Transactions Briefs

Dynamic Output Feedback Stabilization of a Class of Switched Linear Systems

Zhendong Sun and Shuzhi Sam Ge

***Abstract*—In this brief, dynamic output feedback stabilization is investigated for a class of switched linear control systems. Firstly, the switched systems are transformed into the canonical form with a clear system structure. Next, the problem of stabilization via state feedback control is addressed based on the canonical decomposition. Then, a state estimator is introduced to approximate the state vector. Finally, we bring the estimator into the feedback loop and establish the separation property of the overall system. The dynamic output feedback stabilization problem is, thus, solved in a constructive manner.**

***Index Terms*—Hybrid systems, observer, output feedback, stabilization, switched systems, switching path.**

I. Introduction

A switched system is a hybrid system which consists of several subsystems and a rule that orchestrates the switching among them. The study of switched systems is well motivated from several aspects. Firstly, from the practical point of view, switching among different system structures is an essential feature of many engineering systems. Secondly, from the modeling point of view, as complex/intelligent systems are very hard to model/analyze globally and/or in the whole range of operation, a multiple-model approach provides a convenient and efficient way to model these systems. Finally, from the control point of view, multicontroller switching provides an effective mechanism to cope with highly complex systems and/or systems with large uncertainties.

In recent years, there is enormous growth of interest in stability analysis and design of switched systems, see [7] and [9] for surveys of recent development. In particular, there are quite a few works addressing the design issues on how to derive stabilizing switching/control laws for hybrid and switched systems. Among them, stabilizing switching laws were developed in [18] and [20] for switched linear systems; optimal controllers were designed in [2], [5], and [17], and for hybrid and switched systems using various approaches; and a combined switching/controller design procedure was presented for a class of controllable switched linear control systems [15]. For discrete-time switched linear systems, stability/stabilizability issues have been investigated extensively in [8], [12], and [13]. See also [4] and [10] for the computational complexity issues on stability and controllability of hybrid systems.

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In this paper, we address the problem of output feedback stabilization for a class of switched linear control systems. Relevant existing results include the establishment of the controllability and observability criteria in [3], [14], and the development of the observer design procedures in [1]. Based on our previous work [14], we first transform the switched linear systems into a canonical form with a clear system structure. Next, we solve the problem of stabilization via state feedback control and periodic switching. Then, we propose a state estimator to approximate the state vector. The state estimator itself is also a switched linear system. Finally, we establish the separation property of the overall system with the estimator being embedded in the feedback loop as the state observer. The dynamic output feedback stabilization problem is solved constructively.

This paper is arranged as follows. In Section II, we introduce some elementary results. Main results are presented in Section III, while a numerical example is carried out in Section IV, and some concluding remarks are made in Section V.

II. Preliminaries

Consider switched linear control systems described by

$$\begin{cases} \dot{x}(t) = Ax(t) + B_\sigma u(t) \\ y(t) = C_\sigma x(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ are the states, $u(t) \in \mathbf{R}^p$ are the control inputs, $y(t) \in \mathbf{R}^q$ are measured outputs, $\sigma(t) \in M \overset{\text{def}}{=} \{1, 2, \ldots, m\}$ is the switching path to be designed, and A, B_i, and C_i are matrices of compatible dimensions.

System (1) represents a linear plant with multiple control/sensor devices. The description includes the multicontroller switching and the multisensor scheduling as special cases [11].

The objective is to find suitable control inputs and switching laws to make the switched system uniformly asymptotically stable.

Definition 1: System (1) is said to be state feedback stabilizable, if there exist a switching path σ, and a state feedback control law

$$u(t) = l(x(t), \sigma(t))$$

such that the closed-loop system is uniformly asymptotically stable.

Definition 2: System (1) is said to be dynamic output feedback stabilizable, if there exist a switching path σ, and a dynamic output feedback control law

$$\begin{aligned} u(t) &= g(y(t), \hat{x}(t), \sigma(t)) \\ \dot{\hat{x}}(t) &= \bar{f}(\hat{x}(t), y(t), \sigma(t)) \end{aligned}$$

such that the closed-loop system is uniformly asymptotically stable.

The problem of state/output feedback stabilization is to find conditions of stabilizability, and to design stabilizing switching paths as well as state/output feedback control laws for the stabilizable systems.

To address these problems, we need the concepts and criteria of controllability and observability of switched systems. Let us briefly recall the basic results which will be used later, while the details can be found in [14].

For a switched linear control system, its controllable set is formed by all the states which can be transferred to the origin in a finite time by appropriate choices of input u and switching path σ. The system is said to be completely controllable provided that the controllable set is the total state space.

Lemma 1: For switched linear system (1), the controllable set is

$$\mathcal{C} \stackrel{\text{def}}{=} \sum_{i\in M}^{1\le j\le n} \mathrm{Im}(A^{j-1}B_i) \tag{2}$$

where $\mathrm{Im}(B)$ stands for the image space spanned by the columns of matrix B.

It can be seen that (1) is completely controllable iff the matrix pair (A, B) is completely controllable, where $B \stackrel{\text{def}}{=} [B_1, \ldots, B_m]$.

For switched linear system (1), its unobservable set is formed by all the (initial) states which cannot be determined from knowledge of the output and the input in finite time. The system is said to be completely observable if the unobservable set is null.

Lemma 2: For switched linear system (1), the unobservable set is

$$\mathcal{W} \stackrel{\text{def}}{=} \bigcap_{i\in M}^{1\le j\le n} \ker(C_i A^{j-1}) \tag{3}$$

where $\ker(C)$ stands for the kernel space of matrix C.

It can be seen that switched system (1) is completely observable iff matrix pair (C, A) is completely observable with $C \stackrel{\text{def}}{=} \begin{bmatrix} C_1 \\ \vdots \\ C_m \end{bmatrix}$.

In the remainder of this section, we recall some known results on the stabilizability of switched linear (force free) systems.

Definition 3: A switched linear system

$$\dot{x}(t) = A_\sigma x(t) \tag{4}$$

is exponentially stabilizable if there is a switching path σ which makes the system exponentially stable.

A verifiable criterion for the exponential stabilizability of the switched system was given in [18] as follows.

Lemma 3: [18] Suppose that there exists a sequence of nonnegative real numbers $w_i, i \in M$, such that the convex combination matrix $\sum_{i\in M} w_i A_i$ is Hurwitz. Then, (4) is exponentially stabilizable.

A design procedure was provided to construct a state-driven switching law in [18]. Another switching strategy is a time-driven switching law based on the average method as given in [16].

Lemma 4: [16]

Let $A_1, \ldots, A_m$ be matrices in $\mathbf{R}^{n\times n}$ and $w_1, \ldots, w_m$ be nonnegative real numbers. Then, there is a positive real number η, such that, for any $t \le \eta$

$$\begin{aligned}\exp(A_m w_m t)\exp(A_{m-1}w_{m-1}t)\cdots\exp(A_1 w_1 t) \\ = \exp\left(\left(\sum_{i\in M} w_i A_i\right)t + \Upsilon_t t^2\right)\end{aligned} \tag{5}$$

where the entries of matrix Υ_t are analytic and bounded.

III. Main Results

A. Canonical Decomposition

As in the nonswitched linear time-invariant (LTI) case, a switched system can be transformed into the canonical form via a coordinate transformation.

Lemma 5: Switched system (1) is equivalent, via certain state transformation $z = Tx$, to system

$$\begin{aligned}\dot{z}(t) &= \left[\dot{z}_1^T, \dot{z}_2^T, \dot{z}_3^T\right]^T = \bar{A}z(t) + \bar{B}_\sigma u(t) \\ y(t) &= \bar{C}_\sigma z(t)\end{aligned} \tag{6}$$

where

$$\begin{aligned}\bar{A} &= TAT^{-1} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} & \bar{A}_{13} \\ 0 & \bar{A}_{22} & \bar{A}_{23} \\ 0 & 0 & \bar{A}_{33}\end{bmatrix} \\ \bar{B}_i &= TB_i = \begin{bmatrix} \bar{B}_{i,1} \\ \bar{B}_{i,2} \\ 0 \end{bmatrix} \\ \bar{C}_i &= C_i T^{-1} = [0\ \bar{C}_{i,2}\ \bar{C}_{i,3}], \qquad i \in M.\end{aligned} \tag{7}$$

In (6), vector z_1 is controllable but not observable, z_2 is controllable and observable, and z_3 is not controllable.

Proof: The theorem can be proved based on the standard argument (Cf. [6]) and is omitted due to the space limitation.

Note that the dimensions of z_1, z_2 and z_3 are independent of the transition matrix T. In fact, we have

$$\dim z_1 = \dim(\mathcal{C}\cap\mathcal{W}), \dim z_2 = \dim\left(\frac{\mathcal{C}}{\mathcal{W}}\right), \dim z_3 = n - \dim(\mathcal{C}).$$

B. State Feedback Stabilization

In this section, let us consider the problem of stabilization via state feedback control and switching laws.

It can be seen from decomposition (6) that vector z_3 is decoupled from the other vectors and the control inputs. As a consequence, system (6) is stabilizable only if matrix $\bar{A}_{33}$ is Hurwitz, or equivalently, the unstable mode of A is controllable.

Assumption 1: Matrix $\bar{A}_{33}$ is Hurwitz.

For clarity, let $n_2 = \dim(\mathcal{C})$, and

$$\hat{A}_1 = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ 0 & \bar{A}_{22}\end{bmatrix} \quad \hat{B}_i = \begin{bmatrix} \bar{B}_{i,1} \\ \bar{B}_{i,2}\end{bmatrix} \tag{8}$$

and $\hat{B} = [\hat{B}_1, \ldots, \hat{B}_m]$. Note that the matrix pair $(\hat{A}_1, \hat{B})$ is completely controllable. Therefore, for any arbitrarily given set of desired (symmetric) poles $\Lambda = \{\lambda_1, \ldots, \lambda_{n_2}\}$ with negative real parts, we can construct a feedback gain matrix $G \in \mathbf{R}^{mp\times n_2}$, such that matrix $\hat{A}_1 + \hat{B}G$ possesses eigenvalue set Λ. Let us partition G as

$$G = \begin{bmatrix} G_1 \\ \vdots \\ G_m \end{bmatrix}, \qquad G_i \in \mathbf{R}^{p\times n_2}.$$

Fix a set of weighted factors $w_i > 0, i \in M$ with $\sum_{i\in M} w_i = 1$, and define

$$F_i = \frac{1}{w_i}G_i \quad \bar{F}_i = [F_i, 0] \in \mathbf{R}^{p\times n}, \qquad i \in M.$$

For system (6) with feedback control input

$$u(t) = \bar{F}_\sigma z(t) = F_\sigma \begin{bmatrix} z_1(t) \\ z_2(t)\end{bmatrix} \tag{9}$$

the closed-loop system is given by

$$\dot{z}(t) = (\bar{A} + \bar{B}_\sigma \bar{F}_\sigma)z(t). \tag{10}$$

Let $\tilde{A}_i = \bar{A} + \bar{B}_i\bar{F}_i$, $i \in M$, and the average matrix

$$\tilde{A} = \sum_{i\in M} w_i\tilde{A}_i = \begin{bmatrix} \hat{A}_1 + \hat{B}G & \hat{A}_2 \\ 0 & \bar{A}_{33}\end{bmatrix}$$

where $\hat{A}_2 = \begin{bmatrix} \bar{A}_{13} \\ \bar{A}_{23}\end{bmatrix}$. Because both diagonal blocks of $\tilde{A}$ are Hurwitz, matrix $\tilde{A}$ itself is also Hurwitz. From Lemma 3, system (10) is exponentially stabilizable.

In what follows, we propose a switching strategy for system (10) based on Lemma 4. For any matrix E, let $\psi(E)$ denote the largest real part of all its eigenvalues. As matrix $\tilde{A}$ is Hurwitz, we can choose a positive real number $\epsilon < -\psi(\tilde{A})$. By Lemma 4, there is a positive real number η, such that, for any $t \leq \eta$

$$\exp(\tilde{A}_m w_m t) \cdots \exp(\tilde{A}_1 w_1 t) = \exp\left((\tilde{A} + \Upsilon_t t)t\right).$$

Select a $\delta \leq \eta$ such that

$$\psi(\tilde{A} + \Upsilon_\delta \delta) \leq \psi(\tilde{A}) + \epsilon.$$

The existence of such a δ is guaranteed by the continuity of eigenvalues. Note that a smaller ϵ leads to a smaller δ.

Define a periodic switching law

$$\sigma(t) = \begin{cases} 1, & \mathrm{mod}(t, \delta) \in [0, w_1\delta) \\ 2, & \mathrm{mod}(t, \delta) \in [w_1\delta, (w_1 + w_2)\delta) \\ \vdots & \\ m, & \mathrm{mod}(t, \delta) \in \left[\sum_{i=1}^{m-1} w_i\delta, \delta\right) \end{cases} \quad \forall t \tag{11}$$

where $\mathrm{mod}(a, b)$ denotes the remainder of a divided by b. This switching path is periodic with a period of δ.

Simple analysis shows that, under this switching law, system (10) is exponentially stable with the average convergent rate $-\psi(\tilde{A}) - \epsilon$.

Because the property of stabilizability is invariant under state coordinate transformation, the following result follows.

Theorem 1: System (1) satisfying Assumption 1 is state feedback stabilizable.

Remark 1: If system (1) is completely controllable, then we can select the feedback control law and switching frequency, such that the rate of convergence can be arbitrarily assigned for the closed-loop system. Indeed, in this case, the stability margin of the average matrix $\tilde{A}$ can be arbitrarily assigned, and the convergence rate of the switched system can arbitrarily approach that of the average system by switching of high frequency. For an uncontrollable switched system, the convergence rate of the switched system can be assigned arbitrarily near the rate of matrix $\bar{A}_{33}$.

Remark 2: The order of the activated subsystems can be arbitrarily assigned. That is, we can first activate subsystem 1, then switch to subsystem 2, through to subsystem 3, etc. Alternatively, we can first activate subsystem 2, then, subsystem 3, subsystem 1, etc. This feature is very desirable and crucial in practice when some subsystems must be activated before others, as often encountered in the workshops. However, different orders of the switching index may request different switching frequencies to ensure the stability of the switched system.

Remark 3: The ratios among weighted factors can be arbitrarily assigned. The only requirement is that $w_i \neq 0$ and the assumption of $\sum_{i\in M} w_i = 1$ is technical. This flexibility of choosing weighted factors is very beneficial in some circumstances. For example, some control devices may be more reliable than others, in this case the ratios among these devices can set to be high. In particular, if we choose $w_1 = \cdots = w_m = 1/m$, then the switching path (11) is periodic and with the same interval for any subsystem at each period. In this case, the overall system behaves in a multirate control manner which is easily implemented in practice.

C. State Estimator

In this section, we propose a state estimator to approximate the state variables. Note that for stabilization, we only need to estimate the state variables which are used in the feedback loop, that is, the controllable part as illustrated in (9) of Section III-B. For this, it is necessary that this part are completely observable.

Assumption 2: $\mathcal{C} \cap \mathcal{W} = \{0\}$.

Under this assumption, the canonical form (6) can be rewritten as

$$\begin{aligned} \dot{z}(t) &= \begin{bmatrix} \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} \bar{A}_{22} & \bar{A}_{23} \\ 0 & \bar{A}_{33} \end{bmatrix} z(t) + \begin{bmatrix} \bar{B}_{\sigma 2} \\ 0 \end{bmatrix} u(t) \\ y(t) &= [\bar{C}_{\sigma 2} \ \bar{C}_{\sigma 3}] z(t) \end{aligned} \tag{12}$$

where z_2 is controllable and observable and z_3 is uncontrollable.

We propose the following estimator for z_2:

$$\dot{\hat{z}}_2(t) = \bar{A}_{22}\hat{z}_2(t) + L_\sigma[y(t) - \bar{C}_{\sigma 2}\hat{z}_2] + \bar{B}_{\sigma 2}u(t) \tag{13}$$

where $u(t)$, $y(t)$ and σ are the input, output and switching path of system (1), respectively, and matrices $L_1, \ldots, L_m \in \mathbf{R}^{n_2 \times q}$ will be determined later.

The dynamical equation of the estimator can be rewritten as

$$\dot{\hat{z}}_2(t) = (\bar{A}_{22} - L_\sigma \bar{C}_{\sigma 2})\hat{z}_2(t) + L_\sigma y(t) + \bar{B}_{\sigma 2}u(t). \tag{14}$$

Note that the estimator itself is a switched system as well though with the same switching path as system (1). Hence, the switching path is not an independent design variable.

Define the difference between the real state and the estimated state

$$\tilde{z}_2 = z_2 - \hat{z}_2. \tag{15}$$

Subtracting (14) from (12), we obtain

$$\begin{aligned} \dot{\tilde{z}}_2 &= (\bar{A}_{22} - L_\sigma \bar{C}_{\sigma 2})\tilde{z}_2 + (\bar{A}_{23} - L_\sigma \bar{C}_{\sigma 3})z_3 \\ \dot{z}_3 &= \bar{A}_{33} z_3. \end{aligned} \tag{16}$$

Theorem 2: Under Assumptions 1 and 2, system (16) is exponentially stabilizable via periodic switching path (11).

Proof: Denote matrix $\bar{C}_2 = \left[\bar{C}_{12}^T, \ldots, \bar{C}_{m2}\right]^T$. It follows from Assumption 2 that matrix pair $(\bar{C}_2, \bar{A}_{22})$ is completely observable. Therefore, for any arbitrarily given set of desired (symmetric) poles $\Psi = \{\psi_1, \ldots, \psi_{n_2}\}$ with negative real parts, we can construct a feedback gain matrix $\bar{L} \in \mathbf{R}^{n_2 \times mq}$, such that matrix $\bar{A}_{22} - \bar{L}\bar{C}_2$ possesses eigenvalue set Ψ. Now partition $\bar{L}$ as

$$\bar{L} = \left[\bar{L}_1, \ldots, \bar{L}_m\right], \qquad \bar{L}_i \in \mathbf{R}^{n_2 \times q}.$$

For any given set of weighted factors $w_i > 0, i \in M$ with $\sum_{i\in M} w_i = 1$, define

$$L_i = \frac{1}{w_i}\bar{L}_i, \qquad i \in M.$$

Compute

$$\sum_{i\in M} w_i \begin{bmatrix} \bar{A}_{22} - L_i\bar{C}_{i2} & \bar{A}_{23} - L_i\bar{C}_{i3} \\ 0 & \bar{A}_{33} \end{bmatrix} = \begin{bmatrix} \bar{A}_{22} - \bar{L}\bar{C}_2 & \bar{A}_{23} - \bar{L}\bar{C}_3 \\ 0 & \bar{A}_{33} \end{bmatrix} \tag{17}$$

where $\bar{C}_3 \stackrel{\text{def}}{=} \begin{bmatrix} \bar{C}_{13} \\ \vdots \\ \bar{C}_{m3} \end{bmatrix}$. Note that $\bar{A}_{22} - \bar{L}\bar{C}_2$ is Hurwitz, and it follows from Assumption 1 that $\bar{A}_{33}$ is also Hurwitz. Consequently, the average matrix in (17) is Hurwitz. By Lemma 3, (16) is exponentially stabilizable via any periodic switching path with a sufficiently high switching frequency. This completes the proof of the theorem.

Remark 4: Note that the estimator can track the real state asymptotically no matter whether the real state converges or diverges. However, the stability (and the convergence rate) of the error system does depend on the frequency of the switching path. The estimator may not approach the real state if the switching is not fast enough.

Remark 5: From the proof, if the system is completely controllable, then we can assign any pole set for the average system through appropriate selecting the gain matrices L_i, $i \in M$. Furthermore, if the switching frequency is sufficiently high, then the error dynamical system converges exponentially at a rate near that of the average system. Hence the estimator can approximate the real state at any given rate of convergence.

D. Separation Principle

In this section, we establish that the design of the state feedback and the design of the state estimator can be carried out independently for the problem of dynamic output feedback stabilization. This separation property enables us to solve the problem in a clear and constructive way.

Using the estimator of Section III-C to substitute the real state in the feedback controller of Section III-B, we obtain the overall system described by

$$\begin{aligned}\dot{z}(t) &= \begin{bmatrix} \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} \bar{A}_{22} & \bar{A}_{23} \\ 0 & \bar{A}_{33} \end{bmatrix} z(t) + \begin{bmatrix} \bar{B}_{\sigma 2} \\ 0 \end{bmatrix} u(t) \\ y(t) &= [\bar{C}_{\sigma 2}\ \bar{C}_{\sigma 3}] z(t) \\ u(t) &= F_\sigma \hat{z}_2(t) \\ \dot{\hat{z}}_2(t) &= \bar{A}_{22}\hat{z}_2(t) + L_\sigma[y(t) - \bar{C}_{\sigma 2}\hat{z}_2] + \bar{B}_{\sigma 2} u(t). \end{aligned} \tag{18}$$

Substituting the input and output expressions into the differential equations, we obtain the closed-loop system given by

$$\begin{aligned}\dot{z}_2 &= \bar{A}_{22} z_2 + \bar{B}_{\sigma 2} F_\sigma \hat{z}_2 + \bar{A}_{23} z_3 \\ \dot{\hat{z}}_2 &= (\bar{A}_{22} + \bar{B}_{\sigma 2} F_\sigma - L_\sigma \bar{C}_{\sigma 2})\hat{z}_2 + L_\sigma \bar{C}_{\sigma 2} z_2 + L_\sigma \bar{C}_{\sigma 3} z_3 \\ \dot{z}_3 &= \bar{A}_{33} z_3. \end{aligned} \tag{19}$$

Let $\omega = [z_2, \hat{z}_2, z_3]^T$, and, for $i = 1, \ldots, m$

$$\Omega_i = \begin{bmatrix} \bar{A}_{22} & \bar{B}_{i2} F_i & \bar{A}_{23} \\ L_i \bar{C}_{i2} & \bar{A}_{22} + \bar{B}_{i2} F_i - L_i \bar{C}_{i2} & L_i \bar{C}_{i3} \\ 0 & 0 & \bar{A}_{33} \end{bmatrix}. \tag{20}$$

We can rewrite the closed-loop system as

$$\dot{\omega}(t) = \Omega_\sigma \omega(t). \tag{21}$$

The average matrix of Ω_i, $i \in M$ under weighted factors w_i, $i \in M$ can be computed as

$$\Omega = \sum_{i \in M} w_i \Omega_i = \begin{bmatrix} \bar{A}_{22} & \bar{B}_2 G & \bar{A}_{23} \\ \bar{L}\bar{C}_2 & \bar{A}_{22} + \bar{B}_2 G - \bar{L}\bar{C}_2 & \bar{L}\bar{C}_3 \\ 0 & 0 & \bar{A}_{33} \end{bmatrix}. \tag{22}$$

It can be seen that this matrix is similar to a block triangular matrix with stable diagonal blocks. Indeed, simple calculation gives

$$\begin{aligned}&\begin{bmatrix} I_{n_2} & 0 & 0 \\ I_{n_2} & -I_{n_2} & 0 \\ 0 & 0 & I_{n-n_2} \end{bmatrix} \Omega \begin{bmatrix} I_{n_2} & 0 & 0 \\ I_{n_2} & -I_{n_2} & 0 \\ 0 & 0 & I_{n-n_2} \end{bmatrix}^{-1} \\ &= \begin{bmatrix} \bar{A}_{22} + \bar{B}_2 G & -\bar{B}_2 G & \bar{A}_{23} \\ 0 & \bar{A}_{22} - \bar{L}\bar{C}_2 & \bar{A}_{23} - \bar{L}\bar{C}_3 \\ 0 & 0 & \bar{A}_{33} \end{bmatrix}. \end{aligned}$$

Therefore, the characteristic polynomial of the average matrix is the product of those of the state feedback, state estimator and uncontrollable modes. As a consequence, the average system

$$\dot{\omega}(t) = \Omega \omega(t) \tag{23}$$

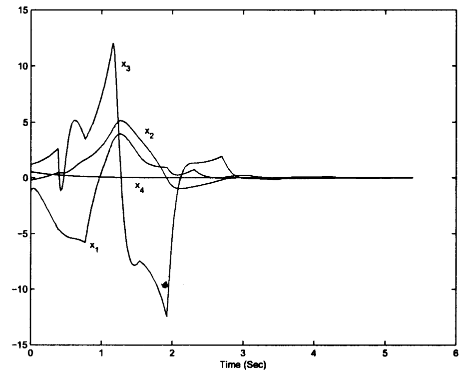

Fig. 1. State trajectories with $\delta = 0.77$.

is asymptotically stable. By Lemma 3, we obtain the main result of this paper as summarized in Theorem 3.

Theorem 3: Under Assumptions 1 and 2, system (1) is dynamical output feedback stabilizable.

Remark 6: From the above analysis, the separation property holds for the average system in terms of eigenvalue assignation. That is, for the average closed-loop system, the design of the average state feedback system and the design of the average state estimator system can be carried out independently. However, there is no such separation property for the design of the switching path. Indeed, the switching law (11) can stabilize the overall system iff it can stabilize both the state feedback system and the state estimator system.

Remark 7: The overall performance of the closed-loop system depends on the gain matrices of the state feedback system and the estimator system, and the switching path. Roughly speaking, the feedback control laws are to achieve specific (local) features of individual subsystems, while the switching law puts these specific features together to produce the desired global performance.

Remark 8: In the above, we only presented a time-driven switching strategy based on the average method. For switched system with a stable convex combination, [18] also proposed a state-feedback switching strategy. That is, switching occurs according to the information of state variables. When the state variables are not available, an important problem naturally arises: how (if possible) to design output-feedback switching strategies for these systems? It seems that this problem is quite involved and we leave it open for further investigation.

IV. Illustrating Example

Consider system (1) with $n = 4$, $m = 2$, and

$$\begin{aligned} A &= \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 3 & 2 & -1 \\ 0 & 0 & 0 & -2 \end{bmatrix} \\ B_1 &= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ B_2 &= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \\ C_1 &= [1\ -1\ 0\ 1] \\ C_2 &= [0\ 1\ 0\ -2]. \end{aligned} \tag{24}$$

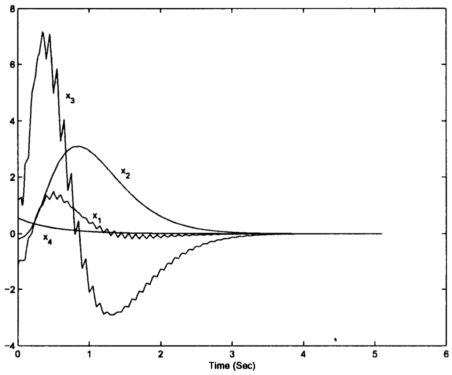

Fig. 2. State trajectories with $\delta = 0.10$.

This system is already in the canonical form (6). It has a one-dimensional uncontrollable mode which is asymptotically stable, and a three-dimensional controllable mode which possesses three unstable poles.

Partition the system matrices as in (18) and fix the equally weighted factors $w_1 = w_2 = 1/2$. Let $\bar{B} = [w_1\bar{B}_{12}, w_2\bar{B}_{22}]$. For the matrix pair $(\bar{A}_{22}, \bar{B})$, assign its poles to $\phi_2 = \{-2.5, -3, -3.5\}$. Similarly, set $\bar{C} = \begin{bmatrix} w_1\bar{C}_{12} \\ w_2\bar{C}_{22} \end{bmatrix}$ and $\phi_3 = \{-4, -4.5, -5\}$. Find a gain matrix $\bar{L}$ such that $\bar{A}_{22} - \bar{L}\bar{C}$ possess pole set ϕ_3.

Let us fix the period at $\delta = 0.77$, and define a periodic switching law as in (11). Fig. 1 shows the state trajectories initially from

$$[-1.140, -0.211, 1.190, -1.116, 0.635, -0.601, 0.551]^T$$

at time $t_0 = 0$. It can be seen that the trajectories are not smooth at the switching instants. However, the uncontrollable mode x_4 is always smooth because this part does not rely on the switching law.

Switching paths with smaller periods would result in system trajectories more resembling that of the average system. Fig. 2 shows the state trajectories with period $\delta = 0.10$. This one has more "chattering" but much more like a linear time-invariant system from the appearance.

V. Conclusion

In this brief, output feedback stabilization has been solved in a constructive manner for a class of switched linear control systems. First, we introduced the canonical form for switched linear systems. Next, we solved the problem of stabilization via state feedback controllers. Then, we proposed a state estimator to approximate the state variables. Finally, we established the separation property of the overall system with the estimator being incorporated into the feedback loop as the state observer.

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