Control of Fully Actuated Ocean Surface Vessels Using a Class of Feedforward Approximators

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***Abstract*—In this brief, we consider the problem of tracking a desired trajectory for fully actuated ocean vessels, in the presence of uncertainties and unknown disturbances. The combination of approximation-based and domination design techniques allows us to handle time-varying disturbances, without the need for explicit knowledge of the bounds. Using backstepping and Lyapunov synthesis, the stable tracking controller is first designed for the full-state feedback case. Subsequently, the output feedback problem is tackled by employing a high-gain observer to estimate the unmeasurable states required by the stable tracking controller. Under the proposed control, semiglobal uniform boundedness of the closed-loop signals is guaranteed for both full-state and output feedback cases.**

***Index Terms*—Function approximation, fuzzy systems, marine vehicle control, neural networks, nonlinear systems, output feedback, tracking.**

I. Introduction

An important issue of model-based control of ocean vessels lies in the handling of *unknown* perturbations to the nominal model, in the form of parametric and functional uncertainties, unmodeled dynamics, and disturbances from the environment. Traditional model-based adaptive controllers may not be applicable since they are generally useful only when dealing with systems in which the dynamics are linear-in-the-parameters, the regressors are exactly known, and the uncertainties are parametric and time-invariant [1]. On the other hand, marine control applications are characterized by time-varying environmental disturbances and widely changing sea conditions. In addition, the presence of nonparametric uncertainties, may excite high-frequency unmodeled dynamics, which could disrupt the function of the adaptive controller and lead to close loop instability [2].

To overcome the limitations of model-based adaptive controllers, we adopt approximation-based control techniques to compensate for functional uncertainties and unknown disturbances from the environment. Such approximators can utilize a standard regressor function whose structure is independent of the ship's dynamic characteristics, thus increasing the portability of the same control algorithm on different ship systems.

Many good results for ship control in the literature rely on model-based approaches. Earlier works include adaptive control of ship heading [3], [4]. More recently, model reference feedforward control was combined with a modified linear quadratic Gaussian (LQG) feedback control for dynamic positioning of ships [5]. Based on a linearized model, vectorial observer backstepping was used to obtain a globally exponentially stable output feedback controller for dynamic positioning of ships [6]. Adaptive vectorial backstepping was employed in [7], but only parametric uncertainties could be handled. Generally speaking, these results require exact knowledge of the ship's parameters, and some restrictions on the disturbances, usually constant or slowly time-varying.

In comparison with model-based approaches, approximation-based methods, which do not require parametric or functional certainty, are gaining recognition. In [8], fuzzy control was used for track keeping of a surface ship, given exact knowledge of model parameters. Model reference fuzzy learning control was applied to cargo ship steering in [9]. In [10], robust fuzzy-based model reference adaptive control (MRAC) was used for course keeping using full-state information. Although functional uncertainties could be handled, the control design considered only the SISO steering equation and neglected the couplings with the surge and sway dynamics. Besides fuzzy systems, neural networks trained under back propagation are also used for intelligent control of ships, in particular the optimal guidance task [11].

From a practical perspective, state-of-the-art actuation systems, such as tunnel thrusters and azipods, are ineffective in providing control action in the sway direction at high speeds, causing the ship to become "underactuated." In this paper, we restrict our attention, without loss of generality, to the simplified case of fully actuated ships at low to moderate speeds, so as to show the effectiveness of approximation-based control for ocean vessels in handling uncertainties. Motivated by results in neural network control of robot manipulators [12], we design approximation-based controller for fully actuated surface vessels using Lyapunov synthesis. Both full-state and output feedback problems are considered, with high-gain observers used to tackle the latter.

II. Problem Formulation

A. Ship Dynamics

Consider the multiple-input–multiple-output (MIMO) dynamics of a 3 degree-of-freedom (3DOF) surface ship in the presence of disturbances:

$$\dot{\eta} = J(\eta)v$$
$$M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau + d(\eta, v, t)$$

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where

$$J = \begin{bmatrix} \cos\eta_\psi & -\sin\eta_\psi & 0 \\ \sin\eta_\psi & \cos\eta_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is the Jacobian transformation matrix; the output $\eta = [\eta_x, \eta_y, \eta_\psi]^T \in R^3$ are the Earth-frame positions and heading, respectively; $v = [v_x, v_y, v_\psi]^T \in R^3$ are the vessel-frame surge, sway, and yaw velocities, respectively; $d(\eta, v, t) \in R^3$ is the unknown disturbance from the environment, and/or unmodeled dynamics, among others; M, $C(v)$, and $D(v)$ are the inertia matrix, the matrix of coriolis' and centripetal terms, and the damping matrix, respectively, all of which are unknown; $g(\eta)$ is an unknown vector of restoring forces due to buoyancy and gravitational forces and moments; and $\tau \in R^3$ is the vector of input signals

Lemma 1: [13] For bounded initial conditions, if there exists a C^1 continuous and positive definite Lyapunov function $V(x)$ satisfying $\gamma_1(\|x\|) \le V(x) \le \gamma_2(\|x\|)$, such that $\dot{V}(x) \le -\rho V(x) + c$, where $\gamma_1, \gamma_2 : R^n \to R$ are class K functions and c is a positive constant, then the solution $x(t)$ is uniformly bounded.

Lemma 2: [14] For the continuous functions $d_i(\eta, v, t) : R^3 \times R^3 \times R \to R$, $i = 1, 2, 3$, there exist positive, smooth, nondecreasing functions $p_i(\eta, v) : R^3 \times R^3 \to R^+$ and $q_i(t) : R \to R^+$ such that

$$|d_i(\eta, v, t)| \le p_i(\eta, v) + q_i(t).$$

Lemma 2 allows one to separate the multivariable disturbance term $d_i(\eta, v, t)$, for $i = 1, 2, 3$, into a bounding function in terms of v, η, the internal states of the ship, and a bounding function in terms of t, which generally includes exogenous effects and uncertainties. As will be seen in Section III-A, this separation is useful because $p_i(\eta, v)$ can be approximated with observable signals η and v. Thus, we are only required to make a mild assumption on the general time-dependent effects.

The control objective is to ensure that all signals are bounded, while the output follows a desired trajectory $\eta_d = [\eta_{xd}, \eta_{yd}, \eta_{\psi d}]^T \in \Omega_{\eta_d}$, such that the tracking errors converge to a small neighborhood of the origin, i.e., $\lim_{t\to\infty} \|\eta - \eta_d\| < \delta$ for some $\delta > 0$.

Assumption 1: For the time-dependent functions $q_i(t)$, $i = 1, 2, 3$, there exist constants $\bar{q}_i \in R^+, \forall\, t > t_0$, such that

$$\|q_i(t)\| \le \bar{q}_i.$$

Assumption 2: For all $t > 0$, there exists constants $N_1, N_2 > 0$ such that $\|\dot{\eta}_d(t)\| \le N_1$ and $\|\ddot{\eta}_d(t)\| \le N_2$.

Remark 1: Assumption 1 is reasonable since the time-dependent component of the disturbance can be largely attributed to the exogenous effects of the environment, which have finite energy and, hence, are bounded. By virtue of the approximation capability of the controller, knowledge of the bound $\bar{q}_i$ is not required—only the assertion of its existence is. Assumption 2 requires that the desired trajectory be sufficiently smooth to avoid actuator saturation induced by sudden jumps of tracking error due to discontinuous command inputs.

B. Feedforward Approximators

Function approximators can be represented as multilayer feedforward networks which may be nonlinearly- or linearly-parameterized. Examples of feedforward approximators include adaptive neural networks [12], [13], [15], [16] and adaptive fuzzy systems [17]. A class of linearly parametrized feedforward approximators used to approximate the continuous function $f(Z) : R^q \to R$ may be represented as follows:

$$f(Z) = \theta^T S(Z) + \varepsilon(Z) \tag{1}$$

where the vector $Z = [z_1, z_2, \ldots, z_q]^T \in R^q$ are the input variables to the approximator, $S(Z) \in R^l$ is a vector of known continuous (linear or nonlinear) basis functions, $\theta \in R^l$ is a vector of adaptable weights, and ε is the approximation error which is bounded over the compact set, i.e., $|\varepsilon(Z)| \le \bar{\varepsilon}, \forall\, Z \in \Omega_Z$, where $\bar{\varepsilon} > 0$ is an unknown constant.

We consider a class of linearly parameterized feedforward approximators, which, according to the universal approximation property [15], can smoothly approximate any continuous function $f(Z)$ over a compact set $\Omega_Z \subset R^q$ to arbitrary any degree of accuracy as

$$f(Z) = \theta^{*T} S(Z) + \varepsilon^*(Z), \qquad \forall\, Z \in \Omega_Z \subset R^q \tag{2}$$

where θ^* are the ideal constant weights in the output layer, and $\varepsilon^*(Z)$ is the approximation error for the special case where $\theta = \theta^*$. The ideal weight vector θ^*, an artificial quantity required for analytical purposes, is defined as the value of θ that minimizes $|\varepsilon(Z)|$ for all $Z \in \Omega_Z \subset R^q$, i.e.,

$$\theta^* := \arg\min_{\theta \in R^l} \left\{ \sup_{Z \in \Omega_Z} |f(Z) - \theta^T S(Z)| \right\}. \tag{3}$$

III. Approximation-Based Control Design

In this paper, we employ approximation-based adaptive backstepping of the ship dynamics (1). Feedforward approximators are used to compensate for unknown nonlinear functions within the known dynamical structure (1), which is in general valid for mechanical systems. Full-state feedback controller will be derived first. Based on this, an output feedback controller will be subsequently designed via certainty equivalence approach, with the unavailable output derivative estimated with a high-gain observer.

A. Full-State Feedback Control

Step 1) Define error variables $z_1 = \eta - \eta_d$ and $z_2 = v - \alpha_1$, and consider Lyapunov function candidate $V_1 = (1/2) z_1^T z_1$. Differentiating z_1 with respect to time yields

$$\dot{z}_1 = J(\eta)(z_2 + \alpha_1) - \dot{\eta}_d. \tag{4}$$

Noting the property $JJ^T = I$, and choosing the virtual control as

$$\alpha_1 = J^T(\eta)(\dot{\eta}_d - K_1 z_1) \tag{5}$$

where $K_1 = K_1^T > 0$, the time derivative of V_1 along the trajectories of (4) is given by

$$\dot{V}_1 = -z_1^T K_1 z_1 + z_1^T J(\eta) z_2. \tag{6}$$

The first term on the right-hand side (RHS) is stabilizing, and the second term will be handled in the next step.

Step 2) Differentiating z_2 with respect to time yields

$$\dot{z}_2 = M^{-1}(-C(v)v - D(v)v - g(\eta) + \tau + d(\eta, v, t)) - \dot{\alpha}_1 \tag{7}$$

where $\dot{\alpha}_1 = (\partial\alpha_1/\partial\eta)\dot{\eta} + (\partial\alpha_1/\partial\dot{\eta}_d)\ddot{\eta}_d + (\partial\alpha_1/\partial z_1)\dot{z}_1$. Consider the Lyapunov function candidate $V_2^* = V_1 + (1/2)z_2^T M z_2$. From Lemma 2 and Assumption 1, we have the following:

$$\begin{aligned}\dot{V}_2^* \leq & - z_1^T K_1 z_1 + z_1^T J(\eta) z_2 \\ & + z_2^T\left(-C(v)v - D(v)v - g(\eta) - M\dot{\alpha}_1 + \tau\right) \\ & + \sum_{i=1}^{3} |z_{2,i}| \left(p_i(\eta, v) + \bar{q}_i\right).\end{aligned} \tag{8}$$

Consider the following desired control law

$$\tau^* = -J^T(\eta)z_1 - K_2 z_2 + C(v)v + D(v)v + g(\eta) + M\dot{\alpha}_1 - \mathrm{Sgn}(z_2)(p(\eta, v) + \bar{q}) \tag{9}$$

where $\mathrm{Sgn}(z_2) := \mathrm{diag}[\mathrm{sgn}(z_{2,i})]$, for $i = 1, 2, 3$, with $\mathrm{sgn}(\cdot)$ as the signum function.

Substituting (9) into (8), the latter can be rewritten as $\dot{V}_2^* \leq -z_1^T K_1 z_1 - z_2^T K_2 z_2$. However, since M, $C(v)$, $D(v)$, $p(\eta, v)$, and $q(t)$ are all unknown, the model-based control law (9) is not feasible. To overcome this problem, we utilize feedforward approximators to estimate the unknown entities in the control law as follows:

$$\tau = - J^T(\eta)z_1 - K_2 z_2 + \hat{\Theta}^T S(Z) \tag{10}$$

$$\dot{\hat{\theta}}_i = - \Gamma_i(S_i(Z)z_{2i} + \sigma_i \hat{\theta}_i) \tag{11}$$

where $\hat{\Theta} := \mathrm{blockdiag}\left[\hat{\theta}_1^T, \hat{\theta}_2^T, \hat{\theta}_3^T\right]$ contains the approximation parameters, $S(Z) = \left[S_1^T(Z), S_2^T(Z), S_3^T(Z)\right]^T$ are the basis functions, and σ_i is a positive constant. The neural network $\hat{\Theta}^T S(Z)$ approximates $\Theta^{*T} S(Z)$ defined by

$$\Theta^{*T} S(Z) = C(v)v + D(v)v + g(\eta) + M\dot{\alpha}_1 - \mathrm{Sgn}(z_2)(p(\eta, v) + \bar{q}) - \varepsilon(Z) \tag{12}$$

where $\varepsilon(Z) \in R^3$ is the approximation error, $\mathrm{Sgn}(z_2) := \mathrm{diag}[\mathrm{sgn}(z_{2,i})]$, for $i = 1, 2, 3$, with $\mathrm{sgn}(\cdot)$ as the signum function, $Z = \left[\eta^T, v^T, \alpha_1^T, \dot{\alpha}_1^T\right]^T$ are the input variables to the feedforward approximators, and $z_{2,i} \in R$, for $i = 1, 2, 3$, are the elements of z_2.

Consider the augmented Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2} z_2^T M z_2 + \frac{1}{2}\sum_{i=1}^{3} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i \tag{13}$$

where $\tilde{\theta}_i := \hat{\theta}_i - \theta_i^*$. Differentiating along (4), (7), and (11) yields

$$\dot{V}_2 \leq - \rho V_2 + C \tag{14}$$

$$\rho := \min\left(2\lambda_{\min}(K_1), \frac{2\lambda_{\min}\left(K_2 - \frac{1}{2}I_{3\times 3}\right)}{\lambda_{\max}(M)} \min_{i=1,2,3}\left(\frac{\sigma_i}{\lambda_{\max}\left(\Gamma_i^{-1}\right)}\right)\right) \tag{15}$$

$$C := \sum_{i=1}^{3} \frac{\sigma_i}{2} \|\theta_i^*\|^2 + \frac{1}{2}\|\bar{\varepsilon}\|^2 \tag{16}$$

where $\lambda_{\min}(\bullet)$ and $\lambda_{\max}(\bullet)$ denote the minimum and maximum eigenvalues of $\bullet$, respectively.

Theorem 1: Consider the ship dynamics (1) with Assumptions 1 and 2, under the action of full-state feedback control law (10) and adaptation law (11). Then, for each compact set Ω_0, where $(\eta(0), v(0), \hat{\theta}_1(0), \hat{\theta}_2(0), \hat{\theta}_3(0)) \in \Omega_0$, the trajectories of the closed-loop system are semiglobally uniformly bounded. The tracking error z_1 converges to a compact set $\Omega_{zs} := \left\{z_1 \in R^3 \mid \|z_1\| \leq \sqrt{(2C/\rho)}\right\}$ asymptotically, where C and ρ are defined in (15) and (16), respectively.

Proof: From (14) and Lemma 1, it is clear that the signals $z_1, z_2, \tilde{\theta}_1, \tilde{\theta}_2, \tilde{\theta}_3$ are semiglobally uniformly bounded. From the boundedness of η_d in Assumption 2, we know that η is bounded. Since $\dot{\eta}_d$ is also bounded, it follows that α_1 is bounded, and in turn v is bounded. With θ_i^* as a constant, we know that $\hat{\theta}_i$ is also bounded, for $i = 1, 2, 3$. Therefore, all signals are bounded. To show that z_1 converges to Ω_{zs} is straightforward using a similar approach as that found in [13]. ∎

B. Output Feedback Control

In the previous section, we have considered the case where full-state measurement is possible, that is, $\eta(t) = [\eta_x, \eta_y, \eta_\psi]^T$ and $v(t) = [v_x, v_y, v_\psi]^T$ are both available. However, in practice, it is not easy to measure the vessel-centered states $v(t)$. In this section, we tackle the output feedback problem for ocean vessels by utilizing high-gain observers.

Lemma 3: [18] Consider the following linear system:

$$\begin{aligned}\epsilon\dot{\pi}_i &= \pi_{i+1}, \quad i = 1, 2, \ldots, n-1 \\ \epsilon\dot{\pi}_n &= - \gamma_1\pi_n - \gamma_2\pi_{n-1} - \cdots - \gamma_{n-1}\pi_2 - \pi_1 + \eta(t)\end{aligned} \tag{17}$$

where ϵ is a small positive constant, and the parameters γ_1 to γ_n are chosen such that the polynomial $s^n + \gamma_1 s^{n-1} + \cdots + \gamma_{n-1}s + 1$ is Hurwitz. Suppose that the system output $\eta(t)$ and its n derivatives are bounded, so that $\|\eta^{(k)}\| < Y_k$, then the following property holds:

$$\xi_k := \frac{\pi_k}{\epsilon^{k-1}} - \eta^{(k-1)} = -\epsilon\zeta^{(k)}, \quad k = 1, 2, \ldots, n \tag{18}$$

where $\zeta := \pi_n + \gamma_1 \pi_{n-1} + \cdots + \gamma_{n-1}\pi_1$ and $\zeta^{(k)}$ denotes the kth derivative of ζ. Furthermore, there exist positive constants h_k (independent of ϵ) and t^* such that $\|\xi_k\| \leq \epsilon h_k$ for $t > t^*$.

Note that $(\pi_{k+1})/\epsilon^k$ asymptotically converges to $\eta^{(k)}$, with a small time constant provided that η and its k derivatives are bounded. Hence, $(\pi_{k+1})/\epsilon^k$ for $k = 1, 2, \ldots, n$ is a suitable observer to estimate the output derivatives up to the nth order.

To prevent peaking [19], saturation functions are employed on the observer signals whenever they are outside the domain of interest Ω, as follows:

$$\begin{aligned} \pi_{i,j}^s &= S_{i,j}\phi\left(\frac{\pi_{i,j}}{S_{i,j}}\right) \\ S_{i,j} &\geq \max_{(z,\tilde{\theta})\in\Omega} (\pi_{i,j}) \\ \phi(a) &= \begin{cases} -1, & \text{for } a < -1 \\ a, & \text{for } |a| \leq 1 \\ 1, & \text{for } a > 1 \end{cases} \end{aligned} \tag{19}$$

for $i = 1, 2$, $j = 1, 2, 3$, where $\xi = [\xi_1, \xi_2]^T$.

Now, we revisit the control law (10) and adaptation law (11) for the full-state feedback case. Via the certainty equivalence approach, we modify them by replacing the unmeasurable state vector z_2 with its estimate $\hat{z}_2 := J^T(\eta)(\pi_2/\epsilon) - \alpha_1$, such that

$$\tau = -J^T(\eta)z_1 - K_2\hat{z}_2 + \hat{\Theta}^T S(\hat{Z}) \tag{20}$$

$$\dot{\hat{\theta}}_i = -\Gamma_i(S_i(\hat{Z}_i)\hat{z}_{2,i} + \sigma_i\hat{\theta}_i). \tag{21}$$

Denote $V_{\text{obs}} = (1/2)\xi^T\xi$, $\bar{K}_2 := K_2 - (1/2)I_{3\times3}$, $\tilde{z}_2 = \hat{z}_2 - z_2$ and $\Lambda = \text{diag}[2l_i/\sigma_i]$, $i = 1, 2, 3$. Taking the time derivative of V_2 along the closed-loop trajectory and using the property $S_i(\hat{Z}_i) - S_i(Z_i) = \epsilon S_{ti}$, where S_{ti} is a bounded vector function [15], yields

$$\begin{aligned} \dot{V}_2 \leq &- z_1^T K_1 z_1 - z_2^T\left(K_2 - \frac{3}{2}I\right)z_2 \\ &- \sum_{i=1}^{3}\frac{\sigma_i}{4}\|\tilde{\theta}_i\|^2 + \lambda_{\max}\left(\bar{K}_2\bar{K}_2^T + \Lambda\right)V_{\text{obs}} \\ &+ \sum_{i=1}^{3}\left(\frac{\epsilon^2\|S_{ti}\|^2}{2} + \frac{\sigma_i}{2}\right)\|\theta_i^*\|^2 + \frac{1}{2}\|\varepsilon\|^2. \end{aligned} \tag{22}$$

Noting the bounds on $V_{\text{obs}}(t)$ as follows:

$$V_{\text{obs}}(t) \leq \frac{1}{2}\epsilon^2(h_1^2 + h_2^2) \tag{23}$$

it can be shown that

$$\dot{V}_2(t) \leq -\rho V_2(t) + C \tag{24}$$

$$\rho := \min\left\{2\lambda_{\min}(K_1), \frac{2\lambda_{\min}\left(K_2 - \frac{3}{2}I\right)}{\lambda_{\max}(M)}, \frac{\min_{i=1,2,3}\left\{\frac{\sigma_i}{2}\|\tilde{\theta}_i\|^2\right\}}{\lambda_{\max}(\Gamma^{-1})}\right\} \tag{25}$$

$$\begin{aligned} C := &\sum_{i=1}^{3}\left(\frac{\epsilon^2\|S_{ti}\|^2}{2} + \frac{\sigma_i}{2}\right)\|\theta_i^*\|^2 \\ &+ \frac{1}{2}\|\varepsilon\|^2 + \frac{1}{2}\lambda_{\max}(\bar{K}_2\bar{K}_2^T + \Lambda)\epsilon^2(h_1^2 + h_2^2) \end{aligned} \tag{26}$$

To ensure that $\rho > 0$, the control gains K_1 and K_2 are chosen to satisfy the following conditions:

$$\lambda_{\min}(K_1) > 0, \quad \lambda_{\min}\left(K_2 - \frac{3}{2}I\right) > 0. \tag{27}$$

Theorem 2: Consider the ship dynamics (1) under Assumptions 1 and 2, with output feedback control law (20), adaptation law (21), and high-gain observer (17) turned on at time t^* in advance. For initial conditions starting in any compact set Ω_0, the solutions of the closed loop system are semiglobally uniformly bounded. Furthermore, the tracking error z_1 converges asymptotically to the compact set $\Omega_{zs} := \left\{z_1 \in R^3 \mid \|z_1\| \leq \sqrt{(2C/\rho)}\right\}$ where C and ρ are defined in (26) and (25), respectively.

Proof: The proof follows the same approach as Theorem 1, and will be omitted for conciseness. ■

Remark 2: In this paper, we proposed a rigorous theoretical treatment of the output feedback problem using high-gain observers, assuming that measurements are perfect. The rationale for choosing a high-gain observer lies in its simplicity and the fact that it does not require a model of the ship or disturbances, in line with the proposed nonmodel-based control philosophy. In practice, the presence of measurement noise necessitates careful implementation, and places a lower limit on the size of ϵ, with possible degradation of transient performance.

Since we do not model the wave disturbances as additive "noise" to low-frequency ship motion [20], but rather as forces that interact with the ship dynamics, hence, by construction, our output measurements are only contaminated with zero mean Gaussian white noise due to the global positioning system (GPS). As noted in [20], modern differential GPS provide high accuracy and precision, such that the zero mean Gaussian white measurement noise is within small, tolerable limits. The main drawback is that the wave disturbances are in the feedback loop, and compensating for them increases wear and tear of the actuators.

IV. Simulation Results

In our simulation study, we consider the model of Cybership II, a 1:70 scale supply vessel replica built in a marine control laboratory in the Norwegian University of Science and Technology. The model can be represented by (1), with parameters obtained from [7]. Without loss of generality, we define the disturbance d as time-varying forces/moment dependent on η and t

$$\begin{aligned} d(\eta, t) &= J^T(\eta)f_e(t) \\ f_e(t) &= [3 + 0.8\sin(0.7t) + 0.2\sin(0.5t) \\ &\quad + 0.2\sin(0.9t), 0, 0]^T. \end{aligned} \tag{28}$$

Note that in general, d can also depend on v, as described in (1). The control objective is to track, using only the measurable information $\eta(t)$, the desired trajectory $\eta_d(t) = [\eta_{xd}(t), \eta_{yd}(t), \eta_{\psi d}(t)]^T$ where $\eta_{xd}(t) = (0.1)\sqrt{2}t$, $\eta_{yd}(t) = 10\cos(0.1\eta_{xd}(t))$, and $\eta_{\psi d}(t) = \tan^{-1}(d\eta_{yd}/d\eta_{xd})$. This gives a maximum desired speed of 0.2 ms^{-1}, which corresponds to 3.1 knots in the full-scale vessel.

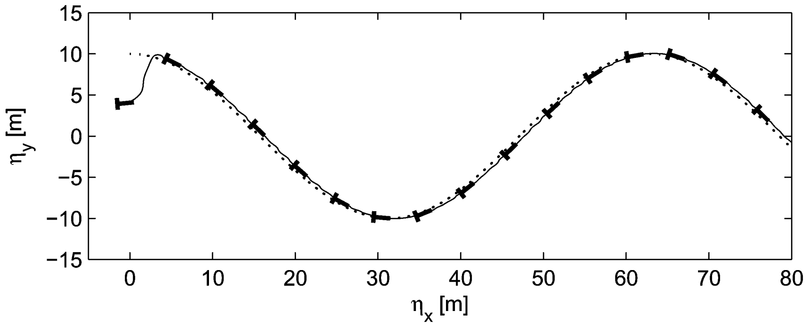

Fig. 1. Tracking performance.

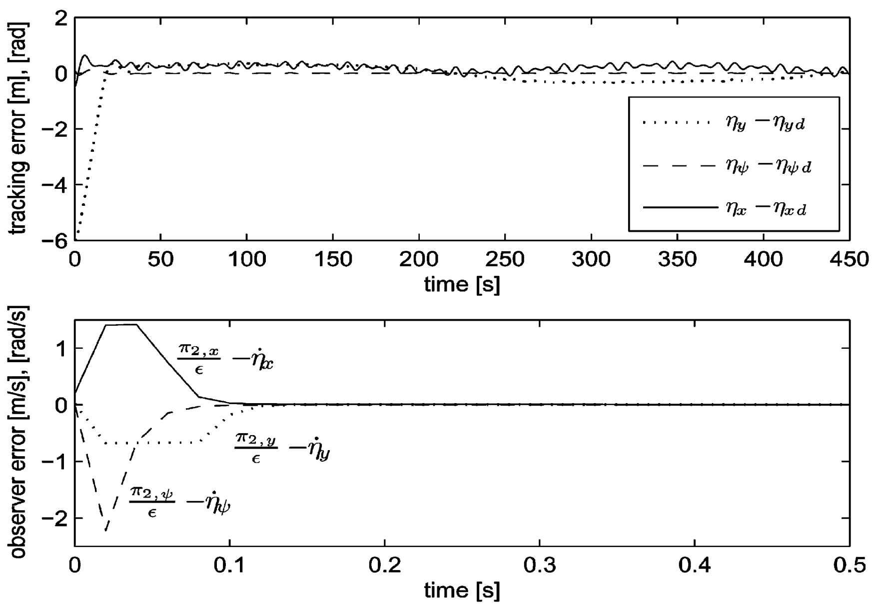

Fig. 2. Top: output tracking errors. Bottom: observer errors.

Linearly parameterized feedforward approximators are used, in which $\hat{\Theta} = \mathrm{diag}\left[\hat{\theta}_1^T, \hat{\theta}_2^T, \hat{\theta}_3^T\right]$ are adaptable parameters, and $\hat{Z} = \left[\eta^T, \hat{v}^T, \alpha_1^T, \dot{\alpha}_1^T\right]^T \in R^{12}$ are the inputs to the approximator. Motivated by [17], the basis functions are defined by

$$s_i(\hat{Z}) := \frac{\mu_i(\hat{Z})}{\sum_{j=1}^{3^{12}} \mu_j(\hat{Z})}, \quad \mu_i(\hat{Z}) := \prod_{j=1}^{12} \nu_j, \quad i = 1, \ldots, 3^{12}, \quad j = 1, \ldots, 12 \tag{29}$$

where ν_j is one of the three functions from the jth set $\left\{(1/(1+e^{a_{1j}(\hat{Z}_j - b_{1j})})), e^{-a_{2j}|\hat{Z}_j - b_{2j}|^2}, (1/(1+e^{-a_{3j}(\hat{Z}_j - b_{3j})}))\right\}$ such that each μ_i is composed of a unique combination of the functions from the 12 sets. The constant parameters a_{kj} and b_{kj}, $k = 1, 2, 3$ are user-defined.

The high-gain observer is designed according to (17) with $n = 2$ and $\gamma_1 = 2$, while the control and adaptation laws are based on (5), (20), and (21), with the control signals saturated whenever $|\tau_x| \geq 5$ N, $|\tau_y| \geq 5$ N, and $|\tau_\psi| \geq 3.5$ Nm. We choose $\Gamma_1 = \mathrm{diag}[12]$, $\Gamma_2 = \mathrm{diag}[6]$, $\Gamma_3 = \mathrm{diag}[40]$, $\sigma_i = 1 \times 10^{-5}$, and $\epsilon = 0.01$. With $K_1 = \mathrm{diag}[0.8]$ and $K_2 = \mathrm{diag}[5.0]$, we know that $\lambda_{\min}(K_1) = 0.8 > 0$ and $\lambda_{\min}(K_2 - (3/2)I) = 3.5 > 0$, which satisfy the conditions of (27). The initial conditions are $\eta(0) = [-0.5, 4, 0.0873]^T$, $v(0) = [0.2, 0, 0]^T$, $\pi_1(0) = \pi_2(0) = [0, 0, 0]^T$, $\hat{\theta}_i = [0, \ldots, 0]^T$ for $i = 1$, 2, 3, where all units are SI.

A. Closed-Loop Performance

From Fig. 1, it can be observed that the tracking performance of the ship is satisfactory, despite the time-varying disturbance. While Fig. 1 shows that the ship remains within a small neighborhood of the desired path, the top graph of Fig. 2 verifies that the ship in fact follows the desired trajectory satisfactorily in time. If the residual error is desired to be lower, it can be reduced by several means, to the effect that C_1/ρ in Theorem 2 decreases. One way is to increase K_1 and/or K_2. An alternative is to increase the density of approximators that span the approximation space, thus reducing the approximation error ε that makes up part of the C_1.

The small time convergence of the high-gain observer estimates to the output derivatives is clearly shown in Fig. 2. Within about 0.2 s, the estimates peak at their respective saturation values and then converge rapidly to the actual output derivatives. Thereafter, the estimates remain in a small neighborhood of the output derivatives.

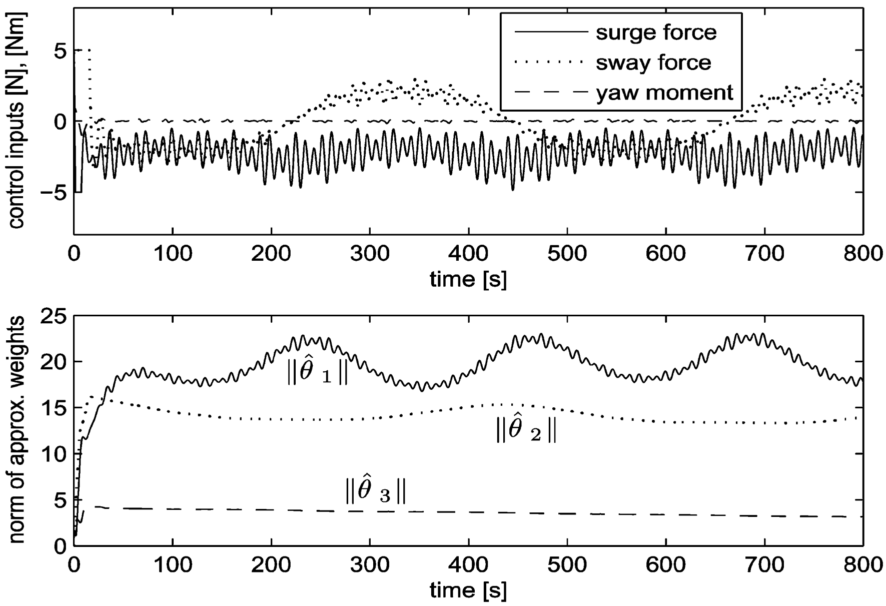

Fig. 3. Top: input forces/moment for ship. Bottom: evolution of norms of online-estimated approximation parameters.

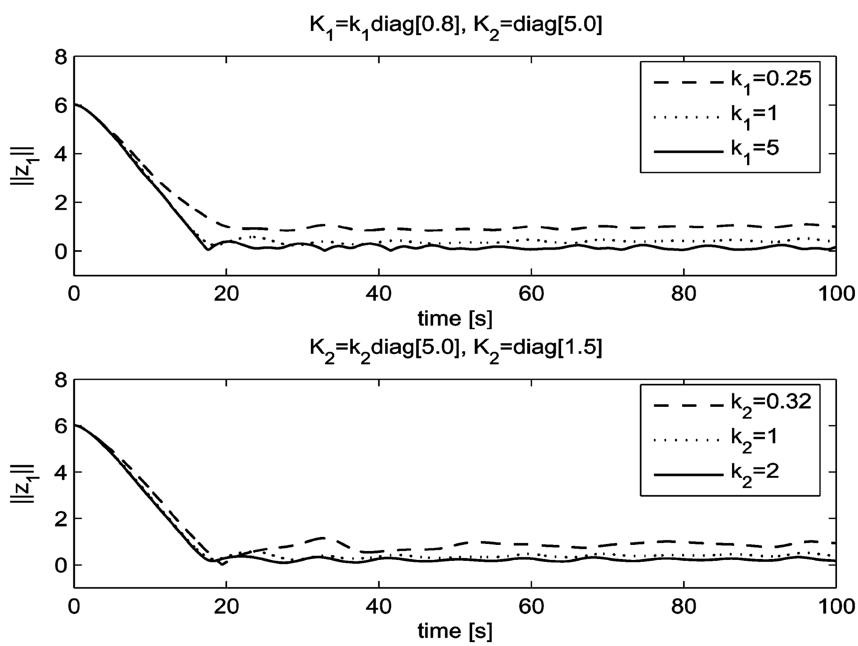

Fig. 4. Comparison of tracking error norm $\|z_1\|$ for different scaling factors k_1 and k_2.

The top graph of Fig. 3 shows that the control input signals are bounded within ± 5 N and ± 3.5 Nm, which we have set as the saturation limits. Initially, both the surge and sway signals are saturated due to the initial tracking error as well as the effects of the high-gain observer and approximation. The norms of the approximation weights are bounded with slight oscillations, as seen from the bottom graph of of Fig. 3.

The sensitivity to control gains K_1 and K_2 is shown in Fig. 4. As K_1 is scaled up, the steady-state tracking error decreases. A similar behavior can be obtained if we scale K_2. This suggests a simple intuitive tuning procedure for the control gains. Indeed, the inverse relationship between the size of the steady state compact set C_1/ρ and the gain K_1 can be seen from (25), since ρ tends to increase with $\lambda_{\min}(K_1)$. However, we do not see the same analytical relationship for K_2, since C_1 depends on K_2.

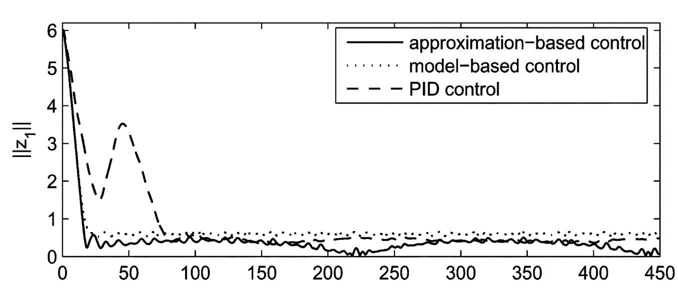

Fig. 5. Comparison of tracking performance.

B. Comparisons With Other Controllers

In this section, we will compare the tracking performance of the proposed approximation-based controller (20) (henceforth named $\tau_{\rm approx}$) with a PID controller $\tau_{\rm pid}$, as well as a model-based nonadaptive backstepping controller τ_{mb}:

$$\begin{aligned}\tau_{\rm pid} =& -K_P z_1 - K_D(J\hat{v} - \dot{\eta}_d) - K_I \int_0^t z_1(\varsigma)d\varsigma \\ \tau_{mb} =& -J^T(\eta)z_1 - K_2\hat{z}_2 + C(\hat{v})\hat{v} + D(\hat{v})\hat{v} \\ &+ M\dot{\alpha}_1(z_1, \hat{z}_2, \eta_d, \dot{\eta}_d, \ddot{\eta}_d)\end{aligned} \tag{30}$$

where $\hat{\nu} = J^T(\pi_2/\epsilon)$; the PID gains $K_P = \mathrm{diag}[0.944, 0.813, 0.372]$, $K_D = \mathrm{diag}[6.296, 6.372, 1.186]$, and $K_I = \mathrm{diag}[0.0632, 0.0485, 0.0485]$ are tuned by linear quadratic regulator (LQR) techniques with $Q = I$ and $R = 250I$. A high R is selected to obtain control magnitudes within the saturation limits. Assuming no available information on the disturbance, we do not include the disturbance compensation term in τ_{mb}.

Fig. 5 shows the comparison of tracking performance between the various controllers. The PID controller $\tau_{\rm pid}$ exhibits good steady-state performance. However, its transient performance is less satisfactory, since the linear control action tends to produce large overshoots, which causes the tracking error to peak initially at about 50 s. Although the PID controller does not explicitly contain any terms from the ship model, the tuning of the PID gains by advanced techniques such as LQR requires knowledge of the ship model. Without the use of such techniques, PID tuning for the MIMO ship system is generally nontrivial, and may require full-scale experiments.

The approximation-based controller performs better than the model-based controller τ_{mb}, with faster decay of tracking error and lower steady-state value. The better performance is due to the fact that the bounded external disturbance $d(\eta, v, t)$ is compensated for by the feedforward approximators, even though no information related to the disturbance is measured or used. In contrast, τ_{mb} does not have any disturbance compensation due to the lack of information and results in a larger mean steady state tracking error. While it is true that one can estimate a conservative bound $p(\eta, v) + \bar{q}$ and augment the term $\mathrm{Sgn}(z_2)(p(\eta, v) + \bar{q})$ in τ_{mb} to dominate the disturbance, this approach may lead to unnecessarily large control signals if $p(\eta, v) + \bar{q}$ is grossly overestimated.

V. Conclusion

In this brief, stable approximation-based tracking control has been designed for surface vessels in the presence of time-varying environmental disturbances, unmodeled dynamics, or parametric/functional uncertainties. Both full-state feedback and output feedback problems have been considered. It has been shown that the closed-loop signals under the proposed control are semiglobally uniformly bounded, and the steady-state compact set to which the error signals converge can be made small through appropriate choice of control design parameters. Simulation results have demonstrated that the surface vessel is able to track a desired trajectory satisfactorily.

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