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Improving Regulation of a Single-Link Flexible Manipulator with Strain Feedback

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Abstract— This paper considers improving the tip regulation performance of a joint-PD controlled single-link flexible manipulator by introducing nonlinear strain feedback. The controller is developed by applying Lyapunov's direct method. The stability of the closed-loop system is theoretically proven based on the partial differential equations (PDE's) which govern the motion of the flexible robot, instead of using the traditional truncated models. The controller is very simple in its form, and only the measurements of joint angle, joint velocity, and strain of the bending beam are needed for implementation. The controller is very robust as well because it is independent of system parameters.

Index Terms—Flexible manipulators, nonlinear strain feedback, regulation.

I. INTRODUCTION

Controller design of flexible manipulators is one of the most challenging problems in control system design. The inherent non-minimum phase behavior of the flexible manipulator system makes it very difficult to achieve high level performance and robustness simultaneously [1].

For the methods of collocating the sensors and actuators at the joint of a flexible manipulator, for example, the joint PD control, only a certain degree of robustness of the system can be guaranteed. Actually, as mentioned in [1] and [2], the robustness of collocated controllers comes directly from the energy dissipating configuration of the resulting system. However, the performance of the flexible system with only a collocated controller, for example, the joint PD controller is often not very satisfactory because the elastic modes of the flexible beam are seriously excited and not effectively suppressed. For this, various kinds of control techniques have been investigated to improve the performance of flexible systems.

Generally speaking, the desired tip regulation performance of a flexible manipulator can be described as

- 1) the joint motion converges to the final position fast;
- 2) the elastic vibrations are effectively suppressed.

Obviously there is a tradeoff between the two requirements. The above description directly leads to the application of the singular perturbation method to the control of flexible robots [4]–[7]. In this method, the dynamics of the system is divided into two parts, i.e., a slow sub-system (corresponding to the joint motion) and a fast sub-system (related to the flexible vibrations), and two sub-controllers are designed accordingly. Since what we care about most in tip regulation is the tip motion of the flexible beam, inverse dynamics (computed torque) method, which has been shown effective in control of rigid-link manipulators, is also investigated for flexible-link robots [15], [16]. This method received increasing attention in recent years due to the rapid development in powerful computing hardware. The method seems to be able to result in better tip motions over other techniques. However, the successful application of this method heavily depends

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on highly accurate models. Other approaches to improve position control performance include linear control [8], robust control [9], sliding mode method [10], feedback control [11], [12], and input pre-shaping approach [13], [14], and among others. It should be noted that the control approaches mentioned above are all based on the truncated models from modal analysis [assumed modes method (AMM)] or finite element method (FEM). It is the truncation of the original infinite dimensional model of a flexible manipulator system to a finite dimensional model that makes the above mentioned control techniques applicable. However, the following problems inevitably arise:

- 1) a relatively high order controller (corresponding to the model with more flexible modes) is often necessary to achieve high accuracy of the performance;
- 2) control and observation spillovers may occur due to the ignored high frequency dynamics [2];
- 3) the controllers may be difficult to implement from the engineering point of view since full states measurements/observers are often required.

An alternative approach is to derive controllers directly from the PDE of the considered flexible systems and thus avoid the undesired model truncation [2], [3], [17]. In [2], *linear* direct strain feedback (DSFB) at the base of the bending beam was considered to enhance the performance of the joint PID controller of a single-link flexible robot. By using some results presented in [23], tracking control of a Euler-Bernoulli beam is discussed in [3]. However, an assumption was made that there is no tip payload and no motor hub inertia. The stability of collocated joint PD controller was discussed for a two-link unloaded rigid-flexible manipulator in [17]. In the three papers, the stability of the closed-loop systems is analyzed directly based on the PDE's which govern the motion of the considered flexible systems. Subsequently, their controllers can avoid the above mentioned problems associated with model truncation.

In this paper, a simple *nonlinear* controller for tip regulation is presented for a single-link flexible manipulator with a point mass tip payload. The controller is totally independent of system parameters and thus robust to system parameters uncertainties. Numerical simulations show that the controller can damp out the flexible vibrations effectively and drives the tip of the flexible beam to a pre-defined position very fast along a smooth trajectory. The closed-loop stability of the system is proven based on the PDE's of the system dynamics. Therefore, the drawbacks of truncated-model-based approaches are avoided. Further, the controller only requires the measurements of joint angle, joint velocity and strain of the beam, which leads to an easy engineering implementation.

The rest of the paper is organized as follows: in Section II, the dynamics of the single-link flexible manipulator is given in the form of PDE's. The new regulation controller is presented in Section III. Numerical simulations are carried out to verify its effectiveness in Section IV, followed by the conclusion and further research in Section V.

II. DYNAMICS OF A SINGLE-LINK FLEXIBLE ROBOT

As shown in Fig. 1, the flexible manipulator is rotating in the horizontal plane and the effect of gravity is not taken into consideration. Frame XOY is the fixed base frame and frame xOy is the local frame rotating with the hub. System parameters and variables are defined as: L , the length of the beam; EI , the uniform flexural rigidity of the beam; M_t , the point mass tip payload; I_h , the hub inertia; τ , the control torque; $\theta(t)$, the joint angle; $y(x, t)$, the elastic deflection measured from the undeformed beam.

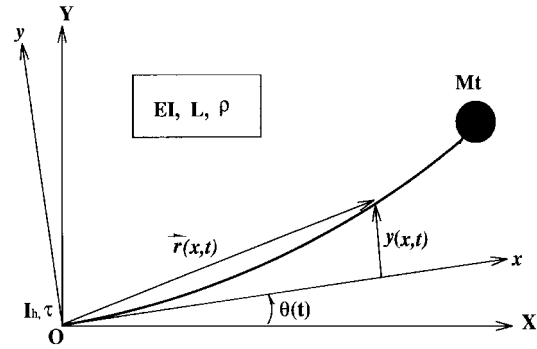


Fig. 1. Flexible manipulator system.

Let $p(x, t) := x\theta(t) + y(x, t)$ represent the position of a point on the flexible beam. The total kinetic energy E_k can be calculated by

$$E_k = \frac{1}{2} I_h \dot{\theta}^2 + \frac{\rho}{2} \int_0^L \dot{p}^2(x, t) dx + \frac{1}{2} M_t \dot{p}^2(L, t) \quad (1)$$

and the total potential energy E_p is given by

$$E_p = \frac{EI}{2} \int_0^L [y''(x, t)]^2 dx. \quad (2)$$

According to the derivation presented in [8], [11], we have the following PDE's of the system dynamics:

$$(I_h + \frac{1}{3}\rho L^3)\ddot{\theta}(t) + \rho \int_0^L x\ddot{y}(x, t) dx + M_t L[L\ddot{\theta}(t) + \ddot{y}(L, t)] = \tau \quad (3)$$

$$\rho[x\ddot{\theta}(t) + \ddot{y}(x, t)] = -EI y''''(x, t). \quad (4)$$

It is noted that the base bending moment of the flexible link can be calculated by

$$EI y''(0, t) = -\frac{1}{3}\rho L^3 \ddot{\theta}(t) - \rho \int_0^L x\ddot{y}(x, t) dx - M_t L[L\ddot{\theta}(t) + \ddot{y}(L, t)].$$

Thus, (3) can be reduced to

$$I_h \ddot{\theta} = \tau + EI y''(0, t).$$

The corresponding boundary conditions are given by the following set of equations:

$$y(0, t) = 0 \quad (5)$$

$$y'(0, t) = 0 \quad (6)$$

$$y''(L, t) = 0 \quad (7)$$

$$EI y'''(L, t) = M_t [L\ddot{\theta}(t) + \ddot{y}(L, t)]. \quad (8)$$

III. CONTROLLER DESIGN

In this paper, we shall only investigate the regulation of the above flexible manipulator. Therefore, the final position of the flexible beam

can be described by $\theta = \theta_f$ and $y(x, t) \equiv 0$. Obviously, the tip position $p(L, t)$ should equal $L\theta_f$ at the final point.

Though simple joint PD controller was shown to be able to stabilize the closed-loop system [17], the vibration of the flexible beam was not effectively controlled. In this paper, nonlinear strain feedback is introduced to improve the performance of the simple joint PD controller. The controller is given by

$$\tau = -k_p[\theta(t) - \theta_f] - k_v\dot{\theta}(t) - k_f y''(x_s, t) - \int_0^t \dot{\theta}(s) y''(x_s, s) ds \quad (9)$$

where k_p, k_v are positive constants and $k_f \geq 0$, and $0 \leq x_s < L$ which gives the location of the strain gauge on the beam. Obviously, $k_f = 0$ leads to the traditionally used joint PD controller of the flexible beam. From (9), one can easily see that only the measurements of joint angle $\theta(t)$, joint velocity $\dot{\theta}(t)$, and strain signal at $x = x_s$, i.e., $y''(x_s, t)$ are needed. Therefore, the controller is very easy to implement from the engineering point of view. The stability of the closed-loop system is stated by the following theorem.

Theorem 1: The closed-loop system described by (3)–(8) and (9) is stable.

Proof: See the Appendix.

Remarks:

- 1) The joint PD controller is a special case of controller (9) by setting k_f to be zero. Accordingly, the Lyapunov function candidate $V(t)$ in (11) is reduced to

$$V(t) = E_k + E_p + \frac{1}{2}k_p[\theta(t) - \theta_f]^2.$$

Although the joint PD controller will not destabilize the system, the system performance, as we will show in the next section, is not satisfactory because the flexible modes of the beam are seriously excited and not effectively suppressed. The introducing of the k_f term into $V(t)$ (11) allows us to explicitly evaluate the vibration of the flexible beam in the Lyapunov function, and subsequently the controller (9) can be expected to have direct control effect on elastic vibration and thus improve the regulation performance.

- 2) The integral type of the k_f item avoids measurements of high order signals, such as $\ddot{\theta}$ and $\dot{y}''(x_s, t)$ in the resulting controller. This is very desirable for easy engineering implementation.
- 3) The controller (9) is independent of system parameters and thus possesses stability robustness to system parameters uncertainties. As a matter of fact, the closed-loop system is stable as long as $k_p, k_v > 0$ and $k_f \geq 0$.
- 4) From the proof in the Appendix, it is easy to see that the possible measurement noise existing in $y''(x_s, t)$ feedback will not destroy the closed-loop stability. This is a very desirable feature in practical applications.
- 5) The controller is easy to implement since only measurements of $\theta, \dot{\theta}$, and strain feedback $y''(x_s, t)$ are required. Unlike the full states feedback controllers suggested in the literature [4]–[9], full states measurements/observers are not necessary.
- 6) The stability proof is based on the dynamics (PDE's) of the system and thus the problems associated with truncated-model-based controllers mentioned in Section I Introduction are avoided.

- 7) As a matter of fact, the controller can be generalized to cases of multiple strain feedbacks as follows:

$$\tau = -k_p[\theta(t) - \theta_f] - k_v\dot{\theta}(t) - \sum_{i=1}^N k_{fi} y''(x_{si}, t) - \int_0^t \dot{\theta}(s) y''(x_{si}, s) ds$$

where $k_{fi} \geq 0, i = 1, 2, \dots, N$, and x_{si} is the location of the i th strain gauge on the flexible beam. The stability of the closed-loop system can be guaranteed by choosing the following Lyapunov function:

$$V(t) = E_k + E_p + \frac{1}{2}k_p[\theta(t) - \theta_f]^2 + \frac{1}{2} \sum_{i=1}^N k_{fi} \left[\int_0^t \dot{\theta}(s) y''(x_{si}, s) ds \right]^2$$

and following the same proving procedure in the Appendix.

- 8) Tip acceleration are often used to improve the performance of the flexible manipulator systems in the literature [18]–[20]. If the tip acceleration feedback is available, a controller can be proposed based on the idea of Theorem 1 as

$$\tau = -k_p[\theta(t) - \theta_f] - k_v\dot{\theta}(t) - k_f a_{tip}(t) - \int_0^t \dot{\theta}(s) a_{tip}(s) ds$$

where a_{tip} is the tip acceleration which can be measured by an accelerometer attached at the tip of the flexible beam. The closed-loop system stability can be guaranteed by using the following Lyapunov function:

$$V(t) = E_k + E_p + \frac{1}{2}k_p[\theta(t) - \theta_f]^2 + \frac{1}{2}k_f \left[\int_0^t \dot{\theta}(s) a_{tip}(s) ds \right]^2.$$

- 9) Depending on the actual instrumentation of the system, a general stable controller can be obtained based on the idea of Theorem 1, remarks 7 and 8 for cases of multiple mixed measurement feedbacks. The controller is summarized in the following Lemma:

Lemma 1: The closed-loop system described by (3)–(8) and the following controller is stable

$$\tau = -k_p[\theta(t) - \theta_f] - k_v\dot{\theta}(t) - \sum_{i=1}^N k_{fi} f_i(x_{si}, t) - \int_0^t \dot{\theta}(s) f_i(x_{si}, s) ds \quad (10)$$

where, depending on the actual instrumentation, $f_i(x_{si}, t)$ can be $y''(x_{si}, t)$ (strain at $x = x_{si}$), $a(x_{si}, t)$ (acceleration at $x = x_{si}$), $y(x_{si}, t)$ (deflection at $x = x_{si}$), $y'(x_{si}, t)$ (rotation at $x = x_{si}$) or $y'''(x_{si}, t)$ (shearing force at $x = x_{si}$), etc. The only restriction on choosing $f_i(x_{si}, t)$ is that it must be zero when the flexible robot is static and undergoes no deformation. This restriction is reasonable since $f_i(x_{si}, t)$ is introduced to evaluate the elastic vibration of the flexible link.

- 10) In Theorem 1 and Lemma 1, only closed-loop stabilities are claimed. Because the systems considered in LaSalle's Theorem [21] is defined in R^n , the theorem is not directly applicable here. Due to the infinite dimensionality of the

system, it is difficult to prove the asymptotic stability, though some results exist [2], [3], using semigroups and among others. Asymptotic control of a flexible robot has been studied in [3], but some unrealistic assumptions were made, e.g., the hub inertia and the tip payload are all zero. Without these assumptions, rigorous proof of asymptotic stability is still difficult to achieve. Interested readers can refer to [23], [24] for discussion of asymptotic stability of some infinite dimensional systems.

In the following, we shall show that practically the asymptotic behavior of the flexible robot can be guaranteed, i.e., the robot can only possibly stop at the final position $\theta = \theta_f$ without vibrating. Consider the general form controller (10). Assume that the joint stops at a position $\theta \equiv \alpha$ (hence $\dot{\theta} \equiv 0$) with $\alpha \neq \theta_f$, thus there is no energy input to the system since $\dot{\theta} \equiv 0$. Due to the existence of internal structural damping in a flexible link in practice (structural damping is neglected in the proof of Theorem 1 and Lemma 1), the flexible robot must tend to stop vibrating and finally be static at the undeformed position. Consequently, the first term in (10) is a nonzero constant, the middle term is zero, and the last term approaches zero [note that $f_i(x_{si}, t)$ is selected such that it is zero when the link is not deformed]. Therefore, the controller in (10) approaches a nonzero constant and thus $\theta \equiv \alpha$ cannot hold. The only possibility is that the flexible link is at the final position $\theta \equiv \theta_f$ without vibrating, which implies the tip regulation is achieved.

Furthermore, it should also be noted that for any damped traditional truncated-model obtained by either AMM or FEM (the effect of internal structural damping has been modeled as a positive definite damping matrix), the controller can be easily shown globally asymptotically stable using the LaSalle's Theorem, since the system in this case has been reduced to a finite dimensional one. Interested readers can refer to [22], in which the asymptotic stability proof of a modified PD controller is given for flexible robots under gravity based on a damped truncated model.

IV. SIMULATION TESTS

In this section, numerical simulations are carried out to show the effectiveness of the controller (9) in suppressing the elastic vibration to obtain better tip regulation performance.

In the simulations, the system parameters are given such values: $L = 1.0$ m, $EI = 2.0$ Nm², $\rho = 0.1$ Kg/m, $I_h = 0.5$ Kgm², and $M_t = 0.05$ Kg. The plant is simulated by a four-mode assumed modes model and the final joint position is $\theta_f = 0.5$ rad. A fourth-order Runge–Kutta program with adaptive steps is used to numerically solve the differential equations. We shall firstly give the simulation results of the following joint PD controller

$$\tau_{PD} = -k_p(\theta - \theta_f) - k_v\dot{\theta}.$$

If the flexible beam is assumed to be rigid [17], i.e., $y(x, t) \equiv 0$, substituting of τ_{PD} into (3) yields the closed-loop joint error equation

$$(I_h + \frac{1}{3}\rho L^3 + M_t L^2)\ddot{e}(t) + k_v\dot{e}(t) + k_p e = 0$$

where $e = \theta - \theta_f$. It should be noted that $\ddot{e} = \ddot{\theta}$ and $\dot{e} = \dot{\theta}$ since the final joint position θ_f is a constant. For this second-order system, if critical damping is assumed ($\xi = 1$) and the natural frequency $\omega_n = 2.5$, we have the following PD controller:

$$\tau_{PD1} = -3.65(\theta - \theta_f) - 2.92\dot{\theta}.$$

For a faster joint motion, $\omega_n = 5.0$, we have

$$\tau_{PD2} = -14.58(\theta - \theta_f) - 5.83\dot{\theta}.$$

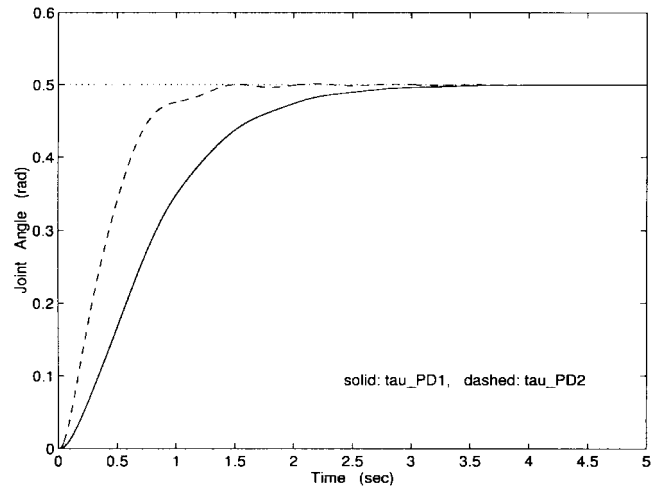


Fig. 2. Joint angle trajectory $\theta(t)$, solid: τ_{PD1} , dashed: τ_{PD2} .

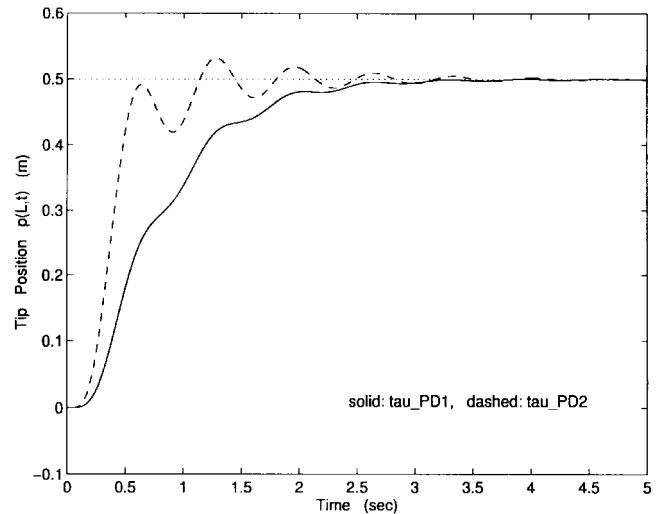


Fig. 3. Tip motion trajectory $p(L, t) = L\theta + y(L, t)$, solid: τ_{PD1} , dashed: τ_{PD2} .

The joint trajectories and tip motion trajectories for τ_{PD1} (solid lines) and τ_{PD2} (dashed lines) are shown in Figs. 2 and 3, while the corresponding control signals are given in Fig. 4. It is seen that faster joint motion (Fig. 2) is obtained at the expense of larger oscillation of the tip trajectory $p(L, t)$. Therefore, although joint PD controller can guarantee the stability of the closed-loop system, the elastic vibration of the flexible beam still needs to be effectively suppressed to achieve high speed and accurate tip regulation.

In the following, we shall show the control results of the controller (9). k_p and k_v are selected to be the same as in τ_{PD2} for comparison. The strain gauge can be attached at $0 \leq x_s < L$ on the beam [note $y''(L, t) = 0$]. Considering the fact that usually the base-strain $y''(0, t)$ is comparatively large, it is used here for feedback control. A comparatively large sensor output will increase the signal to noise ratio and benefit the measurement. The controller is given by

$$\tau = \tau_N = -14.58(\theta - \theta_f) - 5.83\dot{\theta} - k_f y''(0, t) \int_0^t \dot{\theta}(s) y''(0, s) ds.$$

Three simulations are carried out for $k_f = 200, 500$, and 1000 and are shown by dashed, solid and dash-dotted lines, respectively. Joint trajectories, tip motions, control torques and the base-strain feedback

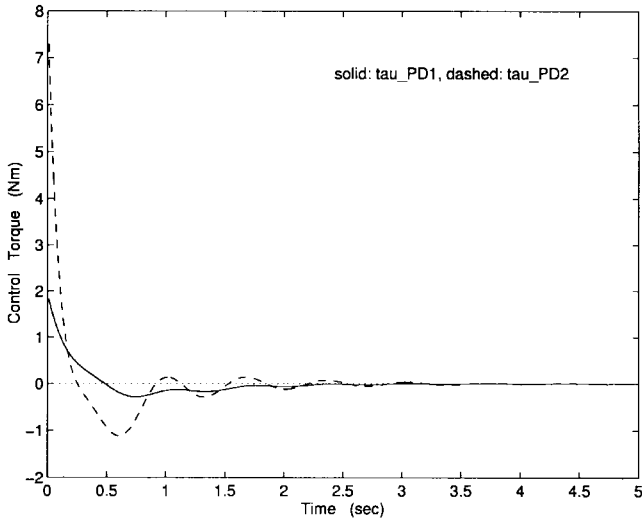


Fig. 4. Control torque τ , solid: τ_{PD1} , dashed: τ_{PD2} .

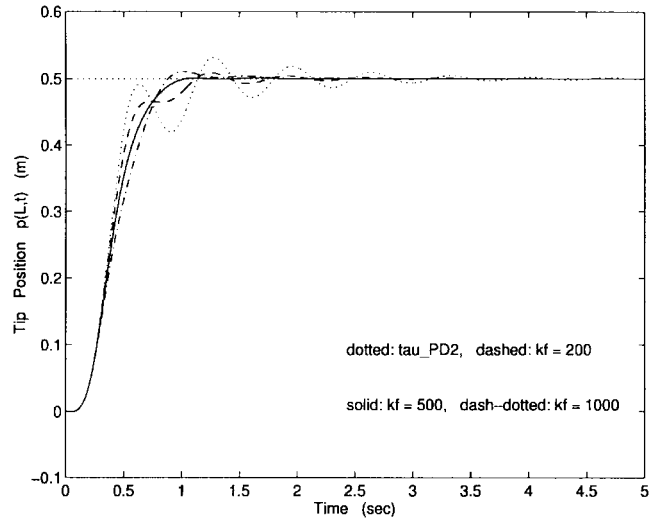


Fig. 6. Tip motion trajectory $p(L, t) = L\theta + y(L, t)$, dotted: τ_{PD2} , dashed: $k_f = 200$, solid: $k_f = 500$, dash-dotted: $k_f = 1000$.

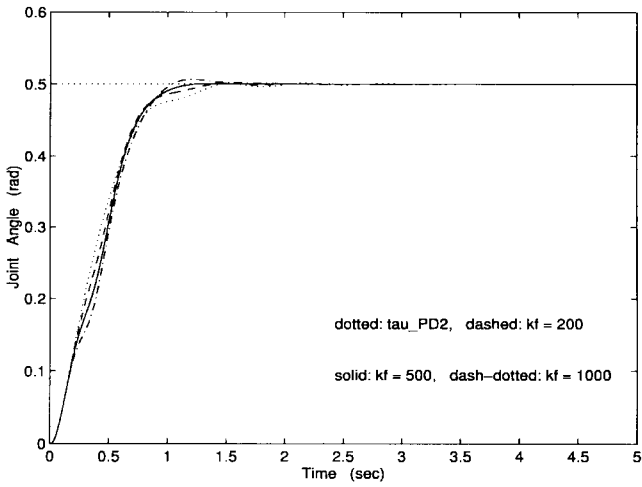


Fig. 5. Joint angle trajectory $\theta(t)$, dotted: τ_{PD2} , dashed: $k_f = 200$, solid: $k_f = 500$, dash-dotted: $k_f = 1000$.

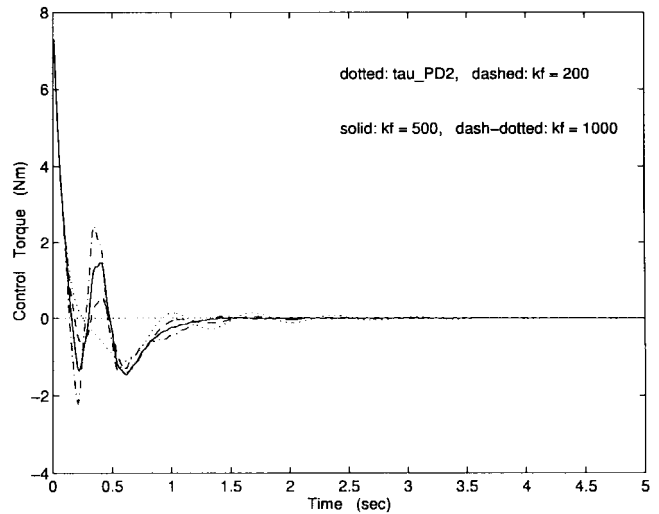


Fig. 7. Control torque τ , dotted: τ_{PD2} , dashed: $k_f = 200$, solid: $k_f = 500$, dash-dotted: $k_f = 1000$.

are shown in Figs. 5–8. The corresponding results of τ_{PD2} are also plotted in these figures by dotted lines for easy comparison.

Comparing with the results of the joint PD controller τ_{PD2} (the dotted lines), it can be seen that the system performance is improved because of the introduction of the k_f item in the sense of smaller tip deflection and less oscillations in the tip motion $p(L, t)$ without reducing the converging speed of joint angle $\theta(t)$.

It was found that the performance is very good when $k_f = 500.0$ shown by the solid lines in comparison with that when $k_f = 200.0$ shown by the dashed lines. The joint motion (Fig. 5) is retained to be fast, and the tip of the flexible beam (what we care about most in tip regulation) moves to the final position along a smooth trajectory very fast with only negligible overshoot (Fig. 6).

When a large gain $k_f = 1000.0$ was used, the system performance (dash-dotted lines) shows no much differences from that of $k_f = 500.0$'s except that the overshoot of $p(L, t)$ becomes visible though it is still very small. The strain feedbacks at the base of the flexible beam, $y''(0, t)$, are given in Fig. 8. As stated above, the strain feedback is introduced to representing the bending of the flexible beam. From Fig. 8, it can be seen that the base bending strain is effectively suppressed after introducing the nonlinear base-strain feedback.

V. CONCLUSION

In this paper, nonlinear base-strain feedback is introduced to improve the tip regulation performance of a flexible manipulator. Closed-loop system stability is proven directly from the PDE's of the system, and thus avoiding the problems of the traditional truncated-model-based approaches. The asymptotic stability can be practically achieved. A general form controller is also presented, in which various kinds of feedback can be used according to actual instrumentation. The controller is robust in terms of parameters uncertainties. Numerical simulations showed that the vibrations of flexible beam are effectively suppressed and the tip of the flexible beam moves to the final position very fast along a smooth trajectory with negligible overshoot.

The introduction of nonlinear feedback term of $f_i(x_{si}, t)$ is motivated by the fact that the traditional joint PD controller does not take into consideration the link's flexibility. However, we currently do not have explicit guidelines on choosing $f_i(x_{si}, t)$ to obtain the most effective control effort and to optimize the system's performance. The base-strain is used in this paper as a most easily obtainable measurement, and just to verify the effectiveness of the presented

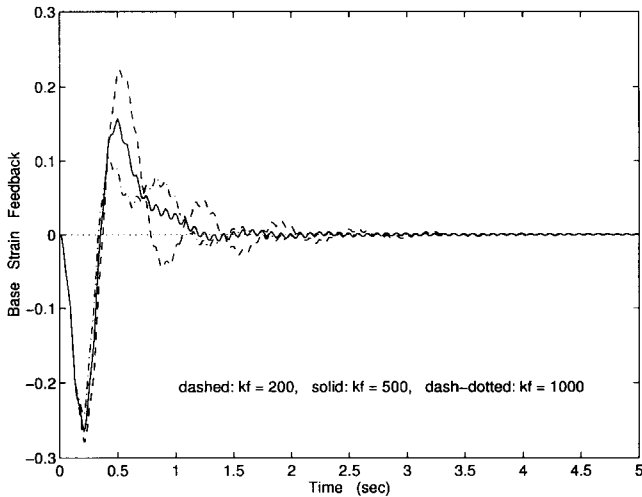


Fig. 8. Strain at the base $y''(0, t)$, dashed: $k_f = 200$, solid: $k_f = 500$, dash-dotted: $k_f = 1000$.

approach. The effect of the $f_i(x_{si}, t)$ feedback on the system dynamics shall be deeply investigated in our future research. Further work shall also be carried out to extend the current results to multi-link flexible manipulators.

APPENDIX PROOF OF THEOREM 1

Consider the following Lyapunov function candidate:

$$V(t) = E_k + E_p + \frac{1}{2}k_p[\theta(t) - \theta_f]^2 + \frac{1}{2}k_f \left[\int_0^t \dot{\theta}(s)y''(x_s, s) ds \right]^2 \quad (11)$$

where E_k and E_p are the total kinetic energy and the total potential energy of the system given in (1) and (2). Obviously, $V(t)$ is positive definite. The time derivative of $V(t)$ is

$$\dot{V}(t) = k_p \dot{\theta}(t)[\theta(t) - \theta_f] + k_f \dot{\theta}(t)y''(x_s, t) \int_0^t \dot{\theta}(s) \cdot y''(x_s, s) ds + \dot{\theta}(t)\tau \quad (12)$$

in which we have used

$$\dot{E}_k + \dot{E}_p = \dot{\theta}\tau \quad (13)$$

which can be proven as follows: Firstly, we assume that the position variable $p(x, t)$ satisfies

$$\frac{d}{dt} \int_0^L \dot{p}^2(x, t) dx = \int_0^L \left[\frac{d}{dt} \dot{p}^2(x, t) \right] dx \quad (14)$$

which was also assumed in [3]. Then, the time derivative of E_k (1) is given by

$$\begin{aligned} \dot{E}_k &= I_h \dot{\theta} \ddot{\theta} + \rho \int_0^L \dot{p}(x, t) \ddot{p}(x, t) dx + M_t \dot{p}(L, t) \ddot{p}(L, t) \\ &= I_h \dot{\theta} \ddot{\theta} + \rho \int_0^L (x \dot{\theta} + \dot{y})(x \ddot{\theta} + \ddot{y}) dx \\ &\quad + M_t (L \dot{\theta} + \dot{y}_L)(L \ddot{\theta} + \ddot{y}_L) \\ &= I_h \dot{\theta} \ddot{\theta} + \frac{1}{3} L^3 \rho \dot{\theta} \ddot{\theta} + \rho \dot{\theta} \int_0^L x \ddot{y} dx \\ &\quad + M_t \dot{\theta} L (L \ddot{\theta} + \ddot{y}_L) + \Delta \end{aligned} \quad (15)$$

where $y_L := y(L, t)$ for simplicity, and Δ is given by

$$\Delta = \rho \int_0^L \dot{y}(x \ddot{\theta} + \ddot{y}) dx + M_t \dot{y}_L (L \ddot{\theta} + \ddot{y}_L).$$

Comparing (3) and (15), we have

$$\dot{E}_k = \dot{\theta}\tau + \Delta. \quad (16)$$

From (4) and (8), Δ can be further written as

$$\begin{aligned} \Delta &= -EI \int_0^L \dot{y} y'''' dx + EI \dot{y}_L y'''' \\ &= EI \left[-(\dot{y}_L y'''' - \dot{y}_0 y_0''') + \int_0^L y'''' d\dot{y} + \dot{y}_L y'''' \right] \\ &= EI \left[\int_0^L y'''' d\dot{y} + \dot{y}_0 y_0'''' \right] \\ &= EI \left[\int_0^L \dot{y}' d y'' + \dot{y}_0 y_0'''' \right] \\ &= EI \left[\dot{y}'_L y''_L - \dot{y}'_0 y''_0 - \int_0^L y'' y'' dx + \dot{y}_0 y_0'''' \right] \end{aligned}$$

where $y_0 := y(0, t)$ for simplicity. From the boundary conditions (5)–(7), we obtain

$$\Delta = -EI \int_0^L y'' y'' dx = -\dot{E}_p.$$

Thus, we arrive at $\dot{E}_k + \dot{E}_p = \dot{\theta}\tau$ and subsequently obtain (13). As a matter of fact, (13) can also be obtained from the analysis of the system energy. Since no damping is considered in the system (3)–(8), we can conclude that the total change in energy of the system must be equal to the work done by the motor torque, i.e.,

$$E_k - E_{k0} + E_p - E_{p0} = \int_0^t \tau \dot{\theta} dt \quad (17)$$

where E_{k0} and E_{p0} are the kinetic energy and the potential energy of the system at the initial moment. Thus, (13) can be easily obtained by taking derivative on both sides of (17) with respect to the time. This fact also verifies that (14) is the properties of $p(x, t)$ in the point-to-point control of the flexible beam.

Now, substituting controller (9) into (12), we have

$$\dot{V}(t) = -k_v \dot{\theta}^2(t)$$

which is negative semi-definite. Thus, the closed-loop system is stable. (QED)

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Controllability of Grasps and Manipulations in Multi-Fingered Hands

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Abstract—Manipulation of objects utilizing gravity and using general equilibrium grasps, which are not necessarily force-closure, is discussed. Examining the controllability of the object's dynamics, in the presence of gravity, leads to the conclusion that almost all equilibrium grasps are locally controllable. This fact is used to show that manipulation from any one equilibrium point, to any other, is possible if there is a continuity of equilibrium points between them. In addition, the equilibrium grasps can be used for changing grasps with Walking Finger Manipulation. The system controllability matrix is also used to test grasp quality, depending not only on kinematic values but also on object orientation and dynamic properties.

Index Terms—Controllability, equilibrium, grasping, gravity, manipulation.

I. INTRODUCTION

A grasp can be described as a combination of contacts between the fingers and the held object. Stability of a grasp has two common definitions, that of force-closure [1]–[4] and that of energetic stability [5]–[8]. The former definition, which is completely static, relates to the ability of the fingers to apply an arbitrary force and moment, on the object, and, therefore, be able to counteract any disturbance. The second definition of stability relates to the ability of the fingers to return the object to an initial grasp after perturbing from it. The question asked in this case, is how the fingers should be controlled so that the forces they exert preserve stability. This definition is more conservative than that of force-closure.

A number of papers have dealt with optimal selection of grasps (see [9] for a review). These analyses are, once again, static, examining only the effects of kinematic parameters on grasp quality.

Studies on object manipulation have dealt more extensively with object and finger dynamics. Li *et al.* [10] developed a computed torque controller for the fingers, that takes into account the dynamics of the fingers and of the object. Gravity is ignored. The controller design assumes that every grasp is both stable and manipulable.

Substantial grasp changes can be obtained using "Walking Finger Manipulation" in which the fingers walk on the object surface until they reach a required grasp. Elci *et al.* [11] examined grasp changes using three fingers that maintain force-closure at all times. This study was extended by Brook *et al.* [12] for four fingers and arbitrarily small friction.

When dealing with force-closure, gravity is considered a disturbing force that can be canceled by the grasping forces. Yet, the relative location of the object's center of mass, with respect to the contact points, does have an effect on the control signal required for stabilizing the object. In addition, for manipulation purposes, gravity can be used as an assisting force [13]–[15].

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