

Erratum

Correction to “Adaptive NN control for a class of strict-feedback discrete-time nonlinear systems”

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The purpose of this note is to correct the typographical errors in Ge, Li, and Lee (2003). According to the definition of $\xi_{if}(k)$, we have $\xi_i(k+n-i+1) = \eta_i(k+n-i+1) + \xi_{if}(k)$, $i = 2, 3, \dots, n$, but the terms $\eta_i(k+n-i+1)$ are missing. While the main results are unchanged, there should be some consequent modifications in the technical derivations, which are detailed as follows.

In Step 1, due to the missing term $\eta_2(k+n-1)$, (A.4) should be updated to

$$\begin{aligned} \xi_2(k+n-1) &= \xi_{2f}(k) + \eta_2(k+n-1) \\ &= \hat{W}_1(k)S_1(z_1(k)) + \eta_2(k+n-1) \end{aligned}$$

and in the following equations (A.6)–(A.8), the missing term, $G_1(k)\eta_2(k+n-1)$, should be added on the right-hand side (R.H.S.) and $2\eta_1(k+1)\eta_2(k)$ should be added on the last equation of (A.10). Noting the fact that

$$2\eta_1(k+1)\eta_2(k) \leq \frac{\bar{\gamma}_1\eta_1^2(k+1)}{\bar{g}_1} + \frac{\bar{g}_1\eta_2^2(k)}{\bar{\gamma}_1} \quad (1)$$

the difference of Lyapunov function candidate (A.10) is updated as follows:

$$\begin{aligned} \Delta V_1(k) &\leq -\frac{\rho_1}{\bar{g}_1}\eta_1^2(k+1) - \frac{1}{\bar{g}_1}\eta_1^2(k) + \beta_1 \\ &\quad - \sigma_1(1 - \sigma_1\bar{\gamma}_1 - \bar{g}_1\sigma_1\bar{\gamma}_1)\|\hat{W}_1(k_1)\|^2 \\ &\quad + \frac{\bar{g}_1\eta_2^2(k)}{\bar{\gamma}_1} \end{aligned} \quad (2)$$

where parameters ρ_1 and $\bar{\gamma}_1$ are updated such that $\rho_1 = 1 - 2\bar{\gamma}_1 - \bar{\gamma}_1l_1 - \bar{g}_1\bar{\gamma}_1l_1$, $\bar{\gamma}_1$ satisfies $\bar{\gamma}_1 < \frac{1}{2+l_1+\bar{g}_1l_1}$ and β_1 is unchanged.

The missing term, $-\Gamma_1S_1(z_1(k_1))G_1(k_1)\eta_2(k)$, should be added on the equation after (A.11) such that the dynamics (A.5) is changed to be

$$\begin{aligned} \tilde{W}_1(k+1) &= A_1(k)\tilde{W}_1(k_1) + \Gamma_1S_1(z_1(k_1))G_1(k_1)\epsilon_{z_1} \\ &= -\sigma_1\Gamma_1W_1^* - \Gamma_1S_1(z_1(k_1))G_1(k_1)\eta_2(k) \end{aligned}$$

where the definition of matrix $A_1(k)$ is unchanged.

The boundedness of $\eta_2(k)$ will be proved in the following steps such that the boundedness of $\eta_1(k)$, $V_1(k)$ and $\hat{W}_1(k)$ can be obtained by the same procedure in the paper.

In Step 2, due to the missing term $\eta_3(k+n-2)$, (A.15) should be updated as follows:

$$\begin{aligned} \xi_3(k+n-2) &= \xi_{3f}(k) + \eta_3(k+n-2) \\ &= \hat{W}_2(k)S_2(z_2(k)) + \eta_3(k+n-2) \end{aligned}$$

and $G_2(k)\eta_3(k+n-2)$ should be added on the R.H.S. of (A.17). Because of the extra term $\frac{\bar{g}_1\eta_2^2(k)}{\bar{\gamma}_1}$ in $\Delta V_1(k)$, decoupled backstepping (Ge, 2004) can be used, and $V_1(k)$ should be removed from (A.18).

Then the difference of $V_2(k)$ is given as

$$\begin{aligned} \Delta V_2(k) &\leq -\frac{\rho_2}{\bar{g}_2}\eta_2^2(k+1) - \frac{1}{\bar{g}_2}\eta_2^2(k) + \beta_2 \\ &\quad - \sigma_2(1 - \sigma_2\bar{\gamma}_2 - \bar{g}_2\sigma_2\bar{\gamma}_2)\|\hat{W}_2(k_2)\|^2 \\ &\quad + \frac{\bar{g}_2\eta_3^2(k)}{\bar{\gamma}_2} \end{aligned} \quad (3)$$

where parameters ρ_2 , β_2 and $\bar{\gamma}_2$ should be updated as $\rho_2 = 1 - 2\bar{\gamma}_2 - \bar{\gamma}_2l_2 - \bar{g}_2\bar{\gamma}_2l_2$, $\beta_2 = \bar{g}_2\epsilon_{z_2}^2/\bar{\gamma}_2 + \sigma_2\|W_2^*\|^2$, and $\bar{\gamma}_2$

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satisfies $\bar{\gamma}_2 < \frac{1}{2+l_2+\bar{g}_2l_2}$. The boundedness of error $\eta_3(k)$ is to be proved in the following steps such that the boundedness of $\eta_2(k)$, $V_2(k)$ and $\hat{W}_2(k)$ can be easily obtained.

In Step i , due to the missing term $\eta_{i+1}(k+n-i)$, (A.20) should be updated as

$$\begin{aligned} \xi_{i+1}(k+n-i) &= \xi_{(i+1)f}(k) + \eta_{i+1}(k+n-i) \\ &= \hat{W}_i(k)S_i(z_i(k)) + \eta_{i+1}(k+n-i) \end{aligned}$$

and $G_i(k)\eta_{i+1}(k+n-i)$ should be added on the R.H.S. of (A.21). To proceed the decoupled backstepping design, remove $\sum_{j=1}^{i-1} V_j(k)$ from (A.23), then the difference of $V_i(k)$ becomes

$$\begin{aligned} \Delta V_i(k) &\leq -\frac{\rho_i}{\bar{g}_i}\eta_i^2(k+1) - \frac{1}{\bar{g}_i}\eta_i^2(k) + \beta_i \\ &\quad - \sigma_i(1 - \sigma_i\bar{\gamma}_i - \bar{g}_i\sigma_i\bar{\gamma}_i)\|\hat{W}_i(k_i)\|^2 \\ &\quad + \frac{\bar{g}_i\eta_{i+1}^2(k)}{\bar{\gamma}_i} \end{aligned} \tag{4}$$

where parameters ρ_i , β_i and $\bar{\gamma}_i$ should be updated such that $\rho_i = 1 - 2\bar{\gamma}_i - \bar{\gamma}_i l_i - \bar{g}_i\bar{\gamma}_i l_i$, $\beta_i = \bar{g}_i\varepsilon_{z_i}^2/\bar{\gamma}_i + \sigma_i\|W_i^*\|^2$ and $\bar{\gamma}_i$ satisfies $\bar{\gamma}_i < \frac{1}{2+l_i+\bar{g}_il_i}$. The proof of boundedness of $\eta_{i+1}(k)$ is left to the next step.

In Step n , to proceed with the decoupled backstepping design (Ge, 2004), we remove $\sum_{j=1}^{n-1} V_j(k)$ from (A.31), and then the difference of $V_n(k)$ becomes

$$\begin{aligned} \Delta V_n(k) &\leq -\frac{\rho_n}{\bar{g}_n}\eta_n^2(k+1) - \frac{1}{\bar{g}_n}\eta_n^2(k) + \beta_n \\ &\quad - \sigma_n(1 - \sigma_n\bar{\gamma}_n - \bar{g}_n\sigma_n\bar{\gamma}_n)\|\hat{W}_n(k)\|^2 \end{aligned} \tag{5}$$

where $\beta_n = \bar{g}_n\varepsilon_{z_n}^2/\bar{\gamma}_n + \sigma_n\|W_n^*\|^2$.

It is obvious that $\Delta V_n \leq 0$ once $|\eta_n(k)| > \sqrt{\bar{g}_n\beta_n}$. This demonstrates that the tracking error $\eta_n(k)$ is bounded and will converge to the compact set denoted by $\Omega_\eta \subset R$, where $\Omega_\eta := \{\chi \mid \chi \leq \sqrt{\bar{g}_n\beta_n}\}$. The boundedness of $\hat{W}_n(k)$ can be proved in the same way as the proof for boundedness of $\hat{W}_1(k)$. From the boundedness of $\eta_n(k)$, the boundedness of the extra term $\frac{\bar{g}_{(n-1)}\eta_n^2(k)}{\bar{\gamma}_{n-1}}$ at Step $(n-1)$ is readily obtained. After $(n-1)$ steps backward, it can be seen from the above iterative design procedures that $V_i(k)$, $\eta_i(k)$, $\hat{W}_i(k)$ and hence $\xi_i(k)$ are bounded, $i = 1, \dots, n-1$.

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