\sigma+l} \tag{19}$$

then $\Delta J(k) \le 0$ once any of the tracking errors $|e_i(k)|, i = 1,\ldots,p$ is larger than $\sqrt{(C_0/b)}$. Furthermore, the tracking error $e(k)$ will converge to the compact set denoted by

$$\Omega_{e_0} \triangleq \left\{ e(k) \middle| |e_i(k)| \le \sqrt{\frac{C_0}{b}}, \qquad i = 1,2,\ldots,p \right\}. \tag{20}$$

Due to negativeness of $\Delta J(k)$, we can conclude that $e(k+\tau)$ must converges to the compact set Ω_{e_0} if $e(k)$ is outside of Ω_{e_0} and all other conditions hold. Thus, $y(k+\tau) \in \Omega_y$ will still hold if $\Omega_{e_0} \subset \Omega_e$.

By subtracting W^* from both sides of weights updating (13), it can be rewritten as

$$\begin{aligned}\tilde{W}(k+1) &= (1-\sigma\gamma)\tilde{W}(k-\tau+1) - \sigma\gamma W^* \\ &\quad - \Gamma S(\bar{z}(k-\tau+1))e^T(k+1)Q(k-\tau+1).\end{aligned}$$

Since $e(k+1)$ converges to the small compact set Ω_{e_0} and all the elements of $S(\bar{z}(k))$ are less than 1, $Q(k-\tau+1)$ and W^* are also bounded, thus, noting Lemma 1 and $0 < 1-\sigma\gamma < 1, \tilde{W}(k)$ will be bounded in a compact set denoted by Ω_{we} by recursive computation if its initial value $\tilde{W}(k_0)$ is bounded. It is obvious that we can initialize $\tilde{W}(k_0)$ to be in the compact set $\Omega_{w_0} \subset \Omega_w$. Hence, according to $\hat{W}(k) = \tilde{W}(k) + W^*$, we conclude $\hat{W}(k)$ is bounded without the need of PE condition. Thus, $u(k) \in \Omega_u$ will still hold if $\Omega_{we} \subset \Omega_w$.

Finally, if we initialize system at time instant k_0 as follows:

$$\begin{aligned} y(k_0 - j) &\in \Omega_{y_0}, \qquad j = 0,\ldots,\max\{n_1,\ldots,n_p\} - 1 \\ u(k_0 - j) &\in \Omega_u, \qquad j = 1,\ldots,\max\{m_1,\ldots,m_p\} + \tau \\ \tilde{W}(k_0 - j) &\in \Omega_{w_0}, \qquad j = 0,\ldots,\tau - 1 \end{aligned}$$

and we choose suitable parameters γ, l and σ according to (19), there exists a constant $k^* > k_0 + \tau$ such that tracking error converge to Ω_{e_0}, and NN weight error converges to Ω_{we} for all $k > k^*$. This implies the closed-loop system is SGUUB. Then, $y(k) \in \Omega_y$ and $u(k) \in \Omega_u$ will hold for all $k > k_0$. ■

Remark 4: It should be noted that the size of Ω_{e_0} indicates the possible maximum bound that the tracking error can reach. Considering Ω_{e_0} defined in (20), we can see that the size of Ω_{e_0} cannot be made arbitrarily small and it cannot be known *a priori* either. Noting that $C_0 = \sigma\|W^*\|_F^2 + \sigma\gamma\alpha_0^T\alpha_0 + (1/\gamma)\beta_0^T\beta_0$, by choosing sufficient small σ, we can see that $C_0(\Omega_{e_0})$ is approximately proportional to $1/\gamma$ provided that b is fixed. Furthermore, noting (19), we know γ is of order $1/l$. Therefore, the larger the approximator size, the larger the error peak maybe be expected, as C_0 grows in proportion to l.

Remark 5: Note that the size of Ω_w is not predetermined, and it is introduced for analytical purpose because NN approximation is only valid on a compact set. In fact, Ω_w can be made arbitrary large to guarantee $\hat{W}(k) \in \Omega_w$, even in the transient period, as we have proved that $\hat{W}(k)$ is bounded. In practical implementation, we can initialize $\hat{W}(0) = 0$ (thus, $u(0) = 0$, which must be within Ω_u), and the corresponding parameter estimation error, $\tilde{W}(0) = \hat{W}(0) - W^* = -W^*$, is obviously bounded, and within the compact set Ω_w as it can be made arbitrarily large. As the control system needs to be initialized for the first τ steps, they could be simply set to be 0. For better performance, especially the transient performance, offline training could be used to initialize the controller [9].

V. Conclusion

In this note, for a class of nonlinear discrete-time MIMO systems with unknown interconnections between subsystems, adaptive direct NN control scheme has been presented using NNs. By finding an orthogonal matrix to update the NN weight matrix, it has been shown that for appropriately chosen controller parameters, stability of the closed-loop adaptive system can be guaranteed.

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