Neural-Network Control of Nonaffine Nonlinear System With Zero Dynamics by State and Output Feedback

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***Abstract*—This paper focuses on adaptive control of nonaffine nonlinear systems with zero dynamics using multilayer neural networks. Through neural network approximation, state feedback control is firstly investigated for nonaffine single-input–single-output (SISO) systems. By using a high gain observer to reconstruct the system states, an extension is made to output feedback neural-network control of nonaffine systems, whose states and time derivatives of the output are unavailable. It is shown that output tracking errors converge to adjustable neighborhoods of the origin for both state feedback and output feedback control.**

***Index Terms*—High-gain observer, neural networks, nonaffine system, output feedback control, zero dynamics.**

I. Introduction

In recent years, control system design for complex nonlinear systems has attracted much attention. Many remarkable results in this area have been obtained, including feedback linearization techniques [1], adaptive backstepping design [2], neural-network (NN) control [3], [4] and fuzzy logic control [5]. Most of these researches are conducted for systems in affine form. Based on differential geometry theory which is a very useful analytical tool for nonlinear control system design, several adaptive schemes have been developed in dealing with the problem of parametric uncertainties [6], [7] for affine nonlinear systems. But there are some practical systems, such as chemical reactions [8], their input variables cannot be expressed in an affine form. Because the input does not appear linearly, which makes the direct feedback linearization difficult, control system design for nonaffine nonlinear systems are not an easy task.

Zero dynamics exist in many practical systems, including isothermal continuous stirred tank reactors (CSTR) [9], field-controlled dc motors [10], controlled van der Pol equation [11], aircraft trajectory tracking control [12], and others. It is necessary to investigate their influence on control system design. Zero dynamics play an important role in the areas of modeling, analysis, and control of linear and nonlinear systems. For linear systems, internal dynamics are defined to be the states that are not observable after a Lie derivative coordinate transformation [13]. By keeping the system output at zero, we obtain the zero dynamics. The stability of the internal dynamics is simply determined by the locations of the zeros, and the stability of zero dynamics implies the global stability of the internal dynamics. For nonlinear systems, intuitions for linear systems are used to define zero dynamics of nonlinear system. They are defined to be the internal dynamics of the systems when the system output is kept at zero. However, unlike the linear case, no results on the global stability or even large range stability can be drawn for the internal dynamics of nonlinear systems and only local stability is guaranteed for the internal dynamics even if the zero dynamics are globally exponentially stable. The zero dynamics of nonlinear system are an intrinsic feature of a nonlinear system, which do not depend on the choice of the control law or the desired trajectories when they are represented in a normal form where the control input u does not explicitly appear in the internal dynamics [11], [13]. But sometimes it is difficult to obtain the normal form because of the difficulty in constructing the transformation functions. With control input u appears in the internal dynamics, different forms may exist [1]. It is not difficult to arrive at similar conclusions and properties as those in a normal form. Much research work has been carried out for systems with zero dynamics [14]–[18].

Recently, NNs have been made particularly attractive and promising for applications to modeling and control of nonlinear systems, owing to its universal approximation abilities, learning and adaptation, parallel distributed abilities. The feasibility of applying NNs to model unknown functions in dynamic systems has been demonstrated in several studies [19], [20]. From these works, it has been shown that for stable and efficient online control using the backpropagation (BP) learning algorithm, the identification must be sufficiently accurate before control action could be initiated. In practical control applications, it is desirable to have systematic methods of ensuring the stability, robustness, and performance properties of the overall system. Recently, several good NN control approaches have been proposed based on Lyapunov analysis [3], [4], [21], [22]. One main advantage of these schemes is that the adaptive laws are derived based on Lyapunov synthesis, therefore, guarantee the stability of the closed-loop systems. However, they can only be applied to relatively simple classes of nonlinear plants in affine forms [3], [4]. For NN control system design of general nonlinear systems, several researchers have suggested to use NNs as emulators of inverse systems. The main idea is that for a system with a finite relative degree, the mapping between a system input and the system output is one-to-one, thus allowing the construction of a "left-inverse" of the nonlinear system using NN. Using the

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implicit function theory, the NN control methods proposed in [20], [23] have been used to emulate the "inverse controller" to achieve the desired control objectives, though no rigorous proof was given in [23]. Based on this idea, adaptive control with rigorous analysis has been investigated for nonaffine nonlinear system by using multilayer NNs in [24], [25] and was applied in [8]. None of the above works considered the zero dynamics, though it plays an important role in nonlinear system control.

In this paper, we are interested in how to control the single-input–single-output (SISO) nonaffine nonlinear system with zero dynamics using multilayer NNs. The problem is not only academically challenging but also of practical interest. Academically, it is very much involved and tedious to extend the results in [24], [25] for nonaffine nonlinear SISO system to nonaffine nonlinear system with zero dynamics. In practice, there are indeed systems that have zero dynamics which include certain types of CSTR systems [9], field-controlled dc motor systems [10] and others. In this paper, based on the implicit function theorem, multilayer NNs are used to approximate the implicit desired feedback control. For the system's zero dynamics, we first assume that the zero dynamics are minimum-phase, i.e., zero dynamics are exponentially stable, then under the Lipschitz condition assumption, by using converse Lyapunov theorem, we can show that the system's internal states do remain in a compact set.

The main contributions of this paper are: 1) the proof of the existence of implicit desired feedback control based on implicit function theorem; 2) state feedback control for nonaffine nonlinear system using NNs; and 3) observer-based NN output control for nonaffine nonlinear system. It should be noted that, although the control schemes are developed for nonaffine systems with zero dynamics, they also can be applied to affine system without zero dynamics, affine system with zero dynamics and nonaffine system without zero dynamics, assuming all the assumptions are satisfied. There is no doubt these kinds of systems cover a wide class of practical processes.

This paper is organized as follows. In Section II, by using Lie derivative, the general form of the SISO nonaffine system is transformed into a normal form in the new coordinates. Then the existence of implicit desired feedback control (IDFC) is proved under some mild assumptions. The state feedback control and the output feedback control are presented in Sections III and IV, respectively. A practical CSTR process simulation shows the effectiveness of the proposed control methods.

II. PROBLEM STATEMENT

Consider SISO nonaffine system

$$\begin{cases} \dot{x} = f(x,u) \\ y = h(x) \end{cases} \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R$ is the input, and $y \in R$ is the output. The mapping $f(\cdot,\cdot) : R^{n+1} \to R^n$ is a partially unknown smooth vector field and $h(\cdot) : R^n \to R$ is a partially unknown smooth function, the degree of uncertainties will be explained later. The control objective is to design a controller such that the system output y follows the desired trajectory y_d. The main difficulty of this control problem is that the system input u does not appear linearly, which makes the direct feedback linearization difficult/impossible.

A. System Transform via Lie Derivative

Let $L_f h$ denote the Lie derivative of the function $h(x)$ with respect to the vector field $f(x,u)$ as

$$L_f h = \frac{\partial[h(x)]}{\partial x} f(x,u).$$

Higher order Lie derivatives can be defined recursively as $L_f^k h = L_f(L_f^{k-1}h)$, $k > 1$.

Let $\Omega_x \subset R^n$ and $\Omega_u \subset R$ be two compact sets such that $x \in \Omega_x$ and $u \in \Omega_u$. System (1) is said to have a strong relative degree ρ in $U = \Omega_x \times \Omega_u$ if there exists a positive integer $1 \le \rho < \infty$ such that

$$\frac{\partial\left[L_f^i h\right]}{\partial u} = 0,\ i = 0,1,\ldots,\rho-1,\ \frac{\partial\left[L_f^\rho h\right]}{\partial u} \neq 0 \tag{2}$$

for all $(x,u) \in U$ [26].

Assumption 2.1: System (1) possesses a strong relative degree $\rho < n$, $\forall (x,u) \in U$.

Define $\phi_j(x) = L_f^{j-1}h$, $j = 1,2,\ldots,n$. Under Assumption 2.1, it was shown in [1], [6] that there exist other $n-\rho$ functions $\phi_{\rho+1}(x),\ldots,\phi_n(x)$, which are independent of u, such that the mapping

$$\Phi(x) = [\phi_1(x), \phi_2(x), \ldots, \phi_n(x)]^T \tag{3}$$

has a Jacobian matrix which is nonsingular for all $x \in \Omega_x$. Therefore, $\Phi(x)$ is a diffeomorphism on Ω_x. By setting

$$\begin{aligned} \xi &= [\phi_1(x), \phi_2(x), \ldots, \phi_\rho(x)]^T \\ \eta &= [\phi_{\rho+1}(x), \phi_{\rho+2}(x), \ldots, \phi_n(x)]^T \end{aligned}$$

system (1) can be transformed into a normal form in the new coordinate $[\xi^T, \eta^T]^T = \Phi(x)$ as follows:

$$\left.\begin{aligned} \dot{\xi}_i &= \xi_{i+1}, \quad i = 1,\ldots,\rho-1 \\ \dot{\xi}_\rho &= b(\xi,\eta,u) \\ \dot{\eta} &= q(\xi,\eta,u) \\ y &= \xi_1 \end{aligned}\right\} \tag{4}$$

where

$$\begin{aligned} b(\xi,\eta,u) &= L_f^\rho h \\ q(\xi,\eta,u) &= [q_1(\xi,\eta,u), q_2(\xi,\eta,u), \ldots, q_{n-\rho}(\xi,\eta,u)]^T \\ q_i(\xi,\eta,u) &= L_f \phi_{\rho+i}(x), \quad i = 1,2,\ldots,n-\rho \\ x &= \Phi^{-1}(\xi,\eta), \quad (\xi,\eta,u) \in \bar{U}. \end{aligned}$$

with the compact set $\bar{U}$ being defined as

$$\bar{U} = \left\{(\xi,\eta,u) \middle| (\xi,\eta) \in \Phi(\Omega_x);\ u \in \Omega_u\right\}.$$

Define the smooth function

$$b_u = \frac{\partial\left[b(\xi,\eta,u)\right]}{\partial u}. \tag{5}$$

According to Assumption 2.1, it can be shown that

$$\frac{\partial[b(\xi,\eta,u)]}{\partial u} \neq 0, \quad \forall (\xi,\eta,u) \in \bar{U}$$

which implies that the smooth function b_u is strictly either positive or negative for all $(\xi,\eta,u) \in \bar{U}$.

Assumption 2.2: There exists a smooth function $b_1(x)$ and a positive constant $d > 0$, such that $b_1(x) \geq |b_u| > d > 0$ holds for all $(x, u) \in U$.

Remark 2.1: From (5), we know that b_u can be viewed as the control gain of the normal system (4). Assumption 2.2 means that the plant input gain is bounded by a positive function of x, which does not pose a strong restriction upon the class of systems. In the following design procedure we only need the existence of Assumption 2.2, and function $b_1(x)$ is not required to be known *a priori*.

Assumption 2.3: There is a positive design constant ε satisfying $|\dot{b}_u/2b_u| \leq b_1(x)/\varepsilon$, $\forall(\xi, \eta, u) \in \Omega_x \times R$.

From now on, without losing generality, we shall assume $b_1(x) \geq b_u > d$.

B. Implicit Desired Feedback Control

Define vectors ξ_d, $\bar{\xi}_d$ and $\tilde{\xi}$ as

$$\begin{aligned} \xi_d &= \left[y_d, \dot{y}_d, \ldots, y_d^{(\rho-1)}\right]^T \in R^\rho \\ \bar{\xi}_d &= \left[\xi_d^T, y_d^{(\rho)}, y_d^{(\rho+1)}\right]^T \in R^{\rho+2} \\ \tilde{\xi} &= \xi - \xi_d = \left[\tilde{\xi}_1, \tilde{\xi}_2, \ldots, \tilde{\xi}_\rho\right]^T \end{aligned} \tag{6}$$

and a filtered tracking error as

$$e_s = \left[\Lambda^T\, 1\right] \tilde{\xi} \tag{7}$$

where $\Lambda = [\lambda_1, \lambda_2, \ldots, \lambda_{\rho-1}]^T$ is an appropriately chosen coefficient vector so that $\tilde{\xi}(t) \to 0$ as $e_s \to 0$, (i.e., $s^{\rho-1} + \lambda_{\rho-1}s^{\rho-2} + \cdots + \lambda_1$ is Hurwitz).

Lemma 2.1: Define $\zeta = [\tilde{\xi}_1, \tilde{\xi}_2, \ldots, \tilde{\xi}_{\rho-1}]^T$ and functions $e_{s_{\max}}(t) = \sup_{0 \leq \tau \leq t} |e_s(\tau)|$ and $\beta_{e_s} = \sup_{T_1 \leq t} |e_s(t)|$. Then, the following equations and inequalities hold:

$$\dot{\zeta}(t) = A\zeta(t) + be_s(t) \tag{8}$$

$$\zeta(t) = \zeta(0)e^{At} + \int_0^t e^{A(t-\tau)}be_s d\tau \tag{9}$$

$$\left\|e^{At}\right\| \leq k_0 e^{-\lambda_0 t} \tag{10}$$

$$\|\zeta(t)\| \leq k_0\|\zeta(0)\| + \frac{k_0}{\lambda_0}e_{s_{\max}}(t) \tag{11}$$

$$\|\zeta(t)\| \leq k_0 e^{-\lambda_0 t}\left(\|\zeta(0)\| + \frac{e^{\lambda_0 T_1}}{\lambda_0}e_{s_{\max}}(T_1)\right) + \frac{k_0}{\lambda_0}\beta_{e_s} \tag{12}$$

$$\|\tilde{\xi}_\rho\| \leq e_{s_{\max}}(t) + d_\lambda\|\zeta\| \tag{13}$$

$$\|\xi\| \leq d_1\|\xi_d\| + d_2 e_{s_{\max}}(t) + d_3 \tag{14}$$

where

$$A = \begin{bmatrix} 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \\ -\lambda_1 & -\lambda_2 & \ldots & -\lambda_{\rho-1} \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{15}$$

with constants $k_0 > 0$, $\lambda_0 > 0$, $d_\lambda = \| \begin{bmatrix} \Lambda \\ 0 \end{bmatrix}^T \|$, $d_1 = 1$, $d_2 = (k_0(1 + d_\lambda)/\lambda_0) + 1$, and $d_3 = k_0(1 + d_\lambda)\|\zeta(0)\|$.

Proof: Considering (4) and (6), (8) and (9) are apparent. From linear system theory, (10) can be established easily.

Considering (9), we can prove inequality (11) as follows:

$$\begin{aligned} \|\zeta(t)\| &\leq k_0 e^{-\lambda_0 t}\|\zeta(0)\| + k_0\int_0^t e^{-\lambda_0(t-\tau)}|e_s|\, d\tau \\ &\leq k_0 e^{-\lambda_0 t}\|\zeta(0)\| + k_0 e^{-\lambda_0 t}e_{s_{\max}}(t)\int_0^t e^{\lambda_0\tau}d\tau \\ &\leq k_0 e^{-\lambda_0 t}\|\zeta(0)\| + k_0 e^{-\lambda_0 t}e_{s_{\max}}(t)\frac{e^{\lambda_0 t} - 1}{\lambda_0} \\ &\leq k_0\|\zeta(0)\| + \frac{k_0}{\lambda_0}e_{s_{\max}}(t). \end{aligned}$$

Noting the above equation and that

$$\begin{aligned} \int_0^t e^{-\lambda_0(t-\tau)}|e_s|\, d\tau = &\int_0^{T_1} e^{-\lambda_0(t-\tau)}|e_s|\, d\tau \\ &+ \int_{T_1}^t e^{-\lambda_0(t-\tau)}|e_s|\, d\tau \end{aligned}$$

we have (12) as follows:

$$\begin{aligned} \|\zeta(t)\| \leq & k_0 e^{-\lambda_0 t}\|\zeta(0)\| + k_0 e^{-\lambda_0 t}\frac{e^{\lambda_0 T_1} - 1}{\lambda_0}e_{s_{\max}}(T_1) \\ &+ k_0 e^{-\lambda_0 t}\frac{e^{\lambda_0 t} - e^{\lambda_0 T_1}}{\lambda_0}\beta_{e_s} \\ \leq & k_0 e^{-\lambda_0 t}\left(\|\zeta(0)\| + \frac{e^{\lambda_0 T_1}}{\lambda_0}e_{s_{\max}}(T_1)\right) \\ &+ \frac{k_0}{\lambda_0}\beta_{e_s},\ \forall t > T_1. \end{aligned} \tag{16}$$

From (6) and (7), we know that $\tilde{\xi}_\rho = e_s - [\Lambda 0]^T\zeta$. Thus, we have (13) as

$$\left\|\tilde{\xi}_\rho\right\| \leq |e_s| + \left\|\begin{bmatrix} \Lambda \\ 0 \end{bmatrix}^T\right\| \cdot \|\zeta\| \leq e_{s_{\max}}(t) + d_\lambda\|\zeta\|.$$

Combining (11) and (13), we arrive at inequality below

$$\begin{aligned} \|\xi\| = \left\|\tilde{\xi} + \xi_d\right\| &\leq \|\zeta\| + \left\|\tilde{\xi}_\rho\right\| + \|\xi_d\| \\ &\leq d_1\|\xi_d\| + d_2 e_{s_{\max}}(t) + d_3 \end{aligned} \tag{17}$$

with $d_1 = 1$, $d_2 = (k_0(1 + d_\lambda)/\lambda_0) + 1$ and $d_3 = k_0(1 + d_\lambda)\|\zeta(0)\|$ being positive constants. Q.E.D

From (4)–(7), the time derivative of the filtered tracking error can be written as

$$\dot{e}_s = b(\xi, \eta, u) - y_d^{(\rho)} + \left[0\, \Lambda^T\right]\tilde{\xi}. \tag{18}$$

Assumption 2.4: The desired trajectory vector $\bar{\xi}_d$ is continuous and available, $\|\bar{\xi}_d\| \leq c$ with c being a known bound.

Adding and subtracting $(b_1(x)/\varepsilon)e_s$ to the right-hand side of (18), we obtain

$$\dot{e}_s = b(\xi, \eta, u) + \nu - \frac{b_1(x)}{\varepsilon}e_s \tag{19}$$

with $\nu(\xi, \eta) = (b_1(x)/\varepsilon e_s - y_d^{(\rho)} + \left[0\, \Lambda^T\right]\tilde{\xi}$. Since $\partial\nu/\partial u = 0$ and $\partial[b(\xi, \eta, u)]/\partial u > d$, $\forall(\xi, \eta, u) \in U$, we can obtain that $\partial[b(\xi, \eta, u) + \nu]/\partial u > d$, and the following lemma.

Lemma 2.2: Assume that $f(x,y) : R^n \times R \rightarrow R$ is continuously differentiable $\forall(x,y) \in R^n \times R$, and there exists a positive constant d such that $\partial f(x,y)/\partial y(x,y) > d > 0$, $\forall(x,y) \in R^n \times R$. Then there exists a continuous (smooth) function $y^* = g(x)$ such that $f(x,y^*) = 0$. For the case $\partial f(x,y)/\partial y(x,y) < -d < 0$, $\forall(x,y) \in R^n \times R$. The result still holds.

Proof: See [25].

Corollary 2.1: If partial derivative $\partial[b(\xi,\eta,u)+\nu]/\partial u > d > 0$, where d is a positive constant. Then, there exists a continuous (smooth) function $u^* = \alpha^c(\xi,\eta,\nu)$ such that $b(\xi,\eta,u^*)+\nu = 0$ holds.

C. Zero Dynamics

If system (4) is controlled by the input u, the state vector η is completely unobservable from the output, then the subsystem

$$\dot{\eta} = q\left(0,\eta,\alpha^c(0,\eta,\nu(0,\eta))\right) \tag{20}$$

is addressed as the *zero dynamics* [1], [27].

Assumption 2.5: System (4) is hyperbolically minimum-phase, i.e., zero dynamics (20) is exponentially stable. In addition, assume that the control input u is designed as a function of the states $x \in \Omega_x$ and the reference signal $\bar{\xi}_d$ satisfying Assumption 2.4, and the function $q(\xi,\eta,u)$ is Lipschitz in ξ, i.e., there exists Lipschitz constants L_ξ and L_q for $q(\xi,\eta,u)$ such that

$$\|q(\xi,\eta,u) - q(0,\eta,u_\eta)\| \le L_\xi\|\xi\| + L_q, \quad \forall(\xi,\eta,u) \in \bar{U} \tag{21}$$

where $u_\eta = \alpha^c(0,\eta,\nu(0,\eta))$.

Under Assumption 2.5, by the converse theorem of Lyapunov [28], there exists a Lyapunov function $V_0(\eta)$ which satisfies

$$\sigma_2\|\eta\|^2 \le V_0(\eta) \le \sigma_1\|\eta\|^2 \tag{22}$$

$$\frac{\partial V_0}{\partial \eta} q(0,\eta,u_\eta) \le -\lambda_a\|\eta\|^2 \tag{23}$$

$$\left\|\frac{\partial V_0}{\partial \eta}\right\| \le \lambda_b\|\eta\| \tag{24}$$

where σ_1, σ_2, λ_a, and λ_b are positive constants.

Lemma 2.3: For the internal dynamics $\dot{\eta} = q(\xi,\eta,u)$ of system (4), if Assumptions 2.4 and 2.5 are satisfied, then there exist positive constants L_a, L_b, L_c and T_0, such that

$$\|\eta(t)\| \le L_a e_{s_{\max}}(t) + L_b\|\xi_d\| + L_c, \quad \forall t > T_0. \tag{25}$$

Proof: According to Assumption 2.5, there exists a Lyapunov function $V_0(\eta)$. Differentiating $V_0(\eta)$ along (4) yields

$$\begin{aligned}\dot{V}_0(\eta) =& \frac{\partial V_0}{\partial \eta} q(\xi,\eta,u) \\ =& \frac{\partial V_0}{\partial \eta} q(0,\eta,u_\eta) + \frac{\partial V_0}{\partial \eta}\left[q(\xi,\eta,u) - q(0,\eta,u_\eta)\right].\end{aligned} \tag{26}$$

Noting (21)–(24), (26) can be written as

$$\dot{V}_0(\eta) \le -\lambda_a\|\eta\|^2 + \lambda_b L_\xi\|\eta\|\|\xi\| + \lambda_b L_q\|\eta\| \tag{27}$$

Noting (14) in Lemma 2.1, we have

$$\begin{aligned}&\dot{V}_0(\eta) \le -\lambda_a\|\eta\|^2 \\ &\quad + \lambda_b\|\eta\|\left(d_1 L_\xi\|\xi_d\| + L_q + d_2 L_\xi e_{s_{\max}}(t) + d_3 L_\xi\right).\end{aligned} \tag{28}$$

Therefore, $\dot{V}_0(\eta) \le 0$, whenever

$$\|\eta\| \ge \frac{\lambda_b}{\lambda_a}\left(d_1 L_\xi\|\xi_d\| + L_q + d_2 L_\xi e_{s_{\max}}(t) + d_3 L_\xi\right) \tag{29}$$

By letting $L_a = \lambda_b L_\xi d_2/\lambda_a$, $L_b = \lambda_b L_\xi d_1/\lambda_a$ and $L_c = \lambda_b(L_q + d_3 L_\xi)/\lambda_a$, we conclude that there exists a positive constant T_0, such that (25) holds. Q.E.D.

III. State Feedback Control

A. Existence of IDFC Control

Lemma 3.1: For system (1), satisfying Assumptions 2.1 and 2.2, there exists a compact subset $\Phi_0 \subset \Phi(\Omega_x)$ and a continuous input $u^* = \alpha^c(\xi,\eta,\nu)$ (which is trajectory-dependent) such that for all $(\xi(0),\eta(0)) \in \Phi_0$, the error (19) can be expressed as

$$\dot{e}_s = -\frac{b_1(x)}{\varepsilon} e_s \tag{30}$$

Subsequently, (30) leads to $\lim_{t\to\infty}|y(t) - y_d(t)| = 0$ asymptotically.

Proof: Considering (19), under Assumptions 2.1–2.2, we know that there exists a desired $u^* = \alpha^c(\xi,\eta,\nu)$ satisfying $b(\xi,\eta,u) + \nu = 0$ from Corollary 2.1.

Since $[\xi^T,\eta^T]^T = \Phi(x)$, the continuous function $\alpha^c(\xi,\eta,\nu)$ is also a function of x and ν. If the IDFC input is chosen as

$$u^*(z) = \alpha^c(\xi,\eta,\nu), z = [x, e_s, \nu_1]^T \in \Omega_z \subset R^{n+2} \tag{31}$$

where $\nu_1 = -y_d^{(\rho)} + \left[0\,\Lambda^T\right]\tilde{\xi}$ and compact set

$$\Omega_z = \left\{(x,e_s,\nu_1)\;\middle|\; x \in \Phi(\Omega_x),\ \|\bar{\xi}_d\| \le c\right\}$$

then

$$b(\xi,\eta,u^*) + \nu = 0. \tag{32}$$

Under the action of $u^*(z)$, (19) and (32) imply that (30) holds. As $b_1(x)/\varepsilon > 0$, (30) is asymptotically stable, i.e., $\lim_{t\to\infty}|e_s| = 0$. Because $s^{n-1} + \lambda_{n-1}s^{n-2} + \cdots + \lambda_1$ is Hurwitz, we have $\lim_{t\to\infty}|y(t) - y_d(t)| = 0$ asymptotically.

It should be noticed that the above result is obtained under the condition of $(\xi,\eta) \in \Phi(\Omega_x)$, $\forall t \ge 0$. We will specify the initial states such that this condition is satisfied. It follows from (30) that $e_s(t) = e_s(0)e^{-\int_0^t (b_1(x(\tau))/\varepsilon)d\tau}$. Because $b_1(x)/\varepsilon > 0$, we have $|e_s(t)| \le |e_s(0)|$. Define the following compact set:

$$\begin{aligned}\Phi_0 = \Bigg\{(\xi(0),\eta(0))\;\Bigg|\;\Big\{(\xi,\eta)\;\big|\;|e_s(t)| < |e_s(0)|,\ \left\|\bar{\xi}_d\right\| \le c\Big\} \\ \subset \Phi(\Omega_x), (\xi(0),\eta(0)) \in \Phi(\Omega_x)\Bigg\}.\end{aligned} \tag{33}$$

Then, for all $(\xi(0),\eta(0)) \in \Phi_0$, we have $(\xi,\eta) \in \Phi(\Omega_x)$, $\forall t \ge 0$. This completes the proof. Q.E.D

Remark 3.1: In Lemma 3.1, we suppose that Ω_u is large enough such that $u^* \in \Omega_u$. If the region of Ω_u is not large enough, some restrictions should be imposed upon the desired trajectory $\bar{\xi}_d$ and design parameter ε.

Remark 3.2: It is shown from Lemma 3.1 that the magnitude c of the desired signal $\bar{\xi}_d$ affects the size of the allowed initial state region Φ_0. This is reasonable because if the reference ξ_d is very large and out of the region $\Phi(\Omega_x)$, the compact set Φ_0

will become an empty set and the tracking problem cannot be solved.

Lemma 3.1 only assumes the existence of IDFC u^*; it does not provide a method to construct it. In this paper, multilayer NNs shall be introduced to construct u^* for achieving tracking control.

B. Control Structure Based on Multilayer Neural Networks

Because the IDFC input $u^*(z)$ defined in (31) is a continuous function on the compact set Ω_z, there exists an integer l (the number of hidden neurons) and ideal constant weight matrices W^* and V^*, such that

$$u^*(z) = W^{*T} S\left(V^{*T}\bar{z}\right) + \varepsilon_u(z),\ \forall z \in \Omega_z \tag{34}$$

where $\bar{z} = [z^T, 1]^T$ and $\varepsilon_u(z)$ represents the approximation error. The following assumption is made for this function approximation.

Assumption 3.1: On the compact set Ω_z, the ideal NN weights W^*, V^* and the NN approximation error are bounded by

$$\|W^*\| \le w_m, \quad \|V^*\|_F \le v_m, \quad |\varepsilon_u(z)| \le \varepsilon_l \tag{35}$$

with w_m, v_m and ε_l being positive constants.

The ideal constant weights W^* and V^* are defined as

$$(W^*, V^*) := arg \min_{(W,V)\in\Omega_w} \left\{ \sup_{z\in\Omega_z} \left| W^T S\left(V^T\bar{z}\right) - u^*(z)\right| \right\} \tag{36}$$

where $\Omega_w = \left\{(W,V) \;\middle|\; \|W\| \le w_m, \|V\|_F \le v_m\right\}$. The magnitude of $\varepsilon_u(z)$ depends on the choices of the number l and the constraint set Ω_w. In general, the larger the weight number l and the constraint set Ω_w are, the smaller the approximation error will be.

Let us consider the robust MNN controller of the form

$$u = u_{nn} + u_b \tag{37}$$

where

$$u_{nn} = \hat{W}^T S\left(\hat{V}^T\bar{z}\right) \tag{38}$$

$$u_b = -\left[\frac{k_p}{\varepsilon}\left(\left\|\bar{z}\hat{W}^T\hat{S}'\right\|_F^2 + \left\|\hat{S}'\hat{V}^T\bar{z}\right\|^2 + 1\right) + k_s|e_s|\right] e_s \tag{39}$$

with k_s and k_p being positive design parameters, $\hat{W}$ and $\hat{V}$ being the estimation of the ideal neural weights W^* and V^*, respectively. The first part $\hat{W}^T S(\hat{V}^T\bar{z})$ in the controller is introduced to approximate the IDFC input u^* to realize tracking control. The second part u_b is a bounding control term, which is introduced to limit the upper bounds of the system states.

Lemma 2.2: The first part $u_{nn} = \hat{W}^T S(\hat{V}^T\bar{z})$ in controller (37) is introduced to approximate the IDFC input u^* to realize tracking control, its approximation error can be expressed as

$$\hat{W}^T S\left(\hat{V}^T\bar{z}\right) - W^{*T} S\left(V^{*T}\bar{z}\right) = \tilde{W}^T\left(\hat{S} - \hat{S}'\hat{V}^T\bar{z}\right) + \hat{W}^T\hat{S}'\tilde{V}^T\bar{z} + d_u \tag{40}$$

where $\hat{S} = S(\hat{V}^T\bar{z})$, $\tilde{W} = \hat{W} - W^*$ and $\tilde{V} = \hat{V} - V^*$ are defined to be the neural weights estimation error, $\hat{S}' = diag\{\hat{s}'_1, \hat{s}'_2, \ldots, \hat{s}'_l\}$ with

$$\hat{s}'_i = s'\left(\hat{v}_i^T\bar{z}\right) = \left.\frac{d\left[s\left(z_a\right)\right]}{dz_a}\right|_{z_a = \hat{v}_i^T\bar{z}},\ i = 1, 2, \ldots, l \tag{41}$$

and the residual term d_u is bounded by

$$|d_u| \le \|V^*\|_F \left\|\bar{z}\hat{W}^T\hat{S}'\right\|_F + \|W^*\| \left\|\hat{S}'\hat{V}^T\bar{z}\right\| + \|W^*\|. \tag{42}$$

Proof: See [25].

Since the function $b(\xi, \eta, u)$ in (4) is nonaffine in the input u, which is difficult to be dealt with directly. By using the mean value theory in [29], there exists a λ $(0 < \lambda < 1)$ such that

$$b(\xi, \eta, u) = b\left(\xi, \eta, u^*\right) + b_{u_\lambda}\left(u - u^*\right) \tag{43}$$

where

$$b_{u_\lambda} = \left.\frac{\partial\left[b\left(\xi, \eta, \bar{u}\right)\right]}{\partial\bar{u}}\right|_{\bar{u} = u_\lambda} \tag{44}$$

with $u_\lambda = \lambda u + (1-\lambda)u^*$. Considering (32) and (43), we can write the error (19) as

$$\dot{e}_s = -\frac{b_1(x)}{\varepsilon}e_s + b_{u_\lambda}\left(u - u^*\right). \tag{45}$$

Since $b_{u_\lambda} > 0$ (Assumption 2.2), the following equation holds:

$$b_{u_\lambda}^{-1}\dot{e}_s = -\frac{b_1(x)}{\varepsilon}b_{u_\lambda}^{-1}e_s + u - u^* \tag{46}$$

Noticing Lemma 3.2, substituting (34), (37), and (40) into (46), we obtain the closed-loop error equation

$$\begin{aligned} b_{u_\lambda}^{-1}\dot{e}_s =& -\frac{b_1(x)}{\varepsilon}b_{u_\lambda}^{-1}e_s + \hat{W}^T S\left(\hat{V}^T\bar{z}\right) + u_b \\ & - W^{*T} S\left(V^{*T}\bar{z}\right) - \varepsilon_u(z) \\ =& -\frac{b_1(x)}{\varepsilon}b_{u_\lambda}^{-1}e_s + \tilde{W}^T\left(\hat{S} - \hat{S}'\hat{V}^T\bar{z}\right) + \hat{W}^T\hat{S}'\tilde{V}^T\bar{z} \\ & - \left[\frac{k_p}{\varepsilon}\left(\left\|\bar{z}\hat{W}^T\hat{S}'\right\|_F^2 + \left\|\hat{S}'\hat{V}^T\bar{z}\right\|^2 + 1\right) + k_s|e_s|\right] e_s \\ & - \varepsilon_u(z) + d_u \end{aligned} \tag{47}$$

which shall be used in stability analysis.

C. Robust Weight Updating Algorithms and Stability Analysis

To updating the MNN weights, the following algorithms are used:

$$\dot{\hat{W}} = -\Gamma_w\left[\left(\hat{S} - \hat{S}'\hat{V}^T\bar{z}\right)e_s + \delta_w\left(1 + |e_s|\right)\hat{W}\right] \tag{48}$$

$$\dot{\hat{V}} = -\Gamma_v\left[\bar{z}\hat{W}^T\hat{S}'e_s + \delta_v\left(1 + |e_s|\right)\hat{V}\right] \tag{49}$$

where $\Gamma_w = \Gamma_w^T > 0$, $\Gamma_v = \Gamma_v^T > 0$, $\delta_w > 0$ and $\delta_v > 0$ are constant design parameters. Because $\tilde{W} = \hat{W} - W^*$ and $\tilde{V} = \hat{V} - V^*$, and W^*, V^* are constant vectors, it is easy to obtain that

$$\dot{\tilde{W}} = \dot{\hat{W}} \quad \text{and} \quad \dot{\tilde{V}} = \dot{\hat{V}}$$

The first terms of the right-hand sides of (48) and (49) are the modified backpropagation algorithms and the last terms

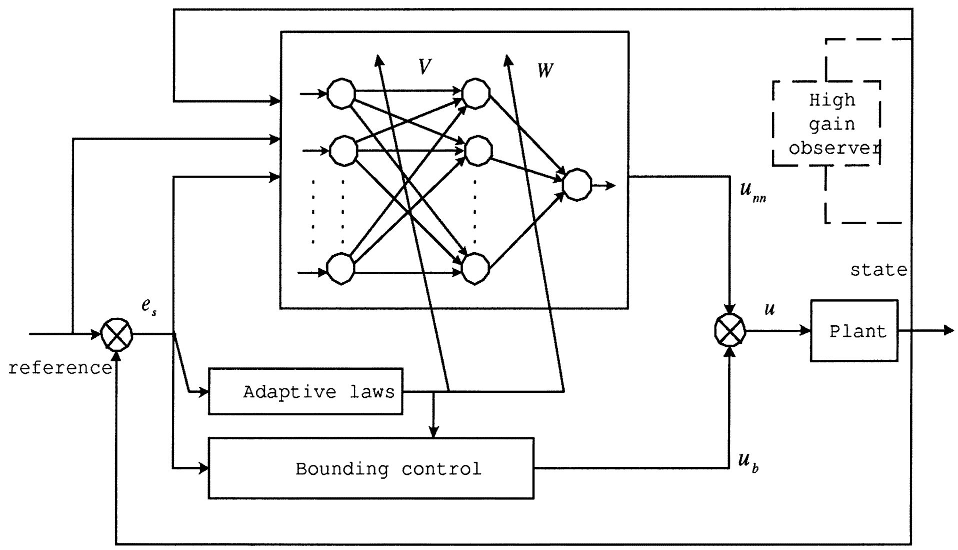

Fig. 1. Control system structure.

of them correspond to a combination of σ-modification [30] and e_1-modification [31], which are introduced to improve the robustness in the presence of the NN approximation error. It should be noted that, (48) and (49) are special classes of several adaptive laws proposed in [25]. The above learning algorithms have a nice property as stated below.

Lemma 3.3: The updated learning algorithms (48) and (49) guarantee that $\hat{W}(t)$, $\hat{V}(t) \in L_\infty$ for bounded initial weights $\hat{W}(0)$ and $\hat{V}(0)$.

Proof: Choose the Lyapunov function candidate $V_w = 1/2\hat{W}^T\Gamma_w^{-1}\hat{W} + 1/2tr\{\hat{V}^T\Gamma_v^{-1}\hat{V}\}$. Using the property $tr\{\hat{V}^T\bar{z}\hat{W}^T\hat{S}'\} = \hat{W}^T\hat{S}'\hat{V}^T\bar{z}$, the time derivative of V_w along (48) and (49) is

$$\begin{aligned}\dot{V}_w =& -\hat{W}^T\left(\hat{S}-\hat{S}'\hat{V}^T\bar{z}\right)e_s - \hat{W}^T\hat{S}'\hat{V}^T\bar{z}e_s\\ &-(1+|e_s|)\left(\delta_w\left\|\hat{W}\right\|^2+\delta_v\left\|\hat{V}\right\|_F^2\right)\\ \le& -\delta_w\left\|\hat{W}\right\|^2-\delta_v\left\|\hat{V}\right\|_F^2\\ &-|e_s|\left(\delta_w\left\|\hat{W}\right\|^2+\delta_v\left\|\hat{V}\right\|_F^2-\left|\hat{W}^T\hat{S}\right|\right).\end{aligned}$$

Since $|\hat{W}^T\hat{S}| \le \delta_w/4\|\hat{W}\|^2 + \|\hat{S}\|^2/\delta_w$, we have

$$\begin{aligned}\dot{V}_w \le& -\delta_w\left\|\hat{W}\right\|^2-\delta_v\left\|\hat{V}\right\|_F^2\\ &-|e_s|\left(\frac{3\delta_w}{4}\left\|\hat{W}\right\|^2+\delta_v\left\|\hat{V}\right\|_F^2-\frac{\left\|\hat{S}\right\|^2}{\delta_w}\right).\end{aligned}$$

Because every element of $\hat{S}$ is not larger than one, we know that $\|\hat{S}\|^2 \le l$ with l being the NN hidden-layer node number. Therefore, $\dot{V}_w < 0$ once $\|\hat{W}\|^2 > 4l/3\delta_w^2$ or $\|\hat{V}\|_F^2 > l/\delta_w\delta_v$. Because l, δ_w and δ_v are positive constants, we conclude that $\hat{W}(t)$, $\hat{V}(t) \in L_\infty$. Q.E.D.

The state feedback control structure is shown in Fig. 1. If the high-gain observer in the dashed box is switched in, we have the output feedback control as will be discussed in Section IV.

Theorem 3.1: For system (1) with Assumptions 2.1–2.5 and 3.1 being satisfied, let the controller be given by (37) and the NN weights be updated by (48) and (49). Then, there exist compact sets Φ_0 and Ω_0, and positive constants c^*, δ_w^*, δ_v^*, ε^*, k_s^*, k_p^* and l^* such that if

1) all initial states $(\xi(0), \eta(0)) \in \Phi_0$, $(\hat{W}(0), \hat{V}(0)) \in \Omega_0$;
2) $l \ge l^*$, $c \le c^*$, $\delta_w \le \delta_w^*$, $\delta_v \le \delta_v^*$, $k_p \ge k_p^*$, $k_s \ge k_s^*$ and $\varepsilon \le \varepsilon^*$, then the trajectory (ξ, η, u) of the system remains in the compact set $\bar{U}$ and the tracking error converges to a neighborhood of the origin which depends on (δ_w^*, δ_v^*, ε^*, k_s^*, k_p^*).

Proof: The proof contains two steps. First, we assume that $(\xi, \eta, u) \in \bar{U}$ holds for all time so that the transformation from system (1) to the normal form (4) and the NN approximation in Assumption 3.1 are valid. With this assumption, we can prove that the tracking error converges to a small neighborhood of the origin. Later, we will show that for a proper reference signal $y_d(t)$ and suitably chosen design parameters, (ξ, η, u) do remain in the compact set $\bar{U}$ if the system starts from a bounded initial set.

Step 1: Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2}\left[b_{u_\lambda}^{-1}e_s^2 + \tilde{W}^T\Gamma_w^{-1}\tilde{W} + tr\left\{\tilde{V}^T\Gamma_v^{-1}\tilde{V}\right\}\right]. \quad (50)$$

Differentiating (50) along (47)–(49), we have

$$\begin{aligned}\dot{V}_1 =& e_s\left[\tilde{W}^T\left(\hat{S}-\hat{S}'\hat{V}^T\bar{z}\right)+\hat{W}^T\hat{S}'\tilde{V}^T\bar{z}+u_b+d_u-\varepsilon_u(z)\right]\\ &-\frac{b_1(x)}{\varepsilon b_{u_\lambda}}e_s^2+\frac{1}{2}\frac{d\left(b_{u_\lambda}^{-1}\right)}{dt}e_s^2+\tilde{W}^T\Gamma_w^{-1}\dot{\hat{W}}+tr\left\{\tilde{V}^T\Gamma_v^{-1}\dot{\hat{V}}\right\}\\ =& -\left[\frac{k_p}{\varepsilon}\left(\left\|\bar{z}\hat{W}^T\hat{S}'\right\|_F^2+\left\|\hat{S}'\hat{V}^T\bar{z}\right\|^2+1\right)+k_s|e_s|\right]e_s^2\\ &-\frac{b_1(x)}{\varepsilon b_{u_\lambda}}e_s^2+\frac{1}{2}\frac{d\left(b_{u_\lambda}^{-1}\right)}{dt}e_s^2+\left[d_u-\varepsilon_u(z)\right]e_s\\ &-\delta_w(1+|e_s|)\tilde{W}^T\hat{W}-\delta_v(1+|e_s|)tr\left\{\tilde{V}^T\hat{V}\right\}.\end{aligned}$$

Noticing Assumption 2.3, we obtain

$$-\left(\frac{b_1(x)}{\varepsilon b_{u_\lambda}}\right)e_s^2+\left(\frac{1}{2}\right)\left(\frac{d\left(b_{u_\lambda}^{-1}\right)}{dt}\right)e_s^2 = \left(-\left(\frac{b_1(x)}{\varepsilon}\right)+\left(\frac{\dot{b}_{u_\lambda}}{2b_{u_\lambda}}\right)\right)\left(\frac{e_s^2}{b_{u_\lambda}}\right)\le 0.$$

Considering (42), Lemma 3.2 and the following inequalities:

$$\begin{aligned}2\tilde{W}^T\hat{W} &= \left\|\tilde{W}\right\|^2+\left\|\hat{W}\right\|^2-\|W^*\|^2\\ &\ge \left\|\tilde{W}\right\|^2-\|W^*\|^2 \end{aligned}\tag{51}$$

$$\begin{aligned}2tr\left\{\tilde{V}^T\hat{V}\right\} &= \left\|\tilde{V}\right\|_F^2+\left\|\hat{V}\right\|_F^2-\|V^*\|_F^2\\ &\ge \left\|\tilde{V}\right\|_F^2-\|V^*\|_F^2 \end{aligned}\tag{52}$$

we obtain

$$\begin{aligned}\dot{V}_1 \le &-\left[\frac{k_p}{\varepsilon}\left(\left\|\bar{z}\hat{W}^T\hat{S}'\right\|_F^2+\left\|\hat{S}'\hat{V}^T\bar{z}\right\|^2+1\right)+k_s|e_s|\right]e_s^2\\ &-\frac{\delta_w}{2}(|e_s|+1)\left(\left\|\tilde{W}\right\|^2-\|W^*\|^2\right)\\ &-\frac{\delta_v}{2}(|e_s|+1)\left(\left\|\tilde{V}\right\|_F^2-\|V^*\|_F^2\right)+|e_s|\\ &\cdot\left(v_m\left\|\bar{z}\hat{W}^T\hat{S}'\right\|_F+w_m\left\|\hat{S}'\hat{V}^T\bar{z}\right\|+w_m+\varepsilon_l\right)\\ \le &-\frac{k_p}{\varepsilon}\left\|\bar{z}\hat{W}^T\hat{S}'\right\|_F^2{e_s}^2+|e_s|v_m\left\|\bar{z}\hat{W}^T\hat{S}'\right\|_F\\ &-\frac{k_p}{\varepsilon}\left\|\hat{S}'\hat{V}^T\bar{z}\right\|^2{e_s}^2+|e_s|w_m\left\|\hat{S}'\hat{V}^T\bar{z}\right\|\\ &+|e_s|(w_m+\varepsilon_l)-k_s|e_s|^3-\frac{k_p}{\varepsilon}{e_s}^2\\ &+\frac{\delta_w}{2}\|W^*\|^2+\frac{\delta_v}{2}\|V^*\|_F^2-\frac{\delta_w}{2}\left\|\tilde{W}\right\|^2\\ &-\frac{\delta_w}{2}|e_s|\left\|\tilde{W}\right\|^2+\frac{\delta_w}{2}|e_s|\|W^*\|^2\\ &-\frac{\delta_v}{2}\left\|\tilde{V}\right\|_F^2-\frac{\delta_v}{2}|e_s|\left\|\tilde{V}\right\|_F^2+\frac{\delta_v}{2}|e_s|\|V^*\|_F^2. \end{aligned}\tag{53}$$

Further, noticing the following inequalities:

$$|e_s|v_m\left\|\bar{z}\hat{W}^T\hat{S}'\right\|_F \le \frac{k_p}{\varepsilon}\left\|\bar{z}\hat{W}^T\hat{S}'\right\|_F^2{e_s}^2+\frac{\varepsilon}{4k_p}{v_m}^2$$

$$|e_s|w_m\left\|\hat{S}'\hat{V}^T\bar{z}\right\| \le \frac{k_p}{\varepsilon}\left\|\hat{S}'\hat{V}^T\bar{z}\right\|^2{e_s}^2+\frac{\varepsilon}{4k_p}{w_m}^2$$

we have

$$\begin{aligned}\dot{V}_1 \le &-k_s|e_s|^3-\frac{k_p}{\varepsilon}e_s^2-\frac{\delta_w}{2}\left\|\tilde{W}\right\|^2-\frac{\delta_v}{2}\left\|\tilde{V}\right\|_F^2\\ &-\frac{\delta_w}{2}|e_s|\left\|\tilde{W}\right\|^2-\frac{\delta_v}{2}|e_s|\left\|\tilde{V}\right\|_F^2+\beta_1|e_s|+\beta_2\\ \le &-k_s|e_s|^3-\frac{k_p}{\varepsilon}e_s^2-\frac{\delta_w}{2}\left\|\tilde{W}\right\|^2-\frac{\delta_v}{2}\left\|\tilde{V}\right\|_F^2\\ &+\beta_1|e_s|+\beta_2 \end{aligned}\tag{54}$$

where

$$\beta_1 = w_m+\varepsilon_l+\frac{\delta_w}{2}{w_m}^2+\frac{\delta_v}{2}{v_m}^2 \tag{55}$$

$$\beta_2 = \frac{\delta_w}{2}{w_m}^2+\frac{\delta_v}{2}{v_m}^2+\frac{\varepsilon}{4k_p}{w_m}^2+\frac{\varepsilon}{4k_p}{v_m}^2. \tag{56}$$

Considering $|e_s|\beta_1 \le k_p/\varepsilon e_s^2+(\varepsilon/4k_p){\beta_1}^2$, inequality (54) can be further written as

$$\dot{V}_1 \le -k_s|e_s|^3-\frac{\delta_w}{2}\left\|\tilde{W}\right\|^2-\frac{\delta_v}{2}\left\|\tilde{V}\right\|_F^2+\beta \tag{57}$$

where constant

$$\beta = \frac{\varepsilon}{4k_p}{\beta_1}^2+\beta_2, \tag{58}$$

Now define

$$\Theta_e = \left\{e_s \,\middle|\, |e_s| \le \sqrt[3]{\frac{\beta}{k_s}}\right\} \tag{59}$$

$$\Theta_w = \left\{\left(\tilde{W},\tilde{V}\right) \,\middle|\, \left\|\tilde{W}\right\| \le \sqrt{\frac{2\beta}{\delta_w}}, \left\|\tilde{V}\right\|_F \le \sqrt{\frac{2\beta}{\delta_v}}\right\} \tag{60}$$

$$G_\beta = \left\{\left(e_s,\tilde{W},\tilde{V}\right) \,\middle|\, k_s|e_s|^3+\frac{\delta_w}{2}\left\|\tilde{W}\right\|^2+\frac{\delta_v}{2}\left\|\tilde{V}\right\|_F^2 \le \beta\right\}. \tag{61}$$

Since β, δ_w, δ_v, ε, and k_s, k_p are positive constants, we know that Θ_e, Θ_w and G_β are compact sets. Equation (57) shows that $\dot{V}_1 \le 0$ once the errors are outside the compact set G_β in (61). According to the standard Lyapunov theorem [32], we conclude that e_s, $\tilde{W}$, and $\tilde{V}$ are bounded. From (57) and (59), it can be seen that $\dot{V}_1$ is strictly negative as long as e_s is outside the compact set Θ_e. Therefore, there exists a constant T_1 such that for $t > T_1$, the filtered tracking error e_s converges to Θ_e, that is to say, $e_s \le \beta_{e_s}$ with $\beta_{e_s}(\delta_w,\delta_v,\varepsilon,k_p,k_s) = \sqrt[3]{\beta/k_s}$. Using Lemma 2.3, the internal dynamic η will converge to

$$\Theta_\eta := \left\{\eta(t) \,\middle|\, \|\eta\| \le L_a e_{s_{\max}}(t)+L_b\|\xi_d\|+L_c\right\} \quad \forall t > T_0. \tag{62}$$

Because $e_s(t)$ converges to β_{e_s}, $e_{s_{\max}}(t)$ is bounded as well. Consequently, $\|\eta\|$ is bounded.

Noting (9) and $\|e^{At}\| \le k_0 e^{-\lambda_0 t}$ with constants $k_0 > 0$ and $\lambda_0 > 0$. Since $|e_s| \le \beta_{e_s}, \forall t > T_1$, from (12), we know that

$$\|\zeta(t)\| \le k_0 e^{-\lambda_0 t}\left(\|\zeta(0)\|+\frac{e^{\lambda_0 T_1}}{\lambda_0}e_{s_{\max}}(T_1)\right)+\frac{k_0}{\lambda_0}\beta_{e_s} \quad \forall t > T_1. \tag{63}$$

Since $e_{s_{\max}}(T_1)$ is bounded, we know that $k_0 e^{-\lambda_0 t}(\|\zeta(0)\|+(e^{\lambda_0 T_1}/\lambda_0)e_{s_{\max}}(T_1))$ decays exponentially. Inequality (63) implies that the tracking error $\tilde{\xi}_1 = y - y_d$ will converge to a neighborhood of the origin which depends on $(\delta_w,\delta_v,\varepsilon,k_p,k_s)$.

In summary, under the assumption of $(\xi,\eta,u) \in \bar{U}$, there exists a constant $T > T_1$ such that 1) for all $t > T$, the tracking error $y - y_d$ converges to a neighborhood of the origin which depends on $(\delta_w,\delta_v,\varepsilon,k_p,k_s)$; 2) the internal dynamics η converges to Θ_η for all $t > T_0$; and 3) the parameter estimate errors $\tilde{W}$ and $\tilde{V}$ are bounded by Θ_w if $(\tilde{W}(0),\tilde{V}(0)) \in \Theta_w$.

Step 2: To complete the proof, we need to show that for a proper choice of the tracking signal $y_d(t)$ and control parameters, the trajectory ξ do remain in the compact set Φ_ξ. From $e_s = \left[\Lambda^T\, 1\right]\tilde{\xi}$ and $\tilde{\xi} = \left[\zeta^T\, \tilde{\xi}_\rho\right]^T$, we can see that $\tilde{\xi}_\rho = e_s - \Lambda^T\zeta$. Therefore

$$\left\|\tilde{\xi}(t)\right\| \le \|\zeta(t)\|+\left|\tilde{\xi}_\rho(t)\right| \le (1+\|\Lambda\|)\|\zeta(t)\|+|e_s(t).|$$

It follows from (63) and the fact that e_s will converge to β_{e_s}, we know that $\|\tilde{\xi}(t)\| \leq k_a\|\zeta(0)\| + k_b\beta_{e_s} + k_c$, $\forall t \geq T_1$, with $k_a = k_0(1+\|\Lambda\|)$, $k_b = (k_a/\lambda_0) + 1$ and $k_c = k_a(e^{\lambda_0 T_1}/\lambda_0)e_{s_{\max}}(T_1)$. Hence

$$\begin{aligned}\|\xi(t)\| &\leq \left\|\tilde{\xi}(t)\right\| + \|\xi_d(t)\| \\ &\leq k_a\|\zeta(0)\| + k_b\beta_{e_s}(\delta_w,\delta_v,\varepsilon,k_p,k_s) + k_c + c,\ \forall t \geq T_1. \end{aligned} \quad (64)$$

We now provide the conditions which guarantees $\xi \in \Phi_\xi$, $\forall t \geq 0$. Define the compact set

$$\Phi_0 := \left\{\xi(0) \;\middle|\; \{\xi \mid \|\xi(t)\| < k_a\|\zeta(0)\|\} \subset \Phi_\xi, |e_s(0)| < \beta_{e_s}\right\} \quad (65)$$

the positive constant

$$c^* := \sup_{c\in R^+}\left\{c \;\middle|\; \left\{\xi \mid \|\xi\| < k_a\|\zeta(0)\| + k_c + c, \xi(0) \in \Phi_0\right\} \subset \Phi_\xi\right\} \quad (66)$$

the positive constants shown in (67)–(71) at the bottom of the page. In summary, for all initial state $\xi(0) \in \Phi_0$, the desired signal $\|\bar{\xi}_d\| \leq c \leq c^*$, if control parameters δ_w, δ_v, k_p, k_s and ε are chosen such that $\delta_w \leq \delta_w^*$, $\delta_v \leq \delta_v^*$, $k_p \geq k_p^*$, $k_s \geq k_s^*$ and $\varepsilon \leq \varepsilon^*$, then the system state ξ will stay in Φ_ξ for all time. Furthermore, because the NN weights have been proven bounded for any bounded $\hat{W}(0)$ and $\hat{V}(0)$ (see Lemma 3.3), we conclude that all signals of the closed-loop system are bounded. This completes the proof. Q.E.D.

Remark 3.3: Compared with the existing exact linearization techniques and NN control methods, the proposed robust adaptive NN controller clearly has some intrinsic advantages. For example, there is no need to search for an explicit controller to cancel the nonlinearities of the system exactly. In fact, even though the functions $f(x,u)$ and $h(x)$ in system (1) are known, it is in general not possible to get an explicit controller for feedback linearization. In addition, the requirements of an off-line training phase and the persistent excitation condition are not needed any more.

Remark 3.4: It is shown in (55) and (56) that smaller β_1 and β_2 might be obtained by choosing a smaller δ_w and δ_v, which may lead to a smaller tracking error. Nevertheless, from (60) it can be seen that smaller δ_w and δ_v may cause larger NN weight errors. In this case the control signal, u, may be increased and out of region U in which Assumptions 2.1, 2.2, and 2.5 hold. On the other hand, if δ_w and δ_v are chosen to be very large, so are β_1 and β_2, which will lead to a large tracking error will happen. Hence, the parameter δ_w and δ_v should be adjusted carefully in practical implementations.

IV. Output Feedback Control

In Section III, the system states and the time derivatives of the outputs $\xi_2, \xi_3, \ldots, \xi_\rho$ are supposed to be available for feedback. This limits the application of the approach, because, in many practical systems, only output y is measurable. In this section, adaptive NN output feedback control is investigated for

$$\delta_w^* := \sup_{\delta_w\in R^+}\left\{\delta_w \middle| \left\{\xi \middle| \|\xi\| < k_a\|\zeta(0)\| + k_b\beta_{e_s}(\delta_w,0,0,0,0) + k_c + c, \xi(0) \in \Phi_0, c \leq c^*\right\} \subset \Phi_\xi\right\} \quad (67)$$

$$\delta_v^* := \sup_{\delta_v\in R^+}\left\{\delta_v \;\middle|\; \left\{\xi \middle| \|\xi\| < k_a\|\zeta(0)\| + k_b\beta_{e_s}(\delta_w,\delta_v,0,0,0) + k_c + c, \xi(0) \in \Phi_0, c \leq c^*, \delta_w \leq \delta_w^*\right\} \subset \Phi_\xi\right\} \quad (68)$$

$$k_p^* := \inf_{k_p\in R^+}\left\{k_p \;\middle|\; \left\{\xi \middle| \|\xi\| < k_a\|\zeta(0)\| + k_b\beta_{e_s}(\delta_w,\delta_v,0,k_p,0) + k_c + c, \xi(0) \in \Phi_0, c \leq c^*, \delta_w \leq \delta_w^*, \delta_v \leq \delta_v^*\right\} \subset \Phi_\xi\right\} \quad (69)$$

$$k_s^* := \inf_{k_s\in R^+}\left\{k_s \;\middle|\; \left\{\xi \middle| \|\xi\| < k_a\|\zeta(0)\| + k_b\beta_{e_s}(\delta_w,\delta_v,0,k_p,k_s) + k_c + c, \xi(0) \in \Phi_0, c \leq c^*, \delta_w \leq \delta_w^*, \delta_v \leq \delta_v^*, k_p \geq k_p^*\right\} \subset \Phi_\xi\right\} \quad (70)$$

and

$$\varepsilon^* := \sup_{\varepsilon\in R^+}\left\{\varepsilon \middle| \left\{\xi \middle| \|\xi\| \leq k_a\|\zeta(0)\| + k_b\beta_{e_s}(\delta_w,\delta_v,\varepsilon,k_p,k_s) + k_c + c, \xi(0) \in \Phi_0, c \leq c^*, \delta_w \leq \delta_w^*, \delta_v \leq \delta_v^*, k_p \geq k_p^*, k_s \geq k_s^*\right\} \subset \Phi_\xi\right\}. \quad (71)$$

nonaffine nonlinear systems with internal dynamics by using a high-gain observer to reconstruct the system states.

A. High-Gain Observer

Since only the output $y = \xi_1$ is measurable and the rest of the output derivatives are not available, we need to estimate $\xi_2, \xi_3, \ldots, \xi_\rho$ to implement the output feedback control. In the following lemma, the high-gain observer used in [27] is presented, which will be used to estimate the output derivatives of system (4).

Lemma 4.1: Suppose the system output $y(t)$ and its first n derivatives are bounded, so that $|y^{(k)}| < Y_k$ with positive constants Y_k. Consider the following linear system:

$$\begin{cases} \epsilon\dot{\pi}_i = \pi_{i+1} \; i = 1, \ldots, n-1 \\ \epsilon\dot{\pi}_n = -\bar{\lambda}_1\pi_n - \bar{\lambda}_2\pi_{n-1} - \cdots - \bar{\lambda}_{n-1}\pi_2 - \pi_1 + y(t) \end{cases} \tag{72}$$

where ϵ is any small positive constant and the parameters $\bar{\lambda}_1$ to $\bar{\lambda}_{n-1}$ are chosen such that the polynomial $s^n + \bar{\lambda}_1 s^{n-1} + \cdots \bar{\lambda}_{n-1}s + 1$ is Hurwitz. Then, we have the following.

1)
$$\frac{\pi_{k+1}}{\epsilon^k} - y^{(k)} = -\epsilon\psi^{(k+1)} \quad k = 1, \ldots, n-1$$
where $\psi = \pi_n + \bar{\lambda}_1\pi_{n-1} +, \ldots, +\bar{\lambda}_{n-1}\pi_1$ with $\psi^{(k)}$ denoting the kth derivative of ψ.

2) There exist positive constants t^* and h_k which only depending on Y_k, ϵ and $\bar{\lambda}_i$, $i = 1, 2, \ldots, n-1$ such that for all $t > t^*$ we have $|\psi^{(k)}| \le h_k$, $k = 2, 3, \ldots, n$.

Proof: The proof can be found in [27]. For completeness, it is given below.

1) From the last equation in (72), we have
$$\frac{\pi_2}{\epsilon} - \dot{y} = \frac{\pi_2}{\epsilon} - \epsilon\ddot{\pi}_n - \bar{\lambda}_1\dot{\pi}_n - \bar{\lambda}_2\dot{\pi}_{n-1} - \cdots - \bar{\lambda}_{n-1}\dot{\pi}_2 - \dot{\pi}_1.$$
Using (72) and the above equation yields
$$\begin{aligned} \frac{\pi_2}{\epsilon} - \dot{y} &= -\epsilon\left(\ddot{\pi}_n + \bar{\lambda}_1\ddot{\pi}_{n-1} + \bar{\lambda}_2\ddot{\pi}_{n-2} + \cdots + \bar{\lambda}_{n-1}\ddot{\pi}_1\right) \\ &= -\epsilon\ddot{\psi}. \end{aligned}$$
Differentiating it and utilizing (72), Item 1) follows.

2) The derivatives of the vector $\pi = [\,\pi_1 \quad \pi_2 \quad \ldots \quad \pi_n\,]^T$ may be computed as follows:
$$\begin{aligned} \pi^{(j)}(t) =& \frac{1}{\epsilon^j}A^j e^{At/\epsilon}\Big[\pi(0) + A^{-1}by(0) + \epsilon A^{-2}b\dot{y}(0) \\ &+ \cdots + \epsilon^{j-1}A^{-j}by^{(j-1)}(0)\Big] \\ &+ \frac{1}{\epsilon}e^{At/\epsilon}\int_0^t e^{A\tau/\epsilon}by^{(j)}(\tau)d\tau, j = 1, 2, \ldots, n \end{aligned} \tag{73}$$
where A is the matrix corresponding to the homogeneous part of (72) and independent of ϵ, and $b = [0 \quad 0 \quad \cdots \quad 1]^T$. Since ξ belongs to the compact set Φ_ξ, and u is bounded, there exist constants $Y_j > 0$ such that $|y^{(j)}| \le Y_j$. Then, for any $\delta > 0$, we may find a constant $t^* > 0$ such that for all $t > t^*$ the first term
$$\begin{aligned} \frac{1}{\epsilon^j}A^j e^{(At/\epsilon)}\Big[&\pi(0) + A^{-1}by(0) \\ &+ \epsilon A^{-2}b\dot{y}(0) + \cdots + \epsilon^{j-1}A^{-j}by^{(j-1)}(0)\Big] \end{aligned}$$
in (73) is bounded by δY_j for each j. Further, since $|y^{(j)}| < Y_j$, there exist constants D_j, which is independent of ϵ, such that the second term $1/\epsilon e^{(At/\epsilon)}\int_0^t e^{(A\tau/\epsilon)}by^{(j)}(\tau)d\tau$ in (73) is bounded by D_jY_j for each j. Fixing an arbitrarily small δ^*, then for $t > t^*$, we have $|\psi^{(j)}| \le h_j$ where $h_j = B(D_j + \delta^*)Y_j$ with B the norm of the vector $[1 \quad \bar{\lambda}_1 \quad \cdots \quad \bar{\lambda}_{n-1}]$. As D_j, δ^*, Y_j, and B are independent of ϵ, the proof is completed. Q.E.D

B. Adaptive Neural Control by Output Feedback

Having the observer (72), we define

$$\begin{aligned} \hat{\xi} &= \left[\xi_1, \frac{\pi_2}{\epsilon}, \frac{\pi_3}{\epsilon^2}, \ldots, \frac{\pi_\rho}{\epsilon^{\rho-1}}\right]^T \\ \hat{\tilde{\xi}} &= \hat{\xi} - \xi_d = \Bigg[\xi_1 - y_d, \frac{\pi_2}{\epsilon} - \dot{y}_d, \\ &\quad \frac{\pi_3}{\epsilon^2} - \ddot{y}_d, \ldots, \frac{\pi_\rho}{\epsilon^{\rho-1}} - y_d^{(\rho-1)}\Bigg]^T \\ \hat{e}_s &= \left[\Lambda^T \; 1\right]\hat{\tilde{\xi}} = e_s + \epsilon e_0 \\ \hat{\nu}_1 &= -y_d^{(\rho)} + \left[0 \, \Lambda^T\right]\hat{\tilde{\xi}} \\ \hat{\bar{z}} &= \left[\hat{\xi}^T, \eta^T, \hat{e}_s, \hat{\nu}_1, 1\right]^T = \bar{z} + \epsilon z_0 \end{aligned}$$

where

$$\begin{aligned} \Psi &= \left[-\ddot{\psi}, -\psi^{(3)}, \ldots, -\psi^{(\rho)}\right]^T \\ e_0 &= [\lambda_2, \lambda_3, \ldots, \lambda_{\rho-1}, 1]\,\Psi \\ z_0 &= \left[0, \Psi^T, 0, e_0, \Lambda^T\Psi, 0\right]^T \end{aligned}$$

Lemma 4.1 shows that $|\psi^{(k)}|$ are bounded by the constants h_k, hence Ψ, e_0 and z_0 are all bounded. Let $\hat{W}$ and $\hat{V}$ be the estimates of W^* and V^*, respectively. The following lemma presents the property of MNN's when the input vector $\bar{z}$ is replaced by the estimation $\hat{\bar{z}}$.

Lemma 4.2: The NN estimation error can be expressed as

$$\begin{aligned} &\hat{W}^T S\left(\hat{V}^T\hat{\bar{z}}\right) - W^{*T}S\left(V^{*T}\bar{z}\right) \\ &\quad = \tilde{W}^T\left(\hat{S}_o - \hat{S}_o'\hat{V}^T\hat{\bar{z}}\right) + \hat{W}^T\hat{S}_o'\tilde{V}^T\hat{\bar{z}} + d_u \end{aligned} \tag{74}$$

where $\tilde{W} = \hat{W} - W^*$, $\tilde{V} = \hat{V} - V^*$, $\hat{S}_o = S(\hat{V}^T\hat{\bar{z}})$, $\hat{S}_o' = diag\{\hat{s}_{o1}', \hat{s}_{o2}', \ldots, \hat{s}_{ol}'\}$ with $\hat{s}_{oi}' = s'(\hat{v}_i^T\hat{\bar{z}}) = d[s(z_a)]/dz_a|_{z_a = \hat{v}_i^T\hat{\bar{z}}}$, $i = 1, 2, \ldots, l$, and the residual term d_u is bounded by

$$|d_u| \le v_m\left\|\hat{\bar{z}}\hat{W}^T\hat{S}_o'\right\|_F + w_m\left(\left\|\hat{S}_o\right\| + \left\|\hat{S}_o'\hat{V}^T\hat{\bar{z}}\right\|\right). \tag{75}$$

Proof: The proof can be obtained by following the similar procedure in Lemma 3.2. It is omitted here.

C. Controller Structure and Stability Analysis

The output feedback controller is designed as follows:

$$u = u_{nn} + u_k + u_b \tag{76}$$

where

$$u_{nn} = \hat{W}^T S\left(\hat{V}^T \hat{\bar{z}}\right) \tag{77}$$

$$u_b = -\left[\frac{k_p}{\varepsilon}\left(\left\|\hat{\bar{z}}\hat{W}^T\hat{S}_o'\right\|_F^2 + \left\|\hat{S}_o\right\|^2 + \left\|\hat{S}_o'\hat{V}^T\hat{\bar{z}}\right\|^2 + 1\right) + k_s\left(|u_{nn}| + 1\right)\right]\hat{e}_s \tag{78}$$

with ε, k_p, $k_s > 0$ being the constant design parameters. The above controller contains three parts for different purposes. The first part u_{nn} is introduced to approximate the IDFC input u^* for achieving adaptive tracking control. The second part u_k is a priori control term based on a nominal model or past control experience to improve the control performance. If no knowledge for the plants is available, u_k can be simply set to zero. The third part u_b is a bounding control term, which is applied for limiting the upper bounds of the system states such that the NN approximation (34) holds on the compact set Ω_z.

The MNN weight updating laws are taken as

$$\dot{\hat{W}} = -\Gamma_w\left[\left(\hat{S}_o - \hat{S}_o'\hat{V}^T\hat{\bar{z}}\right)\hat{e}_s + \delta_w\left(1 + |\hat{e}_s|\right)\hat{W}\right] \tag{79}$$

$$\dot{\hat{V}} = -\Gamma_v\left[\hat{\bar{z}}\hat{W}^T\hat{S}_o'\hat{e}_s + \delta_v\left(1 + |\hat{e}_s|\right)\hat{V}\right] \tag{80}$$

where $\Gamma_w = \Gamma_w^T > 0$, $\Gamma_v = \Gamma_v^T > 0$, $\delta_w > 0$ and $\delta_v > 0$ are constant design parameters. The above learning algorithms have a nice property as stated below.

Lemma 4.3: The updated learning algorithms (79) and (80) guarantee that $\hat{W}(t)$, $\hat{V}(t) \in L_\infty$ for bounded initial weights $\hat{W}(0)$ and $\hat{V}(0)$.

Proof: The proof can be obtained by following the similar procedure in Lemma 3.3. It is omitted here.

Considering (43)–(46), the error system can be written as

$$\dot{e}_s = -\frac{b_1(x)}{\varepsilon}e_s + b_{u_\lambda}\left(u - u^*\right). \tag{81}$$

Applying (34), (74), and (76), we obtain

$$b_{u_\lambda}^{-1}\dot{e}_s = -\frac{b_1(x)}{\varepsilon}b_{u_\lambda}^{-1}e_s + u_b + \tilde{W}^T\left(\hat{S}_o - \hat{S}_o'\hat{V}^T\hat{\bar{z}}\right) + \hat{W}^T\hat{S}_o'\tilde{V}^T\hat{\bar{z}} + d_u - \varepsilon_u(z),\ \forall z \in \Omega_z. \tag{82}$$

Based on (82), closed-loop stability results are summarized in Theorem 4.1.

Theorem 4.1: For system (4) with Assumptions 2.1, 2.2, 2.3, 2.4 and 2.5 being satisfied, high-gain observer (72), controller (76) and adaptive laws (79) and (80), there exist a compact set $\Phi_0 \subset \Phi_x$, and positive constants l^*, c^*, δ_w^*, δ_v^*, k_p^*, ε^* and ϵ^*, such that for any bounded $\hat{W}(0)$ and $\hat{V}(0)$, if

1) the initial state $(\xi(0), \eta(0)) \in \Phi_0$;
2) the observer (72) is turned on at time t^* in advance;
3) $c \le c^*$, $k_s \ge k_s^*$ and $\epsilon \le \epsilon^*$;

then all the signals in the closed-loop system are bounded, the system state $(\xi, \eta) \in \Phi_x, \forall t \ge 0$, and the tracking error converges to a neighborhood of the origin which depends on (k_s, ϵ).

Proof: The proof contains two steps. We first assume that $(\xi, \eta) \in \Phi_x$ holds for all time, which ensures that NN approximation (34), Assumptions 2.1, 2.2, and 2.3 are valid. In this case, we prove the tracking error converging to an (ε, ϵ)-neighborhood of the origin. Later, for a proper choice of the reference signal $y_d(t)$ and controller parameters, we show that (ξ, η) do remain in the compact set Φ_x for all time if the system starts from a bounded initial set.

Step 1: Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2}\left[b_{u_\lambda}^{-1}e_s^2 + \tilde{W}^T\Gamma_w^{-1}\tilde{W} + tr\left\{\tilde{V}^T\Gamma_v^{-1}\tilde{V}\right\}\right]. \tag{83}$$

Differentiating (83) along (82), we have

$$\begin{aligned}\dot{V}_1 = &-\frac{b_1(x)}{\varepsilon}b_{u_\lambda}^{-1}e_s^2 + \tilde{W}^T\left(\hat{S}_o - \hat{S}_o'\hat{V}^T\hat{\bar{z}}\right)e_s \\ &+ \hat{W}^T\hat{S}_o'\tilde{V}^T\hat{\bar{z}}e_s \\ &+ u_b e_s + \left[d_u - \varepsilon_u(z)\right]e_s + \frac{1}{2}\frac{d\left(b_{u_\lambda}^{-1}\right)}{dt}e_s^2 \\ &+ \tilde{W}^T\Gamma_w^{-1}\dot{\tilde{W}} + tr\left\{\tilde{V}^T\Gamma_v^{-1}\dot{\tilde{V}}\right\}.\end{aligned}$$

Noting $\hat{W}^T\hat{S}_o'\tilde{V}^T\hat{\bar{z}} = tr\{\tilde{V}^T\hat{\bar{z}}\hat{W}^T\hat{S}_o'\}$, $\hat{e}_s = e_s + \epsilon e_0$, (79), (80), we obtain

$$\begin{aligned}\dot{V}_1 = &-\left[\frac{b_1(x)}{\varepsilon}b_{u_\lambda}^{-1} + \frac{\dot{b}_{u_\lambda}}{2b_{u_\lambda}^2}\right]e_s^2 + u_b e_s + \left[d_u - \varepsilon_u(z)\right]e_s \\ &-\left(1 + |\hat{e}_s|\right)\left(\delta_w\tilde{W}^T\hat{W} + \delta_v tr\left\{\tilde{V}^T\hat{V}\right\}\right) \\ &-\epsilon e_0\left[\hat{W}^T\hat{S}_o'\tilde{V}^T\hat{\bar{z}} + \tilde{W}^T\left(\hat{S}_o - \hat{S}_o'\hat{V}^T\hat{\bar{z}}\right)\right].\end{aligned} \tag{84}$$

Since $b_u > 0$ and $|\dot{b}_u/(2b_u)| \le b_1(x)/\varepsilon$ (Assumption 2.3), we know that $|\dot{b}_{u_\lambda}|/(2b_{u_\lambda}^2) \le b_1(x)/\varepsilon b_{u_\lambda}^{-1}$. Using $2\tilde{W}^T\hat{W} \ge \|\tilde{W}\|^2 - \|W^*\|^2$ and $2tr\{\tilde{V}^T\hat{V}\} \ge \|\tilde{V}\|_F^2 - \|V^*\|_F^2$, we have

$$\begin{aligned}\dot{V}_1 \le\ & u_b e_s - \frac{\delta_w}{2}\left(1 + |\hat{e}_s|\right)\left(\left\|\tilde{W}\right\|^2 - \|W^*\|^2\right) \\ &-\frac{\delta_v}{2}\left(1 + |\hat{e}_s|\right)\left(\left\|\tilde{V}\right\|_F^2 - \|V^*\|_F^2\right)|e_s|\left(|d_u| + |\varepsilon_l|\right) \\ &+\epsilon|e_0|\left[\left\|\tilde{V}\right\|_F\left\|\hat{\bar{z}}\hat{W}^T\hat{S}_o'\right\|_F\right. \\ &\left.+\left\|\tilde{W}\right\|\left(\left\|\hat{S}_o\right\| + \left\|\hat{S}_o'\hat{V}^T\hat{\bar{z}}\right\|\right)\right].\end{aligned}$$

Considering $\|W^*\| \le w_m$, $\|V^*\|_F \le v_m$, (75) and (78), and the following inequalities:

$$\epsilon|e_0|\left\|\tilde{V}\right\|_F\left\|\hat{\bar{z}}\hat{W}^T\hat{S}_o'\right\|_F \le \frac{\delta_v}{2}\left\|\tilde{V}\right\|_F^2 + \frac{\epsilon^2}{2\delta_v}e_0^2\left\|\hat{\bar{z}}\hat{W}^T\hat{S}_o'\right\|_F^2$$

$$\epsilon|e_0|\left\|\tilde{W}\right\|\left\|\hat{S}_o\right\| \le \frac{\delta_w}{4}\left\|\tilde{W}\right\|^2 + \frac{\epsilon^2}{\delta_w}e_0^2\left\|\hat{S}_o\right\|^2$$

$$\epsilon|e_0|\left\|\tilde{W}\right\|\left\|\hat{S}_o'\hat{V}^T\hat{\bar{z}}\right\| \le \frac{\delta_w}{4}\left\|\tilde{W}\right\|^2 + \frac{\epsilon^2}{\delta_w}e_0^2\left\|\hat{S}_o'\hat{V}^T\hat{\bar{z}}\right\|^2$$

we obtain (85), shown at the bottom of the next page. Since

$$v_m\left\|\hat{\bar{z}}\hat{W}^T\hat{S}_o'\right\|_F|e_s| \le \frac{k_p e_s^2}{2\varepsilon}\left\|\hat{\bar{z}}\hat{W}^T\hat{S}_o'\right\|_F^2 + \frac{\varepsilon v_m^2}{2k_p}$$

$$w_m\left\|\hat{S}_o\right\||e_s| \le \frac{k_p e_s^2}{2\varepsilon}\left\|\hat{S}_o\right\|^2 + \frac{\varepsilon w_m^2}{2k_p}$$

TABLE I
VARIABLES AND PARAMETERS OF THE CSTR SYSTEMS

$t = t'\frac{F_0}{V}$	dimensionless time
$x_1 = \frac{C_A}{C_{A_f}}$	dimensionless composition of reactant A
$x_2 = \frac{C_B}{C_{A_f}}$	dimensionless composition of reactant B
$x_3 = \frac{C_C}{C_{A_f}}$	dimensionless composition of product C
$u = \frac{N_{BF}}{FC_{A_f}}$	dimensionless control input
$c_1 = \frac{k_1 V}{F}$	Damkholer number of the first-order reaction $A \to B$
$c_2 = \frac{k_2 V C_{A_f}}{F}$	Damkholer number of the first-order reaction $A \leftarrow B$
$c_3 = \frac{k_3 V C_{A_f}}{F}$	Damkholer number of the first-order reaction $B \to C$

$$
\begin{aligned}
w_m \left\| \hat{S}_o' \hat{V}^T \hat{\hat{z}} \right\| |e_s| \le & \frac{k_p e_s^2}{2\varepsilon} \left\| \hat{S}_o' \hat{V}^T \hat{\hat{z}} \right\|^2 + \frac{\varepsilon w_m^2}{2k_p} \\
|\varepsilon_l| \, |e_s| \le & \frac{k_p e_s^2}{4\varepsilon} + \frac{\varepsilon \varepsilon_l^2}{k_p} \\
|\hat{e}_s| \left(\frac{\delta_w}{2} w_m^2 + \frac{\delta_v}{2} v_m^2 \right) \le & \frac{k_p e_s^2}{4\varepsilon} + \frac{\varepsilon}{k_p} \left(\frac{\delta_w}{2} w_m^2 + \frac{\delta_v}{2} v_m^2 \right)^2 \\
& + \epsilon |e_0| \left(\frac{\delta_w}{2} w_m^2 + \frac{\delta_v}{2} v_m^2 \right)
\end{aligned}
$$

(85) can be written as

$$
\begin{aligned}
&\dot{V}_1 \\
&\le -\frac{k_p}{\varepsilon} \left(\frac{e_s^2}{2} + \epsilon e_0 e_s \right) \\
&\cdot \left(\left\| \hat{\hat{z}} \hat{W}^T \hat{S}_o' \right\|_F^2 + \left\| \hat{S}_o \right\|^2 + \left\| \hat{S}_o' \hat{V}^T \hat{\hat{z}} \right\|^2 + 1 \right) \\
&- k_s |e_s| \left(|e_s| - \epsilon |e_0| \right) \left(|u_{nn}| + 1 \right) \\
&+ \frac{1}{2} \left(\delta_w + \frac{2\varepsilon}{k_p} + \epsilon \delta_w |e_0| \right) w_m^2 \\
&+ \frac{1}{2} \left(\delta_v + \frac{\varepsilon}{k_p} + \epsilon \delta_v |e_0| \right) v_m^2 + \frac{\varepsilon \varepsilon_l^2}{k_p} \\
&+ \frac{\varepsilon}{k_p} \left(\frac{\delta_w}{2} w_m^2 + \frac{\delta_v}{2} v_m^2 \right)^2 \\
&+ \epsilon^2 e_0^2 \left(\frac{1}{2\delta_v} \left\| \hat{\hat{z}} \hat{W}^T \hat{S}_o' \right\|_F^2 + \frac{1}{\delta_w} \left\| \hat{S}_o \right\|^2 + \frac{1}{\delta_w} \left\| \hat{S}_o' \hat{V}^T \hat{\hat{z}} \right\|^2 \right).
\end{aligned} \tag{86}
$$

It follows from $-\epsilon e_0 e_s \le e_s^2/4 + \epsilon^2 e_0^2$ that

$$
\begin{aligned}
\dot{V}_1 \le & -\frac{k_p e_s^2}{4\varepsilon} \left(\left\| \hat{\hat{z}} \hat{W}^T \hat{S}_o' \right\|_F^2 + \left\| \hat{S}_o \right\|^2 + \left\| \hat{S}_o' \hat{V}^T \hat{\hat{z}} \right\|^2 + 1 \right) \\
& - k_s |e_s| \left(|e_s| - \epsilon |e_0| \right) \left(|u_{nn}| + 1 \right) \\
& + \frac{1}{2} \left(\delta_w + \frac{2\varepsilon}{k_p} + \epsilon \delta_w |e_0| \right) w_m^2 \\
& + \frac{1}{2} \left(\delta_v + \frac{\varepsilon}{k_p} + \epsilon \delta_v |e_0| \right) v_m^2 + \frac{\varepsilon \varepsilon_l^2}{k_p} \\
& + \frac{\varepsilon}{k_p} \left(\frac{\delta_w}{2} w_m^2 + \frac{\delta_v}{2} v_m^2 \right)^2 \\
& + \epsilon^2 e_0^2 \Bigg[\left(\frac{k_p}{\varepsilon} + \frac{1}{2\delta_v} \right) \left\| \hat{\hat{z}} \hat{W}^T \hat{S}_o' \right\|_F^2 + \left(\frac{k_p}{\varepsilon} + \frac{1}{\delta_w} \right) \\
& \qquad \cdot \left(\left\| \hat{S}_o \right\|^2 + \left\| \hat{S}_o' \hat{V}^T \hat{\hat{z}} \right\|^2 \right) + \frac{k_p}{\varepsilon} \Bigg] \\
= & -\left(\frac{k_p e_s^2}{4\varepsilon} - \beta_0 \right) - \left(\frac{k_p e_s^2}{4\varepsilon} - \beta_1 \right) \left\| \hat{\hat{z}} \hat{W}^T \hat{S}_o' \right\|_F^2 \\
& - \left(\frac{k_p e_s^2}{4\varepsilon} - \beta_2 \right) \left(\left\| \hat{S}_o \right\|^2 + \left\| \hat{S}_o' \hat{V}^T \hat{\hat{z}} \right\|^2 \right) \\
& - k_s |e_s| \left(|e_s| - \epsilon |e_0| \right) \left(|u_{nn}| + 1 \right)
\end{aligned} \tag{87}
$$

where

$$
\begin{aligned}
\beta_0 = & \frac{1}{2} \Bigg[\left(\delta_w + \frac{2\varepsilon}{k_p} + \epsilon \delta_w |e_0| \right) w_m^2 \\
& \qquad + \left(\delta_v + \frac{\varepsilon}{k_p} + \epsilon \delta_v |e_0| \right) v_m^2 \Bigg] \\
& + \frac{\varepsilon \varepsilon_l^2}{k_p} + \frac{\varepsilon}{k_p} \left(\frac{\delta_w}{2} w_m^2 + \frac{\delta_v}{2} v_m^2 \right)^2 + \frac{k_p \epsilon^2 e_0^2}{\varepsilon}
\end{aligned} \tag{88}
$$

$$
\beta_1 = \epsilon^2 e_0^2 \left(\frac{k_p}{\varepsilon} + \frac{1}{2\delta_v} \right), \quad \beta_2 = \epsilon^2 e_0^2 \left(\frac{k_p}{\varepsilon} + \frac{1}{\delta_w} \right). \tag{89}
$$

Now, let

$$
\beta_{e_s}(\varepsilon, \epsilon, \delta_w, \delta_v, k_p) = \max \left\{ 2\sqrt{\frac{\varepsilon \beta_0}{k_p}},\ 2\sqrt{\frac{\varepsilon \beta_1}{k_p}},\ 2\sqrt{\frac{\varepsilon \beta_2}{k_p}},\ \epsilon |e_0| \right\}
$$

and define

$$
\Theta_e := \left\{ e_s \mid |e_s| \le \beta_{e_s} \right\}. \tag{90}
$$

$$
\begin{aligned}
\dot{V}_1 \le & -e_s \left(e_s + \epsilon e_0 \right) \left[\frac{k_p}{\varepsilon} \left(\left\| \hat{\hat{z}} \hat{W}^T \hat{S}_o' \right\|_F^2 + \left\| \hat{S}_o \right\|^2 + \left\| \hat{S}_o' \hat{V}^T \hat{\hat{z}} \right\|^2 + 1 \right) + k_s \left(|u_{nn}| + 1 \right) \right] \\
& + \left(1 + |\hat{e}_s| \right) \left(\frac{\delta_w}{2} w_m^2 + \frac{\delta_v}{2} v_m^2 \right) + |e_s| \left(v_m \left\| \hat{\hat{z}} \hat{W}^T \hat{S}_o' \right\|_F + w_m \left\| \hat{S}_o \right\| + w_m \left\| \hat{S}_o' \hat{V}^T \hat{\hat{z}} \right\| + |\varepsilon_l| \right) \\
& + \epsilon^2 e_0^2 \left(\frac{1}{2\delta_v} \left\| \hat{\hat{z}} \hat{W}^T \hat{S}_o' \right\|_F^2 + \frac{1}{\delta_w} \left\| \hat{S}_o \right\|^2 + \frac{1}{\delta_w} \left\| \hat{S}_o' \hat{V}^T \hat{\hat{z}} \right\|^2 \right).
\end{aligned} \tag{85}
$$

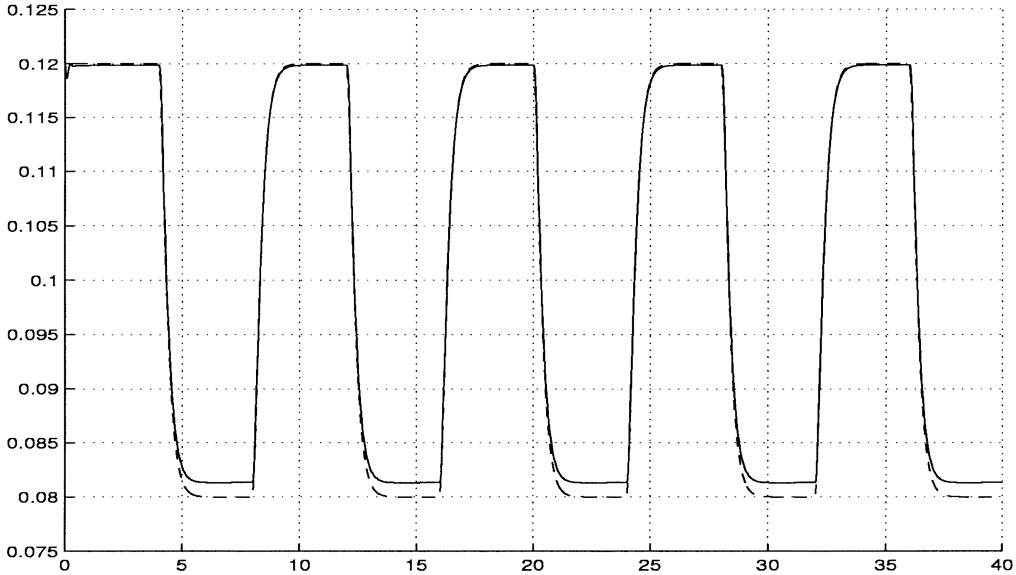

Fig. 2. ξ_1 follows ξ_{1d} ("- -") (state feedback).

Since $\delta_w, \delta_v, v_m, w_m, k_p, k_s, \epsilon$, and ε are positive constants and e_0 and ε_l are bounded, it follows that β_{e_s} is also bounded, which means that Θ_e is a compact set. From (87)–(90), it is shown that $\dot{V}_1$ is strictly negative as long as e_s is outside the set Θ_e. Therefore, the filtered error e_s is bounded, and there exists a constant $T_1 \geq t^*$ such that for $t \geq T_1$, the filtered tracking error e_s converges to Θ_e which is a neighborhood of the origin that depends on $(\varepsilon, \epsilon, \delta_w, \delta_v, k_p)$.

The mapping of $e_s = \left[\Lambda^T \; 1\right] \tilde{\xi}$ can be expressed in state space eqn (9). The solution for ζ can be written as $\zeta(t) = e^{At}\zeta(0) + \int_0^t e^{A(t-\tau)} b e_s d\tau$. It follows that

$$\|\zeta(t)\| \leq k_0 \|\zeta(0)\| e^{-\lambda_0 t} + k_0 \int_0^t e^{-\lambda_0 (t-\tau)} \left|e_s(\tau)\right| d\tau. \quad (91)$$

Because $|e_s(t)| \leq \beta_{e_s}, \forall t \geq T_1$, from (12), we have

$$\|\zeta(t)\| \leq k_0 e^{-\lambda_0 t} \left[\|\zeta(0)\| + \frac{e^{\lambda_0 T_1}}{\lambda_0} e_{s_{\max}}(T_1)\right] + \frac{k_0}{\lambda_0} \beta_{e_s}, \quad \forall t \geq T_1. \quad (92)$$

Since $e_s(t)$ is bounded, we know that $k_0 e^{-\lambda_0 t}\left[\|\zeta(0)\| + e^{\lambda_0 T_1}/\lambda_0 e_{s_{\max}}(T_1)\right]$ decays exponentially. Inequality (92) implies that the tracking error $\tilde{\xi}_1 = y - y_d$ will converge to a neighborhood of the origin which depends on $(\varepsilon, \epsilon, \delta_w, \delta_v, k_p)$.

Step 2: To complete the proof, we need to show that for a proper choice of the tracking signal $y_d(t)$ and control parameters, the trajectory ξ do remain in the compact set Φ_ξ. Considering a positive function $V_b = e_s^2/2$ and controller (76), the time derivative of V_b along (81) is

$$\dot{V}_b = -\frac{b_1(x)}{\varepsilon} e_s^2 + b_{u_\lambda}(u - u^*) e_s. \quad (93)$$

Using (34), (76)–(78), we have

$$\begin{aligned}\dot{V}_b = &-\frac{b_1(x)}{\varepsilon} e_s^2 \\ &+ b_{u_\lambda}\Bigg[-k_s\left(|u_{nn}| + 1\right)(e_s + \epsilon e_0) \\ &\qquad + u_{nn} - W^{*T} S\left(V^{*T}\bar{z}\right) - \varepsilon_u(z)\Bigg] e_s \\ &- \frac{b_{u_\lambda} k_p}{\varepsilon} e_s (e_s + \epsilon e_0) \\ &\cdot \left(\left\|\hat{\bar{z}} \hat{W}^T \hat{S}_o'\right\|_F^2 + \left\|\hat{S}_o\right\|^2 + \left\|\hat{S}_o' \hat{V}^T \hat{\bar{z}}\right\|^2 + 1\right). \end{aligned} \quad (94)$$

Since every element of $S(V^{*T}\bar{z})$ is not larger than one, we know that

$$W^{*T} S\left(V^{*T}\bar{z}\right) \leq \|W^*\| \left\|S\left(V^{*T}\bar{z}\right)\right\| \leq w_m \sqrt{l}. \quad (95)$$

Therefore

$$\begin{aligned}\dot{V}_b \leq &-\frac{b_1(x)}{\varepsilon} e_s^2 - b_{u_\lambda} k_s \left(|u_{nn}| + 1\right) |e_s| \\ &\cdot \left[\frac{9|e_s|}{10} - \frac{|u_{nn}| + w_m\sqrt{l} + |\varepsilon_l|}{k_s\left(|u_{nn}| + 1\right)}\right] \\ &- \frac{b_{u_\lambda} k_p}{\varepsilon} |e_s| \left(|e_s| - \epsilon |e_0|\right) \\ &\cdot \left(\left\|\hat{\bar{z}} \hat{W}^T \hat{S}_o'\right\|_F^2 + \left\|\hat{S}_o\right\|^2 + \left\|\hat{S}_o' \hat{V}^T \hat{\bar{z}}\right\|^2 + 1\right) \\ &- b_{u_\lambda} k_s |e_s| \left(\frac{|e_s|}{10} - \epsilon |e_0|\right) \left(|u_{nn}| + 1\right). \end{aligned}$$

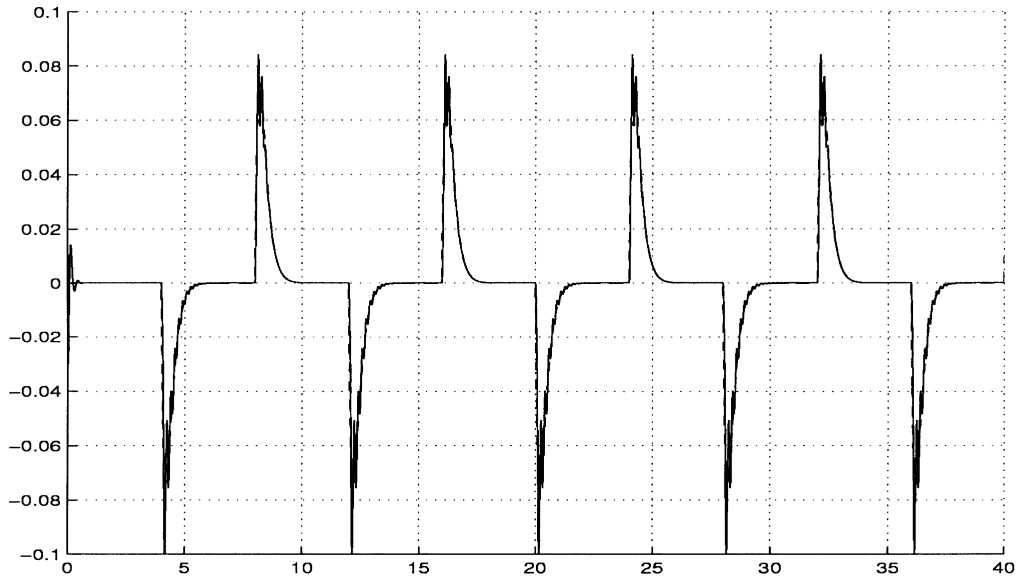

Fig. 3. ξ_2 follows ξ_{2d} ("- -") (state feedback).

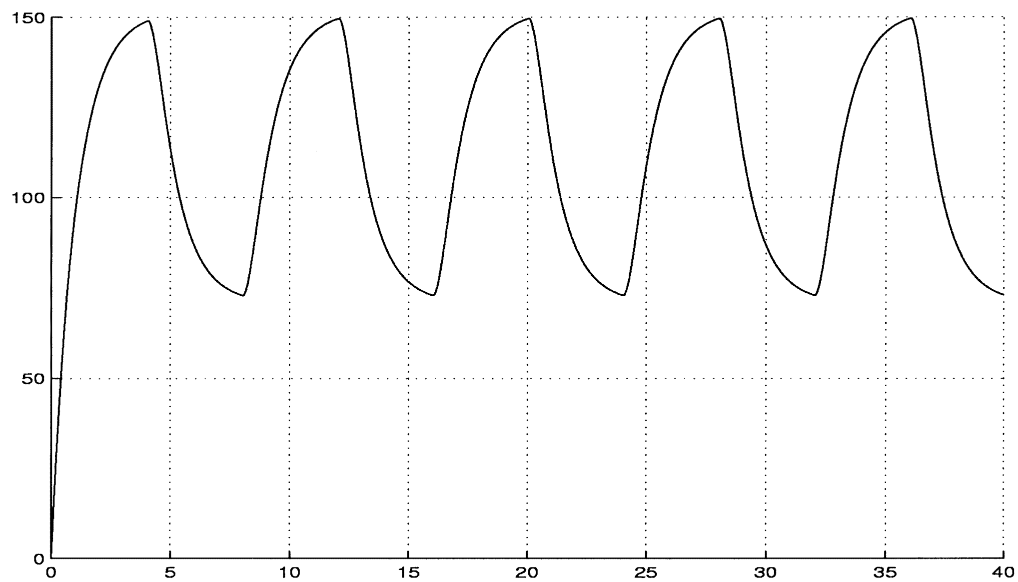

Fig. 4. Internal dynamics η (state feedback).

Since $(|u_{nn}| + w_m\sqrt{l} + |\varepsilon_l|)/(k_s(|u_{nn}|+1)) \leq 1/k_s(w_m\sqrt{l} + |\varepsilon_l| + 1)$, $b_{u_\lambda} > 0$ and $b_1(x)/\varepsilon > 0$, it is shown that $\dot{V}_b \leq 0$ once $|e_s| \geq R_0(\epsilon, k_s)$ with

$$R_0(\epsilon, k_s) = \max\left\{10\epsilon\,|e_0|\,, \frac{10}{9k_s}\left(w_m\sqrt{l} + |\varepsilon_l| + 1\right)\right\}. \tag{96}$$

We know that if $|e_s(0)| \leq R_0(\epsilon, k_s)$, then there exists a constant $T_r > 0$, such that

$$|e_s(t)| \leq R_0(\epsilon, k_s), \quad \forall t \geq T_r. \tag{97}$$

From $e_s = \left[\Lambda^T\, 1\right]\tilde{\xi}$ and $\tilde{\xi} = \left[\zeta^T\,\tilde{\xi}_\rho\right]^T$, we can see that $\tilde{\xi}_\rho = e_s - \Lambda^T\zeta$. Therefore

$$\left\|\tilde{\xi}(t)\right\| \leq \|\zeta(t)\| + \left|\tilde{\xi}_\rho(t)\right| \leq (1 + \|\Lambda\|)\|\zeta(t)\| + |e_s(t)|\,.$$

It follows from (91) and (97) that $\|\zeta(t)\| \leq k_0\left[\|\zeta(0)\| + e^{\lambda_0 T_r}/\lambda_0 e_{s_{\max}}(T_r)\right] + k_0/\lambda_0 R_0(\epsilon, k_s)$, $\forall t \geq T_r$. Hence

$$\begin{aligned}\|\xi(t)\| &\leq \left\|\tilde{\xi}(t)\right\| + \|\xi_d(t)\| \\ &\leq k_a\|\zeta(0)\| + k_b R_0(\epsilon, k_s) + k_c + c, \forall t \geq 0\end{aligned} \tag{98}$$

with $k_a = k_0(1 + \|\Lambda\|)$, $k_b = k_a/\lambda_0 + 1$ and $k_c = k_a e^{\lambda_0 T_r}/\lambda_0 e_{s_{\max}}(T_r)$. We now provide the conditions which guarantees $\xi \in \Phi_\xi$, $\forall t \geq 0$. Define the compact set

$$\Phi_0 := \left\{\xi(0) \,\middle|\, \left\{\xi \mid \|\xi(t)\| < k_a\|\zeta(0)\|\right\} \subset \Phi_\xi,\ |e_s(0)| < R_0(\epsilon, k_s)\right\} \tag{99}$$

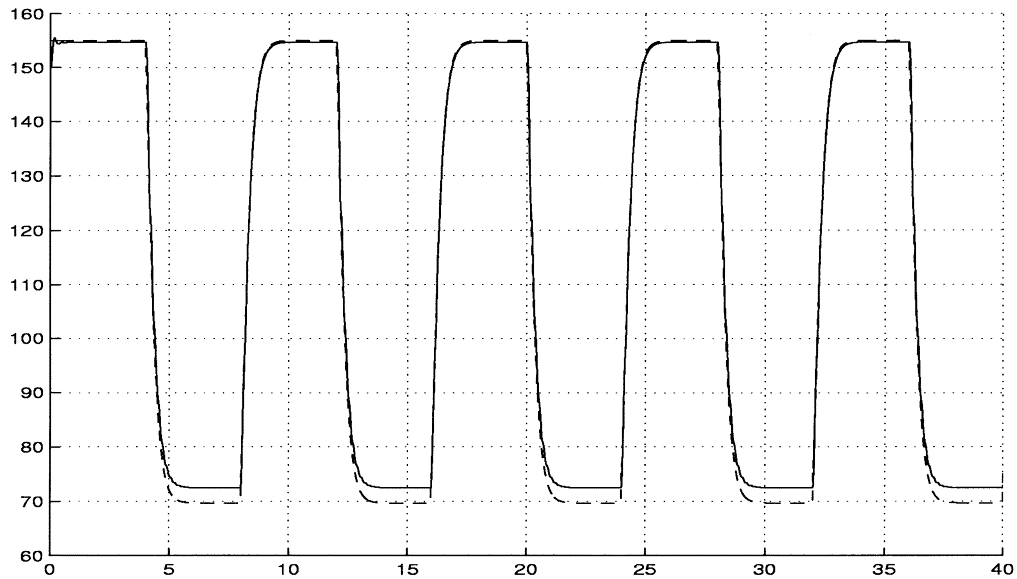

Fig. 5. u follow IDFC u_d ("- -") (state feedback).

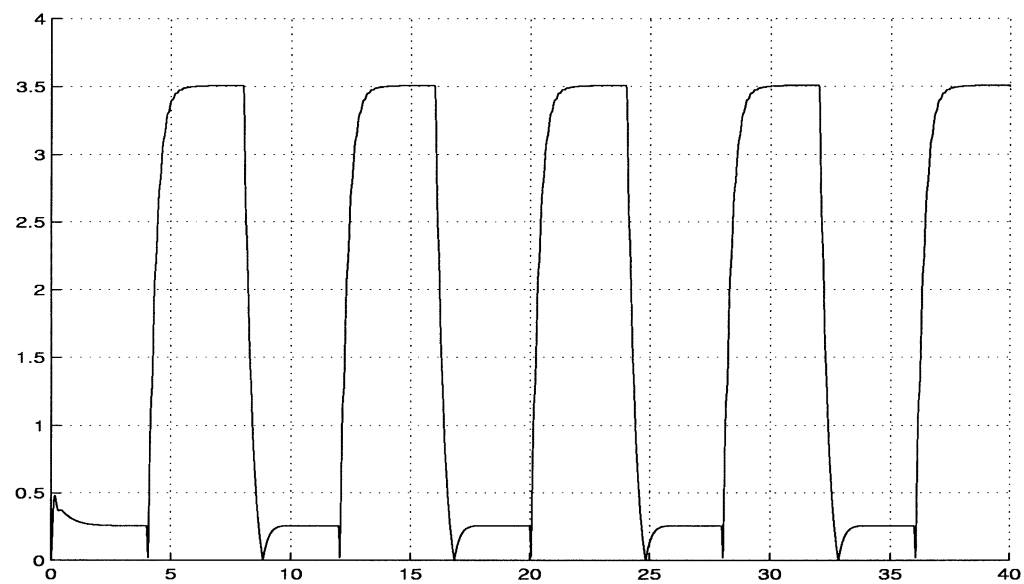

Fig. 6. NN weight $\|\hat{W}\|$ (state feedback).

the positive constant

$$c^* := \sup_{c \in R^+} \left\{ c \,\middle|\, \left\{ \xi \mid \|\xi\| < k_a \|\zeta(0)\| + k_c + c, \; \xi(0) \in \Phi_0 \right\} \subset \Phi_\xi \right\} \quad (100)$$

the positive constant

$$k_s^* := \inf_{k_s \in R^+} \left\{ k_s \,\middle|\, \left\{ \xi | \; \|\xi\| < k_a \|\zeta(0)\| + k_b R_0(0, k_s) + k_c + c, \xi(0) \in \Phi_0, c \le c^* \right\} \subset \Phi_\xi \right\} \quad (101)$$

and

$$\epsilon^* := \sup_{\epsilon \in R^+} \left\{ \epsilon \,\middle|\, \left\{ \xi | \; \|\xi\| \le k_a \|\zeta(0)\| + k_b R_0(\epsilon, k_s) + k_c + c, \xi(0) \in \Phi_0, c \le c^*, k_s \ge k_s^* \right\} \subset \Phi_\xi \right\}. \quad (102)$$

In summary, for all initial state $\xi(0) \in \Phi_0$, the desired signal $\|\bar{\xi}_d\| \le c \le c^*$, if control parameters k_s and ϵ are chosen such that $k_s \ge k_s^*$ and $\epsilon \le \epsilon^*$, then the system state ξ will stay in Φ_ξ for all time. The boundedness of ξ guarantees that the observer state $\hat{\xi}$ is bounded (see Lemma 4.1). Since the NN weights have been proven bounded for any bounded $\hat{W}(0)$ and $\hat{V}(0)$ (see Lemma 4.3), we conclude that all signals of the closed-loop system are bounded. This completes the proof. Q.E.D.

Remark 4.1: In the adaptive NN controller (76), two additional control terms, the bounding control term u_b and the prior

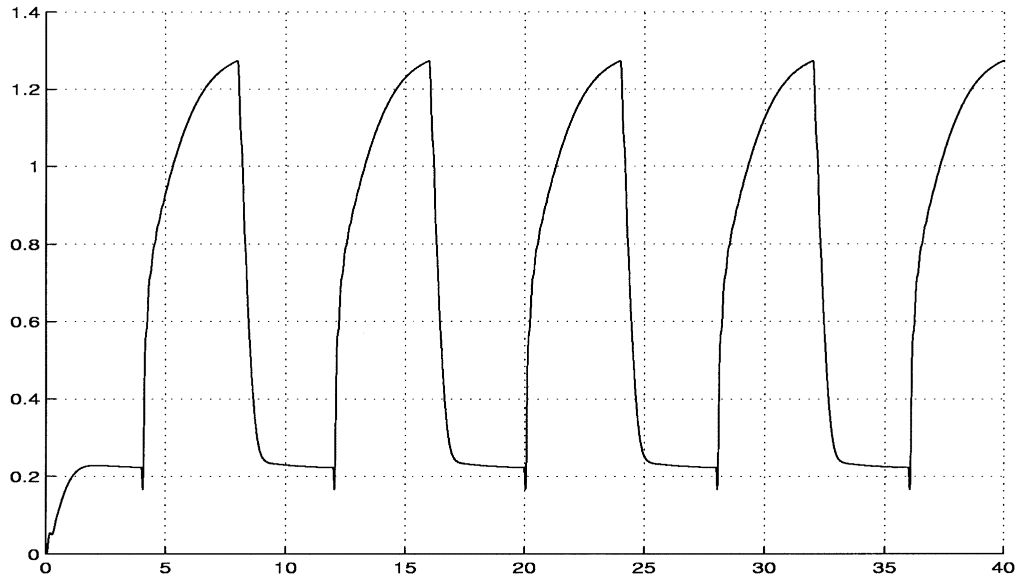

Fig. 7. NN weight $\|\hat{V}\|_F$ (state feedback).

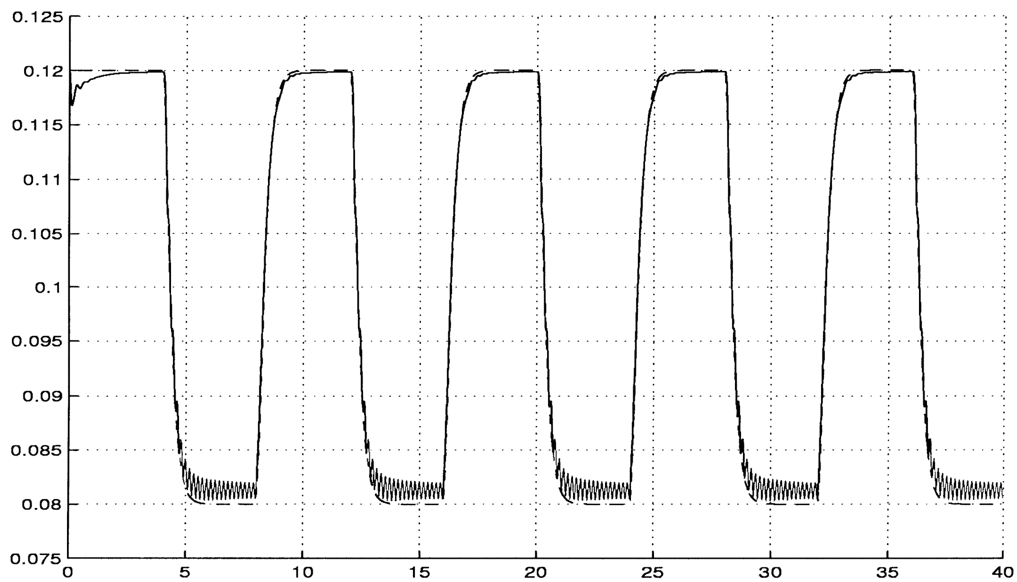

Fig. 8. ξ_1 follows ξ_{1d} ("- -") (output feedback).

control term u_k, are provided. The first one can be viewed as a supervisory control, which is introduced for limiting the upper bounds of the system variables such that $\xi \in \Phi_\xi$ holds. The second one provides a chance that control engineers can use conventional techniques to design an initial controller and then add the adaptive NNs to work in parallel to achieve high tracking accuracy. From (36), it can be seen that the closer u_k and u^* is, the smaller the ideal weight W^* and V^* will be. Considering (88)–(90), one can see that smaller W^* and V^* will lead to smaller output tracking error. Therefore, if u_k is designed adequately, the control performance can be improved. On the other hand, even though u_k is inadequate, the use of the above adaptive NN controller still results in a stable tracking.

Remark 4.2: In Theorem 4.1, it requires that the observer (72) to be turned on at time t^* in advance. This is because the high-gain observer may exhibit a peaking phenomenon and the estimated state errors might be very large in the initial transient period. If the observer is turned on at time t^* before the controller is put into operation, Lemma 4.1 guarantees that the state estimation $\pi_{k+1}/\epsilon^k - y^{(k)}$ is bounded by the constant ϵh_k which only depends on Y_k, ϵ and $\bar{\lambda}_i$ and, therefore, the peaking of the controller can be avoided. Another method to overcome the peaking problem is to introduce an estimate saturation or control input saturation [33]–[35]. Thus, during the short transient period when the state estimates exhibit peaking, the saturation prevents the peaking from being transmitted to the plant.

V. Simulation Study

In this section, a practical isothermal continuous stirred tank reactor is simulated to illustrate the proposed state feedback and output feedback controllers.

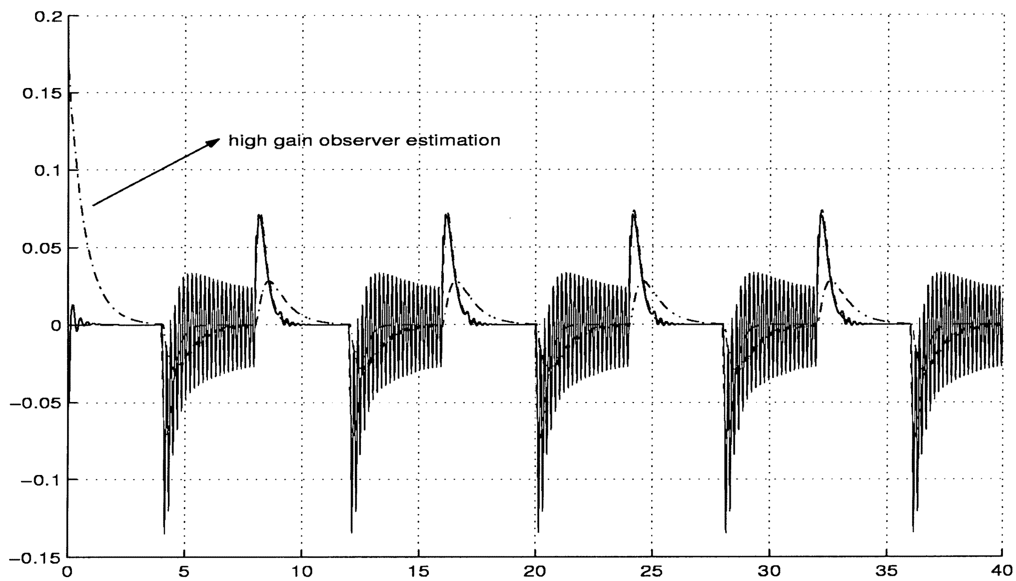

Fig. 9. ξ_2 follows ξ_{2d} ("- -") (output feedback).

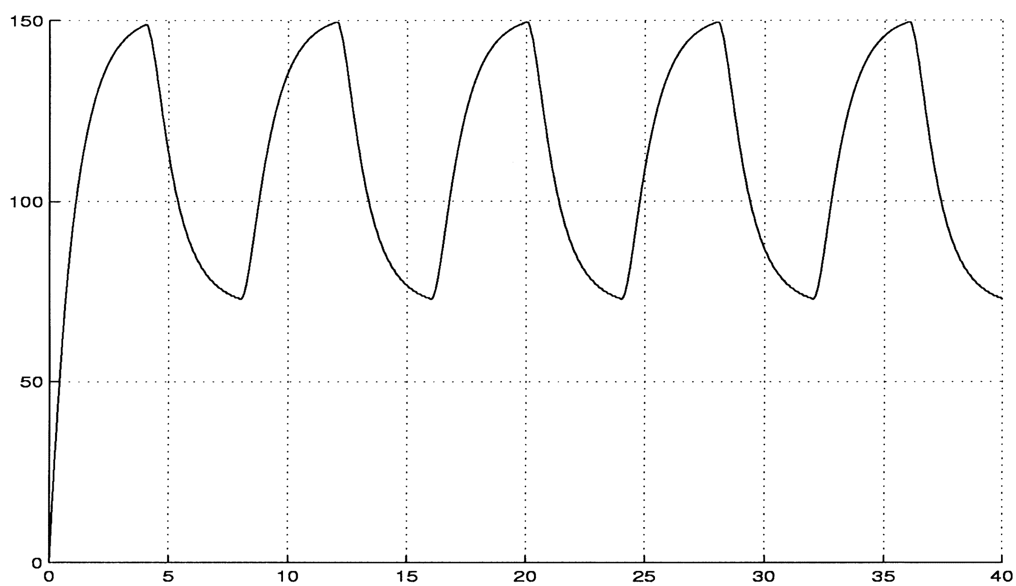

Fig. 10. Internal dynamics η (output feedback).

In [8], we have showed the effectiveness of adaptive NN control for a class of nonaffine nonlinear systems without zero dynamics both theoretically and numerically. Though it is easy for us to cook up a nonaffine nonlinear system with zero dynamics, it is more meaningful to work on physical models of real systems. Owing to the difficulty in finding a practical nonaffine nonlinear system with zero dynamics, an affine nonlinear system with zero dynamics is used here to verify the effectiveness of the proposed controller. The reasons are as follows: 1) for this affine CSTR model, the IDFC controller can be computed, thus, we can verify the effectiveness of the proposed controllers; 2) although this is an affine system, it is also a special kind of nonaffine system; and 3) it is a practical physical system.

Consider a class of multicomponent reaction $A \rightleftharpoons B \rightarrow C$ taken place in a CSTR [9]. The output of the process is the concentration of C and the manipulated variable is the molar feed flow rate of B, N_{BF}. A mass balance gives the modeling equations

$$\begin{aligned} V\frac{dC_A}{dt'} &= F\left(C_{A_f} - C_A\right) - Vk_1C_A + Vk_2C_B^2 \\ V\frac{dC_B}{dt'} &= -FC_B + Vk_1C_B^2 - Vk_3C_B^2 + N_{BF} \\ V\frac{dC_C}{dt'} &= -FC_C + Vk_3C_B^2 \\ y &= C_C. \end{aligned}$$

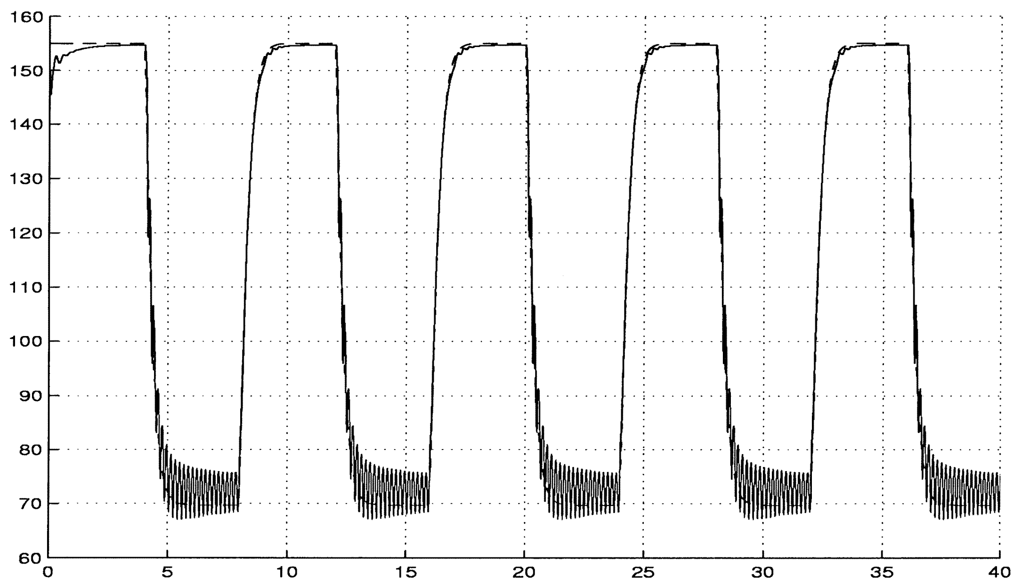

Fig. 11. u follow IDFC u_d ("- -") (output feedback).

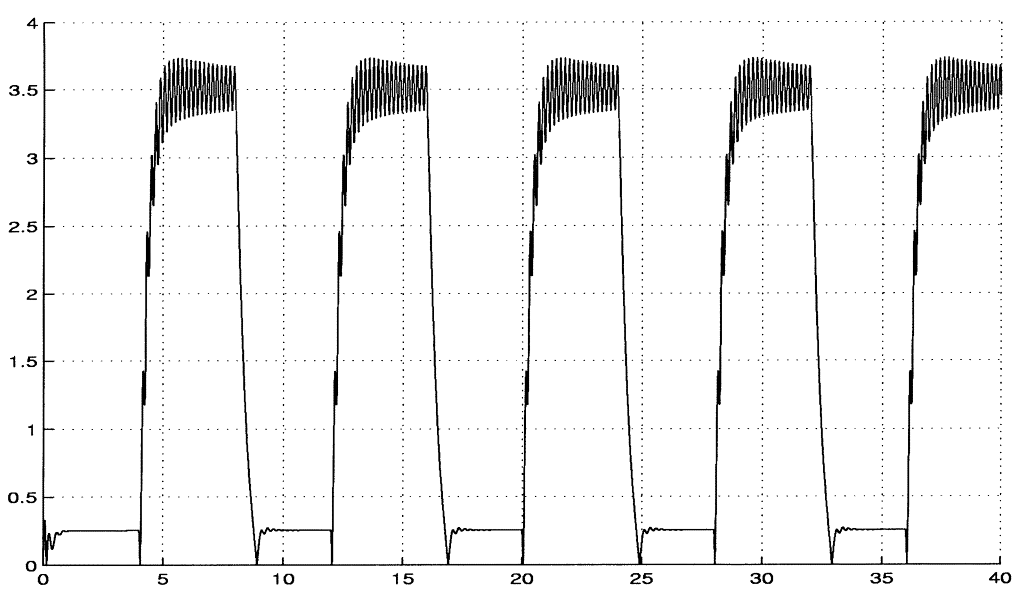

Fig. 12. NN weight $\|\hat{W}\|$ (output feedback).

With the dimensionless variables given in Table I, we can obtain the dimensionless state–space model description

$$\begin{aligned}\dot{x}_1 &= 1 - x_1 - c_1 x_1 + c_2 x_2^2\\ \dot{x}_2 &= -x_2 + c_1 x_1 - c_2 x_2^2 - c_3 x_2^2 + u\\ \dot{x}_3 &= -x_3 + c_3 x_2^2\\ y &= h(x) = x_1.\end{aligned}$$

It is easy to check that the relative degree of this system is 2. By define diffeomorphism

$$\begin{aligned}\xi_1 &= x_1\\ \xi_2 &= 1 - x_1 - c_1 x_1 + c_2 x_2^2\end{aligned}$$

and a temporary variable $f_t = [(1+c_1)\xi_1 + \xi_2 - 1]/c_2$ the system can be transformed into

$$\begin{aligned}\dot{\xi}_1 &= \xi_2\\ \dot{\xi}_2 &= f_0(\xi_1,\xi_2) + g_0(\xi_1,\xi_2)\,u\\ \dot{\eta} &= -\eta + c_3 f_t\\ y &= \xi_1\end{aligned} \tag{103}$$

where $f_0(\xi_1,\xi_2) = 2c_1c_2\sqrt{f_t}\xi_1 - (c_1+1)\xi_2 - 2c_2[1 + (c_2 + c_3)\sqrt{f_t}]f_t$ and $g_0(\xi_1,\xi_2) = 2c_2\sqrt{f_t}$.

Assuming the Damkholer numbers are chosen as follows: $c_1 = 20$, $c_2 = 0.1$ and $c_3 = 10$. Furthermore, considering the operation range of x_1 and x_2, Assumption 2.2 holds. Thus, the existence of the IDFC controller is guaranteed, which is also verified in the simulations.

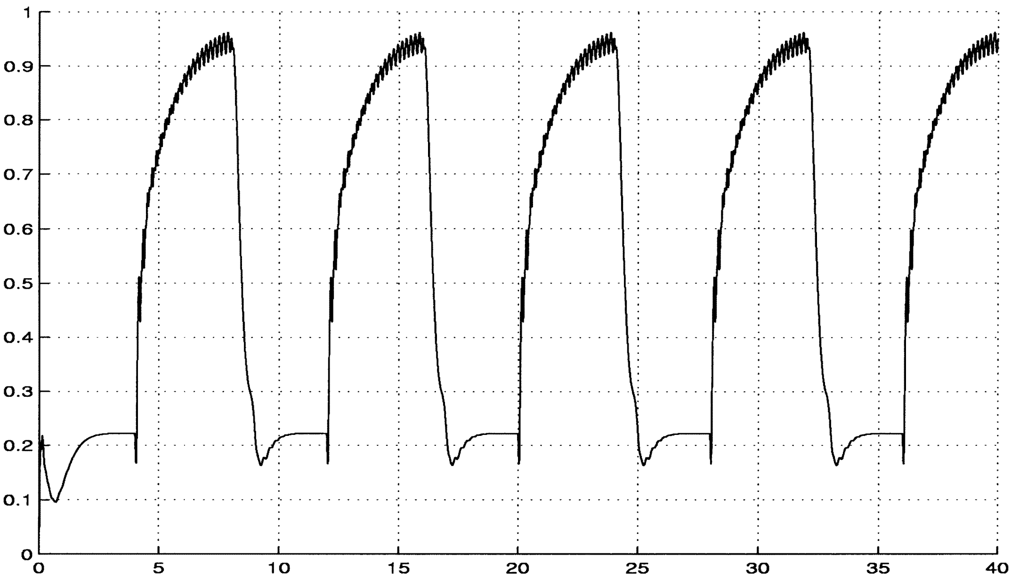

Fig. 13. NN weight $\|\hat{V}\|_F$ (output feedback).

The control objective is to make the concentration y track the set-point step change signal $y_d(t) = 0.1 \pm 0.02$. In order to obtain a smooth reference signal, a linear reference model is used to shape the discontinuous reference signal for providing the desired signals y_d. The following reference model is to be implemented:

$$\frac{y_d(s)}{r(s)} = \frac{\omega_n^2}{s^2 + 2\zeta_n \omega_n s + \omega_n^2}$$

where the natural frequency $\omega_n = 5.0$ rad/min and the damping ratio $\zeta_n = 1.0$.

State Feedback Control: The closed-loop system structure is shown in Fig. 1. When all the system states are available, the proposed state feedback control method is applicable. Control input is $u = u_{nn} + u_b$, with

$$\begin{aligned} u_{nn} =& \hat{W}^T S\left(\hat{V}^T \bar{z}\right) \\ u_b =& -\left[\frac{k_p}{\varepsilon}\left(\left\|\bar{z}\hat{W}^T \hat{S}'\right\|_F^2 + \left\|\hat{S}'\hat{V}^T\bar{z}\right\|^2 + 1\right) + k_s\left|e_s\right|\right] e_s \end{aligned}$$

where $z = [\xi, \eta, e_s, \nu_1]^T$, $\bar{z} = [z^T, 1]^T$ and $\nu_1 = -\ddot{y} + \left[0\,\Lambda^T\right]\tilde{\xi}$. $\hat{W}$ and $\hat{V}$ are tuned by update laws in (48) and (49).

Simulation parameters are chosen as follows: neural number $l = 500$; $b_1(x) = 10$; $\epsilon = 0.1$; $\lambda_1 = 70$; $\varepsilon = 0.01$; $k_s = 0.1, k_p = 0.01$; $\hat{W}(0) = \hat{V}(0) = 0$; $\delta_w = 0.5$; $\delta_v = 0.25$; $\Gamma_w = 5.0$; $\Gamma_v = 5.0$. System initial status are $\xi_1 = 0.12$ and $\xi_2 = 0.0$.

Simulation results are shown in Figs. 2–7. We can see that the output trajectory follows the step changes of reference signal. Meanwhile, the derivative of output $\dot{y}$ also follows the reference signal $\dot{y}_d$. Internal dynamics and NN weights are all bounded. It should be noted that the control input follows the ideal IDFC control trajectory, which verifies the existence of the IDFC controller.

Output Feedback Control: When system states are not available, the high gain observer can be used to reconstruct them. In this part, the proposed high gain observer based output feedback control method is applied to the same CSTR system in state feedback control. The closed-loop system structure is shown in Fig. 1. Assume that state ξ_2 is unavailable.

The control input is $u = u_{nn} + u_b + u_k$ with

$$\begin{aligned} u_{nn} =& \hat{W}^T S\left(\hat{V}^T \hat{\bar{z}}\right) \\ u_b =& -\left[\frac{k_p}{\varepsilon}\left(\left\|\hat{\bar{z}}\hat{W}^T\hat{S}_o'\right\|_F^2 + \left\|\hat{S}_o\right\|^2 + \left\|\hat{S}_o'\hat{V}^T\hat{\bar{z}}\right\|^2 + 1\right)\right. \\ &+ \left. k_s\left(\left|u_{nn}\right| + 1\right)\right]\hat{e}_s \\ u_k =& 0 \end{aligned}$$

where $z = [\hat{\xi}, \eta, e_s, \nu_1]^T$, $\bar{z} = [z^T, 1]^T$ and $\nu_1 = -\ddot{y} + \begin{bmatrix} 0 \\ \Lambda^T \end{bmatrix}\hat{\tilde{\xi}}$. $\hat{W}$ and $\hat{V}$ are tuned by update laws in (79) and (80). It should be noted that $\hat{\xi} = [\xi_1, \hat{\xi}_2]^T$ replaces $\xi = [\xi_1, \xi_2]^T$ (state feedback control) here, because ξ_2 is not available. The high gain observer is applied to estimate ξ_2.

Simulation parameters are the same as those used in state feedback control. For the high gain observer used, its parameter are chosen as follows: $\epsilon = 0.01$; the initial observer states $[\pi_1(0), \pi_2(0)] = [0, 0]$; the initial system states $\xi_1(0) = 0.12$ and $\xi_2(0) = 0.0$.

Simulation results are shown in Fig. 8–13. We can see that the output follows the desired trajectories. The control input follows the IDFC. The internal dynamics and neural weights are all bounded. But $\hat{\xi}_2$ does not follow the desired signal ξ_{2d} very well. Furthermore, oscillation appear in control and output trajectories.

VI. CONCLUSION

In this paper, we have presented state feedback and output feedback control scheme for a class of single-input-single-output nonaffine system with zero dynamics. Its zero dynamics are assumed to be exponentially stable. Based on implicit function theory, stable adaptive NN controllers are developed for both state and output feedback control. The proposed design guarantees the stability of the closed-loop adaptive system and the convergence of the tracking errors.

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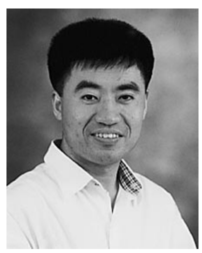

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