

Stable Adaptive Control for a Class of Nonlinear Systems Using a Modified Lyapunov Function

T. Zhang, S. S. Ge, and C. C. Hang

Abstract—This paper investigates the adaptive control design for a class of nonlinear systems using Lyapunov's stability theory. The proposed method is developed based on a novel Lyapunov function, which removes the possible controller singularity problem in some of the existing adaptive control schemes using feedback linearization techniques. The resulting closed-loop system is proven to be globally stable, and the output tracking error converges to an adjustable neighborhood of zero.

Index Terms—Adaptive nonlinear control, Lyapunov stability.

I. INTRODUCTION

In this work, we consider the adaptive control problem for nonlinear systems in the following normal form:

$$\begin{cases} \dot{x}_i = x_{i+1}, & i = 1, 2, \dots, n-1 \\ \dot{x}_n = a(x) + b(x)u, \\ y = x_1, \end{cases} \quad (1)$$

where

$$\begin{array}{ll} x = [x_1, x_2, \dots, x_n]^T \in R^n & \text{state variables;} \\ u \in R & \text{system input;} \\ y \in R & \text{system output;} \\ a(x) \text{ and } b(x) & \text{smooth functions.} \end{array}$$

In controller design based on the feedback linearization technique, the most commonly used control structure is $u = [-a(x) + v]/b(x)$, with v being a new control variable. When the nonlinearities $a(x)$ and $b(x)$ are unknown, many adaptive control schemes have been developed (e.g., [2]–[8] and the references therein), in which the unknown function $b(x)$ is usually approximated by a function approximator $\hat{b}(x, \hat{\theta})$ (where $\hat{\theta}$ is an estimated weight or parameter vector). Consequently, the estimate $\hat{b}(x, \hat{\theta})$ must be away from zero for avoiding a possible singularity problem. Several attempts have been made to deal with such a problem, as follows:

- i) choosing the initial parameter $\hat{\theta}(0)$ sufficiently close to the ideal value by off-line training before the controller is put into operation [3];
- ii) using projection algorithms to guarantee the estimate $\hat{\theta}$ inside a feasible set, in which $\hat{b}(x, \hat{\theta}) \neq 0$ (some *a priori* knowledge for the studied systems is required for constructing the projection algorithms) [2], [5]–[7];
- iii) modifying the adaptive controller by introducing a sliding mode control portion to keep the control magnitude bounded [4], [8];
- iv) applying neural networks or fuzzy systems to approximate the inverse of $b(x)$ in [7] and [9], which requires the upper bound of the first time derivative of $b(x)$ being known *a priori*.

In this correspondence, by introducing a modified Lyapunov function, a novel Lyapunov-based adaptive controller is presented. The singularity issue mentioned above is completely avoided, and at the same time, the stability and control performance of the closed-loop system are guaranteed. In Section II, we present the plant, assumptions, and some notations used in the paper. In Section III, a novel Lyapunov func-

tion is developed to construct a desired Lyapunov-based controller. A direct adaptive controller and its stability analysis are provided in Section IV.

II. PRELIMINARIES

We study the adaptive control problem for nonlinear system (1) with unknown smooth functions $a(x)$ and $b(x)$. The control objective is to design a globally stable adaptive controller such that the system output y follows a desired trajectory y_d as close as possible.

Assumption 1: The sign of $b(x)$ is known, and a known continuous function $\bar{a}(x) \geq 0$ and a constant $b_0 > 0$ exist such that $|a(x)| \leq \bar{a}(x)$ and $|b(x)| \geq b_0, \forall x \in R^n$.

The above assumption implies that smooth function $b(x)$ is strictly either positive or negative. From now on, without losing generality, we shall assume $b(x) \geq b_0 > 0, \forall x \in R^n$. Define

$$e = [x_1 - y_d, x_2 - \dot{y}_d, \dots, x_n - y_d^{(n-1)}]^T$$

and a filtered tracking error s as

$$s = \left(\frac{d}{dt} + \lambda \right)^{n-1} e_1 = [\Lambda^T \quad 1]e, \quad \text{with } \lambda > 0 \quad (2)$$

where $\Lambda = [\lambda^{n-1}, (n-1)\lambda^{n-2}, \dots, (n-1)\lambda]^T$.

Remark 2.1: It has been shown in [11] that definition (2) has the following properties: i) $s = 0$ defines a time-varying hyperplan in R^n on which the tracking error e_1 converges to zero asymptotically, ii) if $e(0) = 0$ and $|s(t)| \leq C, \forall t \geq 0$ with constant $C > 0$, then $e(t) \in \Omega_c, \forall t \geq 0$ with

$$\Omega_c = \left\{ e \mid |e_i| \leq 2^{i-1} \lambda^{i-n} C, \quad i = 1, 2, \dots, n \right\} \quad (3)$$

and iii) if $e(0) \neq 0$ and $|s(t)| \leq C$, then $e(t)$ will converge to Ω_c within a time-constant $(n-1)/\lambda$. In addition, the tracking error can be expressed as $e_1 = H(s)s$, with $H(s)$ a proper stable transfer function.

Considering system (1) and definition (2), the time derivative of s can be written as

$$\dot{s} = a(x) + b(x)u + \nu \quad (4)$$

where $\nu = -y_d^{(n)} + [0 \quad \Lambda^T]e$.

Assumption 2: A desired trajectory vector $x_d = [y_d, \dot{y}_d, \dots, y_d^{(n)}]^T$ is continuous and available, and $x_d \in \Omega_d \subset R^{n+1}$ with Ω_d a compact set.

III. LYAPUNOV-BASED CONTROL STRUCTURE

From (2), it is shown that $x_n = s + y_d^{(n-1)} - [\Lambda^T \quad 0]e$. For notational convenience, we denote $b(\psi, s + \nu_1) = b(x)$ with $\psi = [x_1, x_2, \dots, x_{n-1}]^T$ and $\nu_1 = y_d^{(n-1)} - [\Lambda^T \quad 0]e$. Define a smooth scalar function

$$V_s = \int_0^s \frac{\sigma}{b(\psi, \sigma + \nu_1)} d\sigma \quad (5)$$

which is a function of s, ψ , and ν_1 . By mean value theory [13], V_s can be rewritten as $V_s = \lambda_s s^2 / b(\psi, \lambda_s s + \nu_1)$ with $\lambda_s \in (0, 1)$. Because $1/b(x) > 0, \forall x \in R^n$, it is shown that V_s is positive definitive with respect to s .

Lemma 3.1: For system (1) satisfying Assumptions 1 and 2, if $x \in L_\infty$, and a desired Lyapunov-based controller is chosen as

$$u^* = -k(t)s - \frac{a(x)}{b(x)} - g(z) \quad (6)$$

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where bounded continuous function $k(t) \geq k^* > 0$ with constant k^* , and

$$g(z) = \frac{1}{s} \int_0^s \left[\sigma \sum_{i=1}^{n-1} \frac{\partial b^{-1}(\psi, \sigma + \nu_1)}{\partial x_i} x_{i+1} + \frac{\nu}{b(\psi, \sigma + \nu_1)} \right] d\sigma, \quad (7)$$

$$z = [x^T, s, \nu, \nu_1]^T;$$

then, the system tracking error converges to zero.

Proof: For Lyapunov function candidate V_s , its time derivative

$$\begin{aligned} \dot{V}_s &= \frac{\partial V_s}{\partial s} \dot{s} + \frac{\partial V_s}{\partial \psi} \dot{\psi} + \frac{\partial V_s}{\partial \nu_1} \dot{\nu}_1 \\ &= \frac{s}{b(x)} \dot{s} + \int_0^s \sigma \left[\frac{\partial b^{-1}(\psi, \sigma + \nu_1)}{\partial \psi} \dot{\psi} \right] d\sigma \\ &\quad + \dot{\nu}_1 \int_0^s \sigma \left[\frac{\partial b^{-1}(\psi, \sigma + \nu_1)}{\partial \nu_1} \right] d\sigma. \end{aligned} \quad (8)$$

Because $\partial b^{-1}(\psi, \sigma + \nu_1)/\partial \nu_1 = \partial b^{-1}(\psi, \sigma + \nu_1)/\partial \sigma$ and $\nu = -\dot{\nu}_1$, it is shown that

$$\begin{aligned} &\dot{\nu}_1 \int_0^s \sigma \left[\frac{\partial b^{-1}(\psi, \sigma + \nu_1)}{\partial \nu_1} \right] d\sigma \\ &= -\nu \int_0^s \sigma \frac{\partial b^{-1}(\psi, \sigma + \nu_1)}{\partial \sigma} d\sigma \\ &= -\nu \left[\frac{\sigma}{b(\psi, \sigma + \nu_1)} \Big|_0^s - \int_0^s \frac{1}{b(\psi, \sigma + \nu_1)} d\sigma \right] \\ &= -\frac{\nu s}{b(x)} + \int_0^s \frac{\nu}{b(\psi, \sigma + \nu_1)} d\sigma. \end{aligned}$$

Substituting (4) and the above equation into (8) and applying (7), we obtain

$$\dot{V}_s = s \left[u + \frac{a(x)}{b(x)} + g(z) \right]. \quad (9)$$

Let

$$h(s) = \int_0^s \left[\sigma \sum_{i=1}^{n-1} \frac{\partial b^{-1}(\psi, \sigma + \nu_1)}{\partial x_i} x_{i+1} + \frac{\nu}{b(\psi, \sigma + \nu_1)} \right] d\sigma;$$

then, $h(0) = 0$. Hence, by the fundamental theorem of calculus, we have

$$\lim_{s \rightarrow 0} g(z) = \frac{\partial h(s)}{\partial s} \Big|_{s=0} = \frac{\nu}{b(\psi, \nu_1)}.$$

Therefore, $g(z)$ is smooth and well defined, and u^* is continuous. Substituting (6) into (9) leads to $\dot{V}_s = -k(t)s^2 \leq -k^*s^2 \leq 0$. Integrating it, we have

$$\int_0^\infty k^* s^2(\tau) d\tau \leq V_s(0) - V_s(\infty) \leq V_s(0)$$

and hence, $s \in L_2$. Because $x, x_d \in L_\infty$, it is clear from (2) that $s \in L_\infty$. Using the conditions of $x, x_d, s \in L_\infty$, and the smoothness of system functions $a(x), b(x)$, and $g(z)$, it can be seen from (4) that $\dot{s} \in L_\infty$. According to Barbalat's lemma [10], $s \in L_2$ and $\dot{s} \in L_\infty$, we conclude that $s \rightarrow 0$ as $t \rightarrow \infty$. Using property i) of Remark 2.1, we have $\lim_{t \rightarrow \infty} e_1(t) = 0$. Q.E.D.

IV. ADAPTIVE CONTROLLER DESIGN AND STABILITY ANALYSIS

In the case of unknown nonlinearities $a(x)$ and $b(x)$, the desired controller u^* given in (6) is not available. A linearly parameterized approximator shall be used to approximate the unknown nonlinearities in (6). Several function approximators can be applied for this purpose, e.g., radial basis function (RBF) neural networks [8], [9], high-order

neural networks [12] or fuzzy systems [6], which can be described as $W^T S(z)$ with input vector $z \in R^{n+3}$, weight vector $W \in R^l$, node number l , and basis function vector $S(z) \in R^l$. Universal approximation results indicate that, if l is chosen sufficiently large, then $W^T S(z)$ can approximate any continuous function to any desired accuracy over a compact set [6], [9], [12]. We first define a compact set

$$\Omega_z = \{(x, s, \nu, \nu_1) | x \in \Omega_\varepsilon, x_d \in \Omega_d\} \subset R^{n+3} \quad (10)$$

with compact subset Ω_ε to be specified later. As function $a(x)/b(x) + g(z)$ in (6) is smooth, the following approximation holds:

$$\frac{a(x)}{b(x)} + g(z) = W^{*T} S(z) + \mu_l, \quad \forall z \in \Omega_z \quad (11)$$

with bounded function approximation error μ_l and the ideal weight W^* defined as

$$W^* := \arg \min_{W \in \Omega_w} \left\{ \sup_{z \in \Omega_z} \left| W^T S(z) - \frac{a(x)}{b(x)} - g(z) \right| \right\} \quad (12)$$

where $\Omega_w = \{W | \|W\| \leq w_m\}$ with constant $w_m > 0$ chosen by the designer.

The magnitude of μ_l depends on the choices for basis function $S(z)$, node number l , and constraint set Ω_w . In general, the larger the weight number l and the constraint set Ω_w , the smaller the approximation error.

With the function approximation (11), we present the adaptive controller

$$u = -k(t)s - \hat{W}^T S(z) \quad (13)$$

where \hat{W} is the estimate of W^* , and choose the adaptive law as

$$\dot{\hat{W}} = \begin{cases} \gamma S(z)s, & \text{if } \|\hat{W}\| < w_m \text{ or} \\ & \|\hat{W}\| = w_m \text{ and } \hat{W}^T S(z)s \leq 0, \\ \gamma S(z)s - \gamma \frac{\hat{W} \hat{W}^T S(z)}{\|\hat{W}\|^2} s, & \text{if } \|\hat{W}\| = w_m \text{ and } \hat{W}^T S(z)s > 0 \end{cases} \quad (14)$$

with adaptive gain $\gamma > 0$. The above weight tuning algorithm is a standard learning law with projection algorithm [5], [10], which guarantees that $\|\hat{W}(t)\| \leq w_m, \forall t > 0$ for the initial condition satisfying $\|\hat{W}(0)\| \leq w_m$ (see [5] and [10] for the proof).

Theorem 4.1: For the closed-loop adaptive system consisting of plant (1), controller (13), and adaptive law (14), if the initial weight $\|\hat{W}(0)\| \leq w_m$, and the gain

$$k(t) = \frac{1}{\varepsilon} \sqrt{1 + \frac{1}{b_0^2} [\bar{a}^2(x) + \nu^2] + [\hat{W}^T S(z)]^2}, \quad \varepsilon > 0 \quad (15)$$

then

i) all of the closed-loop signals are bounded, and the state vector x remains in

$$\Omega_\varepsilon = \left\{ x(t) \mid |e_i(t)| \leq 2^i \lambda^{i-n} \varepsilon, \quad i = 1, 2, \dots, n, x_d \in \Omega_d \right\} \quad (16)$$

$$\forall t \geq T;$$

ii) the mean square of output tracking error satisfies

$$\lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t e_1^2(\tau) d\tau \leq \varepsilon^2 c_1 \lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t \mu_l^2(\tau) d\tau \quad (17)$$

with computable constants $T, c_1 \geq 0$.

Proof: The proof is similar to that of standard adaptive control schemes in [2]–[10], except for the particular choice of the novel Lya-

punov function. It should be noticed that condition $x \in L_\infty$ plays an important role in achieving the tracking control in Lemma 3.1. In the following, we first establish the boundedness of the states.

i) Let $V_1 = s^2/2$. Its time derivative along (4) is

$$\begin{aligned} \dot{V}_1 &= b(x) \left[-k(t)s - \hat{W}^T S(z) + \frac{a(x) + \nu}{b(x)} \right] s \\ &\leq -b(x)k(t) \left\{ s^2 - \frac{|s|}{k(t)} \left[\frac{|a(x)|}{b(x)} + \frac{|\nu|}{b(x)} + |\hat{W}^T S(z)| \right] \right\}. \end{aligned}$$

Noting (15), $b(x) \geq b_0 > 0$, and the fact that

$$\begin{aligned} &\left[\frac{|a(x)|}{b(x)} + \frac{|\nu|}{b(x)} + |\hat{W}^T S(z)| \right]^2 \\ &\leq 3 \left\{ \frac{\bar{a}^2(x)}{b_0^2} + \frac{\nu^2}{b_0^2} + [\hat{W}^T S(z)]^2 \right\} \end{aligned}$$

we have $\dot{V}_1 \leq -b(x)k(t)(s^2 - \sqrt{3}\varepsilon|s|)$. From $2\sqrt{3}\varepsilon|s| \leq s^2 + 3\varepsilon^2$ and $s^2 = 2V_1$, it follows that

$$2\dot{V}_1 \leq -b(x)k(t)[2V_1(t) - 3\varepsilon^2]. \quad (18)$$

Using the comparison principle in [14], we have

$$2V_1(t) - 3\varepsilon^2 \leq [2V_1(0) - 3\varepsilon^2] e^{-\int_0^t b(x(\tau))k(\tau) d\tau}.$$

Because $b(x)k(t) \geq b_0/\varepsilon$ [as $k(t) \geq 1/\varepsilon$ obtained from (15)], the above inequality implies that

$$2V_1(t) \leq 2V_1(0)e^{-b_0 t/\varepsilon} + 3\varepsilon^2.$$

Therefore, the filtered error s is bounded by

$$s^2(t) \leq s^2(0)e^{-b_0 t/\varepsilon} + 3\varepsilon^2. \quad (19)$$

Remark 2.1 shows that the boundedness of s implies the boundedness of the system state x . Because $\|\hat{W}(t)\| \leq w_m$ has been guaranteed by projection algorithm (14), we conclude that all of the closed-loop signals are bounded. It can be calculated from (19) that $|s(t)| \leq 2\varepsilon$, $\forall t \geq T_1 = \max[0, (2\varepsilon/b_0)\ln(|s(0)|/\varepsilon)]$. Remark 2.1 indicates the state errors $|e_i(t)| \leq 2^i \lambda^{i-n} \varepsilon$, $i = 1, 2, \dots, n$, $\forall t \geq T = T_1 + (n-1)/\lambda$. Hence, $x \in \Omega_\varepsilon$, $\forall t \geq T$ with T a computable constant.

ii) Considering a Lyapunov function candidate $V = V_s + \gamma^{-1} \hat{W}^T \hat{W}/2$ with $\tilde{W} = \hat{W} - W^*$, its time derivative $\dot{V} = \dot{V}_s + \gamma^{-1} \hat{W}^T \dot{\hat{W}}$. For system (1) with $t \geq T$, function approximation (11) holds for all $z \in \Omega_\varepsilon$ (because $x \in \Omega_\varepsilon$ and $x_d \in \Omega_d$). From (9), (11), and (13), we have

$$\dot{V} = s[-k(t)s - \tilde{W}^T S(z) + \mu_l] + \gamma^{-1} \hat{W}^T \dot{\hat{W}}, \quad \forall t \geq T.$$

Utilizing adaptive law (14) and along the similar arguments of applying projection algorithm in [5] and [10], we obtain

$$\dot{V} \leq s[-k(t)s + \mu_l], \quad \forall t \geq T.$$

By noting $k(t) \geq 1/\varepsilon$ and $\mu_l s \leq s^2/2\varepsilon + \varepsilon\mu_l^2/2$, it follows that

$$\dot{V} \leq -\frac{s^2}{\varepsilon} + \mu_l s \leq -\frac{s^2}{2\varepsilon} + \frac{\varepsilon}{2} \mu_l^2, \quad \forall t \geq T. \quad (20)$$

Integrating (20) over $[T, t]$ leads to

$$\int_T^t s^2(\tau) d\tau \leq -\int_T^t 2\varepsilon \dot{V} d\tau + \varepsilon^2 \int_T^t \mu_l^2(\tau) d\tau$$

$$\leq 2\varepsilon V(T) + \varepsilon^2 \int_T^t \mu_l^2(\tau) d\tau, \quad \forall t \geq T. \quad (21)$$

Integrating (19) over $[0, T]$, we obtain

$$\int_0^T s^2(\tau) d\tau \leq \frac{\varepsilon}{b_0} s^2(0)(1 - e^{-b_0 T/\varepsilon}) + 3\varepsilon^2 T. \quad (22)$$

Because $V_s = \int_0^s \sigma/b(\psi, \sigma + \nu_1) d\sigma \leq s^2/2b_0$ (for $b(x) \geq b_0 > 0$) and $\|\hat{W}(t)\| \leq 2w_m$ (as $\|\hat{W}(t)\| \leq w_m$, $\forall t \geq 0$), it is shown that

$$\begin{aligned} V(T) &= V_s(T) + \gamma^{-1} \|\hat{W}(T)\|^2/2 \\ &\leq s^2(T)/2b_0 + 2\gamma^{-1} w_m^2. \end{aligned}$$

Combining (21) and (22) and applying the inequality $s^2(T) \leq s^2(0)e^{-b_0 T/\varepsilon} + 3\varepsilon^2$ [obtained from (19)], we have

$$\begin{aligned} \int_0^t s^2(\tau) d\tau &\leq \varepsilon^2 \int_0^t \mu_l^2(\tau) d\tau + \frac{\varepsilon}{b_0} [s^2(0) + 3\varepsilon^2] \\ &\quad + 4\varepsilon\gamma^{-1} w_m^2 + 3\varepsilon^2 T, \quad \forall t \geq 0. \quad (23) \end{aligned}$$

Clearly, the last three terms of the above inequality are computable nonnegative constants. Because the tracking error $e_1 = H(s)s$, with $H(s)$ a proper stable transfer function (Remark 2.1), using [10, Lemma 4.8.2], we have

$$\int_0^t e_1^2(\tau) d\tau \leq \varepsilon^2 c_1 \int_0^t \mu_l^2(\tau) d\tau + c_2, \quad \forall t \geq 0, \quad (24)$$

with computable constants $c_1, c_2 \geq 0$. It follows that (17) holds.

Q.E.D.

Remark 4.1: From Theorem 4.1, we can see that adaptive controller (13) contains two parts for different purposes. The first part $-k(t)s$ can be viewed as a bounding control term used to guarantee the system states exponentially converging to a designed compact set Ω_ε for any initial states, such that, after a computable transient time T , function approximation (11) is valid on $z \in \Omega_\varepsilon$, $\forall t \geq T$. The second part in controller (13) is an adaptive term applied for reducing the effects of system uncertainties.

Remark 4.2: From (19), we have $s(t)^2 \leq s^2(0) + 3\varepsilon^2$, $\forall t \geq 0$. Remark 2.1 implies that the tracking error satisfies $|e_1(t)| \leq \lambda^{1-n} \sqrt{s^2(0) + 3\varepsilon^2}$, $\forall t \geq (n-1)/\lambda$. Therefore, the upper bound of the tracking error depends on initial state errors and design parameters λ and ε . In addition, (17) reveals that the mean square error can be reduced by choosing smaller ε or larger weight numbers l (which is helpful for reducing μ_l in function approximation).

Remark 4.3: Although decreasing ε may improve the tracking performance significantly, we do not suggest using a very small ε in practical applications because this may lead to a high-gain controller and increase the bandwidth of the closed-loop system. The main purpose of setting parameter ε in controller (13) is for adjusting the size of Ω_ε . From a practical perspective, the size of the compact set Ω_ε (depending on Ω_ε and Ω_d) is very important in the construction of function approximator. For example, when applying RBF neural networks, selection of the number and centers of basis functions is mainly based on the region of interest that corresponds to Ω_ε . In general, the larger the set Ω_ε is, the more the neural networks nodes are needed for achieving the same approximation accuracy.

V. CONCLUSION

A direct adaptive control scheme has been presented for a class of nonlinear systems. The key point of the proposed approach lies in introducing the novel Lyapunov function for constructing the Lyapunov-based adaptive controller such that the possible controller singularity

problem is avoided. The resulting closed-loop system is guaranteed to be globally stable, and the bounds of the system states and tracking error are obtained explicitly. Some possible methods have been provided for improving the control performance of the adaptive system.

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Decentralized Stabilization of a Class of Interconnected Stochastic Nonlinear Systems

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Abstract—This paper focuses on a class of large-scale interconnected stochastic nonlinear systems. The interconnections are bounded by strong nonlinear functions that contain first-order and higher order polynomials as special cases. The problem we address is to design a decentralized controller such that the closed-loop, large-scale, interconnected stochastic nonlinear system is globally asymptotically stable in probability for all admissible interconnections. It is shown that the decentralized global stabilization via both state feedback and output feedback can be solved by a Lyapunov-based recursive design method.

Index Terms—Decentralized control, dynamic output feedback, interconnected systems, nonlinear stochastic systems, state feedback.

I. INTRODUCTION

A number of large-scale systems founded in the real world are composed of a set of small interconnected subsystems, such as power systems, digital communication networks, economic systems, and urban traffic networks. It is generally impossible to incorporate many feedback loops into the controller design and is too costly even if they can be implemented. These difficulties motivate the development of decentralized control theory, in which each subsystem is controlled independently of its locally available information.

The decentralized stabilization problem for deterministic interconnected linear systems with uncertainties satisfying the so-called strict matching conditions has been investigated in [1], [11]–[13], and references therein. The interconnections among subsystems treated in these papers are mostly bounded by first-order polynomials of state. It was pointed in [11] and [12] that a decentralized control system based on the first-order bounded interconnections may become unstable when the interconnections are of higher order. Recently, following the development of centralized control of deterministic nonlinear systems [4], [7], [8], a decentralized adaptive stabilization for a class of large-scale deterministic interconnected nonlinear systems was proposed in [5], in which the strict matching condition was relaxed and higher order polynomial interconnections among subsystems were considered.

In this paper, we shall investigate a global decentralized stabilization problem for a class of large-scale stochastic nonlinear systems with strong interconnections, which involve first-order and higher order polynomial interconnections as special cases. Inspired by the recent work of centralized stochastic nonlinear control [2], [3], [10], we show that the decentralized global robust stabilization can be achieved for the interconnected, large-scale, stochastic nonlinear systems by employing a Lyapunov-based recursive controller design method. Both the state feedback control and dynamic output feedback control are considered. Our results extend existing centralized global stabilization results for stochastic systems to decentralized control of large-scale interconnected stochastic nonlinear systems.

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