



## Brief Paper

Nonregular feedback linearization for a class of second-order nonlinear systems<sup>☆</sup>

S. S. Ge\*, Zhendong Sun, T. H. Lee

*Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117576*

Received 16 June 2000; revised 17 January 2001; received in final form 9 April 2001

**Abstract**

In this paper, a class of second-order nonlinear cascade control systems is considered. Under certain structural assumptions, it is proven that these systems are exactly linearizable via nonregular static state feedbacks and state diffeomorphisms. Linearizing input transformations and the corresponding state diffeomorphisms are presented. Finally, nonregular static feedback linearization is applied to a class of flexible joint robots, and the controller constructed is globally asymptotically stabilizing. © 2001 Elsevier Science Ltd. All rights reserved.

*Keywords:* Nonlinear systems; Feedback linearization; Nonregular state feedback; Robot control; Global stability

**1. Introduction**

During the last decades, nonlinear systems and control theory have witnessed tremendous development. As one of the most active research areas, feedback linearization is a powerful tool for control and synthesis of nonlinear systems, and has been widely applied to many engineering systems, for example, electrical drivers (Taylor, 1994; Bodson & Chiasson, 1998), rigid and flexible joint robots (Spong & Vidyasagar, 1989; De Luca, 1998a), spacecrafts (Sheen & Bishop, 1994; Lee & Cochran, 1997), to list a few.

Feedback linearization involves transforming a nonlinear system into a controllable linear one by using state feedback and coordinate transformations. This problem has been studied using increasingly more general feedback transformations. Regular static state feedback linearization was solved in Brockett (1978) for single-input systems and in Jakubczyk and Respondek (1980) for multi-input systems. Regular dynamic state feedback linearization was firstly proposed in Cheng (1987) and then developed in Guay, McLellan, and Bacon (1997) and the references therein. Recently, nonregular static/

dynamic state feedback linearization was introduced and addressed in Sun and Xia (1997).

Most existing results on feedback linearization are based on the (first-order) state-space representation of the plant. However, many practical engineering models, such as mechanical systems, are derived by Euler–Lagrange equations or Newton–Euler formulation which usually lead to second- or higher-order representation. For these high-order systems, the transformation to a state-space format, though straightforward, may obscure some relevant model structural properties and lead to complicated expressions (De Luca, 1998b). To this end, there is a demand to address feedback design problems based directly on the original high-order models rather than on the transformed state-space representation.

In this paper, we will present a new criterion for nonregular static state feedback linearization of a class of second-order systems. Under some structural assumptions, it is shown that these systems are nonregular static state feedback linearizable, and a linearizing state feedback as well as the corresponding coordinate transformations can be obtained. As an application, globally asymptotically stabilizing controllers are presented for a class of flexible joint robots.

This paper is organized as follows. Section 2 presents a linearizability criterion for a class of second-order systems. Its application to flexible joint robots is addressed in Section 3. The last section contains some concluding remarks.

<sup>☆</sup>This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor In-Joong Ha under the direction of Editor Hassan Khalil.

\*Corresponding author. Tel: + 65-874-6821; fax: + 65-779-1103.

E-mail address: eleges@nus.edu.sg (S. S. Ge).

**2. Main results**

**Definition 1.** An affine nonlinear control system

$$\dot{x} = f(x) + \sum_{i=1}^m g_i(x)u_i = f(x) + G(x)u, \quad x \in \mathfrak{R}^n \quad (1)$$

with  $f(0) = 0$  and  $\text{rank } G(0) = m$  is said to be (locally) nonregular static state feedback linearizable (at the origin), if it can be transformed into a controllable linear system

$$\dot{z} = Az + Bv$$

via a locally defined state diffeomorphism

$$z = \phi(x), \quad \phi(0) = 0 \quad (2)$$

and a locally defined nonregular static state feedback

$$u = \alpha(x) + \beta(x)v, \quad \alpha(0) = 0, \beta(x) \in \mathfrak{R}^{m \times l}, \quad v \in \mathfrak{R}^l, \quad l \leq m. \quad (3)$$

In nonregular static feedback (3), the gain matrix  $\beta(x)$  is not necessarily square, and even if it is square, it is not necessarily nonsingular at the origin. In other words, “nonregular” means “not necessarily regular”. It is readily seen that a regular static state feedback is also a nonregular static state feedback. Accordingly, the concept of nonregular state feedback linearization is a generalization of its regular counterpart.

For nonregular static state feedback linearization, some necessary conditions and sufficient conditions were presented in Sun and Xia (1997). It is also shown that regular dynamic state feedback linearizability does not imply nonregular static state feedback linearizability, but the reverse does not necessarily hold. In the sequel, a new linearizability criterion for a class of second-order nonlinear cascade systems will be presented.

Consider a second-order nonlinear system given by

$$\begin{aligned} \ddot{x} &= f(x, \dot{x}) + p(x, \dot{x})y + g(x, \dot{x})u, \\ \ddot{y} &= u, \end{aligned} \quad (4)$$

where  $x, \dot{x}, y, \dot{y} \in \mathfrak{R}^n$  are the states,  $u \in \mathfrak{R}^n$  are the inputs,  $f(x, \dot{x})$  is a smooth vector field,  $p(x, \dot{x})$  and  $g(x, \dot{x})$  are  $n \times n$  matrices of real-valued functions.

Let us study the linearizability of system (4) around an equilibrium point. Without loss of generality, assume the origin is an equilibrium point of the unforced system, that is  $f(0, 0) = 0$ . We made the following two assumptions on the system structures:

**Assumption 1.** Matrix  $p(x, \dot{x})$  has the upper triangular structure and is nonsingular at the origin, i.e.,

$$p(x, \dot{x}) = \begin{bmatrix} p_{1,1}(x, \dot{x}) & p_{1,2}(x, \dot{x}) & \cdots & p_{1,n}(x, \dot{x}) \\ 0 & p_{2,2}(x, \dot{x}) & \cdots & p_{2,n}(x, \dot{x}) \\ & & \ddots & \\ 0 & 0 & \cdots & p_{n,n}(x, \dot{x}) \end{bmatrix}, \quad p_{i,i}(0, 0) \neq 0, \quad i = 1, \dots, n. \quad (5)$$

**Assumption 2.** Matrix  $g(x, \dot{x})$  has the strictly upper triangular structure and is of rank  $n - 1$  at the origin, i.e.,

$$g(x, \dot{x}) = \begin{bmatrix} 0 & g_{1,2}(x, \dot{x}) & \cdots & g_{1,n}(x, \dot{x}) \\ 0 & 0 & \cdots & g_{2,n}(x, \dot{x}) \\ & & \ddots & \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad g_{i,i+1}(0, 0) \neq 0, \quad i = 1, \dots, n - 1. \quad (6)$$

**Theorem 1.** System (4) satisfying Assumptions 1 and 2 is nonregular static state feedback linearizable.

**Proof.** Denote  $\xi = [x^T, \dot{x}^T, y^T, \dot{y}^T]^T$ . Let us design the last  $n - 1$  input channels as

$$u^2 = \begin{bmatrix} u_2 \\ \vdots \\ u_n \end{bmatrix} = \begin{bmatrix} \alpha_2(\xi) \\ \vdots \\ \alpha_n(\xi) \end{bmatrix} \quad (7)$$

such that states  $x$  and  $\dot{x}$  can be linearized to be a single chain of integrator, i.e.,

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \vdots \\ \ddot{x}_{n-1} \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}. \quad (8)$$

Bearing Assumptions 1 and 2 in mind, substituting (7) and (8) into (4), leads to

$$\begin{aligned} f_i(x, \dot{x}) + \sum_{j=i}^n p_{i,j}(x, \dot{x})y_j + \sum_{j=i+1}^n g_{i,j}(x, \dot{x})\alpha_j &= x_{i+1}, \\ i &= 1, \dots, n - 1. \end{aligned} \quad (9)$$

From which, we can solve for  $\alpha_i$  backwardly as follows:

$$\begin{aligned} \alpha_n &= c_n(x, \dot{x}, y^n) + d_n(x, \dot{x})y_{n-1}, \\ \alpha_{n-1} &= c_{n-1}(x, \dot{x}, y^{n-1}) + d_{n-1}(x, \dot{x})y_{n-2}, \\ &\vdots \\ \alpha_2 &= c_2(x, \dot{x}, y^2) + d_2(x, \dot{x})y_1, \end{aligned}$$

where  $y^i = [y_i, \dots, y_n]^T, i = 1, \dots, n$ , and

$$\begin{aligned} c_n(x, \dot{x}, y^n) &= g_{n-1,n}^{-1}(x, \dot{x})(x_n - f_{n-1}(x, \dot{x}) - p_{n-1,n}(x, \dot{x})y_n), \\ d_n(x, \dot{x}) &= -g_{n-1,n}^{-1}(x, \dot{x})p_{n-1,n-1}(x, \dot{x}), \\ &\vdots \\ c_2(x, \dot{x}, y^2) &= g_{1,2}^{-1}(x, \dot{x})\left(x_2 - f_1(x, \dot{x}) - \sum_{j=2}^n p_{1,j}(x, \dot{x})y_j \right. \\ &\quad \left. - \sum_{j=3}^n g_{1,j}\alpha_j\right), \\ d_2(x, \dot{x}) &= -g_{1,2}^{-1}(x, \dot{x})p_{1,1}(x, \dot{x}). \end{aligned}$$

Note that the real-valued functions  $d_2(x, \dot{x}), \dots, d_n(x, \dot{x})$  are non-zero at the origin.

The overall system of (4) and (7) becomes

$$\begin{bmatrix} \ddot{x}_1 \\ \vdots \\ \ddot{x}_{n-1} \\ \ddot{x}_n \\ \ddot{y}_1 \\ \ddot{y}_2 \\ \vdots \\ \ddot{y}_n \end{bmatrix} = \begin{bmatrix} x_2 \\ \vdots \\ x_n \\ f_n(x, \dot{x}) + p_{n,n}(x, \dot{x})y_n \\ 0 \\ c_2(x, \dot{x}, y^2) + d_2(x, \dot{x})y_1 \\ \vdots \\ c_n(x, \dot{x}, y^n) + d_n(x, \dot{x})y_{n-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_1. \quad (10)$$

It is routine to verify that it satisfies the criterion for regular static state feedback linearizability (cf. e.g., Isidori, 1989, Theorem 4.2.6). Accordingly, system (10) is regular static state feedback linearizable. Furthermore, it is readily seen that  $h(\xi) = x_1$  is a linearizing output. The corresponding linearizing coordinate and input transformations are

$$\begin{aligned} z &= \left[ h, \frac{dh}{dt}, \dots, \frac{d^{4n-1}h}{dt^{4n-1}} \right]^T \\ &= [x_1, \dot{x}_1, \dots, x_n, \dot{x}_n, \varphi_1 + \psi_1 y_n, \varphi_2 \\ &\quad + \psi_2 \dot{y}_n, \dots, \varphi_{2n-1} + \psi_{2n-1} y_1, \varphi_{2n} + \psi_{2n} \dot{y}_1]^T \end{aligned} \quad (11)$$

and

$$u_1 = \alpha_1(\xi) + \beta_1(\xi)v, \quad v \in \mathfrak{R}, \quad (12)$$

where

$$\begin{aligned} \varphi_1 &= f_n(x, \dot{x}), \\ \psi_1 &= p_{n,n}(x, \dot{x}), \\ \varphi_{2i} &= L_{\bar{f}}\varphi_{2i-1} + (L_{\bar{f}}\psi_{2i-1})y_{n+1-i}, \\ \psi_{2i} &= \psi_{2i-1}, \quad i = 1, \dots, n, \\ \varphi_{2i+1} &= L_{\bar{f}}\varphi_{2i} + (L_{\bar{f}}\psi_{2i})\dot{y}_{n+1-i} + \psi_{2i}c_{n+1-i}, \\ \psi_{2i+1} &= \psi_{2i}d_{n+1-i}, \quad i = 1, \dots, n-1, \\ \alpha_1 &= -\frac{1}{\psi_{2n}}(L_{\bar{f}}\varphi_{2n} + (L_{\bar{f}}\psi_{2n})\dot{y}_1), \\ \beta_1 &= \frac{1}{\psi_{2n}}, \end{aligned}$$

with  $L_{\bar{f}}\varphi$  denoting the derivative of  $\varphi$  along

$$\begin{aligned} \bar{f} &= [\dot{x}_1, x_2, \dot{x}_2, \dots, x_n, \dot{x}_n, f_n + p_{n,n}y_n, \dot{y}_1, 0, \dot{y}_2, c_2 \\ &\quad + d_2y_1, \dots, \dot{y}_n, c_n + d_ny_{n-1}]^T. \end{aligned}$$

To sum up, under nonregular static state feedback

$$u = \begin{bmatrix} \alpha_1(\xi) \\ \alpha_2(\xi) \\ \vdots \\ \alpha_n(\xi) \end{bmatrix} + \begin{bmatrix} \beta_1(\xi) \\ 0 \\ \vdots \\ 0 \end{bmatrix} v = \alpha(\xi) + \beta(\xi)v \quad (13)$$

and state diffeomorphism (11), system (4) is transformed into

$$\dot{z} = Az + Bv, \quad (14)$$

where  $(A, B)$  is the Brunovsky canonical matrix pair. By Definition 1, system (4) is nonregular static state feedback linearizable.  $\square$

**Remark 1.** Assumptions 1 and 2 of system (4) can be further relaxed to one assumption, Assumption 3 as follows.

**Assumption 3.** There exists a positive definite matrix  $M(x, \dot{x})$  such that

$$p(x, \dot{x}) = M(x, \dot{x})\bar{p}(x, \dot{x}), \quad g(x, \dot{x}) = M(x, \dot{x})\bar{g}(x, \dot{x})$$

where matrix  $\bar{p}(x, \dot{x})$  has an upper triangular structure and is nonsingular at the origin, and matrix  $\bar{g}(x, \dot{x})$  has a strictly upper triangular structure and is of rank  $n - 1$  at the origin.

Under this relaxed assumption, system (4) is still linearizable via nonregular static state feedback. This fact can be proven along the same line as in the proof of Theorem 1. In fact,  $h(\xi) = x_1$  is a linearizing output and the corresponding linearizing input and coordinate transformations can be obtained accordingly.

**Remark 2.** Theorem 1 can be readily extended to higher-order systems

$$x^{(k)} = f(\bar{x}) + p(\bar{x})y + g(\bar{x})u, \quad (15)$$

$$y^{(k)} = u,$$

where  $k > 2$ , and  $\bar{x} = [x^T, \dot{x}^T, \dots, (x^{(k-1)})^T]^T$ . Furthermore, they can be extended to systems in nonhomogeneous orders

$$\begin{aligned} [x_1^{(k_1)}, \dots, x_n^{(k_n)}]^T &= f(\bar{x}) + p(\bar{x})y + g(\bar{x})u, \\ [y_1^{(l_1)}, \dots, y_n^{(l_n)}]^T &= u, \end{aligned} \quad (16)$$

where  $k_i, l_i \geq 1, i = 1, \dots, n$ , and  $\bar{x} = [x_1, \dots, x_1^{(k_1-1)}, \dots, x_n, \dots, x_n^{(k_n-1)}]^T$ .

### 3. Application to flexible joint robots

Position control of flexible joint robot manipulators has been discussed by many researchers in the literature. Although numerous control and design strategies from diverse disciplines were developed based on simplified models (see e.g., Lewis, Abdallah, & Dawson, 1993; Ge, 1996; Readman, 1994; Canudas de Wit, Siciliano, & Bastin, 1996; Ge, Lee, & Harris, 1998; Sage, De Mathelin, & Ostertag, 1999 and references therein), only a few references studied the regulation issues based on the complete dynamic model derived in Tomei (1990, 1991). Among these, a proportional-derivative (PD) controller is developed to globally asymptotically stabilize the

robot to a reference position in Tomei (1991). Several variations of this controller can be found in Kelly, Ortega, Ailon, and Loria (1994), which replaced the derivative operator by a high pass filter, and in Ortega,

represents the friction forces, and  $\Gamma = [\tau^0]$ , with  $\tau$  being the torques supplied by the motors.

Matrix  $M_2(q_1)$  has the strictly upper triangular structure given by

$$M_2(q_1) = \begin{bmatrix} 0 & m_{1,2}(q_{1,1}) & m_{1,3}(q_{1,1}, q_{1,2}) & \cdots & m_{1,n}(q_{1,1}, \dots, q_{1,n-1}) \\ 0 & 0 & m_{2,3}(q_{1,2}) & \cdots & m_{2,n}(q_{1,2}, \dots, q_{1,n}) \\ & & & \ddots & \\ 0 & 0 & 0 & \cdots & m_{n-1,n}(q_{1,n-1}) \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}. \quad (18)$$

Loria, Kelly, and Praly (1995), which presented a globally stabilizing output feedback controller following the passivity approach.

In this paper, globally stabilizing controllers are presented for a class of full-order flexible joint robots within the framework of nonregular static state feedback linearization. The controllers require full state measurements which is more restrictive than the aforementioned controllers for this class of systems.

### 3.1. Mathematical model

Consider a robot with  $n + 1$  rigid links interconnected by  $n$  flexible revolute joints. Under the assumptions that the joint flexibility are modeled as linear torsional springs and the rotors of the actuators are modeled as uniform bodies of revolution, the robot's dynamics are given by (Tomei, 1991)

$$M(q_1)\ddot{q} + C(q, \dot{q})\dot{q} + K_E q + G(q_1) + F(q, \dot{q}) = \Gamma, \quad (17)$$

where  $q = [q_1, q_2]$  with  $q_1$  and  $q_2$  representing  $n \times 1$  vectors of the positions of the links and the actuators, respectively,

$$M(q) = \begin{bmatrix} M_1(q_1) & M_2(q_1) \\ M_2^T(q_1) & M_3 \end{bmatrix}$$

is the inertia matrix which is positive definite,  $C(q, \dot{q})$  is the centripetal and Coriolis matrix,

$$K_E = \begin{bmatrix} K_e & -K_e \\ -K_e & K_e \end{bmatrix},$$

with  $K_e = \text{diag}[k_1, \dots, k_n]$  being the diagonal stiffness matrix whose entries are flexible constants of the joints,

$$G(q_1) = \begin{bmatrix} G_1(q_1) \\ 0 \end{bmatrix}$$

is the gravity term,

$$F(q, \dot{q}) = \begin{bmatrix} F_1(q_1, \dot{q}_1) \\ F_2(q_2, \dot{q}_2) \end{bmatrix}$$

Matrix  $C(q, \dot{q})$  can be decomposed as

$$C(q, \dot{q}) = \begin{bmatrix} C_1(q_1, \dot{q}_1) + C_2(q_1, \dot{q}_2) & C_3(q_1, \dot{q}_1) \\ C_4(q_1, \dot{q}_1) & 0 \end{bmatrix},$$

where

$$C_{1,ij}(q_1, \dot{q}_1) = \frac{1}{2} \left[ \dot{q}_1^T \frac{\partial M_{1,ij}}{\partial q_1} + \left( \frac{\partial M_1^i}{\partial q_{1,j}} - \frac{\partial M_1^j}{\partial q_{1,i}} \right) \dot{q}_1 \right],$$

$$C_{2,ij}(q_1, \dot{q}_2) = \frac{1}{2} \left( \frac{\partial M_2^i}{\partial q_{1,j}} - \frac{\partial M_2^j}{\partial q_{1,i}} \right) \dot{q}_2,$$

$$C_{3,ij}(q_1, \dot{q}_1) = \frac{1}{2} \left( \dot{q}_1^T \frac{\partial M_{2,ij}}{\partial q_1} - \frac{\partial (M_2^T)^j}{\partial q_{1,i}} \dot{q}_1 \right),$$

$$C_{4,ij}(q_1, \dot{q}_1) = \frac{1}{2} \left( \dot{q}_1^T \frac{\partial M_{2,ji}}{\partial q_1} - \frac{\partial (M_2^T)^i}{\partial q_{1,j}} \dot{q}_1 \right),$$

with  $M^i$  denoting the  $i$ th row of matrix  $M$ .

When  $M_2(q_1) = 0$ , there is no inertial coupling between the links and the actuators and model (17) is reduced to the well known simplified model (Spong, 1987). This simplified model is always regular static state feedback linearizable. To avoid this trivial case, we assume that  $M_2(q_1) \neq 0$ . Furthermore, for ease of handling, we make the following assumption:

**Assumption 4.** Matrix  $M_2(q_1)$  is constant, and  $m_{i,i+1} \neq 0$ ,  $i = 1, \dots, n - 1$ .

The constant assumption for  $M_2(q_1)$  is valid for spatial three-link elbow manipulators and for planar robots with any number of rotational joints (De Luca, 1998b).

Under Assumption 4, system (17) is reduced to

$$M_1(q_1)\ddot{q}_1 + M_2\ddot{q}_2 + C_1(q_1, \dot{q}_1)\dot{q}_1 + K_e(q_1 - q_2) + G_1(q_1) + F_1(q_1, \dot{q}_1) = 0,$$

$$M_2^T\ddot{q}_1 + M_3\ddot{q}_2 + K_e(q_2 - q_1) + F_2(q_2, \dot{q}_2) = \tau. \quad (19)$$

Note that system (19) is not regular static state feedback linearizable (De Luca, 1998b).

3.2. Nonregular feedback linearizability

Denote

$$N(q_1, \dot{q}_1) = -M_1^{-1}(q_1)(C_1(q_1, \dot{q}_1)\dot{q}_1 + K_e q_1 + G_1(q_1) + F_1(q_1, \dot{q}_1)).$$

Consider the control law

$$\tau = M_2^T(N(q_1, \dot{q}_1) + M_1^{-1}(q_1)K_e q_2) + K_e(q_2 - q_1) + F_2(q, \dot{q}) + (M_3 - M_2^T M_1^{-1}(q_1)M_2)u. \tag{20}$$

Note that matrix  $M_3 - M_2^T M_1^{-1}(q_1)M_2$  is nonsingular (and positive definite) from the positive definiteness of  $M(q_1)$ .

System (19) can be rewritten in the form of partial feedback linearization (Spong, 1997)

$$\begin{aligned} \ddot{q}_1 &= N(q_1, \dot{q}_1) + M_1^{-1}(q_1)(K_e q_2 - M_2 u), \\ \ddot{q}_2 &= u. \end{aligned} \tag{21}$$

It is clear that system (21) is of form (4) and satisfies Assumption 3. According to Remark 1, this system is nonregular static state feedback linearizable.

Following the proof of Theorem 1, we can compute a linearizing state coordinate and its corresponding input transformations as

$$z = \phi(q, \dot{q}) = [q_{1,1}, \dot{q}_{1,1}, \dots, q_{1,n}, \dot{q}_{1,n}, \eta_1, \dots, \eta_{2n}]^T \tag{22}$$

and

$$u = \alpha + \beta v = \begin{bmatrix} -\frac{L_X \eta_{2n}}{\gamma_1} \\ A^{-1} \theta \end{bmatrix} + \begin{bmatrix} 1 \\ \gamma_1 \\ 0 \end{bmatrix} v, \tag{23}$$

respectively, where

$$A = \begin{bmatrix} m_{1,2} & \dots & m_{1,n} \\ & \ddots & \\ 0 & \dots & m_{n-1,n} \end{bmatrix},$$

$$\theta = \begin{bmatrix} k_1 q_{2,1} - D_1 \rho \\ \vdots \\ k_{n-1} q_{2,n-1} - D_{n-1} \rho \end{bmatrix},$$

$$\eta_1 = a_n + \delta_n, \quad \eta_{i+1} = L_X \eta_i, \quad i = 1, \dots, 2n - 1,$$

$$\gamma_1 = \frac{\partial \eta_{2n}}{\partial \dot{q}_{2,1}},$$

with  $D_i = [d_{i,1}, \dots, d_{i,n}]$  being the  $i$ th row of matrix  $M_1(q_1)$ , and

$$\delta = [\delta_1, \dots, \delta_n]^T = N(q_1, \dot{q}_1),$$

$$a_n = \frac{1}{d_{n,n}} \left( k_n q_{2,n} - \sum_{i=1}^{n-1} d_{n,i} (q_{1,i+1} - \delta_i) \right),$$

$$\rho = [q_{1,2} - \delta_1, \dots, q_{1,n} - \delta_{n-1}, \eta_1 - \delta_n]^T,$$

$$X = [\dot{q}_{1,1}, q_{1,2}, \dots, \dot{q}_{1,n}, a_n - \delta_n, \dot{q}_{2,1}, 0, \dot{q}_{2,2}, \alpha_2, \dots, \dot{q}_{2,n}, \alpha_n]^T.$$

Combining (20) and (23) gives

$$\begin{aligned} \tau &= M_2^T(N(q_1, \dot{q}_1) + M_1^{-1}(q_1)K_e q_2) + K_e(q_2 - q_1) \\ &\quad + F_2(q, \dot{q}) + (M_3 - M_2^T M_1^{-1}(q_1)M_2) \begin{bmatrix} -\frac{L_X \eta_{2n}}{\gamma_1} \\ \Gamma^{-1} \theta \end{bmatrix} \\ &\quad + M_3 \begin{bmatrix} 1 \\ \gamma_1 \\ 0 \end{bmatrix} v, \end{aligned} \tag{24}$$

which is the overall input transformation for system (17).

The transformed system is of Brunovsky canonical form

$$\dot{z} = [z_2, \dots, z_{4n}, v]^T. \tag{25}$$

Note that the linearizing transformations (22) and (24) are globally defined. Accordingly, system (17) is globally static state feedback linearizable.

3.3. Global stabilization

The feedback linearizability of robot system (17) enable us to design globally asymptotically stabilizing controllers by using standard linear design technique.

Suppose  $p(\lambda) = \lambda^{4n} + l_{4n} \lambda^{4n-1} + \dots + l_2 \lambda + l_1$  is a Hurwitz polynomial. Let

$$v = -Lz = -\sum_{i=1}^{4n} l_i z_i. \tag{26}$$

The closed-loop system (25) and (26) is globally exponential stable.

Accordingly, the controller

$$\begin{aligned} \tau &= M_2^T(N(q_1, \dot{q}_1) + M_1^{-1}(q_1)K_e q_2) + K_e(q_2 - q_1) \\ &\quad + F_2(q, \dot{q}) + (M_3 - M_2^T M_1^{-1}(q_1)M_2) \\ &\quad \times \begin{bmatrix} -\frac{L_X \eta_{2n}}{\gamma_1} \\ \Gamma^{-1} \theta \end{bmatrix} - M_3 \begin{bmatrix} \sum_{i=1}^{4n} l_i \phi_i(q, \dot{q}) \\ \gamma_1 \\ 0 \end{bmatrix} \end{aligned} \tag{27}$$

is globally asymptotically stabilizing system (17).

4. Conclusion

In this paper, a new criterion of nonregular static state feedback linearization has been presented for a class of second-order nonlinear cascade systems. Some extensions of this criterion to more general systems have also been considered. As an application, controllers have been designed to asymptotically stabilize a class of flexible joint robots.

Acknowledgements

The authors would like to thank the anonymous reviewers for their constructive and insightful comments for further improving the quality of this work.

## References

- Bodson, M., & Chiasson, J. (1998). Differential-geometric methods for control of electric motors. *International Journal of Robust and Nonlinear Control*, 8(11), 923–954.
- Brockett, R. W. (1978). Feedback invariants for nonlinear systems. In *Proceedings of Sixth IFAC World Congress* (pp. 1115–1120).
- Canudas de Wit, C., Siciliano, B., & Bastin, G. (1996). *Theory of robot control*. London: Springer.
- Cheng, D. Z. (1987). Linearization with dynamic compensation. *Systems Science and Mathematics Science*, 7(2), 200–204.
- Ge, S. S. (1996). Adaptive controller design for flexible joint manipulators. *Automatica*, 32(2), 273–278.
- Ge, S. S., Lee, T. H., & Harris, C. J. (1998). *Adaptive neural network control of robotic manipulators*. NJ: World Scientific.
- Guay, M., McLellan, P. J., & Bacon, D. W. (1997). Condition for dynamic feedback linearization of control-affine nonlinear systems. *International Journal of Control*, 68(1), 87–106.
- Isidori, A. (1989). *Nonlinear control systems*. Berlin: Springer.
- Jakubczyk, B., & Respondek, W. (1980). On linearization of control systems. *Bulletin de l'Academie Polonaise des Sciences, Serie des Sciences Matematyki*, 28, 517–522.
- Kelly, R., Ortega, R., Ailon, A., & Loria, A. (1994). Global regulation of flexible joint robots using approximate differentiation. *IEEE Transactions on Automatic Control*, 39(6), 1222–1224.
- Lee, S., & Cochran, J. E. (1997). Orbital maneuvers via feedback linearization and bang-bang control. *Journal of Guidance, Control and Dynamics*, 20(1), 104–110.
- Lewis, F. L., Abdallah, C. T., & Dawson, D. M. (1993). *Control of robot manipulators*. New York: Macmillan.
- De Luca, A. (1998a). Decoupling and feedback linearization of robots with mixed rigid/elastic joints. *International Journal of Robust and Nonlinear Control*, 8(11), 965–977.
- De Luca, A. (1998b). A general algorithm for dynamic feedback linearization of robots with elastic joints. In *Proceedings of 37th Conference on Decision and Control*, Leuven, Belgium (pp. 504–510).
- Ortega, R., Loria, A., Kelly, R., & Praly, L. (1995). On passivity-based output feedback global stabilization of Euler–Lagrange systems. *International Journal of Robust and Nonlinear Control*, 5(4), 313–323.
- Readman, M. C. (1994). *Flexible joint robots*. Boca Raton, FL: CRC Press.
- Sage, H. G., De Mathelin, M. F., & Ostertag, E. (1999). Robust control of robot manipulators: a survey. *International Journal of Control*, 72(16), 1498–1522.
- Sheen, J. J., & Bishop, R. (1994). Spacecraft nonlinear control. *Journal of Astronautics Science*, 42(3), 361–377.
- Spong, M. W. (1987). Modeling and control of elastic joint robots. *Journal of Dynamic Systems, Measurement and Control*, 109(4), 310–319.
- Spong, M. W. (1997). Underactuated mechanical systems. In B. Siciliano, & K. P. Valavanis (Eds.), *Control problems in robotics and automation*. London: Springer.
- Spong, M. W., & Vidyasagar, M. (1989). *Robot dynamics and control*. New York: Wiley.
- Sun, Z., & Xia, X. (1997). On nonregular feedback linearization. *Automatica*, 33(7), 1339–1344.
- Taylor, D. (1994). Nonlinear control of electric machines: an overview. *IEEE Control Systems*, 14(6), 41–51.
- Tomei, P. (1990). An observer for flexible joint robots. *IEEE Transactions on Automatic Control*, 35(6), 739–743.
- Tomei, P. (1991). A simple PD controller for robots with elastic joints. *IEEE Transactions on Automatic Control*, 36(10), 1208–1213.



**S. S. Ge** received the B.Sc. degree from Beijing University of Aeronautics and Astronautics (BUAA), Beijing, China, in 1986, and the Ph.D. degree and the Diploma of Imperial College (DIC) from Imperial College of Science, Technology and Medicine, University of London, in 1993. From May 1992 to June 1993, he did his postdoctoral research at Leicester University, England. He has been with the Department of Electrical & Computer Engineering, the National University of Singapore since 1993, and is currently as an Associate Professor. He was a visiting staff in Laboratoire de Automatique de Grenoble, France in 1996, the University of Melbourne, Australia in 1998, 1999, and University of Petroleum, China in 2001.

He has authored and co-authored over 100 international journal and conference papers, one monograph and co-invented one patent. He served as an Associate Editor on the Conference Editorial Board of the IEEE Control Systems Society in 1998 and 1999, has been serving as an Associate Editor, IEEE Transactions on Control Systems Technology since June 1999, and a Member of the Technical Committee on Intelligent Control of the IEEE Control System Society since 2000. He was the winner of the 1999 National Technology Award, Singapore. He serves as a technical consultant in local industry. He is currently a Senior Member of IEEE. His current research interests are Control of Nonlinear Systems, Neural Networks and Fuzzy Logic, Robot Control, Real-Time Implementation, Path Planning and Sensor Fusion.



**Zhendong Sun** received the B.S. degree in Applied Mathematics from Ocean University of Qingdao (China) in 1990, the M.S. in Systems Science from Xiamen University (China) in 1993, and the Ph.D. degree in Electrical Engineering from Beijing University of Aeronautics and Astronautics (China) in 1996.

During 1996–1998, he was a postdoctoral research associate in Department of Automation, Tsinghua University, China.

In 1998, he joined the faculty of Science, Beijing University of Aeronautics and Astronautics, as an Associate Professor. Since 2000, he has been with the National University of Singapore as a Research Fellow.

Dr. Sun's current research interests are in the fields of nonlinear control systems, switched and hybrid systems, and sampled data systems. He was awarded the Guan Zhao-Zhi Prize at the Chinese Control Conference (Hong Kong, China) in 2000.



**T. H. Lee** received the B.A. degree with First Class Honours in the Engineering Tripos from Cambridge University, England, in 1980; and the Ph.D. degree from Yale University in 1987. He is a tenured Professor in the Department of Electrical and Computer Engineering at the National University of Singapore. He is also currently Head of the Drives, Power and Control Systems Group in this Department, and the Vice-Dean (Research) in the Faculty of Engineering.

Dr. Lee's research interests are in the areas of adaptive systems, knowledge-based control and intelligent mechatronics. He has published extensively in these areas, and currently holds Associate Editor appointments in Automatica; the IEEE Transactions in Systems, Man and Cybernetics; Control Engineering Practice (an IFAC journal); the International Journal of Systems Science (Taylor and Francis, London); and Mechatronics journal (Oxford, Pergamon Press). Dr. Lee was a recipient of the Cambridge University Charles Baker Prize in Engineering.