Adaptive Repetitive Control for a Class of Nonlinearly Parametrized Systems

Mingxuan Sun and Shuzhi Sam Ge

***Abstract*—In this note, Lyapunov-based adaptive repetitive control is presented for a class of nonlinearly parametrized systems. Through the use of an integral Lyapunov function, the controller singularity problem is elegantly solved as it avoids the nonlinear parametrization from entering into the adaptive control and repetitive control. Global stability of the adaptive system and asymptotic convergence of the tracking error are established, and tracking error bounds are provided to quantify the control performance. Both partially and fully saturated learning laws are analyzed in detail, and compared analytically.**

***Index Terms*—Adaptive control, nonlinear parametrization, repetitive control, saturated-learning.**

I. Introduction

Adaptive control has been extensively studied in the literature for nonlinear systems that are linear-in-the-parameters. However, only few results are available for nonlinear systems that are nonlinearly parametrized due to its difficulty in analysis and design even though nonlinear parametrization is common in many control applications such as friction dynamics [1] and fermentation processes [2]. In [3], globally stable output-feedback control was developed using high-gain adaptation for nonlinearly parametrized systems with known and constant relative degree. Applying min–max optimization strategy, novel control was presented for nonlinear systems with convex/concave parametrization [4]. Though feedback linearization control has been extensively investigated for large classes of nonlinear systems, the well-known controller singularity problem in its adaptive version was not fully addressed. In [5], integral Lyapunov function based control was used to avoid the control singularity in feedback linearization-based designs, and to design the direct adaptive controller for a class of nonlinearly parametrized systems. Recently, fractional parametrization was used in controller designs for uncertain systems [6]. Through practical stability analysis, it is shown that this novel method can be employed with parametrization in terms of either system dynamics or their bounding functions.

In practice, many operations are repeatable tasks and the systems are commonly subjected to periodic/repeated disturbances and uncertainties [7]. Though the aforementioned adaptive controls can guarantee closed-loop stability, perfect tracking for such tasks may not be achieved, despite the fact that it is highly desirable for high precision engineering. Learning control and repetitive control are the alternatives to address this problem. Fundamentally, learning control is the same as repetitive control in the sense that both approaches exploit the repetitive nature of processes, except that the former requires initial repositioning, and the latter starts from where it has left, which could be regarded as the existence of repositioning errors for the former. For more details, refer to [8]–[11]. The early effort made by Lyapunov's direct method was found in [12], where a repetitive controller was designed without requiring any explicit knowledge of dynamics of the manipulators, and saturated-learning was suggested to ensure the boundedness of estimates. Additional features of saturated-learning were further discussed in [13]. The repetitive control presented in [14] was used with discontinuous projection modifications ensuring that all the estimates for the unknown repeatable nonlinearities are within the known bounds, by exploiting properties of the projection mapping. By use of Lyapunov synthesis, robust control techniques are well developed for nonlinear learning control of robot manipulators [15]. Indeed there are formulations that aim for learning periodic uncertainties, rather than for following periodic references. For an in-depth comparison of adaptive, learning and robust controls to address the problem of learning time functions, either periodic or not, readers may refer to [16], and more recent developments [17] and [18].

The aforementioned repetitive control results are all for systems that are linear-in-the-parameters. For the case where parameters enter the system dynamics nonlinearly, additional precautions have to be made for avoiding the possible control singularity if the same Lyapunov-based synthesis is applied. For robotic manipulators, different update laws were introduced for parameter adaptation, e.g., in iteration domain [19], [21] and in time domain [20]. Under mild assumption, it is shown that perfect tracking performance is retained by partially-saturated learning in [22]. Motivated by the success of integral Lyapunov functions [5], [23] and [24] in the context of adaptive control to avoid controller singularity, in this note, such a Lyapunov function is used for repetitive control design. The developed adaptive repetitive controller is applicable to a class of nonlinearly parameterized systems. Compared with the previous works, the main contributions of the note lie in: i) the utilization of the integral Lyapunov function in avoiding the nonlinear parametrization from entering into repetitive control, consequently avoiding the possible singularity problem; ii) the adoption of fully saturated learning in ensuring the estimate within a prespecified region; and iii) the combination of repetitive control and adaptive control in leading to the globally stable adaptive system in the presence of both unknown constants and unknown time functions of known period.

II. Preliminaries

Consider a class of single-input–single-output (SISO) nonlinear systems described by

$$\begin{cases} \dot{x}_i = x_{i+1}, \qquad i = 1, 2, \ldots, n-1 \\ \dot{x}_n = \frac{1}{b(x,\theta_b)}\left(f(x,\theta_f) + g\left(x, \delta(t)\right) + u\right) \\ y = x_1 \end{cases} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^T = [y, \dot{y}, \ldots, y^{n-1}]^T \in R^n$, $u \in R$ and $y \in R$ are the state vector, system input and output, respectively, $\theta_f \in R^{nf}$ and $\theta_b \in R^{n_b}$ are the vectors of unknown constant parameters, $\delta(t) \in R^{n_g}$ is the vector of unknown time functions, $b(x, \theta_b)$, $f(x, \theta_f)$ and $g(x, \delta(t))$ are nonlinear functions with unknown parameters. Define $x_d(t) = [y_d(t), \dot{y}_d(t), \ldots, y_d^{(n-1)}(t)]^T$, where $y_d(t)$ is the desired trajectory with bounded derivatives up to nth order.

Assumption 1: The sign of $b(x, \theta_b)$ is known and there exist constant $b_{\min}$ and bound function $\bar{b}(x) > 0$ such that $0 < b_{\min} \le |b(x, \theta_b)| \le \bar{b}(x) < \infty$, for all $x \in R^n$ and $\theta_b \in R^{n_b}$.

Remark 1: Assumption 1 is reasonable because $|b(x, \theta_b)|$ being away from zero avoids system (1) to be singular, while the existence of the bounded function $\bar{b}(x)$ is a controllable condition of system (1). It should be noted that $b_{\min}$ and $\bar{b}(x)$ are required only for analytical purposes, their true values are not necessarily known. Assumption

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1 implies that function $b(x, \theta_b)$ is strictly either positive or negative. Without losing generality, it is assumed that $0 < b_{\min} \leq b(x, \theta_b) \leq \bar{b}(x) < \infty$, for all $x \in R^n$ and $\theta_b \in R^{n_b}$.

Assumption 2: The time-varying uncertainty $\delta(t)$ is continuous in time and periodic with known period T, i.e., $\delta(t) = \delta(t - T)$.

Remark 2: The continuity assumed in Assumption 2, implies the boundedness of $\delta(t)$ on $[0, T]$. By the periodicity, $\delta(t)$ is bounded for $t \in [0, \infty)$ with an appropriate bound. For simplicity, this note focuses on handling periodic uncertainties using integral Lyapunov functions. In practice, there may exist nonperiodic exogenous signals to be learned, for which, robust control can be used elegantly for learning nonperiodic but smooth time functions [16]. It shows that the necessary condition for learning is not periodicity, but smooth time functions. As for nonperiodic uncertainties, it is out of the scope of the current note.

Assumption 3: Nonlinear functions $b(x, \theta_b)$, $f(x, \theta_f)$ and $g(x, \delta(t))$ satisfy

$$\begin{aligned} b(x, \theta_b) &= \theta_b^T w_b(x) + b_0(x) \\ f(x, \theta_f) &= \theta_f^T w_f(x) + f_0(x) \\ g(x, \delta(t)) &= \delta^T(t) v(x) \end{aligned} \tag{2}$$

where $w_f(x) \in R^{n_f}$, $v(x) \in R^{n_g}$, and $f_0(x) \in R$ are known continuous functions, $w_b(x) \in R^{n_b}$ and $b_0(x) \in R$ are known continuously differentiable functions.

Remark 3: System functions in (1) are parametrized as in (2). From (2), we know that θ_f and $\delta(t)$ appear linearly in system (1), but θ_b enters the system nonlinearly through the parametrized term of $b(x, , \theta_b)$.

As is well known, repetitive control applies to situation in which a system follows a periodic trajectory. Our work will lie in the traditional framework of repetitive control if $y_d(t)$ is periodic.

In this note, we will use $\text{sat} : R \to R$ to denote the saturation function defined as, for a scalar a

$$\text{sat}_{\bar{a}}(a) = \begin{cases} \bar{a}^1, & a < \bar{a}^1 \\ a, & \bar{a}^1 \leq a \leq \bar{a}^2 \\ \bar{a}^2, & a > \bar{a}^2 \end{cases} \tag{3}$$

where $\bar{a} = \{\bar{a}^1, \bar{a}^2\}$ represent the lower and upper bounds of a, satisfying that $\bar{a}^1 < \bar{a}^2$. Case $\bar{a}^1 \neq -\bar{a}^2$ indicates that the saturation function is asymmetric. For a vector $a \in R^m$, the vector-valued saturation function is defined as $\text{sat}_{\bar{a}}(a) = [\text{sat}_{\bar{a}_1}(a_1), \text{sat}_{\bar{a}_2}(a_2), \ldots, \text{sat}_{\bar{a}_m}(a_m)]^T$ with $\bar{a} = \{\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_m\}$.

Lemma 1: For $a, b \in R^m$, if a satisfies that $\bar{b}_i^1 \leq a_i \leq \bar{b}_i^2$, $i = 1, 2, \ldots, m$, then

$$[(\gamma + 1)a - (\gamma b + \text{sat}_{\bar{b}}(b))]^T \Lambda [b - \text{sat}_{\bar{b}}(b)] \leq 0$$

where $\gamma \geq 0$ is a scalar and $\Lambda < 0$ is a diagonal matrix with appropriate dimension.

Proof: See the Appendix. ■

III. Main Results

In this section, we will first investigate the impact due to presence of the periodic uncertainty, and then present analysis for two fully saturated learning laws.

A. Impact of Periodic Uncertainty

Define the filtered error as $e_f = [a^T 1]e$, $e = [e_1, e_2, \ldots, e_n] = x - x_d$, where $a = [a_1, a_2, \ldots, a_{n-1}]^T$ is chosen such that the polynomial $s^{n-1} + a_{n-1}s^{n-2} + \cdots + a_1$ is Hurwitz. To facilitate the control design, consider the integral Lyapunov function $V_f = (1/2)\int_0^{e_f} b(\bar{x}_{n-1}, \sigma + \nu_1, \theta_b) d\sigma^2$, $\bar{x}_{n-1} = [x_1, x_2, \ldots, x_{n-1}]^T$, and $\nu_1 = y_d^{(n-1)} - [a^T, 0]e$. It has been shown that such a Lyapunov function candidate can effectively solve the so-called controller singularity problem that is usually encountered in adaptive feedback linearization control. The necessity for introduction of $b(x, \theta_b)$ in V_f will be further clarified in Remark 7 later, and readers are referred to [23] for further more details. To show the problem clearly, let us follow the treatment detailed in [5] for the nonlinearly parametrized system with periodic uncertainty. Defining $\nu = -y_d^{(n)} + [0 \ \Lambda^T]e$, the time derivative of e_f can be calculated as

$$\dot{e}_f = \frac{1}{b(x, \theta_b)} (f(x, \theta_f) + g(x, \delta(t)) + u) + \nu. \tag{4}$$

Differentiating V_f along (4) yields

$$\dot{V}_f = e_f \left[\theta^T w(z) + \delta^T(t) v(x) + h(z) + u\right] \tag{5}$$

where $z = [x^T, x_d^T, y_d^{(n)}]^T$, $\theta = [\theta_f^T, \theta_b^T]^T$, and

$$\begin{aligned} w(z) = \Bigg[w_f^T(x), \frac{1}{e_f} \int_0^{e_f} \Bigg(\sigma \sum_{i=1}^{n-1} \frac{\partial w_b^T(\bar{x}_{n-1}, \sigma + \nu_1)}{\partial x_i} x_{i+1} \\ + \nu w_b^T(\bar{x}_{n-1}, \sigma + \nu_1) \Bigg) d\sigma \Bigg]^T \\ h(z) = f_0(x) + \frac{1}{e_f} \int_0^{e_f} \Bigg[\sigma \sum_{i=1}^{n-1} \frac{\partial b_0(\bar{x}_{n-1}, \sigma + \nu_1)}{\partial x_i} x_{i+1} \\ + \nu b_0(\bar{x}_{n-1}, \sigma + \nu_1) \Bigg] d\sigma. \end{aligned}$$

It is easy to check that both $w(z)$ and $h(z)$ are well defined even if e_f approaches zero. From (5), we observe that the design difficulties arise from two uncertainties: Unknown constant parameters and unknown time-varying functions. If the constant parameter vector θ is available, to highlight the effect of $\delta(t)$, let us consider the same uncertainty equivalent control

$$u = u_f = -\kappa e_f - \theta^T w(z) - h(z), \qquad \kappa > 0 \tag{6}$$

which renders (5) to

$$\dot{V}_f = -\kappa e_f^2 + \delta^T(t) v(x) e_f. \tag{7}$$

Remark 4: If there is no disturbance, i.e., $\delta(t) = 0$, asymptotic stability follows naturally as it has been established for unknown constant θ in [5]. When there exists disturbance signal, $\delta(t) \neq 0$, no stability conclusion can be drawn from (7), and additional measures have to be taken to handle the unknown time varying uncertainty. One effective method is to incorporate feedback controller (6) with compensation in an effort to handle the periodic uncertainty, in which repetitive update is made for the estimate of $\delta(t)$ by taking advantage of its periodic nature.

Consider the following repetitive control:

$$u = u_f + \hat{u}_r \tag{8}$$

where the feedback control u_f is given by (6), and $\hat{u}_r$ is given by

$$\hat{u}_r = -\hat{\delta}^T(t)v(x) \tag{9}$$

with the estimate for $\delta(t)$ being updated by the partially saturated learning law

$$\hat{\delta}(t) = \begin{cases} \text{sat}_{\bar{\delta}_T}\left(\hat{\delta}_T(t)\right) + \Gamma v\left(x(t)\right)e_f(t), & t > 0 \\ 0, & t \in [-T, 0] \end{cases} \tag{10}$$

where $\Gamma > 0$ is diagonal, $\hat{\delta}_T(t) = \hat{\delta}(t-T)$, and $\bar{\delta}_T = [\bar{\delta}_{T_1}, \bar{\delta}T_1]$ contains the lower and upper bounds, which can be chosen by the designer. Assume that $\delta(t) = \text{sat}_{\bar{\delta}_T}(\delta(t))$, i.e., without losing generality, we assume that $\delta(t)$ lies within the saturation bounds, $\bar{\delta}_T$.

Substituting the repetitive control (8) into (5) results in, by denoting $\tilde{\delta}(t) = \delta(t) - \hat{\delta}(t)$

$$\dot{V}_f = -\kappa e_f^2 + \tilde{\delta}^T(t)v(x)e_f. \tag{11}$$

Define the Lyapunov function candidate $V_1(t) = V_f(t) + V_r(t)$ with $V_r(t) = (1/2)\int_{t-T}^{t}\tilde{\delta}^T(\tau)\Gamma^{-1}\tilde{\delta}(\tau)d\tau$. From Assumption 2, $\delta(t-T) = \delta(t)$. Then, the time derivative of V_r can be given as

$$\begin{aligned}\dot{V}_r(t) = \frac{1}{2}\Big\{&\left[\delta(t) - \hat{\delta}(t)\right]^T\Gamma^{-1}\left[\delta(t) - \hat{\delta}(t)\right] \\ &- \left[\delta(t) - \text{sat}_{\bar{\delta}_T}\left(\hat{\delta}_T(t)\right)\right]^T \\ &\times \Gamma^{-1}\left[\delta(t) - \text{sat}_{\bar{\delta}_T}\left(\hat{\delta}_T(t)\right)\right] \\ &+ \left[2\delta(t) - \hat{\delta}_T(t) - \text{sat}_{\bar{\delta}_T}\left(\hat{\delta}_T(t)\right)\right]^T \\ &\times \Gamma^{-1}\left[\hat{\delta}_T(t) - \text{sat}_{\bar{\delta}_T}\left(\hat{\delta}_T(t)\right)\right]\Big\}.\end{aligned}$$

By Lemma 1, for the case $\gamma = 1$, $[2\delta(t) - \hat{\delta}_T(t) - \text{sat}_{\bar{\delta}_T}(\hat{\delta}_T(t))]^T\Gamma^{-1}[\hat{\delta}_T(t) - \text{sat}_{\bar{\delta}_T}(\hat{\delta}_T(t))] \leq 0$, which leads to

$$\begin{aligned}\dot{V}_r(t) \leq &-\frac{1}{2}\left[\hat{\delta}(t) - \text{sat}_{\bar{\delta}_T}\left(\hat{\delta}_T(t)\right)\right]^T \\ &\times \Gamma^{-1}\left[\hat{\delta}(t) - \text{sat}_{\bar{\delta}_T}\left(\hat{\delta}_T(t)\right)\right] \\ &- \tilde{\delta}^T(t)\Gamma^{-1}\left[\hat{\delta}(t) - \text{sat}_{\bar{\delta}_T}\left(\hat{\delta}_T(t)\right)\right].\end{aligned}$$

Applying learning law (10) yields

$$\dot{V}_r \leq -\frac{1}{2}v^T(x)\Gamma v(x)e_f^2 - \tilde{\delta}^T(t)v(x)e_f. \tag{12}$$

Note that term $\tilde{\delta}^T(t)v(x)e_f$ appears with opposite sign on the right-hand sides of both (11) and (12). Combining (11) and (12), we can derive that

$$\dot{V}_1 \leq -\kappa e_f^2 - \frac{1}{2}v^T(x)\Gamma v(x)e_f^2. \tag{13}$$

We can readily conclude the boundedness and convergence of the repetitive control.

Remark 5: It can be seen that the repetitive control can solve the problem due to the presence of the periodic uncertainty, and at the same time achieve asymptotic tracking effectively. Repetitive control is thus capable of learning unknown time functions of known period. Saturating $\hat{\delta}_T(t)$ in (10) is sufficient to ensure the boundedness of $\hat{\delta}(t)$, whenever e_f is bounded. Though the saturated term in (10) ensures the boundedness of $\hat{\delta}(t)$, learning law (10) cannot ensure the estimate $\hat{\delta}(t)$ to be within a prespecified region, due to the unsaturated term in the learning law. As such, fully saturated learning is to be investigated next to address the problem where $\hat{\delta}(t)$ is confined within a known region.

B. Fully Saturated Learning

For fully saturated learning, the entire right-hand side of the learning law is saturated. In this subsection, we will establish stability of the closed-loop system and convergence of the tracking error, in which fully saturated learning laws ensure the estimate $\hat{\delta}(t)$ to be within a prespecified region. Consider the following adaptive repetitive control for unknown constant θ:

$$u = \hat{u}_f + \hat{u}_r \tag{14}$$

with $\hat{u}_f = -\kappa e_f - \hat{\theta}^T w(z) - h(z)$ and $\hat{u}_r = -\hat{\delta}^T(t)v(x)$.

To handle the periodic uncertainty $\delta(t)$, we apply the fully saturated learning law

$$\hat{\delta}(t) = \begin{cases} \text{sat}_{\bar{\delta}_e}\left(\hat{\delta}_e(t)\right), & t > 0 \\ 0, & t \in [-T, 0] \end{cases} \tag{15}$$

where $\Gamma > 0$ is diagonal, $\hat{\delta}_e(t) = \hat{\delta}(t-T) + \Gamma v(x(t))e_f(t)$, and $\bar{\delta}_e$ represents the lower and upper bounds of $\hat{\delta}_e(t)$. Without losing generality, we also assume that $\delta(t)$ lies within the saturation bounds, $\bar{\delta}_e$.

Though fully saturated learning can be used to handle both constant and periodic unknowns, we shall only use it to handle the periodic ones as the constant unknowns can be easily solved using the following adaptation law:

$$\dot{\hat{\theta}} = \Phi w(z)e_f \quad \hat{\theta}(0) = 0 \tag{16}$$

where $\Phi > 0$ is diagonal. It should be noted that the adaptive repetitive control could still work well as a modified adaptation law with leakage is applied [25].

Choose the Lyapunov function candidate $V_2 = V_f + V_a + V_r$ with $V_a = (1/2)\tilde{\theta}^T\Phi^{-1}\tilde{\theta}$, $\tilde{\theta} = \theta - \hat{\theta}$ and V_f and V_r being the same as those in V_1. The time derivative of V_2 can be derived as

$$\dot{V}_2 = -\kappa e_f^2 + \tilde{\theta}^T w e_f + \tilde{\delta}^T(t)v(x)e_f + \dot{V}_a + \dot{V}_r.$$

By the definition of V_a, $\dot{V}_a = -\tilde{\theta}^T\Phi^{-1}\dot{\hat{\theta}}$. Employing (16) yields

$$\dot{V}_2 = -\kappa e_f^2 + \tilde{\delta}^T(t)v(x)e_f + \dot{V}_r. \tag{17}$$

From Assumption 2, $\delta(t-T) = \delta(t)$. Accordingly, we know that

$$\begin{aligned}\dot{V}_r(t) = &-\tilde{\delta}^T(t)\Gamma^{-1}\left[\hat{\delta}(t) - \hat{\delta}(t-T)\right] \\ &-\frac{1}{2}\left[\hat{\delta}(t) - \hat{\delta}(t-T)\right]^T\Gamma^{-1}\left[\hat{\delta}(t) - \hat{\delta}(t-T)\right].\end{aligned}$$

Applying learning law (15), we obtain

$$\dot{V}_r(t) = \left[\delta(t) - \mathrm{sat}_{\bar{\delta}_e}\left(\hat{\delta}_e(t)\right)\right]\Gamma^{-1}\left[\hat{\delta}_e(t) - \mathrm{sat}_{\bar{\delta}_e}\left(\hat{\delta}_e(t)\right)\right] - \tilde{\delta}^T(t)v\left(x(t)\right)e_f(t) - \frac{1}{2}\left[\hat{\delta}(t) - \hat{\delta}(t-T)\right]^T \times \Gamma^{-1}\left[\hat{\delta}(t) - \hat{\delta}(t-T)\right].$$

By Lemma 1, for the case $\gamma = 0$, $[\delta(t) - \mathrm{sat}_{\bar{\delta}_e}(\hat{\delta}_e(t))]\Gamma^{-1}[\hat{\delta}_e(t) - \mathrm{sat}_{\bar{\delta}_e}(\hat{\delta}_e(t))] \leq 0$, which results in

$$\dot{V}_r \leq -\tilde{\delta}^T(t)v(x)e_f - \frac{1}{2}\left[\hat{\delta}(t) - \hat{\delta}(t-T)\right]^T \times\Gamma^{-1}\left[\hat{\delta}(t) - \hat{\delta}(t-T)\right]. \quad (18)$$

Combining (17) and (18) yields

$$\dot{V}_2 \leq -\kappa e_f^2 - \frac{1}{2}\left[\hat{\delta}(t) - \hat{\delta}(t-T)\right]^T\Gamma^{-1}\left[\hat{\delta}(t) - \hat{\delta}(t-T)\right]. \quad (19)$$

From (19), $\int_0^t \dot{V}_2(\tau)d\tau \leq 0$, which implies $V_2(t) \leq V_2(0)$. Since $V_2(0)$ is bounded, then $V_2 \in L_\infty$. $V_f \in L_\infty$ and $\tilde{\theta} \in L_\infty$ as well. By Assumption 1, $e_f \in L_\infty$. It follows that $e \in L_\infty$ from the definition of e_f, and $x \in L_\infty$ from the boundedness of x_d. From (14) and (15), it is easy to check that $\hat{u}_f \in L_\infty$, $\hat{\delta}(t) \in L_\infty$, $\hat{u}_r \in L_\infty$, and $u \in L_\infty$. Consequently, from (4), $\dot{e}_f \in L_\infty$. Integrating (19) leads to $\int_0^t e_f^2(\tau)d\tau \leq (1/\kappa)V_2(0)$, which implies $e_f \in L_2$. It follows by Barbalat's lemma that $\lim_{t\to\infty} e_f(t) = 0$, which in turn implies that $\lim_{t\to\infty} e(t) = 0$.

Theorem 1: Under Assumptions 1, 2 and 3, all the signals in the closed-loop system consisting of plant (1) and adaptive repetitive control (14) are globally uniformly bounded, and the error between the actual and the desired trajectories converges to zero asymptotically as $t \to \infty$, i.e., $\lim_{t\to\infty} e(t) = 0$.

Theorem 2: For the closed-loop adaptive system consisting of (1) and (14) satisfying Assumptions 1–3, there exist computable constants $k_0 > 0$ and $a_0 > 0$ such that, for $n \geq 2$

i) the root-mean-square tracking error bound is given by

$$\sqrt{\frac{1}{t}\int_0^t e_1^2(\tau)d\tau} \leq \frac{k_0}{\sqrt{2a_0 t}}\|\zeta(0)\|\sqrt{1 - e^{-2a_0 t}} + \frac{k_0}{\sqrt{2a_0\kappa}}\sqrt{\bar{V}_2(0)} \quad (20)$$

where $\zeta(0) = [e_1(0), e_2(0), \ldots, e_{n-1}(0)]^T$, $\bar{V}_2(0) = (1/2)(b_{\max}e_f^2(0) + \theta^T\Phi^{-1}\theta + \int_{-T}^0 \delta^T(\tau)\Gamma^{-1}\delta(\tau)d\tau)$ and $b_{\max}$ is the upper bound of $\bar{b}(x)$;

ii) the L_∞ tracking error bound is given by

$$|e_1(t)| \leq k_0\|\zeta(0)\|e^{-a_0 t} + \frac{k_0}{\sqrt{2a_0\kappa}}\sqrt{\bar{V}_2(0)}. \quad (21)$$

Proof: The proof, similar to that for [5, Th. 4.1], is omitted here.■

Remark 6: In Theorem 2, the error bounds are obtained for $n \geq 2$. For the case $n = 1$, $e_f = e_1$. From (19), the L_2 bound for e_1 can be expressed as $\int_0^\infty e_1^2(\tau)d\tau \leq (1/\kappa)\bar{V}_2(0)$. By Assumption 1, $V_f \geq (1/2)b_{\min}e_1^2$. Since $V_f(t) \leq V_2(t) \leq V_2(0) \leq \bar{V}_2(0)$, then the L_∞ bound for e_1 can be given as $|e_1(t)| \leq (2/b_{\min})\bar{V}_2(0)$.

We emphasize the lexibility of choice for fully saturated learning laws, and that the learning law (15) is by no means exclusive. An alternative learning law, similar to the conventional repetitive control, where the update term is formed by signals from the last period, can be given as

$$\hat{\delta}(t) = \begin{cases} \mathrm{sat}_{\bar{\delta}_e}\left(\hat{\delta}_e(t)\right), & t > T \\ 0, & t \in [0, T] \end{cases} \quad (22)$$

where $\hat{\delta}_e(t) = \hat{\delta}(t-T) - \Gamma v(x(t-T))e_f(t-T)$.

Choose $V_r(t) = (1/2)\int_t^{t+T}\tilde{\delta}^T(\tau)\Gamma^{-1}\tilde{\delta}(\tau)d\tau$. Its time derivative is

$$\dot{V}_r(t) = \frac{1}{2}\left\{\left[2\delta(t) - \hat{\delta}_e(t+T) - \mathrm{sat}_{\bar{\delta}_e}\left(\hat{\delta}_e(t+T)\right)\right]^T \times\Gamma^{-1}\left[\hat{\delta}_e(t+T) - \mathrm{sat}_{\bar{\delta}_e}\left(\hat{\delta}_e(t+T)\right)\right] + \left[\delta(t) - \hat{\delta}_e(t+T)\right]^T\Gamma^{-1}\left[\delta(t) - \hat{\delta}_e(t+T)\right] - \left[\delta(t) - \hat{\delta}(t)\right]^T\Gamma^{-1}\left[\delta(t) - \hat{\delta}(t)\right]\right\}. \quad (23)$$

By Lemma 1, for the case $\gamma = 1$, $[2\delta(t+T) - \hat{\delta}_e(t+T) - \mathrm{sat}_{\bar{\delta}_e}(\hat{\delta}_e(t+T))]^T\Gamma^{-1}[\hat{\delta}_e(t+T) - \mathrm{sat}_{\bar{\delta}_e}(\hat{\delta}_e(t+T))] \leq 0$, which leads to

$$\dot{V}_r(t) \leq \frac{1}{2}\left[\hat{\delta}_e(t+T) - \hat{\delta}(t)\right]^T\Gamma^{-1}\left[\hat{\delta}_e(t+T) - \hat{\delta}(t)\right] - \left[\delta(t) + \hat{\delta}(t)\right]^T\Gamma^{-1}\left[\hat{\delta}_e(t+T) - \hat{\delta}(t)\right]\Big\}.$$

Employing learning law (22), we obtain

$$\dot{V}_r(t) \leq \frac{1}{2}v^T(x)\Gamma v(x)e_f^2 - \tilde{\delta}^T(t)v(x)e_f. \quad (24)$$

Again applying (14), the feedback control $\hat{u}_f$ should be replaced with

$$\hat{u}_f = -\kappa e_f - \frac{1}{2}v^T(x)\Gamma v(x)e_f - \hat{\theta}^T w(z) - h(z) \quad (25)$$

which results in $\dot{V}_2 \leq -\kappa e_f^2$.

Remark 7: The update term $\Gamma v(x)e_f$ in the presented learning laws is independent of $b(x, \theta_b)$. Therefore, the parametrization of $b(x, \theta_b)$ does not enter into the learning laws. This is due to the utilization of the integral Lyapunov function. The situation may be different if other V_f function is used, e.g., $V_f = (1/2)e_f^2$. If θ_b and θ_f are known, the repetitive controller (6) with $u_f = -b(x, \theta_b)\kappa e_f - f(x, \theta_f) - b(x, \theta_b)\nu$ leads to

$$\dot{V}_f = -\kappa e_f^2 + \frac{1}{b(x, \theta_b)}\tilde{\delta}^T(t)v(x)e_f. \quad (26)$$

To cancel the second term on the right-hand side of (26), we have to choose the update term as $(1/b(x, \theta_b))\Gamma v(x)e_f$. The parameter θ_b consequently enters into the learning law, which may result in a possible singularity problem when we employ its certainty-equivalence adaptive controller for unknown θ_b and θ_f, due to the estimated $b(x, \hat{\theta}_b)$ becoming zero.

IV. Conclusion

In this note, Lyapunov based adaptive repetitive control has been proposed for the class of nonlinear parametrized systems. The controller singularity problem is effectively solved by using the integral Lyapunov function in avoiding the nonlinear parametrization from entering into the adaptive control and repetitive control. Asymptotic convergence of the tracking error is established in the presence of periodic uncertainty, while global stability of the closed-loop system is ensured. Error bounds have been provided to characterize the control performance.

APPENDIX

Note that for $a, b \in R^m$ and $\Lambda \in R^{m \times m}$, if $a_i b_i \leq 0$, $i = 1, 2, \ldots, m$, and Λ is diagonal and positive definite, then $a^T \Lambda b \leq 0$, where a_i and b_i are the components of a and b, respectively. Thus, we need only to prove for each component

$$\left[(\gamma+1)a_i - \left(\gamma b_i + \mathrm{sat}_{\bar{b}_i}(b_i)\right)\right]\left[b_i - \mathrm{sat}_{\bar{b}_i}(b_i)\right] \leq 0 \tag{27}$$

where $\gamma \geq 0$, and a_i satisfies that $\bar{b}_i^1 \leq a_i \leq \bar{b}_i^2$. There are three possible cases which we should consider in order to prove (27). Case $\bar{b}_i^1 \leq b_i \leq \bar{b}_i^2$: It follows that $(\gamma+1)a_i - (\gamma b_i + \mathrm{sat}_{\bar{b}_i}(b_i)) = (\gamma+1)(a_i - b_i)$ and $b_i - \mathrm{sat}_{\bar{b}_i}(b_i) = 0$. Hence, (27) is true for this case. Case $b_i < \bar{b}_i^1$: It follows that $\gamma b_i + \mathrm{sat}_{\bar{b}_i}(b_i)) = \gamma b_i + \bar{b}_i^1 < (\gamma+1)\bar{b}_i^1 \leq (\gamma+1)a_i$. Since $b_i - \mathrm{sat}_{\bar{b}_i}(b_i) = b_i - \bar{b}_i^1 < 0$, then $(\gamma b_i + \bar{b}_i^1)(b_i - \bar{b}_i^1) > (\gamma+1)a_i(b_i - \bar{b}_i^1)$. Hence, (27) also holds for this case. Case $b_i > \bar{b}_i^2$: It follows that $\gamma b_i + \mathrm{sat}_{\bar{b}_i}(b_i)) = \gamma b_i + \bar{b}_i^2 > (\gamma+1)\bar{b}_i^2 \geq (\gamma+1)a_i$. Since $b_i - \mathrm{sat}_{\bar{b}_i}(b_i) = b_i - \bar{b}_i^2 > 0$, then $(\gamma b_i + \bar{b}_i^2)(b_i - \bar{b}_i^2) > (\gamma+1)a_i(b_i - \bar{b}_i^2)$. In summary, (27) is true for all three cases.

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A Decomposition Algorithm for Feedback Min–Max Model Predictive Control

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***Abstract*—An algorithm for solving feedback min–max model predictive control for discrete-time uncertain linear systems with constraints is presented in this note. The algorithm is based on applying recursively a decomposition technique to solve the min–max problem via a sequence of low complexity linear programs. It is proved that the algorithm converges to the optimal solution in finite time. Simulation results are provided to compare the proposed algorithm with other approaches.**

***Index Terms*—Optimization algorithms, predictive control for linear systems, uncertain systems.**

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