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Multi-Robot Formations based on the Queue-Formation Scheme with Limited Communications

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***Abstract*— In this paper, we investigate the operation of the Queue-formation structure (or Q-structure) in multi-robot teams with limited communications. Information flow is divided into two time scales: (i) the fast time scale where the robots' reactive actions are determined based only on local communications, and (ii) the slow time scale, where information required is less demanding, can be collected over a longer time with intermittent information loss. Therefore, there is no need for global information at all times, reducing the overall communication load. In addition, a dynamic target determination algorithm, based on the Q-structure, is used to produce a series of targets that incrementally guide each robot into formation. It provides greater control over the distance between robots on the same queue for better formation scaling. An analysis of the convergence of the system of robots is provided. Simulation studies verify the effectiveness of the scheme.**

***Index Terms*— Multi-Robot Formations, Limited Communication Ranges, Deliberative Coordination, Convergence**

I. INTRODUCTION

Robust multi-robot collaboration in dynamic and uncertain environments has been intensively studied in recent years. Methods such as virtual leaders [1], social potentials [2] and formation constrained functions [3] can be used to guide robots into formations. Individual robots may also be allocated predefined positions in a formation [4]. A decentralized scheme based on the virtual structure approach has also been proposed for effective formation maintenance [5]. General methods for the controller design for the formation maintenance of multiple vehicles tracking a desired path have also been proposed in [6]. A number of studies have also been carried out on the stability and convergence of formation schemes (largely based on the nodes-and-edges representation) [7]–[9].

Most of the above seminal works use representations based on connectivity graphs where each robot tracks a specific node as a target. Such a representation is also implicit in more reactive approaches, such as those that require a robot to choose and follow a neighbor at a pre-specified distance and orientation. When team size changes, the graph representation will change and become difficult to track dynamically. Since a large class of problems (e.g. robotic search and surveillance, convoy movements) only require robots to maintain the overall appearance of the formation, the Q-structure has been recently introduced [10] to improve scalability and flexibility. Two drawbacks of the scheme is the reliance on persistent global communications and that the convergence of robots into formations based on the scheme had only been established via experimentation.

The main contributions of this paper are as follows:

(i) The Q-structure in [10] is extended by considering finite communication ranges, and separating the decision making process into two time scales – a fast time scale for reactive decision making based only on local communications, and a slower time scale which allows less time critical information to propagate through a weakly connected network. Persistent global communications is not required, reducing overall communications load. It also permits intermittent information losses as information is collected over a longer time.

(ii) A rigorous proof of convergence for a decentralized control law that guides robots into formations represented by the Q-structure is presented. It extends the work in [11] as follows: (a) Inter-agent potentials are designed to exhibit a different set of properties to reflect limited communication ranges, with a more general controller design, (b) A different, more concise convergence proof for the more general controller is presented.

(iii) A dynamic target determination algorithm is proposed to incrementally guide robots into their queues. This improves upon the original scheme [10] by separating agent decisions regarding positions on queues from reactive inter-agent repulsive forces, to an even distribution of agents along queues.

II. FORMATION REPRESENTATION AND DYNAMIC TARGET DETERMINATION

A. Division of Information Flow

Information flow is separated into slow and fast time scales. The control of the formation takes place on these two levels based on the information available on each time scale.

(i) Fast-time scale: This facilitates time critical and reactive decision making, such as collision avoidance and getting into formation. It only involves local communications

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between robots. Explicit controls governing the actual movements and paths of the agents occur at this level. Such decisions take place at a higher frequency when information is available.

(ii) Slow-time scale: This refers to the gradual multi-hop transfer of information, through a weakly connected communication network, between robots not in direct range of each other. The collection of information over a longer time period allows for intermittent information losses between links. Formation control on this level involves low frequency decisions regarding the (re)allocation of robots to different vertices or queues.

Interactions between the agents are mostly local since agents respond reactively to data it obtains from others around itself based on direct communication. This is not equivalent to requiring global information at all times for all decisions. (Re)Allocation based on long term information flows occurs at fixed periodic intervals. This information might not be the most current and subjected to time delays. Hence, there is no need for constant global communications between all robots. In addition, while information regarding out-of-range agents may be available, these are not taken into consideration while making pathing decisions other than for (re)allocation.

B. Formations and Queues

For completeness, we briefly state the concepts regarding 'queues' and formations as detailed in [10].

Definition 1 (Formations [10]): A formation is a desired overall appearance of the agent team, consisting of relative positioning constraints and acceptable positions for each agent. The constraints are realized in the form of the vertices and queues. A formation is denoted by $\mathcal{F} = (\mathcal{Q}, \mathcal{V}_F(N))$, where $\mathcal{Q}$ is the set of queues, and $\mathcal{V}_F(N)$ represents the set of formation vertices, V_i $(i = 1, \ldots, N_v)$, where N is the total number of robots[1], around the target.

Definition 2 (Queues [10]): A queue, $\mathcal{Q}_j \in \mathcal{Q}$, is denoted as $\mathcal{Q}_j = (\mathcal{V}_j, \mathcal{S}_j, \mathcal{C}_j)$. The main elements characterizing a queue are described as follows:

(i) $\mathcal{V}_j \subseteq \mathcal{V}_F(N)$ (Queue Vertices): a list of either one or two formation vertices through which $\mathcal{Q}_j$ passes.

(ii) $\mathcal{S}_j$ (Shape): a set of points following an equation in $\mathbb{R}^3$ that describes the spatial appearance of $\mathcal{Q}_j$, and is specified in the coordinate frame of the first formation vertex in the list $\mathcal{V}_j$.

(iii) $\mathcal{C}_j$ (Capacity): a fraction that refers to the proportion of all the robots in the formation it can hold, i.e., $\sum_{j=1}^{N_q} \mathcal{C}_j = 1$, where N_q is the total number of queues in the formation.

Through the partitioning of information flows into short term and long term flows, we allow reshuffling and refinement of robots between queues (i.e. as per the queue change algorithm presented in [10]) to occur based on long term information gathered from the entire network. The need for robots to react in a timely manner is not compromised by the long term information propagation through the network. Please refer to [12] for an analysis of the Q-structure using its graphical representation.

[1]Each formation vertex is represented by its position relative to the coordinate frame of the target.

C. Determination of Target on Queue

This section describes an algorithm that each robot r_i, associated with a queue $\mathcal{Q}(i)$, uses for target determination. The algorithm also governs the distance between robots within the same queue. Compared to the purely reactive scheme in [10], it improves the scaling of formations through an adaptation of the parameter d_{ir} (acceptable inter-robot distance for robots on the same queue).

Algorithm 1 Determining Target on Queue (by agent r_i)

Let $R_{c,i} \in R_N$ be an ordered set of agents (according to increasing Euclidean distance from $\mathcal{V}_{\mathcal{Q}(i)}(1)$) within communication range of r_i and belonging to the same queue as r_i, i.e., belonging to $\mathcal{Q}(i)$.
Suppose r_i is the n-th agent in the list $R_{c,i}$.
if n=1 **then**
  Set $q_{tg,i} = \mathcal{V}_{\mathcal{Q}(i)}(1)$.
else
  Let $r_j \in R_{c,i}$ be the $(n-1)$-th agent in the list.
  Set $q_{tg,i} = \arg\min_{q \in Q} \|q - \mathcal{V}_{\mathcal{Q}(i)}(1)\|$ where $Q = \{q \in \mathcal{Q}(i) \mid \|q - q_{tg,j}\| = d_{ir}$ and $\|q - \mathcal{V}_{\mathcal{Q}(i)}(1)\| > \|q_{tg,j} - \mathcal{V}_{\mathcal{Q}(i)}(1)\|\}$.

The algorithm is executed when $R_{c,i}$ changes. It works by considering the agents within communication range of r_i and which also belong to the same queue as r_i. The target of r_i is set to be a point on $\mathcal{Q}(i)$ and at a distance of d_{ir} away from the target of r_j. If r_i is the agent in $R_{c,i}$ that is closest to the queue vertex $\mathcal{V}_{\mathcal{Q}(i)}(1)$, its target will be set to be the queue vertex. The target changes in response to the information it has of other robots within communication range and which are of the same queue.

The common objective ($\mathcal{F}_N$) will result in a weakly connected communication network for each subset of agents within the same queue. Although an agent may not be in direct communications with some others within the same queue, the decisions of preceding agents will be reflected by the decisions made by others within communication range.

Lemma 2.1: Given a set of agents and considering only direct communications between an agent and those in its neighborhood, Algorithm 1, together with the common objective given in the form of the desired formation $\mathcal{F}_N$, will result in constant targets for each agent on each queue.

Proof: Let r_i and r_j be the n-th and $(n-1)$-th furthest agents in $R_{c,i}$ from the queue vertex $\mathcal{V}_{\mathcal{Q}(i)}(1)$. According to Algorithm 1, if $q_{tg,j}$ is constant, $q_{tg,i}$ will be constant too, and at a distance of d_{ir} along the queue from $q_{tg,j}$. Consider a queue $\mathcal{Q}_*$ where all agents belonging to this queue have converged into a weakly connected net due to the common objective. Let $R_{Q*} = \{r_{q1}, r_{q2}, \ldots, r_{qN_q}\}$ be this set of N_q agents, ordered in ascending order according to their distance from the queue vertex $\mathcal{V}_{\mathcal{Q}*}(1)$. For the set $R_{c,q1}$, r_{q1} will be the closest to the vertex, and from Algorithm 1, its target will be constant and locked to $q_{tg,q1} = \mathcal{V}_{Q*}(1)$. From the argument in the preceding paragraph, the target of the second agent in R_{Q*}, $q_{tg,q2}$ will be constant because $q_{tg,q1}$ is fixed. Therefore, by induction, the target of the $n-th$ agent will be fixed and constant, once the agents have converged into a weakly connected net around their respective queues. ■

III. Navigation of Robots into Position

From Lemma 2.1, the targets of each agent will become constant within finite time, and the control laws presented in this section will first bring each agent to converge to their queues and onto their desired targets.

Consider the following potential function:

$$U = U_{tg} + U_{ob} \quad (1)$$

where U_{tg} is the attractive potential between the robots and their target, written as:

$$U_{tg} = \frac{1}{2}\sum_{i=1}^{N} \|q_i - q_{tg,i}\|^2 \quad (2)$$

Let a robot, r_i, be able to reliably communicate with only N_i robots (comprising the set $R_i \in R$). U_{ob} reflects the collision avoidance behavior, and is chosen to be:

$$U_{ob} = \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} U_{ob,ij} \quad (3)$$

where $U_{ob,ij}$ is a function of U_{ij} and $U_{tg,ij}$, which are given by

$$U_{ij} = \frac{1}{2}\|q_i - q_j\|^2 \quad (4)$$

$$U_{tg,ij} = \frac{1}{2}\|q_{tg,i} - q_{tg,j}\|^2 \quad (5)$$

and $U_{ob,ij}$ is chosen such that

(a) $U_{ob,ij} = \infty$, if $U_{ij} = 0$
(b) $U_{ob,ij} > 0$, if $U_{ij} \neq 0$
(c) $U'_{ob,ij} = \frac{\partial U_{ob,ij}}{\partial U_{ij}} = 0$, if $U_{ij} = U_{tg,ij}$
(d) $U''_{ob,ij} = \frac{\partial^2 U_{ob,ij}}{\partial U_{ij}^2} \geq 0$, if $U_{ij} = U_{tg,ij}$
(e) $U_{ob,ij} \approx 0$, if $U_{ij} \geq 0.5 d_{ij}^2$

Based on the above properties, $U_{ob,ij}$ may be chosen as

$$U_{ob,ij} = f_{ij}\left(\frac{U_{ij}}{U_{tg,ij}^2} + \frac{1}{U_{ij}}\right) \quad (6)$$

where

$$f_{ij} = \frac{1}{1 + \exp(a_t(U_{ij} - U_{tg,ij})^3)} \quad (7)$$

where a_t is a user-defined constant.

At each time instant, each robot moves along the negative gradient of the potential function U, given by

$$\begin{aligned}
\dot{U} &= \sum_{i=1}^{N}(q_i - q_{tg,i})^T u_i \\
&\quad + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} U'_{ob,ij}(q_i - q_j)^T(u_i - u_j) \\
&= \sum_{i=1}^{N}\left((q_i - q_{tg,i})^T + \sum_{j\neq i}^{N} U'_{ob,ij} q_{ij}^T\right) u_i \\
&= \sum_{i=1}^{N}\Omega_i^T u_i \quad (8)
\end{aligned}$$

where $q_{ij} = q_i - q_j$ and Ω_i is defined as

$$\Omega_i = (q_i - q_{tg,i}) + \sum_{j\neq i}^{N} U'_{ob,ij} q_{ij} \quad (9)$$

This implies that a choice of

$$u_i = -C\Omega_i \quad (10)$$

where $C \in \mathbb{R}_{+}^{n_w \times n_w}$ is a symmetric, positive definite matrix, which is chosen as $C = \mathbf{I}_{n_w \times n_w} c$ where $c > 0$, will result in

$$\dot{U} = -\sum_{i=1}^{N}\Omega_i^T C \Omega_i \quad (11)$$

and the closed loop dynamics of a single robot r_i in the team is then given by

$$\dot{q}_i = -C\Omega_i \quad (12)$$

If the robots are at non-colliding positions at initial time t_0, and the target of each robot is different as well, these conditions may be written as

$$\|q_i(t_0) - q_j(t_0)\| \geq \epsilon_1 \quad (13)$$

where ϵ_1 is a strictly positive constant, and R is the set of robots comprising the team. In addition, Algorithm 1 guarantees that if the condition in (13) is satisfied, the targets for each cycle do not collide, i.e., $\|q_{tg,i} - q_{tg,j}\| \geq \epsilon_2, \quad \forall i, j \in R$, where ϵ_2 is strictly positive. It is thus desired that, under such conditions, each robot will converge toward their targets, and at the same time avoiding collisions, i.e.

$$\begin{aligned}
\lim_{t\to\infty}(q_i(t) - q_{tg,i}) &= 0 \\
\|q_i(t) - q_j(t)\| &\geq \epsilon_3, \quad \forall i, j \in R \text{ and } \forall t \geq t_0 \geq 0
\end{aligned} \quad (14)$$

where ϵ_3 is a strictly positive number representing the minimum acceptable inter-robot distance.

Theorem 3.1: Under the conditions (13), the common formation objective, $\mathcal{F}_N$, and Algorithm 1, the control input to each robot, given in (10), will result in the convergence of each robot to their desired targets, and such that:

(i) The target at q_{tg} is located at an asymptotically stable equilibrium point of (12), and
(ii) The critical points of the system *other than that at* q_{tg} are unstable equilibrium points.

Proof: Integrating both sides of (11) from t_0 to t, we obtain

$$\begin{aligned}
&U_{tg}(t) + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} U_{ob,ij}(t) \\
&\leq U_{tg}(t_0) + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} U_{ob,ij}(t_0) \quad (15)
\end{aligned}$$

where

$$U_{tg}(t) = \frac{1}{2}\sum_{i=1}^{N}\|q_i(t)-q_{tg,i}\|^2$$

$$U_{ob,ij}(t) = f_{ij}(t)\left(\frac{U_{ij}(t)}{U_{tg,ij}^2}+\frac{1}{U_{ij}(t)}\right) \quad (16)$$

From the conditions in (13), $U_{ij}(t_0)$ and $U_{tg,ij}$ are strictly larger than some positive constants. Furthermore, since f_{ij} is also bounded ($0 < f_{ij} < 1$), the right hand side of (15) is bounded by some positive constant (the value of which depends on the initial conditions at t_0). Hence, the left hand side is also bounded, which in turn implies that $U_{ij}(t)$ must be strictly larger than some positive constant for all $t \geq t_0 \geq 0$. From (16), $\|q_i(t) - q_j(t)\|$ will therefore always be larger than some strictly positive constant, and there will be no collisions. The boundedness of the left hand side of (15) also implies that of $\|q_i(t)\|$ for all $t \geq t_0 \geq 0$, and the solutions of the closed loop system in (12) exist.

By setting $\Omega_i = 0$, we obtain the root sets (critical points) of the system in (12), which are given by $q = q_{tg}$ (due to Property (c) of $U_{ob,ij}$) and $q = q_c$ (representing the remaining critical points), where $q = [q_1^T, \ldots, q_N^T]^T$ and $q_{tg} = [q_{tg,1}^T, \ldots, q_{tg,N}^T]^T$ and $q_c = [q_{c,1}^T, \ldots, q_{c,N}^T]^T$.

The behavior of the equilibrium points is examined by considering the relative distances between agents. To convert the dynamics of each agent (given in (12)) to inter-agent dynamics, we define $q_{ij} = q_i - q_j$ and $q_{tg,ij} = q_{tg,i} - q_{tg,j}$ for all $i, j \in R$ for each i, and arranging i and j such that $i < j$. This yields the dynamics of q_{ij} as

$$\dot{q}_{ij} = -C\Omega_{ij} \quad (17)$$

where

$$\Omega_{ij} = \Omega_i - \Omega_j \quad (18)$$

$$= (q_{ij} - q_{tg,ij}) + \sum_{\ell\neq i}^{N} U'_{ob,i\ell}q_{i\ell} - \sum_{\ell\neq j}^{N} U'_{ob,j\ell}q_{i\ell}$$

$$= (q_{ij} - q_{tg,ij}) + 2U'_{ob,ij}q_{ij} + \sum_{\ell\neq i,\ell\neq j}^{N}\left(U'_{ob,i\ell}q_{i\ell} - U'_{ob,j\ell}q_{j\ell}\right) \quad (19)$$

We define:

$$\bar{q} = [q_{12}^T, q_{13}^T, \ldots, q_{ij}^T, \ldots, q_{N-1N}^T]^T \quad (20)$$

$$\bar{q}_{tg} = [q_{tg,12}^T, q_{tg,13}^T, \ldots, q_{tg,ij}^T, \ldots, q_{tg,N-1N}^T]^T \quad (21)$$

$$\bar{q}_c = [q_{12c}^T, q_{13c}^T, \ldots, q_{ijc}^T, \ldots, q_{(N-1)(N)c}^T]^T \quad (22)$$

$$\bar{C} = \mathrm{diag}(C, \ldots, C), \quad (23)$$

$$F(\bar{q}, \bar{q}_{tg}) = [\Omega_{12}^T, \Omega_{13}^T, \ldots, \Omega_{ij}^T, \ldots, \Omega_{N-1N}^T]^T \quad (24)$$

where $\bar{C}$ comprises E number of C along its diagonal, and E is the total number of communication links that can exist between robots if global communications exist[2]. The closed loop system in (17) may then be written as

$$\dot{\bar{q}} = -\bar{C}F(\bar{q}, \bar{q}_{tg}) \quad (25)$$

Furthermore, given the common formation objective, the system of agents will converge into a weakly connected net, which implies that the maximum distance between any two agents is given by $(N-1)d_{ij}$, and that $\bar{q}$ is bounded. Therefore, we have the compact set given by

$$\Upsilon = \{\bar{q} \mid \|\bar{q}\| \leq N(N-1)d_{ij}\} \quad (26)$$

upon which LaSalle's Invariance Principle will be applied to examine the stability of the system around the equilibrium points.

To proceed, we linearize (25) at the critical points $\bar{q}_e$, which can be $\bar{q}_{tg}$ or $\bar{q}_c$. This results in

$$\frac{d(\bar{q} - \bar{q}_e)}{dt} = -\bar{C}\left.\frac{\partial F(\bar{q}, \bar{q}_{tg})}{\partial \bar{q}}\right|_{\bar{q}=\bar{q}_e}(\bar{q} - \bar{q}_e) \quad (27)$$

where the general gradient of $F(\bar{q}, \bar{q}_{tg})$ with respect to $\bar{q}$ is

$$\frac{\partial F(\bar{q}, \bar{q}_{tg})}{\partial \bar{q}} = \begin{bmatrix} \frac{\partial\Omega_{12}}{\partial q_{12}} & \frac{\partial\Omega_{12}}{\partial q_{13}} & \cdots & \cdots & \frac{\partial\Omega_{12}}{\partial q_{N-1N}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial\Omega_{ij}}{\partial q_{12}} & \cdots & \frac{\partial\Omega_{ij}}{\partial q_{ij}} & \cdots & \frac{\partial\Omega_{ij}}{\partial q_{N-1N}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial\Omega_{N-1N}}{\partial q_{12}} & \cdots & \cdots & \cdots & \frac{\partial\Omega_{N-1N}}{\partial q_{N-1N}} \end{bmatrix} \quad (28)$$

where $i, j \in R$, and

$$\frac{\partial\Omega_{ij}}{\partial q_{ij}} = \mathbf{I}_{n_w\times n_w} + 2U'_{ob,ij} + 2U''_{ob,ij}q_{ij}q_{ij}^T \quad (29)$$

$$\frac{\partial\Omega_{ij}}{\partial q_{i_*j_*}} = \sigma U'_{ob,i_*j_*} + \sigma U''_{ob,i_*j_*}q_{i_*j_*}q_{i_*j_*}^T \quad (30)$$

in which $\mathbf{I}_{(n_w\times n_w)}$ is an n_w-dimensional identity matrix, and $q_{i_*j_*}$ is defined such that $(i_*, j_*) \neq (i, j)$, $i_* \neq j_*$ and σ can either be 1 or -1 depending on the values of i, j, i_* and j_*. The second and third term in (29) are obtained with the product rule on the second term in (19). Equation (30) is similarly obtained from the third term in (19).

To investigate the properties of the equilibrium $\bar{q}_e$, consider the following Lyapunov function candidate

$$V_{\bar{q}_e} = (\bar{q} - \bar{q}_e)^T(\bar{q} - \bar{q}_e) \quad (31)$$

whose derivative along the solution of (31) satisfies

$$\dot{V}_{\bar{q}_e} = -2c\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(q_{ij} - q_{e,ij})^T \left(\mathbf{I}_{n_w\times n_w} + N\mathbf{I}_{n_w\times n_w}\, U'_{ob,ij}\big|_{q_{ij}=q_{e,ij}} + N\, U''_{ob,ij}\big|_{q_{ij}=q_{e,ij}}\, q_{e,ij}q_{e,ij}^T\right)(q_{ij} - q_{e,ij}) \quad (32)$$

[2]Hence, E may be seen as the number of edges in a fully connected net, with each robot represented as a node.

Since $U'_{ob,ij}\big|_{q_{ij}=q_{tg,ij}} = 0$ and $U''_{ob,ij}\big|_{q_{ij}=q_{tg,ij}} \geq 0$, substituting $\bar{q}_e = \bar{q}_{tg}$ into (32) gives

$$\begin{aligned}\dot{V}_{\bar{q}_{tg}} &= -2c \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (q_{ij} - q_{tg,ij})^T \\ &\quad \left(\mathbf{I}_{n_w \times n_w} + N\, U''_{ob,ij}\big|_{q_{ij}=q_{tg,ij}} q_{tg,ij} q_{tg,ij}^T\right) \\ &\quad (q_{ij} - q_{tg,ij}) \\ &\leq -2c \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (q_{ij} - q_{tg,ij})^T (q_{ij} - q_{tg,ij})\end{aligned} \quad (33)$$

which clearly indicates that $\bar{q}_{tg}$ is asymptotically stable.

To show that the remaining critical points of the system ($\bar{q}_c$) are unstable equilibrium points, consider the following.

$$\bar{q}_c^T F(\bar{q}_c, \bar{q}_{tg}) = 0 \quad (34)$$

$$\begin{aligned}\Rightarrow\ & \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \Big(q_{c,ij}^T (q_{c,ij} - q_{tg,ij}) \\ & + N\, U'_{ob,ij}\big|_{q_{ij}=q_{c,ij}} q_{c,ij}^T q_{c,ij}\Big) = 0 \\ \Rightarrow\ & \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(1 + N\, U'_{ob,ij}\big|_{q_{ij}=q_{c,ij}}\right) q_{c,ij}^T q_{c,ij} \\ & = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} q_{c,ij}^T q_{tg,ij}\end{aligned} \quad (35)$$

Consider the term $q_{c,ij}^T q_{tg,ij}$ and the agents i and j. The agent j can be seen as an obstacle situated at $q_{ij} = 0$. Similarly, agent i is an obstacle with respect to j at $q_{ji} = 0$. At $q_{ij} = q_{c,ij}$, both agents are at their critical points. For this to hold, both critical points must lie along a straight line along the vector $q_{tg,ij}$ and between $q_{tg,i}$ and $q_{tg,j}$. That is, the point $q_{ij} = 0$ must lie between the points $q_{ij} = q_{tg,ij}$ and $q_{ij} = q_{c,ij}$, and such that these 3 points are colinear. Thus, the term $\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} q_{c,ij}^T q_{tg,ij}$ is strictly negative and there exists at least one pair (i, j) denoted by (i^*, j^*) such that

$$1 + N\, U'_{ob,i^*j^*}\big|_{q_{i^*j^*}=q_{c,i^*j^*}} \leq -b \quad (36)$$

where b is a strictly positive constant. Substituting $\bar{q}_e = \bar{q}_c$ into (32) gives

$$\begin{aligned}\dot{V}_{\bar{q}_c} &= -2c \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (q_{ij} - q_{c,ij})^T \\ &\quad \Big(\mathbf{I}_{n_w \times n_w} + N\mathbf{I}_{n_w \times n_w}\, U'_{ob,ij}\big|_{q_{ij}=q_{c,ij}} \\ &\quad + N\, U''_{ob,ij}\big|_{q_{ij}=q_{c,ij}} q_{c,ij} q_{c,ij}^T\Big) (q_{ij} - q_{c,ij}) \\ &\geq 2cb(q_{i^*j^*} - q_{c,i^*j^*})^T (q_{i^*j^*} - q_{c,i^*j^*}) \\ &\quad -2c \sum_{i=1, i\neq i^*}^{N-1} \sum_{j=i+1, j\neq j^*}^{N} (q_{ij} - q_{c,ij})^T \\ &\quad \left(\mathbf{I}_{n_w \times n_w} + N\mathbf{I}_{n_w \times n_w}\, U'_{ob,ij}\big|_{q_{ij}=q_{c,ij}}\right) (q_{ij} - q_{c,ij}) \\ &\quad -2c \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (q_{ij} - q_{c,ij})^T \\ &\quad \left(N\, U''_{ob,ij}\big|_{q_{ij}=q_{c,ij}} q_{c,ij} q_{c,ij}^T\right) (q_{ij} - q_{c,ij})\end{aligned} \quad (37)$$

Considering a subspace such that $q_{ij} = q_{c,ij}\ \forall (i, j) \in \{1, \ldots, N\}, (i, j) \neq (i^*, j^*)$ and $(q_{ij} - q_{c,ij})^T q_{c,ij} q_{c,ij}^T (q_{ij} - q_{c,ij}) = 0,\ \forall (i, j) \in \{1, \ldots, N\}$. In this subspace, the following holds

$$V_{\bar{q}_c} = (q_{i^*j^*} - q_{c,i^*j^*})^T (q_{i^*j^*} - q_{c,i^*j^*}) \quad (38)$$

$$\dot{V}_{\bar{q}_c} \geq 2bc(q_{i^*j^*} - q_{c,i^*j^*})^T (q_{i^*j^*} - q_{c,i^*j^*}) \quad (39)$$

which indicates that $\bar{q}_c$ is unstable. ■

For practical implementation, a robot r_i may not have information from robot out of its communications range and can only compute an approximate value of Ω_i, given by

$$\hat{\Omega}_i = (q_i - q_{tg,i}) + \sum_{j\neq i, j\in R_i} U'_{ob,ij} q_{ij} \quad (40)$$

where R_i is the set of robots within the d_i-neighborhood of r_i, and the control law becomes

$$\dot{\hat{u}} = -C\hat{\Omega} \quad (41)$$

The approximation error for each robot is

$$\begin{aligned}e_\Omega &= \Omega - \hat{\Omega} \\ &= \sum_{j\neq i, j\in R_{ni}} U'_{ob,ij} q_{ij}\end{aligned} \quad (42)$$

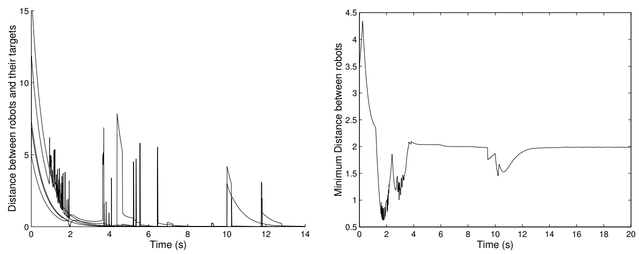
where $R_{ni} = R \setminus R_i$ is the set of robots that r_i cannot communicate with. From property (e) of $U_{ob,ij}$, we know that for $j \in R_{ni}$, $U_{ob,ij}, U_{ob,ij} \approx 0$. In addition, assuming that $\|q_{ij}(0)\|$ is bounded, since the robots converge to their targets on the queues and $\|q_{tg,ij}\|$ is also bounded, the value of e_Ω is bounded by some small positive real value, and the error that arises due incomplete information from robots out of communication range can be kept relatively small through the use of f_{ij} to weight the importance of repulsive forces between robots.

IV. Simulation Studies

Simulations consist of five circular, omni-directional robots, each of diameter 0.3m, with control input u_i in (41) using $\hat{\Omega}_i$ in (40). The parameters a_t, d_{ir} and C are chosen to be 10, 2 and the identity matrix respectively. It is assumed that each robot is able to localize itself in the global frame. Furthermore, each robot is equipped with a laser scanner (180°) and 16 sonar range sensors arranged in a ring around the circular robots for obstacle avoidance. The sensor noise introduced into the range sensing has a normal distribution of 0.2 variance. The communication range of the robots is set to 3m.

A. *Formation Convergence and Scaling*

This section examines the convergence of robots to a wedge formation, and how it scales when 2 robots are removed (deactivated) at $t = 10s$. In the final formation, robots are to be 2m apart. The robots are initialized at random positions in a 20m×20m square around the point (10m,10m). Figure 1(a) shows how the distance of the robots from their targets vary over time.



(a) Convergence of Robots (b) Minimum Inter-Robot Separation

Fig. 1. Robot convergence to formation with robot deactivation/removal at $t = 10$s.

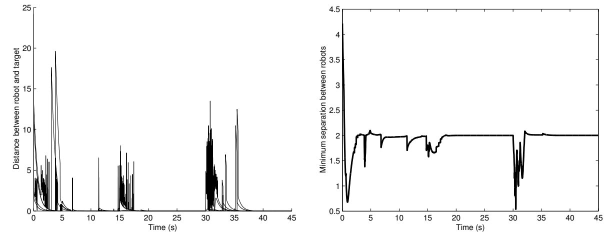
Figure 1(b) shows the minimum center-to-center distance between any two robots at each time. The minimum distance between any two robots is always greater than 0.5m and no collisions occur. Figure 1(a) shows the distance of each robot from their target at each time instant. The spikes in the graphs are the result of changes in the targets for each robot (according to Algorithm 1) as they interact with others within communication range. These spikes, however, cease to appear when the robots get within communication range of each other and their targets reach a constant state. This is further evidenced by the absence of spikes when scaling occurs at $t = 10s$, and the robots converge to their new targets.

B. *Changing Formations*

This section examines the effect of formation changes from a wedge, to a column (perpendicular to the orientation of the target), and finally to a line (parallel to the target's orientation). The results are shown in Fig. 2. Spikes are observed in the graphs at the times when formation changes is initiated, occurring due to the abrupt change in targets. Comparing the second and third clusters of spikes, it can be seen that changing from a column to a line is more disruptive due to the further distances to the new targets. On the whole, the team requires an average of transition time of 4 to 6 seconds.

V. Conclusions

In this paper, the original scheme using the Q-structure has been extended to improve the performance when only local



(a) Convergence of Robots (b) Minimum Inter-Robot Separation

Fig. 2. Robot convergence to formation with formation switching. Wedge: $t = [0s, 15s)$, Column: $t = [15s, 30s)$, Line: $t = [30s, 45s)$

communication is present in a weakly connected network. This is a more realistic environment compared to approaches that require persistent global communications, which is seldom achievable in real world applications. In addition, a dynamic target assignment strategy has been proposed, based on Q-structures, that aims to guide robots into appropriate positions in the required formations. Lastly, we examined the convergence properties of the proposed approach, and further verified the effectiveness of our approach with realistic simulations.

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