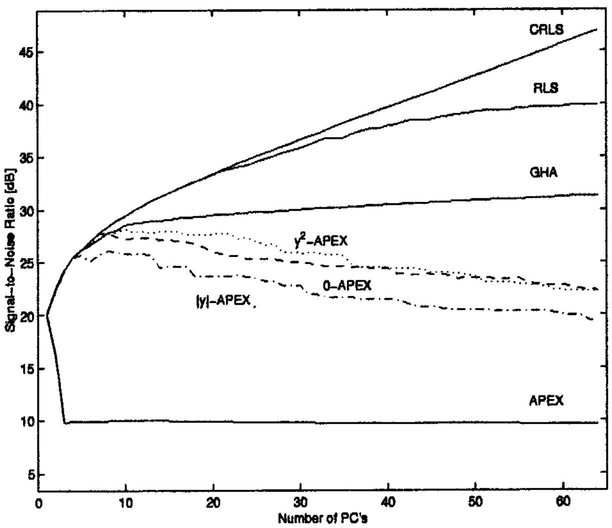

Fig. 1. Sequence of SNR values computed extracting sequentially 64 PCs from 8×8 image blocks.

(512 × 512 pixels) are sequentially extracted. The maximum number of epochs per each PC has been fixed to 40 while a threshold on the $\Delta\mathbf{W}$ (2×10^{-4}) allows the algorithm to jump to the next PC when a reasonable convergence has been achieved. The SNR is computed after that eight PCs have been extracted. The step size η has been fixed to 0.01, and the magnifying factor ρ has been set to 8.

Using a 256 × 256 pixel image, and the same setup used in the previous experiment, SNR values have been computed by sequentially extracting all the 64 PCs of the image. Fig. 1 reports the results obtained in this case. Both Table I and Fig. 1 show how the proposed algorithms perform always better than the original APEX, although their performance is lower with respect to GHA and other more complex approaches.

V. Conclusion

The aim of this brief was to give a clear theoretical framework for APEX-like PCA algorithms derived by an optimization formulation based on a pair of objective functions defined in order to make the Rubner–Tavan's laterally connected neural network able to perform sequential principal component analysis of stationary random signals. The gradient-based stochastic optimization of these criteria yields a class of learning rules which depend on free functions that affect their dynamics. A theoretical study has also been carried out in order to analytically prove the convergence (in the mean sense) of the associated dynamical systems to the expected solutions; the theoretical analysis had also provided us with some useful hints for choosing the shape of the free functions. The experimental results performed on real-world signals (gray-level images) illustrated the performances of some members of the new class with respect to the original APEX algorithm.

Acknowledgment

The first author wishes to gratefully thank Dr. S. Costa who kindly prepared the experimental setup for Section IV.

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Adaptive Control of Uncertain Chua's Circuits

S. S. Ge and C. Wang

***Abstract*—In this brief, we consider the problem of adaptive backstepping control of the Chua's circuits with all the parameters unknown. First, we show that several types of Chua's circuits, including the Chua's oscillator, Chua's circuit with cubic nonlinearity, and Murali–Lakshmanan–Chua circuit, can be transformed into a class of nonlinear systems in the nonautonomous "strict-feedback" form. Second, an adaptive backstepping with tuning functions method is extended to this nonautonomous "strict-feedback" system, and then employed to control the output of the Chua's circuit to asymptotically track an arbitrarily given reference signal generated from a known, bounded and smooth nonlinear reference model. Both global stability and asymptotic tracking of the closed-loop system are guaranteed. Simulation results are presented to show the effectiveness of the approach.**

***Index Terms*—Adaptive backstepping, Chua's circuits, nonautonomous systems.**

I. Introduction

Controlling chaotic systems has recently been in the focus of attention in the nonlinear dynamics literature ([1], [2] and the references therein). In particular, many adaptive control schemes have been successfully applied to the control and synchronization of chaotic systems

Manuscript received April 14, 1999; revised November 10, 1999 and February 23, 2000. This paper was recommended by Associate Editor B. E. Shi.

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Publisher Item Identifier S 1057-7122(00)07063-X.

[3]–[7]. All these methods are based on rigorous Lyapunov stability theorem and Lyapunov function methods. But the construction of the Lyapunov functions remains to be a difficult task.

In the past decade, adaptive control of nonlinear systems has undergone rapid developments ([10] and the references therein). Using backstepping approach, a systematic design method of globally stable adaptive controller was developed for parametric strict-feedback systems [8]. The overparametrization problem was soon successfully eliminated through the tuning functions by Krstiĉ *et al.* [9]. Backstepping with tuning functions has become one of the most popular design methods in the nonlinear adaptive control area.

In this brief, we consider the problem of adaptive backstepping control of the Chua's circuits with all the parameters unknown. First, it is noticed that several types of Chua's circuits [11], including the Chua's oscillator [12] , Chua's circuit with cubic nonlinearity [13], and Murali–Lakshmanan–Chua circuit [14], can be transformed into the following nonautonomous strict-feedback form

$$\begin{aligned}\dot{x}_i &= b_i g_i(\overline{x}_i, t)x_{i+1} + \theta^T F_i(\overline{x}_i, t) + f_i(\overline{x}_i, t),\\ &\qquad 1 \le i \le n-1\\ \dot{x}_n &= b_n g_n(\overline{x}_n, t)u + \theta^T F_n(\overline{x}_n, t) + f_n(\overline{x}_n, t)\\ y &= x_1\end{aligned} \tag{1}$$

where $\overline{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i, i = 1, \ldots, n, u \in R$, and $y \in R$ are the states, input and output, respectively; $b = [b_1, b_2, \ldots, b_n]^T \in R^n$ and $\theta = [\theta_1, \theta_2, \ldots, \theta_p] \in R^p$ are the vectors of unknown constant parameters of interest; $g_i(\cdot) \neq 0, F_i(\cdot), f_i(\cdot), i = 1, \ldots, n-1$ are known, smooth nonlinear functions, with their jth derivatives $(j = 0, \ldots, n-i)$ uniformly bounded in $t; g_n(\cdot) \neq 0, F_n(\cdot), f_n(\cdot)$ are known continuous nonlinear functions, uniformly bounded in t. Assume that the signs of parameters $b_i, i = 1, \ldots, n$ are known.

Second, we extend the adaptive backstepping with tuning functions method to this nonautonomous strict-feedback system (2) where all coefficients b_i are unknown. Note that in the literature, adaptive backstepping control was developed for the case where only one of coefficients b_i is unknown due to the complexity of derivation. Finally, this design method is applied to the control of the Chua's circuit with all of the key parameters unknown. With the extended backstepping design method, we can control the output of the Chua's circuit, originally in equilibrium state, periodic state or chaotic state, to a given equilibrium, or to track any smooth, bounded reference signal, including chaotic signals. With these properties, this method will be of interest to researchers in the area of chaotic systems control. Simulation studies are conducted to show the effectiveness of this approach.

II. Chua's Circuits in Strict-Feedback Form

The Chua's circuit contains three linear energy storage elements (one inductor L and two capacitors C_1 and C_2), one linear resistor R, and one nonlinear resistor called Chua's diode $g(v_{C_1})$. Its dynamic equations are described by

$$\begin{aligned}C_1 \frac{dv_{C_1}}{dt} &= \frac{1}{R}(v_{C_2} - v_{C_1}) - g(v_{C_1})\\ C_2 \frac{dv_{C_2}}{dt} &= \frac{1}{R}(v_{C_1} - v_{C_2}) + i_L\\ L\frac{di_L}{dt} &= -v_{C_2}\end{aligned} \tag{2}$$

where C_1, C_2, L, and R are all circuit parameters, i_L is the current through the inductor L, v_{C_1}, and v_{C_2} are the voltages across C_1 and C_2, respectively, and the piecewise linear function $g(v_{C_1})$ describes the V–i characteristics of the Chua's diode g as follows:

$$g(v_{C_1}) = G_b v_{C_1} + \tfrac{1}{2}(G_a - G_b)(|v_{C_1} + 1| - |v_{C_1} - 1|) \tag{3}$$

with $G_a < 0$ and $G_b < 0$ being some appropriately chosen constants.

By defining $b_1 = 1/L > 0, b_2 = 1/RC_2 > 0, \theta_1 = 1/C_2, \theta_2 = 1/RC_2, \theta_3 = 1/RC_1, \theta_4 = 1/RC_1 + G_b/C_1$, and $\theta_5 = (G_a - G_b)/2C_1$, and defining the state variables as

$$x_1 = i_L, \quad x_2 = v_{C_2}, \quad x_3 = v_{C_1} \tag{4}$$

(3) can be reformulated in the following form:

$$\begin{aligned}\dot{x}_1 &= -b_1 x_2\\ \dot{x}_2 &= b_2 x_3 + \theta_1 x_1 - \theta_2 x_2\\ \dot{x}_3 &= u + \theta_3 x_2 - \theta_4 x_3 - \theta_5[|x_3 + 1| - |x_3 - 1|]\end{aligned} \tag{5}$$

where the control $u(\cdot)$ is assumed to be introduced into the third equation of (5) to form the controlled Chua's circuit.

In comparison with the strict-feedback system form (2), and in the case when all the system parameters are unknown constants, i.e., $\theta = [\theta_1, \theta_2, \ldots, \theta_5]^T, b_1$, and b_2 are unknown (except that the signs of b_1 and b_2 are assumed to be known), we have

$$\begin{aligned}g_1(x_1) &= -1, \quad g_2(x_1, x_2) = 1, \quad g_3(x_1, x_2, x_3) = 1\\ f_1(x_1) &= 0, \quad f_2(x_1, x_2) = 0, \quad f_3(x_1, x_2, x_3) = 0\\ F_1(x_1) &= [0 \quad 0 \quad 0 \quad 0 \quad 0]^T\\ F_2(x_1, x_2) &= [x_1 \quad -x_2 \quad 0 \quad 0 \quad 0]^T\\ F_3(x_1, x_2, x_3) &= [0 \;\; 0 \;\; x_2 \;\; -x_3 \;\; -(|x_3 + 1| - |x_3 - 1|)]^T.\end{aligned}$$

Following the same procedure, it can be verified that several other kinds of Chua's circuits, such as the Chua's Oscillator, the Chua's circuit with cubic nonlinearity and the Murali–Lakshmanan–Chua circuit, can all be transformed into the nonautonomous strict-feedback form (2).

In the next section, we will extend the adaptive backstepping with tuning functions method [9], [10] to the class of nonautonomous strict-feedback system in form (2).

III. Adaptive Backstepping with Tuning Functions

Consider a known, bounded and smooth reference model as follows:

$$\begin{aligned}\dot{x}_{ri} &= f_{ri}(x_r, t), \quad 1 \le i \le m\\ y_r &= x_{r1}\end{aligned} \tag{6}$$

where $x_r = [x_{r1}, x_{r2}, \ldots, x_{rm}]^T \in R^m (m \ge n), y_r \in R$ are the states and output, respectively; $f_{ri}(\cdot), i = 1, \ldots, m$ are known smooth nonlinear functions, with their jth derivatives $(j = 0, \ldots, n-i)$ uniformly bounded in t.

Let us design an adaptive state-feedback controller which can guarantee global stability of the closed-loop system and asymptotic tracking of the output $y = x_1(t)$ of system (2) to the output $y_r = x_{r1}(t)$ of the reference model, i.e.,

$$y(t) - y_r(t) \to 0, \quad \text{as } t \to \infty. \tag{7}$$

In the literature, adaptive backstepping control was developed for the case where only one of coefficients b_i is unknown due to the complexity of derivation. In this brief, adaptive backstepping control is developed and presented in an easy-to-use format for the case where all coefficients b_i are unknown.

The backstepping design procedure is recursive. At the ith step, the ith-order subsystem is stabilized with respect to a Lyapunov function V_i by the design of a stabilizing function α_i, and tuning functions $\tau_i, \pi_i^{b_1}, \ldots, \pi_i^{b_i}$. For the unknown b_i, two estimates $\hat{b}_i$ and $\hat{\varrho}_i$ are introduced, where $\hat{\varrho}_i$ is the estimate of $\varrho_i = 1/b_i$ and is introduced to avoid the division by $\hat{b}_i(t)$. The control law u and the update laws $\dot{\hat{\theta}}, \dot{\hat{b}}_i$, and $\dot{\hat{\varrho}}_i$ are designed as

1) Adaptive Control Law:

$$u = \frac{\hat{\varrho}_n}{g_n}\left(-c_n z_n - \hat{b}_{n-1} g_{n-1} z_{n-1} + \sum_{k=1}^{n-2} z_{k+1} \frac{\partial \alpha_k}{\partial \hat{\theta}} \Gamma F_{ns} - \sum_{j=1}^{n-3}\sum_{k=1}^{n-j-2}\left(z_{k+2}\frac{\partial \alpha_{k+1}}{\partial \hat{b}_j}\right)\gamma \frac{\partial \alpha_{n-3}}{\partial x_j} g_j x_{j+1} - \hat{\theta}^T F_{ns} - f_{ns}\right). \quad (8)$$

2) Parameter Update Laws:

$$\begin{aligned}
\dot{\hat{\theta}} &= \tau_n = \tau_{n-1} + \Gamma F_{ns} z_n \\
\dot{\hat{b}}_i &= \pi_n^{b_i} = \pi_{n-1}^{b_i} - \gamma z_n \frac{\partial \alpha_{n-1}}{\partial x_i} g_i x_{i+1}, \quad 1 \le i \le n-1 \\
\dot{\hat{\varrho}}_i &= -\mathrm{sgn}(b_i)\gamma z_i\left(\frac{g_i}{\hat{\varrho}_i}\alpha_i + g_i x_{r(i+1)}\right), \quad 1 \le i \le n-1 \\
\dot{\hat{\varrho}}_n &= -\mathrm{sgn}(b_n)\gamma \frac{g_n}{\hat{\varrho}_n} u
\end{aligned} \quad (9)$$

with *coordinate transformation:*

$$\begin{aligned}
z &= [z_1, z_2, \ldots, z_n]^T \\
z_1 &= x_1 - x_{r1} \\
z_{i+1} &= x_{i+1} - \hat{\varrho}_i x_{r(i+1)} - \alpha_i, \quad 1 \le i \le n-1.
\end{aligned} \quad (10)$$

3) Regressor:

$$\begin{aligned}
F_{1s} &= F_1, \quad F_{is} = F_i - \sum_{k=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_k} F_k, \quad 2 \le i \le n \\
f_{1s} &= f_1 + g_1 x_{r2} - f_{r1} \\
f_{is} &= f_i + g_i x_{r(i+1)} - f_{ri} - \sum_{k=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_k} f_k \\
&\quad - \sum_{k=1}^{i-1}\hat{b}_k \frac{\partial \alpha_{i-1}}{\partial x_k} g_k x_{k+1} - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\tau_i \\
&\quad - \sum_{k=1}^{i-2}\frac{\partial \alpha_{i-1}}{\partial \hat{b}_k}\pi_i^{b_k} - \sum_{k=1}^{n}\frac{\partial \alpha_{i-1}}{\partial x_{rk}} f_{rk} - \hat{\varrho}_{i-1} f_{ri} \\
&\quad - \sum_{k=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial \hat{\varrho}_k}\dot{\hat{\varrho}}_k - \dot{\hat{\varrho}}_{i-1}\left(x_{ri} + \frac{\partial \alpha_{i-1}}{\partial \hat{\varrho}_{i-1}}\right) \\
&\quad - \frac{\partial \alpha_{i-1}}{\partial t}, \quad 2 \le i \le n-1 \\
f_{ns} &= f_n - f_{rn} - \sum_{k=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_k} f_k - \sum_{k=1}^{n-1}\hat{b}_k\frac{\partial \alpha_{n-1}}{\partial x_k} g_k x_{k+1} \\
&\quad - \sum_{k=1}^{n-2}\frac{\partial \alpha_{n-1}}{\partial \hat{b}_k}\pi_n^{b_k} - \sum_{k=1}^{n}\frac{\partial \alpha_{n-1}}{\partial x_{rk}} f_{rk} \\
&\quad - \sum_{k=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial \hat{\varrho}_k}\dot{\hat{\varrho}}_k - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\tau_n - \hat{\varrho}_{n-1} f_{rn} \\
&\quad - \dot{\hat{\varrho}}_{n-1}\left(x_{rn} + \frac{\partial \alpha_{n-1}}{\partial \hat{\varrho}_{n-1}}\right) - \frac{\partial \alpha_{n-1}}{\partial t}.
\end{aligned} \quad (11)$$

4) Tuning Functions for $\hat{\theta}$:

$$\tau_1 = \Gamma F_{1s} z_1, \quad \tau_i = \tau_{i-1} + \Gamma F_{is} z_i, \qquad 2 \le i \le n. \quad (12)$$

5) Tuning Functions for $\dot{\hat{b}}_i$:

$$\begin{aligned}
\pi_1^{b_1} &= \gamma g_1 z_1 z_2 \\
\pi_i^{b_1} &= \pi_{i-1}^{b_1} - \gamma z_i \frac{\partial \alpha_{i-1}}{\partial x_1} g_1 x_2, \quad 2 \le i \le n \\
&\vdots \\
\pi_i^{b_{i-1}} &= \pi_{i-1}^{b_{i-1}} - \gamma z_i \frac{\partial \alpha_{i-1}}{\partial x_{i-1}} g_{i-1} x_i, \quad 2 \le i \le n \\
\pi_i^{b_i} &= \gamma g_i z_i z_{i+1}, \quad 2 \le i \le n-1.
\end{aligned} \quad (13)$$

6) Virtual Control Functions:

$$\begin{aligned}
\alpha_1 &= \frac{\hat{\varrho}_1}{g_1}\left(-c_1 z_1 - \hat{\theta}^T F_{1s} - f_{1s}\right) \\
\alpha_i &= \frac{\hat{\varrho}_i}{g_i}\left(-c_i z_i - \hat{b}_{i-1} g_{i-1} z_{i-1} - \hat{\theta}^T F_{is} - f_{is} + \sum_{k=1}^{i-2} z_{k+1}\frac{\partial \alpha_k}{\partial \hat{\theta}}\Gamma F_{is} - \sum_{j=1}^{i-3}\sum_{k=1}^{i-j-2} \times z_{k+2}\frac{\partial \alpha_{k+1}}{\partial \hat{b}_j}\gamma\frac{\partial \alpha_{i-1}}{\partial x_j} g_j x_{j+1}\right), \\
&\qquad 2 \le i \le n-1
\end{aligned} \quad (14)$$

where $c_i > 0, \gamma > 0, \Gamma = \Gamma^T > 0$, and $\alpha_0 = 0$.

The stability and asymptotic tracking of the closed-loop system is summarized in Theorem 1.

Theorem 1: The closed-loop adaptive system consisting of plant (2), reference model (6), controller (8), and parameter update law (9) has a globally uniformly stable equilibrium at $z = [z_1, z_2, \ldots, z_n]^T = 0$. This guarantees the global boundedness of all the signals in the closed-loop system, including the states $x = [x_1, x_2, \ldots, x_n]^T$, the control u and parameter estimates $\hat{\theta}, \hat{b}_1, \ldots, \hat{b}_{n-1}$ and $\hat{\varrho}_1, \ldots, \hat{\varrho}_n$, and $\lim_{t\to\infty} z(t) = 0$, i.e., subsequently,

$$\lim_{t\to\infty}[y(t) - y_r(t)] = 0.$$

Proof: See Appendix A for detailed procedures.

Remark 1: The strict-feedback system (2) only has parametric uncertainties which appear linearly with respect to the known nonlinear functions. For the case when both parametric uncertainty and unknown nonlinear functions are present in the system, where these unknown nonlinear functions could be due to modeling errors, external disturbances, time variations in the system, robust adaptive control design can be used to guarantee robustness with respect to bounded uncertainties, and exogenous disturbances (see, e.g., [16] and the references therein). But in general it cannot achieve convergence of the tracking error to zero without using high gain.

Remark 2: Reference model (6) is very general, and covers many chaotic systems, such as the Rossler system and Lorenz system. It should be noted that one limitation of (6) is that only smooth functions are included. Accordingly, the Chua's circuit (5) has to be eliminated from the list due to its nonsmooth function (3). However, following the same procedure in the Appendix A, it can be shown that the proposed method is still applicable to the reference model containing nonsmooth, continuous functions in the following form:

$$\begin{aligned}
\dot{x}_{ri} &= f_{ri}\left(\overline{x}_{r(i+1)}, t\right), \quad 1 \le i \le n-1 \\
\dot{x}_{rj} &= f_{rj}(\overline{x}_{rm}, t), \quad n \le j \le m \\
y_r &= x_{r1}
\end{aligned} \quad (15)$$

where $\overline{x}_{ri} = [x_{r1}, x_{r2}, \ldots, x_{ri}]^T \in R^i, i = 1, \ldots, m, y_r \in R$ are the states and output, respectively; $f_{ri}(\cdot), i = 1, 2, \ldots, n-1$

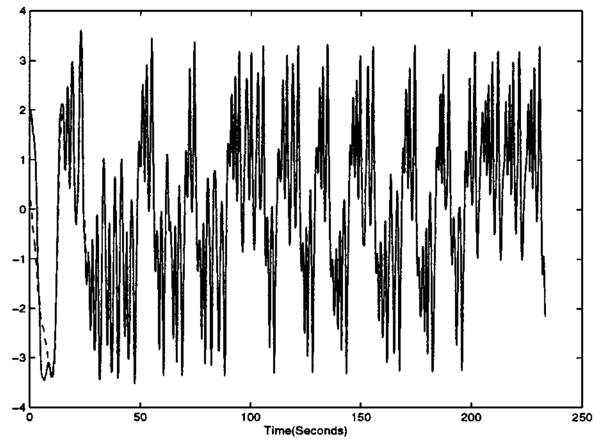

Fig. 1. Output tracking performance (x_1—solid line and x_{r1}—dash line).

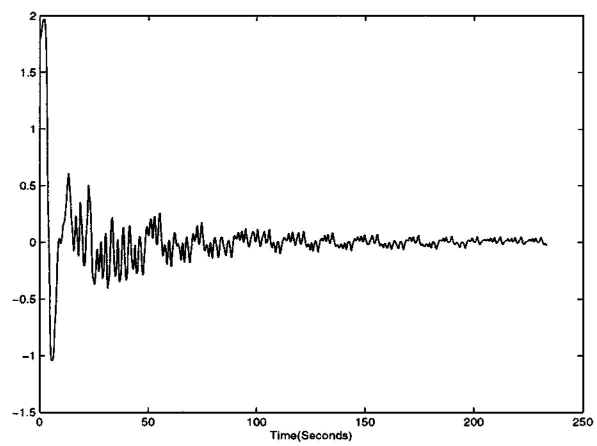

Fig. 2. Tracking error $x_1(t) - x_{r1}(t)$.

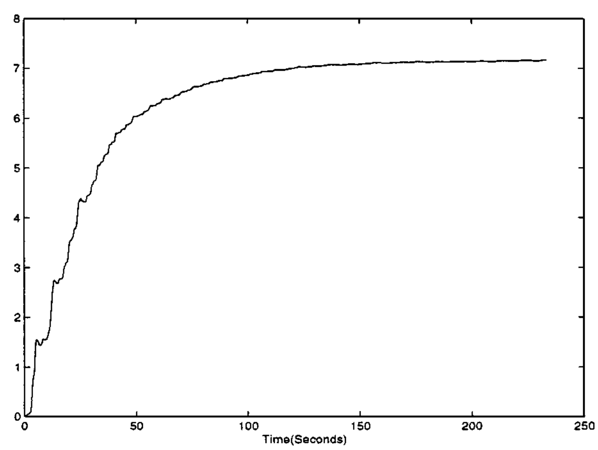

Fig. 3. L_2 norm of all parameter estimates.

are known smooth nonlinear functions, with their jth derivatives $(j = 0, \ldots, n-i)$ uniformly bounded in t; while $f_{rj}(\cdot), n \leq j \leq m$ are known continuous functions, uniformly bounded in t. This is feasible because there is no need for differentiation of $f_{rj}(\cdot), n \leq j \leq m$ in the design procedures.

It can be easily seen that the Chua's circuits mentioned in Section II can all be transformed into (15). Thus, the Chua's circuits satisfy the conditions for both the plant (2) and the reference model (15). We will take the Chua's circuit as an example in simulation studies to show the effectiveness of this approach.

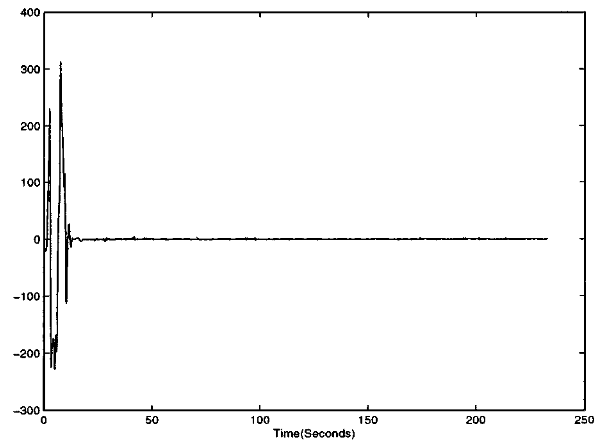

Fig. 4. Control signal u.

IV. Simulation Studies

Assume that the controlled Chua's circuit (5) is originally $(u = 0)$ in the periodic state, period-1 attractor [15], with parameters $C_1 = 0.11364, C_2 = 1, L = 0.0625, R = 1, G_a = -1.143$, and $G_b = -0.714$, i.e., $b_1 = 16, b_2 = 1$ and $\theta = [1.0000, 1.0000, 8.7997, 2.5167, -1.8875]^T$. The objective is to force the output $y = x_1(t)$ of the controlled Chua's circuit (5) to asymptotically track the chaotic reference signal $y_r = x_{r1}(t)$ generated from another uncontrolled Chua's circuit (5) $(u = 0)$ in chaotic state, double-scroll attractor [15], with parameters $C_1 = 0.10204, C_2 = 1, L = 0.0625, R = 1, G_a = -1.143$, and $G_b = -0.714$.

The initial conditions are chosen that $x_1(0) = 2, x_2(0) = 0.3, x_3(0) = 0.4, x_{r1}(0) = 0.2, x_{r2}(0) = 0.5$, and $x_{r3}(0) = 0.3$. The design parameters of controller (8) and parameter update law (9) are chosen as $c_1 = 10, c_2 = 20, c_3 = 50, \gamma = 0.1$, and $\Gamma = \mathrm{diag}[0.03, 0.1, 0.1, 0.02, 0.07]$. These gains are chosen by trial and error for better performance. Through intensive simulation studies, it was shown that when the gains are too small, the convergence of tracking error is very slow; when the gains become too large, the control signal will become inhibitedly high.

Numerical simulation results are shown in Figs. 1–4. As shown in Figs. 1 and 2, the output $y = x_1(t)$ of the controlled Chua's circuit (5) asymptotically track the chaotic reference signal $y_r = x_{r1}(t)$. The boundedness of parameter estimates and control signal u are shown in Figs. 3 and 4, respectively. It can be shown that at the same time the states $x_2(t)$ and $x_3(t)$ of the controlled Chua's oscillator (5) remain bounded.

V. Conclusion

In this brief, first, we showed that several kinds of Chua's circuits can be transformed into the nonautonomous strict-feedback system. Then, an adaptive backstepping with tuning functions method has been extended to the nonautonomous strict-feedback system. Global stability of the closed-loop system is guaranteed and asymptotic tracking of any smooth, bounded reference signal (including chaotic signals) is achieved. Along with the advantages, the backstepping design procedure has certain disadvantages. One of them is that, for high-order system, the nonlinear expression of the controller becomes increasingly complex. This disadvantage can be relaxed by modular design methods [10].

APPENDIX

A. Proof of Theorem 1

The backstepping design procedure is recursive. At the ith step, the ith-order subsystem is stabilized with respect to a Lyapunov function V_i by the design of a stabilizing function α_i, and tuning functions $\tau_i, \pi_i^{b_1}, \ldots, \pi_i^{b_i}$. The update law for the parameter estimates $\hat{\theta}(t)$ and $\hat{b}_i$, and the feedback control u are designed in the final step.

Step 1) The derivative of $z_1 = x_1 - x_{r1}$ is given by

$$\begin{aligned}\dot{z}_1 &= \hat{b}_1 g_1 z_2 + \frac{g_1}{\hat{\varrho}_1}\alpha_1 + \hat{\theta}^T F_{1s} + f_{1s} - (\hat{\theta}-\theta)^T F_{1s} \\ &\quad - (\hat{b}_1 - b_1) g_1 z_2 + (b_1\hat{\varrho}_1 - 1)\left(\frac{g_1}{\hat{\varrho}_1}\alpha_1 + g_1 x_{r2}\right) \qquad (16)\end{aligned}$$

where z_2, F_{1s}, f_{1s}, and α_1 are defined in (9), (11), and (14), respectively.

Virtual control α_1 is used to stabilize (16) with respect to the Lyapunov function candidate

$$\begin{aligned}V_1 &= \frac{1}{2}z_1^2 + \frac{1}{2}(\hat{\theta}-\theta)^T\Gamma^{-1}(\hat{\theta}-\theta) + \frac{1}{2\gamma}(\hat{b}_1 - b_1)^2 \\ &\quad + \frac{|b_1|}{2\gamma}(\hat{\varrho}_1 - \varrho_1)^2. \qquad (17)\end{aligned}$$

Its derivative is

$$\begin{aligned}\dot{V}_1 &= \hat{b}_1 g_1 z_1 z_2 + z_1\left(\frac{g_1}{\hat{\varrho}_1}\alpha_1 + \hat{\theta}^T F_{1s} + f_{1s}\right) \\ &\quad + (b_1\hat{\varrho}_1 - 1)\,\mathrm{sgn}(b_1)\gamma^{-1}\Bigg(\dot{\hat{\varrho}}_1 + \mathrm{sgn}(b_1)\gamma z_1 \\ &\quad \cdot \left(g_1 x_{r2} + \frac{g_1}{\hat{\varrho}_1}\alpha_1\right)\Bigg) + (\hat{\theta}-\theta)^T\Gamma^{-1}\left(\dot{\hat{\theta}} - \Gamma F_{1s} z_1\right) \\ &\quad + (\hat{b}_1 - b_1)\gamma^{-1}\left(\dot{\hat{b}}_1 - \gamma g_1 z_1 z_2\right). \qquad (18)\end{aligned}$$

Define the tuning functions τ_1 and $\pi_1^{b_1}$ for $\hat{\theta}$ and $\hat{b}_1$, respectively, as in (12) and (13), and choose the parameter update law for $\hat{\varrho}_1$ as in (9), then $\dot{V}_1$ becomes

$$\begin{aligned}\dot{V}_1 &= -c_1 z_1^2 + \hat{b}_1 g_1 z_1 z_2 + (\hat{\theta}-\theta)\Gamma^{-1}\left(\dot{\hat{\theta}} - \tau_1\right) \\ &\quad + (\hat{b}_1 - b_1)\gamma^{-1}\left(\dot{\hat{b}}_1 - \pi_1^{b_1}\right). \qquad (19)\end{aligned}$$

Step 2) The derivative of z_2 is expressed as

$$\begin{aligned}\dot{z}_2 &= \dot{x}_2 - \hat{\varrho}_1\dot{x}_{r2} - \dot{\hat{\varrho}}_1 x_{r2} - \dot{\alpha}_1 \\ &= \hat{b}_2 g_2 z_3 + \frac{g_2}{\hat{\varrho}_2}\alpha_2 + \hat{\theta}^T F_{2s} + f_{2s} + \frac{\partial\alpha_1}{\partial\hat{\theta}}(\tau_2 - \dot{\hat{\theta}}) \\ &\quad - (\hat{b}_2 - b_2) g_2 z_3 + (\hat{b}_1 - b_1)\frac{\partial\alpha_1}{\partial x_1} g_1 x_2 \\ &\quad - (\hat{\theta}-\theta)^T F_{2s} + (b_2\hat{\varrho}_2 - 1)\left(g_2 x_{r3} + \frac{g_2}{\hat{\varrho}_2}\alpha_2\right) \qquad (20)\end{aligned}$$

where z_3, F_{2s}, f_{2s}, and α_2 are defined in (10), (11), and (14), respectively.

Virtual control α_2 is used to stabilize the (z_1, z_2)-subsystem with respect to the Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2\gamma}(\hat{b}_2 - b_2)^2 + \frac{|b_2|}{2\gamma}(\hat{\varrho}_2 - \varrho_2)^2. \qquad (21)$$

The derivative of V_2 is

$$\begin{aligned}\dot{V}_2 &= -c_1 z_1^2 + \hat{b}_2 g_2 z_2 z_3 \\ &\quad + z_2\left(\hat{b}_1 g_1 z_1 + \frac{g_2}{\hat{\varrho}_2}\alpha_2 + \hat{\theta}^T F_{2s} + f_{2s}\right) \\ &\quad + z_2\frac{\partial\alpha_1}{\partial\hat{\theta}}\left(\tau_2 - \dot{\hat{\theta}}\right) + (\hat{\theta}-\theta)^T\Gamma^{-1}(\dot{\hat{\theta}}_1 - \tau_1 - \Gamma F_{2s} z_2) \\ &\quad + (b_2\hat{\varrho}_2 - 1)\,\mathrm{sgn}(b_2)\gamma^{-1}\Bigg(\dot{\hat{\varrho}}_2 + \mathrm{sgn}(b_2)\gamma z_2 \\ &\quad \cdot \left(g_2 x_{r3} + \frac{g_2}{\hat{\varrho}_2}\alpha_2\right)\Bigg) + (\hat{b}_1 - b_1)\gamma^{-1} \\ &\quad \cdot \left(\dot{\hat{b}}_1 - \pi_1^{b_1} + \gamma\frac{\partial\alpha_1}{\partial x_1} g_1 x_2 z_2\right) + (\hat{b}_2 - b_2)\gamma^{-1} \\ &\quad \times \left(\dot{\hat{b}}_2 - \gamma g_2 z_2 z_3\right). \qquad (22)\end{aligned}$$

Define tuning functions $\tau_2, \pi_2^{b_1}$, and $\pi_2^{b_2}$ for $\hat{\theta}, \hat{b}_1$, and $\hat{b}_2$, respectively, as in (12) and (13), and choose the parameter update law for $\hat{\varrho}_2$ as in (9), then $\dot{V}_2$ becomes

$$\begin{aligned}\dot{V}_2 &= -c_1 z_1^2 - c_2 z_2^2 + \hat{b}_2 g_2 z_2 z_3 + z_2\frac{\partial\alpha_1}{\partial\hat{\theta}}(\tau_2 - \dot{\hat{\theta}}) \\ &\quad + (\hat{\theta}-\theta)\Gamma^{-1}\left(\dot{\hat{\theta}} - \tau_2\right) + (b_1 - \hat{b}_1)\gamma^{-1}\left(\dot{\hat{b}}_1 - \pi_2^{b_1}\right) \\ &\quad + (b_2 - \hat{b}_2)\gamma^{-1}\left(\dot{\hat{b}}_2 - \pi_1^{b_2}\right). \qquad (23)\end{aligned}$$

Step $i, 3 \le i \le n-1$. The derivative of z_i is expressed as

$$\begin{aligned}\dot{z}_i &= \hat{b}_i g_i z_{i+1} + \frac{g_i}{\hat{\varrho}_i}\alpha_i + \hat{\theta}^T F_{is} + f_{is} - (\hat{\theta}-\theta)^T F_{is} \\ &\quad + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}(\tau_i - \dot{\hat{\theta}}) + \sum_{k=1}^{i-2}\frac{\partial\alpha_{i-1}}{\partial\hat{b}_k}\left(\pi_i^{b_k} - \dot{\hat{b}}_k\right) \\ &\quad + \sum_{k=1}^{i-1}(\hat{b}_k - b_k)\frac{\partial\alpha_{i-1}}{\partial x_k} g_k x_{k+1} - (\hat{b}_i - b_i) g_i z_{i+1} \\ &\quad + (b_i\hat{\varrho}_i - 1)\left(g_i x_{r(i+1)} + \frac{g_i}{\hat{\varrho}_i}\alpha_i\right) \qquad (24)\end{aligned}$$

where z_{i+1}, F_{is}, f_{is}, and α_i are defined in (10), (11), and (14), respectively.

Virtual control α_i is used to stabilize the $(z_1, \ldots, z_i)$-subsystem with respect to the Lyapunov function candidate

$$\begin{aligned}V_i &= V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2\gamma}(\hat{b}_i - b_i)^2 \\ &\quad + \frac{|b_i|}{2\gamma}(\hat{\varrho}_i - \varrho_i)^2. \qquad (25)\end{aligned}$$

The derivative of V_i is

$$\begin{aligned}\dot{V}_i &= -\sum_{k=1}^{i-1} c_k z_k^2 + z_i\left(\hat{b}_{i-1} g_{i-1} z_1 + \frac{g_i}{\hat{\varrho}_i}\alpha_i + \hat{\theta}^T F_{is} + f_{is}\right) \\ &\quad + \hat{b}_i g_i z_i z_{i+1} + \sum_{k=1}^{i-2}\left(z_{k+1}\frac{\partial\alpha_k}{\partial\hat{\theta}}\right)\left(\tau_{i-1} - \dot{\hat{\theta}}\right) \\ &\quad + \sum_{j=1}^{i-3}\sum_{k=1}^{i-j-2} z_{k+2}\frac{\partial\alpha_{k+1}}{\partial\hat{b}_j}\left(\pi_{i-1}^{b_j} - \dot{\hat{b}}_j\right) \\ &\quad + (\hat{\theta}-\theta)^T\Gamma^{-1}\left(\dot{\hat{\theta}} - \tau_{i-1} - \Gamma F_{is} z_i\right) \\ &\quad + \sum_{k=1}^{i-1}(\hat{b}_k - b_k)\gamma^{-1}\left(\dot{\hat{b}}_k - \pi_{i-1}^{b_k} + \gamma z_i\frac{\partial\alpha_{i-1}}{\partial x_k} g_k x_{k+1}\right) \\ &\quad + (\hat{b}_i - b_i)\gamma^{-1}\left(\dot{\hat{b}}_i - \gamma g_i z_i z_{i+1}\right) + z_i\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\left(\tau_i - \dot{\hat{\theta}}\right) \\ &\quad + (b_i\hat{\varrho}_i - 1)\,\mathrm{sgn}(b_i)\gamma^{-1}\Bigg[\dot{\hat{\varrho}}_i + \mathrm{sgn}(b_i)\gamma z_i \\ &\quad \times \left(g_i x_{r(i+1)} + \frac{g_i}{\hat{\varrho}_i}\alpha_i\right)\Bigg] + z_i\sum_{k=1}^{i-2}\frac{\partial\alpha_{i-1}}{\partial\hat{b}_k}\left(\pi_i^{b_k} - \dot{\hat{b}}_k\right). \qquad (26)\end{aligned}$$

Define tuning functions $\tau_i, \pi_i^{b_1}, \ldots, \pi_i^{b_i}$ for $\hat\theta, \hat b_1, \ldots, \hat b_i$, respectively, as in (12) and (13), and choose the parameter update law for $\hat\varrho_2$ as in (9). Noting that $\tau_{i-1} - \dot{\hat\theta} = \tau_i - \dot{\hat\theta} + \tau_{i-1} - \tau_i = \tau_i - \dot{\hat\theta} - \Gamma F_{is} z_i$ and $\pi_{i-1}^{b_j} - \dot{\hat b}_j = \pi_i^{b_j} - \dot{\hat b}_j + \gamma z_i (\partial\alpha_{i-1}/\partial x_j) g_j x_{j+1}$, we rewrite $\dot V_i$ as

$$\begin{aligned} \dot V_i = &-\sum_{k=1}^{i-1} c_k z_k^2 + \hat b_i g_i z_i z_{i+1} + \sum_{k=1}^{i-1}\left(z_{k+1}\,\frac{\partial\alpha_k}{\partial\hat\theta}\right)\left(\tau_i - \dot{\hat\theta}\right) \\ &+ (\hat\theta-\theta)^T\Gamma^{-1}\left(\dot{\hat\theta} - \tau_i\right) + \sum_{j=1}^{i}(\hat b_j - b_j)\gamma^{-1}\left(\dot{\hat b}_j - \pi_i^{b_j}\right) \\ &+ \sum_{j=1}^{i-2}\sum_{k=1}^{i-j-1}\left(z_{k+2}\,\frac{\partial\alpha_{k+1}}{\partial\hat b_j}\right)\left(\pi_{i-1}^{b_j} - \dot{\hat b}_j\right). \end{aligned} \tag{27}$$

Step n. Since this is our last step, the derivative of z_n is expressed as

$$\begin{aligned} \dot z_n = &\frac{g_n}{\hat\varrho_n}\,u + \hat\theta^T F_{ns} + f_{ns} + \frac{\partial\alpha_{n-1}}{\partial\hat\theta}\left(\tau_n - \dot{\hat\theta}\right) \\ &+ \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat b_k}\left(\pi_n^{b_k} - \dot{\hat b}_k\right) - (\hat\theta-\theta)^T F_{ns} \\ &+ \sum_{k=1}^{n-1}(\hat b_k - b_k)\,\frac{\partial\alpha_{n-1}}{\partial x_k}\,g_k x_{k+1} + (b_n\hat\varrho_n - 1)\left(\frac{g_n}{\hat\varrho_n}u\right) \end{aligned} \tag{28}$$

where F_{ns} and f_{ns} are defined in (11).

Physical control u is to stabilize the $(z_1, \ldots, z_n)$-system with respect to the Lyapunov function candidate

$$V_n = V_{n-1} + \frac{1}{2}\,z_n^2 + \frac{|b_n|}{2\gamma}\,(\hat\varrho_n - \varrho_n)^2. \tag{29}$$

The derivative of V_n is

$$\begin{aligned} \dot V_n = &-\sum_{k=1}^{n-1} c_k z_k^2 + z_n\Bigg(\hat b_{n-1} g_{n-1} z_n + \frac{g_n}{\hat\varrho_n}\,u \\ &+ \hat\theta^T F_{ns} + f_{ns}\Bigg) + \sum_{k=1}^{n-1} z_{k+1}\,\frac{\partial\alpha_k}{\partial\hat\theta}\left(\tau_{n-1} - \dot{\hat\theta}\right) \\ &+ (\hat\theta-\theta)^T\Gamma^{-1}\left(\dot{\hat\theta} - \tau_{n-1} - \Gamma F_{ns} z_n\right) \\ &+ z_n\,\frac{\partial\alpha_{n-1}}{\partial\hat\theta}\left(\tau_n - \dot{\hat\theta}\right) + z_n\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat b_k}\left(\pi_n^{b_k} - \dot{\hat b}_k\right) \\ &+ \sum_{k=1}^{n-1}(\hat b_k - b_k)\gamma^{-1}\left(\dot{\hat b}_k - \pi_{n-1}^{b_k} + \gamma z_n\,\frac{\partial\alpha_{n-1}}{\partial x_k}\,g_k x_{k+1}\right) \\ &+ \sum_{j=1}^{n-3}\sum_{k=1}^{n-j-2}\left(z_{k+2}\,\frac{\partial\alpha_{k+1}}{\partial\hat b_j}\right)\left(\pi_{n-1}^{b_j} - \dot{\hat b}_j\right) \\ &+ (b_n\hat\varrho_n - 1)\,\mathrm{sgn}(b_n)\gamma^{-1}\left[\dot{\hat\varrho}_n + \mathrm{sgn}(b_n)\gamma z_n\left(\frac{g_n}{\hat\varrho_n}\,u\right)\right]. \end{aligned} \tag{30}$$

Choose the parameter update laws for $\hat\theta, \hat b_1, \ldots, \hat b_{n-1}$ and $\hat\varrho_n$ as in (9). Noting that $\tau_{n-1} - \dot{\hat\theta} = \tau_{n-1} - \tau_n = -\Gamma F_{ns} z_n$ and $\pi_{n-1}^{b_j} - \dot{\hat b}_j = \gamma z_n (\partial\alpha_{n-1}/\partial x_j) g_j x_{j+1}$, (30) can be written as

$$\begin{aligned} \dot V_n = &-\sum_{k=1}^{n-1} c_k z_k^2 + z_n\Bigg[\hat b_{n-1} g_{n-1} z_n + \frac{g_n}{\hat\varrho_n}\,u + \hat\theta^T F_{ns} \\ &+ f_{ns} - \sum_{k=1}^{n-2} z_{k+1}\,\frac{\partial\alpha_k}{\partial\hat\theta}\,\Gamma F_{ns} + \sum_{j=1}^{n-3}\sum_{k=1}^{n-j-2} \\ &\times\left(z_{k+2}\,\frac{\partial\alpha_{k+1}}{\partial\hat b_j}\right)\gamma\,\frac{\partial\alpha_{n-3}}{\partial x_j}\,g_j x_{j+1}\Bigg]. \end{aligned} \tag{31}$$

By choosing the control u as in (8) such that the bracketed term multiplying z_n in (31) equals $-c_n z_n^2$, we have

$$\dot V_n = -\sum_{k=1}^{n} c_k z_k^2 \tag{32}$$

which proves that 1) equilibrium $z = 0$ is globally uniformly stable; 2) $\hat\theta, \hat b_1, \ldots, \hat b_{n-1}$ and $\hat\varrho_1, \ldots, \hat\varrho_n$ are bounded. Since $z_1 = x_1 - x_{r1}$ and x_{r1} is bounded, we see that x_1 is also bounded. The boundedness of x_i, $i = 2, \ldots, n$ follows from the boundedness of α_{i-1} and $\hat\varrho_{i-1}$, $i = 2, \ldots, n$ and x_{ri}, and the fact that $x_i = z_i + \hat\varrho_{i-1} x_{ri} + \alpha_{i-1}$, $i = 2, \ldots, n$. Using (8), we conclude that the control u is also bounded.

From the LaSalle–Yoshizawa theorem [10], it further follows that, all the solutions of the $(z_1, \ldots, z_n)$-system converge to the manifold $z = 0$ as $t \to \infty$. From the definition $z_1 = x_1 - x_{r1} = y - y_r$, we conclude that $[y(t) - y_r(t)] \to 0$ as $t \to \infty$. Q.E.D.

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