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Brief Paper

Adaptive stabilization of uncertain nonholonomic systems by state and output feedback☆

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Abstract

In this paper, adaptive state feedback and output feedback control strategies are presented for a class of nonholonomic systems in chained form with drift nonlinearity and parametric uncertainties. Both control laws are developed using state scaling and backstepping techniques. In particular, novel adaptive switching is proposed to overcome the uncontrollablity problem associated with $x_0(t_0)=0$. Observer-based output feedback design is developed when only partial system states are measurable, and a filtered observer rather than the traditional linear observer is used to handle the technical problem due to the presence of unavailable states in the regressor matrix. The proposed control strategies can steer the system globally converge to the origin, while the estimated parameters maintain bounded.

Keywords: Adaptive; Stabilization; Nonholonomic system; State feedback; Output feedback

1. Introduction

Due to Brockett's theorem (Brockett, 1983), it is well known that nonholonomic systems with restricted mobility cannot be stabilized to a desired configuration (or posture) via differentiable, or even continuous, pure-state feedback, although it is controllable. A number of approaches have been proposed for the problem, which can be classified as (i) discontinuous time-invariant stabilization (Astolfi, 1996), (ii) time-varying stabilization (Walsh & Bushnell, 1995; Samson, 1993) and (iii) hybrid stabilization (Sordalen & Canudas de Wit, 1995; Canudas de Wit, Berghuis, & Nijmeijer, 1994). See the survey paper Kolmanovsky and McClamroch (1995) for more details and references therein.

One commonly used approach for controller design of nonholonomic systems is to convert, with appropriate state and input transformations, the original systems into some canonical forms for which controller design can be carried out easier (Murray & Sastry, 1993; M'Closkey & Murray, 1992; Huo & Ge, 2001). Using the special algebra structures of the canonical forms, various feedback strategies have been proposed to stabilize nonholonomic systems in the literature (Astolfi, 1996; Ge, Sun, Lee, & Spong, 2001a; Murray, 1993; Sun, Ge, Huo, & Lee, 2001; Jiang & Nijmeijer, 1999). Recently, adaptive control strategies were proposed to stabilize the dynamic nonholonomic systems with modeling or parametric uncertainties (Colbaugh, Barany, & Glass, 1996; Ge, Wang, Lee, & Zhou, 2001b). Neural network control was applied to obtain practical point stabilization solution for a nonholonomic mobile robot with uncertainty (Fierro & Lewis, 1995). Hybrid control based on supervisory adaptive control was presented to globally asymptotically stabilize a wheeled mobile robot (Hespanha, Liberzon, & Morse, 1999). Adaptive state feedback control was considered in Do and Pan (2002) using input-to-state scaling. It should be noticed that all these papers are concerned with state-feedback control. Output feedback tracking and regulation were presented in Dixon, Dawson, Zergeroglu, and Behal (2001) for practical wheeled mobile robots. In Jiang (2000), robust exponential regulation for

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nonholonomic systems with input and state-driven disturbances was presented under the assumption that the bounds of the disturbances are known.

This paper addresses the problem of stabilization of a class of nonholonomic systems in chained form with drift nonlinearity and parameter uncertainties. The main contributions of this paper are listed as follows:

(i) adaptive state feedback stabilization using state scaling and backstepping is developed without imposing any restriction on the system order and the growth of the drift nonlinearities;
(ii) new adaptive switching is designed to handle the technical problem of uncontrollability at $x_0(t_0)=0$, which prevents the possible finite escape of system states, and at the same time guarantees the boundedness of all the signals in the system; and
(iii) output feedback stabilization of a class of uncertain nonholonomic systems is provided when only partial system states are measurable with an adaptive nonlinear observer to solve the problem of the regressor matrix involving unmeasured states.

2. Problem formulation

The nonholonomic systems in a chained form have been formulated as a canonical form for many nonlinear mechanical systems with nonholonomic constraints on velocities. The purpose of this paper is to deal with a class of perturbed canonical nonholonomic systems as follows:

$$\dot{x}_0 = u_0 + \phi_0^{\mathrm{T}}(x_0)\theta,$$
$$\dot{x}_i = u_0 x_{i+1} + \phi_i^{\mathrm{T}}(u_0,x_0,\bar{x}_i)\theta \quad 1 \leqslant i < n, \ \ n \geqslant 2,$$
$$\dot{x}_n = u_1 + \phi_n^{\mathrm{T}}(u_0,x_0,x)\theta \tag{1}$$

where $[x_0,x^{\mathrm{T}}]^{\mathrm{T}} \triangleq [x_0,x_1,\ldots,x_n]^{\mathrm{T}} \in R^{n+1}$ are system states, $\bar{x}_i \triangleq [x_1,\ldots,x_i]^{\mathrm{T}} \in R^i$, u_0 and u_1 are control inputs, $\phi_0(x_0) \in R^l$ and $\phi_i(u_0,x_0,\bar{x}_i) \in R^l$, $1 \leqslant i \leqslant n$ are vectors of smooth nonlinear functions of x_0, u_0 and $\bar{x}_i$, and $\theta \in R^l$ is a vector of unknown bounded constant parameters.

The control objective is to design adaptive control laws of the form $u_0=u_0(x_0,\mu)$, $u_1=u_1(\chi,\mu)$ and $\dot{\mu}=v(\chi,\mu)$, such that $(x_0(t),x(t))$ converge to zero as $t \to \infty$ and all other signals in the closed-loop system are bounded. When all the system states are available for controller design, $\chi=(x_0,x)$. When only the system output is available for controller design, $\chi=(x_0,x_1)$, a flat output of undisturbed system (1) (Fliess, Levine, Martin, & Rouchon, 1995). By a flat output, we mean that: (i) its dimension equals that of the control input, and (ii) it is a function of the state, of the control input, and of the derivatives of the control input such that the state and the control input can be expressed as functions of the output and the derivatives of the output.

3. State feedback control

When full state (x_0,x) is available, the class of systems which can be handled is much larger than that for output feedback control. A full characterization of the class of systems (1) is given by the following assumption on $\phi_0(x_0)$ and $\phi_i(x_0,u_0,\bar{x}_i)$, $1 \leqslant i \leqslant n$.

Assumption 1. For ϕ_0, there is a known smooth function vector φ_0 such that

$$\phi_0(x_0) = x_0\varphi_0(x_0).$$

For $1 \leqslant i \leqslant n$, there are some known smooth function vectors φ_j, $1 \leqslant j \leqslant i$, such that

$$\phi_i(u_0,x_0,\bar{x}_i) = \sum_{j=1}^{i} x_j\varphi_j(u_0,x_0,\bar{x}_i).$$

The nonlinearities ϕ_i, $0 \leqslant i \leqslant n$ satisfy the *triangularity structure*. Assumption 1 implies that the origin is an equilibrium point of system (1). For clarity, the case that $x_0(t_0) \neq 0$ is considered in Section 3.1 first, then the case that $x_0(t_0)=0$ is dealt with in Section 3.2 later.

3.1. State scaling and controller design

The inherently triangular structure of system (1) suggest that we should design the control inputs u_0 and u_1 in two separate stages.

Consider the control

$$u_0(x_0,\hat{\theta}_0) = x_0 g_0(x_0,\hat{\theta}_0), \tag{2}$$

$$g_0(x_0,\hat{\theta}_0) = -\varphi_0^{\mathrm{T}}(x_0)\hat{\theta}_0 - \sqrt{k_0^2 + (\varphi_0^{\mathrm{T}}(x_0)\hat{\theta}_0)^2}, \tag{3}$$

where $k_0 > 0$ and $\hat{\theta}_0$ is the first estimate of θ.

Remark 1. Note that control u_0 is an adaptive version of Sontag formula (Sontag, 1989), and is used to stabilize x_0 with uncertainties. Because of the particular choice of (3), $g_0(x_0,\hat{\theta}_0) \neq 0$ is guaranteed irregardless of the values of x_0 and $\hat{\theta}_0$. Hence, $1/g_0$ is well-defined.

Consider the Lyapunov function candidate

$$V_0 = \frac{1}{2}x_0^2 + \frac{1}{2}\tilde{\theta}_0^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}_0 \tag{4}$$

with $\tilde{\theta}_0 = \theta - \hat{\theta}_0$ and the adaptation law for $\hat{\theta}_0$ as

$$\dot{\hat{\theta}}_0 = \Gamma x_0 \phi_0(x_0), \quad \Gamma = \Gamma^{\mathrm{T}} > 0. \tag{5}$$

Its time derivative is given by $\dot{V}_0 \leqslant -k_0 x_0^2$. Accordingly, we can conclude that $\hat{\theta}_0$ is bounded, $x_0 \to 0$ as $t \to \infty$ by using LaSalle's Invariant Theorem.

Using control law (2), the closed-loop dynamics of the x_0-subsystem is

$$\dot{x}_0 = -x_0\left(\sqrt{k_0^2 + (\varphi_0^{\mathrm{T}}\hat{\theta}_0)^2} - \varphi_0^{\mathrm{T}}\tilde{\theta}_0\right). \tag{6}$$

Since $x_0(t)$ and $\tilde{\theta}_0(t)$ are bounded, the solution of (6) is $x_0(t) = x_0(t_0)e^{-\int_{t_0}^{t}\lambda(s)\,ds}$, $\lambda(s) = -\varphi_0^{\mathrm{T}}\tilde{\theta}_0(s) + \sqrt{k_0^2 + (\varphi_0^{\mathrm{T}}\hat{\theta}_0(s))^2}$. Consequently, the two possible cases for $x_0(t) = 0$ are: (i) $x_0(t_0) = 0$ and (ii) $t = \infty$. Since $x_0(t_0) \neq 0$ is assumed, it is concluded that u_0 can guarantee that x_0 does not cross zero for all $t \in (t_0, \infty)$ provided that $x_0(t_0) \neq 0$.

Remark 2. If $x_0(t_0) \neq 0$, control u_0 and the adaptation law (5) guarantee that x_0 does not cross zero for all $t \in (t_0, \infty)$ and is independent of the x-subsystem.

From the above analysis, we can see that the x_0-state in (1) can be globally regulated to zero via u_0 in (2) as $t \to \infty$. This phenomenon causes serious trouble in controlling the x-subsystem via the control input u_1 because, in the limit (i.e. $u_0 = 0$), the x-subsystem is uncontrollable. It can be avoided by utilizing the following discontinuous state scaling transformation (Astolfi, 1996; Astolfi & Schaufeberger, 1996)

$$z_i = \frac{x_i}{x_0^{n-i}} \quad 1 \leqslant i \leqslant n. \tag{7}$$

The discontinuous coordinates transformation (7) is an application of σ process (Arnold, 1987). It mainly consists of a rational (discontinuous) coordinates transformation, possessing the property of increasing the resolution around a given point.

Under the new z-coordinates, with the choice of u_0 as in (2), the x-system is transformed into

$$\begin{aligned}\dot{z}_i &= g_0(x_0,\hat{\theta}_0)z_{i+1} + f_i(x_0,z_i,\hat{\theta}_0) + \psi_i^{\mathrm{T}}(x_0,\bar{z}_i,\hat{\theta}_0)\theta,\\ \dot{z}_n &= u_1 + \psi_n^{\mathrm{T}}(u_0,x_0,x)\theta,\end{aligned} \tag{8}$$

where

$$\psi_i(x_0,\bar{z}_i,\hat{\theta}_0) = \frac{\phi_i(u_0,x_0,\bar{x}_i)}{x_0^{n-i}} - \frac{(n-i)z_i\phi_0(x_0)}{x_0}, \tag{9}$$

$$f_i(x_0,z_i,\hat{\theta}_0) = -(n-i)g_0(x_0,\hat{\theta}_0)z_i \tag{10}$$

are computable and known for controller design and $\bar{z}_i \triangleq [z_1,\dots,z_i]^{\mathrm{T}}$, $1 \leqslant i \leqslant n-1$.

By following the standard procedures, stable adaptive control can be obtained.

Step 1: Define $e_1 = z_1$, $e_2 = z_2 - \alpha_1$, α_1 is referred to be the first virtual control to stabilize the e_1-subsystem. Consider

$$V_1 = \frac{1}{2}e_1^2 + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta} \tag{11}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ and $\hat{\theta}$ is another estimate of θ.

Choosing the virtual control α_1 and the tuning function τ_1 as

$$\begin{aligned}\alpha_1(x_0,z_1,\hat{\theta}_0,\hat{\theta}) = \frac{1}{g_0(x_0,\hat{\theta}_0)}[&-k_1e_1 - f_1(x_0,z_1,\hat{\theta}_0)\\ &-\psi_1^{\mathrm{T}}(x_0,z_1,\hat{\theta}_0)\hat{\theta}], \quad k_1 > 0,\end{aligned} \tag{12}$$

$$\tau_1(x_0,z_1,\hat{\theta}_0) = \Gamma e_1\psi_1(x_0,z_1,\hat{\theta}_0) \tag{13}$$

yields

$$\dot{V}_1 \leqslant -k_1e_1^2 + g_0e_1e_2 - \tilde{\theta}^{\mathrm{T}}\Gamma^{-1}(\dot{\hat{\theta}} - \Gamma e_1\psi_1(x_0,z_1,\hat{\theta}_0)).$$

Note that the term with $\tilde{\theta}$ would have been eliminated if we had chosen the following update law $\dot{\hat{\theta}} = \tau_1$. Since this is not the last step, we postpone the choice of update law and tolerate the presence of $\tilde{\theta}$ in $\dot{V}_1$.

Step i ($2 \leqslant i \leqslant n$): Consider $V_i = V_{i-1} + \frac{1}{2}e_i^2$, let $h_1(x_0,z_1,\hat{\theta}_0,\hat{\theta}) = 0$, and define

$$\begin{aligned}w_i(x_0,\bar{z}_i,\hat{\theta}_0,\hat{\theta}) &= \psi_i(x_0,\bar{z}_i,\hat{\theta}_0) - \frac{\partial\alpha_{i-1}(x_0,\bar{z}_{i-1},\hat{\theta}_0,\hat{\theta})}{\partial x_0}\\ &\quad\times\phi_0(x_0) - \cdots - \frac{\partial\alpha_{i-1}(x_0,\bar{z}_{i-1},\hat{\theta}_0,\hat{\theta})}{\partial z_{i-1}}\psi_{i-1}(x_0,\bar{z}_{i-1},\hat{\theta}_0)\end{aligned}$$

$$\eta_i(x_0,\bar{z}_i,\hat{\theta}_0,\hat{\theta}) = \frac{\partial\alpha_{i-1}(x_0,\bar{z}_{i-1},\hat{\theta}_0,\hat{\theta})}{\partial\hat{\theta}}$$

$$h_i(x_0,\bar{z}_i,\hat{\theta}_0,\hat{\theta}) = h_{i-1}(x_0,\bar{z}_{i-1},\hat{\theta}_0,\hat{\theta}) + e_i\eta_i(x_0,\bar{z}_i,\hat{\theta}_0,\hat{\theta})$$

which are computable for $2 \leqslant i \leqslant n$.

By choosing the virtual control α_i, the tuning function τ_i, $2 \leqslant i \leqslant n-1$, as

$$\begin{aligned}\alpha_i(x_0,\bar{z}_i,\hat{\theta}_0,\hat{\theta}) &= \frac{1}{g_0(x_0,\hat{\theta}_0)}\Big\{-g_0(x_0,\hat{\theta}_0)e_{i-1} - k_ie_i - w_i^{\mathrm{T}}(x_0,\bar{z}_i,\hat{\theta}_0,\hat{\theta})\hat{\theta}\\ &\quad+\frac{\partial\alpha_{i-1}(x_0,\bar{z}_{i-1},\hat{\theta}_0,\hat{\theta})}{\partial x_0}u_0(x_0,\hat{\theta}_0) + \frac{\partial\alpha_{i-1}(x_0,\bar{z}_{i-1},\hat{\theta}_0,\hat{\theta})}{\partial\hat{\theta}_0}\tau_0(x_0) + \frac{\partial\alpha_{i-1}(x_0,\bar{z}_{i-1},\hat{\theta}_0,\hat{\theta})}{\partial z_1}[g_0(x_0,\hat{\theta}_0)z_1\\ &\quad+f_1(x_0,z_1,\hat{\theta}_0)] + \cdots + \frac{\partial\alpha_{i-1}(x_0,\bar{z}_{i-1},\hat{\theta}_0,\hat{\theta})}{\partial z_{i-1}}[g_0(x_0,\hat{\theta}_0)z_i + f_{i-1}(x_0,z_{i-1},\hat{\theta}_0)] + h_{i-1}(x_0,\bar{z}_{i-1},\hat{\theta}_0,\hat{\theta})\\ &\quad\times\Gamma w_i(x_0,\bar{z}_i,\hat{\theta}_0,\hat{\theta}) + \eta_i(x_0,\bar{z}_i,\hat{\theta}_0,\hat{\theta})\tau_i(x_0,\bar{z}_i,\hat{\theta}_0,\hat{\theta})\Big\}\end{aligned}$$

$$\tau_i(x_0,\bar{z}_i,\hat{\theta}_0,\hat{\theta}) = \tau_{i-1}(x_0,\bar{z}_{i-1},\hat{\theta}_0,\hat{\theta}) + \Gamma e_iw_i(x_0,\bar{z}_i,\hat{\theta}_0,\hat{\theta})$$

and the actual control u_1, update law for $\hat\theta$ as

$$\begin{aligned} u_1&(x_0,z,\hat\theta_0,\hat\theta)\\ &= -g_0(x_0,\hat\theta_0)e_{n-1}-k_ne_n-w_n^{\mathrm T}(x_0,z,\hat\theta_0,\hat\theta)\hat\theta\\ &\quad+\frac{\partial\alpha_{n-1}(x_0,\bar z_{n-1},\hat\theta_0,\hat\theta)}{\partial x_0}u_0(x_0,\hat\theta_0)\\ &\quad+\frac{\partial\alpha_{n-1}(x_0,\bar z_{n-1},\hat\theta_0,\hat\theta)}{\partial\hat\theta_0}\tau_0(x_0)\\ &\quad+\frac{\partial\alpha_{n-1}(x_0,\bar z_{n-1},\hat\theta_0,\hat\theta)}{\partial z_1}[g_0(x_0,\hat\theta_0)z_2+f_1(x_0,z_1,\hat\theta_0)]\\ &\quad+\cdots+\frac{\partial\alpha_{n-1}(x_0,\bar z_{n-1},\hat\theta_0,\hat\theta)}{\partial z_{n-1}}[g_0(x_0,\hat\theta_0)z_n\\ &\quad+f_{n-1}(x_0,z_{n-1},\hat\theta_0)]+\eta_n(x_0,z,\hat\theta_0,\hat\theta)\tau_n(x_0,z,\hat\theta_0,\hat\theta)\\ &\quad+h_{n-1}(x_0,\bar z_{n-1},\hat\theta_0,\hat\theta)\Gamma w_n(x_0,z,\hat\theta_0,\hat\theta),\ k_n>0, \end{aligned} \tag{14}$$

$$\dot{\hat\theta}=\tau_n(x_0,z,\hat\theta_0,\hat\theta) \tag{15}$$

then, at the last step, we can show that

$$\dot V_n\leqslant -k_0x_0^2-\sum_{j=1}^n k_je_j^2. \tag{16}$$

Note that $u_1(x_0,z,\hat\theta_0,\hat\theta)$ and $\alpha_i(x_0,\bar z_i,\hat\theta_0,\hat\theta)$ are smooth functions and satisfy $u_1(x_0,0,\hat\theta_0,\hat\theta)=0$, $\forall x_0,\hat\theta_0,\hat\theta$ and $\alpha_i(x_0,0,\hat\theta_0,\hat\theta)=0$, $\forall x_0,\hat\theta_0,\hat\theta$.

We have thus far completed the controller design procedure for $x_0(t_0)\neq 0$. Without loss of generality, we can assume that $t_0=0$.

3.2. Switching control strategy design

For $x_0(0)=0$, different schemes can be used for different classes of systems. The most commonly used control strategy is to use constant control for u_0 and u_1 under certain strict conditions. However, for systems with non-Lipschitz nonlinearities, the choice of constant feedbacks for u_0 and u_1 may lead to a finite escape for the systems in question. A simple example is given by

$$\dot x=u+kx^2. \tag{17}$$

Given any constant control $u=u_c$, the explicit solution of x is

$$x(t)=\tan\left(t\sqrt{ku_c}\right)\frac{\sqrt{ku_c}}{k}. \tag{18}$$

It can be seen that for any $t_s>0$, we can always find a finite k such that the solution $x(t)$ blows up before t_s. In fact, finite escape exists for $\dot x=u+kx^n, n\geqslant 2$. Thus, care must be taken to avoid the possibility of finite escape for such classes of systems.

3.2.1. New adaptive switching

In this paper, a novel adaptive switching is presented to solve the problem of finite escape for the class of systems where the uncertainties do not satisfy the Lipschitz condition in general.

When $x_0(t_0)=0$, choose u_0 as

$$u_0=x_0g_0+u_0^* \tag{19}$$

where g_0 is given by (3), $\hat\theta_0$ is updated by (5), and constant $u_0^*>0$.

Choosing the same Lyapunov function (4), its time derivative is given by

$$\dot V_0\leqslant -k_0x_0^2+u_0^*x_0 \tag{20}$$

which leads to the boundedness of x_0, and consequently the boundedness of $\hat\theta_0$ as well.

Using the control law (19), the closed-loop dynamics of the x_0-subsystem is

$$\dot x_0=-x_0\left(\sqrt{k_0^2+(\varphi_0^{\mathrm T}\hat\theta_0)^2}-\varphi_0^{\mathrm T}\tilde\theta_0\right)+u_0^*. \tag{21}$$

Since $x_0(t)$ and $\tilde\theta_0(t)$ are bounded, the solution of (21) is

$$x_0(t)=\mathrm e^{-\int_{t_0}^t\lambda(s)\,\mathrm ds}\int_{t_0}^t u_0^*\mathrm e^{\int_{t_0}^s\lambda(\tau)\,\mathrm d\tau}\,\mathrm ds +x_0(t_0)\mathrm e^{-\int_{t_0}^t\lambda(s)\,\mathrm ds},$$

where $\lambda=\sqrt{k_0^2+(\varphi_0^{\mathrm T}\hat\theta_0)^2}-\varphi_0^{\mathrm T}\tilde\theta_0$. Consequently, we have x_0 does not escape and $x_0(t_s)\neq 0$ for any given finite $t_s>0$. Thus, state scaling for the control design can be carried out.

During the time period $[0,t_s]$, using u_0 defined in (19), backstepping-based feedback $u_1=u_1^*(x_0,x,\hat\theta)$ and a new update law $\dot{\hat\vartheta}=\dot{\hat\vartheta}^*(x_0,x,\hat\vartheta)$ can be obtained by following the control design procedure described in Section 3.1 to the original x-system in (1). Then we conclude that the x-state of (1) do not blow up during the time period $[0,t_s]$. Since $x_0(t_s)\neq 0$ at t_s, we can switch the control input u_0 and u_1 to (2) and (4), respectively.

Theorem 1. *Under Assumption* 1, *if the control law* (2) *and the full state feedback control law* (14) *are applied to* (1) *with adaptation laws* (5) *and* (15) *along with the new adaptive control based switching strategy*, *uncertain system* (1) *is globally regulated at origin*, *and the estimated parameters are bounded.*

Proof. Choose the Lyapunov function $V=V_0+\sum_{i=1}^n\frac12 e_i^2+\frac12\tilde\theta^{\mathrm T}\Gamma^{-1}\tilde\theta$. Its time derivative is given by $\dot V\leqslant -k_0x_0^2-\sum_{j=1}^n k_je_j^2$, which means $E=(x_0,e_1,\ldots,e_n)\in L_\infty^{n+1}$ and $\tilde\theta_0,\tilde\theta\in L_\infty^l$. Since θ is a constant vector, we have $\hat\theta_0,\hat\theta$ are bounded. From LaSalle's Invariant Theorem, it further concludes that $(E,\hat\theta_0,\hat\theta)$ converges to the largest invariant set M contained in the set where $\dot V=0$, which implies that $E(t)\to 0$ as $t\to\infty$.

Note that all the virtual controls $\alpha_i(0,\dots,0,\hat{\theta}_0,\hat{\theta}) = 0$, $1 \leqslant i \leqslant n-1$ and the actual control $u_1(0,\dots,0,\hat{\theta}_0,\hat{\theta})=0$. These properties along with $E(t) \to 0$ as $t \to \infty$ imply that $(x_0(t), z(t)) \to 0$ as $t \to \infty$. Accordingly, we conclude that $(x_0(t), x(t)) \to 0$ as $t \to \infty$.

3.2.2. Constant control based switching

For a class of systems with its nonlinearities satisfying the Lipschitz condition $|\phi_0^{\mathrm{T}}\theta| \leqslant c_0|x_0|$ (which is a special case of the problem setting of the paper), the constant control based switching strategy can also be used to solve the problem elegantly (Huo & Ge, 2001; Jiang, 2000).

When $x_0(0) = 0$, choose u_0 as a constant as follows

$$u_0 = u_0^*, \quad u_0^* > 0. \tag{22}$$

It is easy to obtain that x_0 does not escape since the first drift term is global Lipschitz in the special case. Given any finite $t_s > 0$, during the time period $[0, t_s]$, substituting u_0 defined in (22) instead of (2) into the original x-system in (1), we have

$$\begin{aligned} \dot{x}_i &= u_0^* x_{i+1} + \phi_i^{\mathrm{T}}(x_0, \bar{x}_i)\theta \quad 1 \leqslant i \leqslant n, 2 \leqslant n, \\ \dot{x}_n &= u_1 + \phi_n^{\mathrm{T}}(x_0, x)\theta. \end{aligned} \tag{23}$$

As system (23) is a special case of system (8), the proposed control design can be directly applied. Since $x_0(t_s) \neq 0$ at t_s, we can switch the control input u_0 and u_1 to (2) and (14), respectively.

4. Output feedback control

When only the system output is measurable and the rest of the system states are not available for feedback, we need to estimate them. It is found that when the system uncertainties are in the linear-in-parameters (LIP) form, a more stringent condition has to be imposed to make the output feedback stabilization problem solvable.

Assumption 2. x_0-subsystem in system (1) owns a special structure, i.e. $\dot{x}_0 = u_0 + c_0 x_0$, where c_0 is known.

For each $1 \leqslant i \leqslant n$, there is a known smooth function vector $\bar{\phi}_i$ such that

$$\phi_i(u_0, x_0, \bar{x}_i) = x_1 \bar{\phi}_i(u_0, x_0, x_1)$$

which is a function of u_0 and the available states x_0 and x_1 only.

Remark 3. In comparison with Assumption 1, Assumption 2 is more strict. Without loss of generality (change of input), it can be assumed that $c_0 = 0$, i.e., $\dot{x}_0 = u_0$. For consistence, c_0 is kept in the following derivation.

As in Section 3, let us consider $x_0(0) \neq 0$ first. The fact that c_0 is a known constant leads us to choose the following control law

$$u_0 = -k_0 x_0, \quad k_0 > c_0 \tag{24}$$

with respect to the following Lyapunov function candidate $V_0 = \frac{1}{2}x_0^2$, whose time derivative is given by $\dot{V}_0 = -(k_0 - c_0)x_0^2 \leqslant 0$. As a consequence, we have $x_0 \to 0$ as $t \to \infty$.

Using the state scaling (7), the x-system is transformed into

$$\dot{z} = Az + bu_1 + \psi^{\mathrm{T}}(x_0, x)\theta \tag{25}$$

where

$$A = \begin{bmatrix} a_{11} & -k_0 & 0 & \cdots & 0 \\ 0 & a_{22} & -k_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -k_0 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{26}$$

and $a_{ii} = (n-i)(k_0 - c_0)$, $\psi^{\mathrm{T}} = [\psi_1 \;\; \psi_2 \;\; \dots \;\; \psi_n]^{\mathrm{T}} = \left[\frac{\phi_1(-k_0x_0, x_0, x_1)}{x_0^{n-1}} \;\; \frac{\phi_2(-k_0x_0, x_0, \bar{x}_2)}{x_0^{n-2}} \;\; \dots \;\; \phi_n(-k_0x_0, x_0, x) \right]^{\mathrm{T}}$.

Lemma 1. *There exists a smooth function matrix $\bar{\psi}$ such that*

$$\psi(x_0, x) = z_1 \bar{\psi}(x_0, z_1). \tag{27}$$

Proof. The proof is straightforward from Assumption 2.

Denote $C=[1,\dots,0]$ and consider $y=Cz=z_1$ as the output of the transformed system (25). It is easy to verify that the pair (A, C) is observable and the pair (A, b) is controllable. In the following, an observer is to be designed to estimate the unmeasurable states $[z_2, \dots, z_n]$.

To facilitate the following adaptive backstepping control design, the regressor matrix of the drift nonlinearities needs to be known. If traditional linear observers were used to estimate the unmeasured states, the regressor matrix would unavoidably become unknown. In order to avoid the uncertain regressor matrix in parameter adaptation, an adaptive nonlinear observer is presented to overcome this difficulty.

The adaptive observer is given by

$$\dot{\hat{z}} = A\hat{z} + bu_1 + L(y - C\hat{z}) + \psi^{\mathrm{T}}\hat{\theta} - \zeta^{\mathrm{T}}\dot{\hat{\theta}}, \tag{28}$$

where $L = [l_1, l_2, \dots, l_n]^{\mathrm{T}}$ is chosen so that $A - LC$ is a stable matrix, and $\zeta^{\mathrm{T}} = [\zeta_1 \; \cdots \; \zeta_n]^{\mathrm{T}} \in R^{n \times l}$ is defined by the following $n \times l$ matrix differential equation

$$\dot{\zeta}^{\mathrm{T}} = (A - LC)\zeta^{\mathrm{T}} - \psi^{\mathrm{T}} \tag{29}$$

as discussed in the literature of parametric adaptive control (Krstic, Kanellakopoulos, & Kokotovic, 1995).

Let $e = z - \hat{z}$, $\tilde{\theta} = \theta - \hat{\theta}$, thus $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$. Multiplying on the right hand side of (29) by $\tilde{\theta}$ and rearranging it, we obtain

$$\psi^{\mathrm{T}}\tilde{\theta} = (A - LC)\zeta^{\mathrm{T}}\tilde{\theta} - \dot{\zeta}^{\mathrm{T}}\tilde{\theta}. \tag{30}$$

Define

$$\tilde{e} = e + \zeta^{\mathrm{T}}\tilde{\theta}. \tag{31}$$

The overall system to be controlled is given by

$$\dot{\tilde{e}} = (A - LC)\tilde{e}, \tag{32}$$

$$\dot{z}_1 = -k_0\hat{z}_2 + (n-1)(k_0 - c_0)z_1 + \psi_1^{\mathrm{T}}\theta - k_0 e_2, \tag{33}$$

$$\dot{\hat{z}}_i = a_{ii}\hat{z}_i - k_0\hat{z}_{i+1} + l_i e_1 + \psi_i^{\mathrm{T}}\hat{\theta} - \zeta_i^{\mathrm{T}}\dot{\hat{\theta}}, \tag{34}$$

$$\dot{\hat{z}}_n = u_1 + l_n e_1 + \psi_n^{\mathrm{T}}\hat{\theta} - \zeta_n^{\mathrm{T}}\dot{\hat{\theta}}. \tag{35}$$

In the following, output feedback control is presented.

Step 1: Define

$$\xi_1 = z_1, \tag{36}$$

$$\xi_2 = \hat{z}_2 - \alpha_1 \tag{37}$$

where α_1 is viewed as the first virtual control input to stabilize the $(\tilde{e}, z_1)$-subsystem

$$\begin{aligned} \dot{\tilde{e}} &= (A - LC)\tilde{e} \\ \dot{z}_1 &= -k_0\hat{z}_2 + (n-1)(k_0 - c_0)z_1 + \psi_1^{\mathrm{T}}\theta - k_0 e_2. \end{aligned} \tag{38}$$

Choose the following Lyapunov function candidate

$$V_1 = \tilde{e}^{\mathrm{T}}P\tilde{e} + \frac{1}{2}\xi_1^2 + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta} \tag{39}$$

where P is the solution of the Lyapunov equation

$$P(A - LC) + (A - LC)^{\mathrm{T}}P = -I_n. \tag{40}$$

By completing the squares, we have

$$-\xi_1 k_0\tilde{e}_2 \leqslant \frac{1}{2}\tilde{e}^{\mathrm{T}}\tilde{e} + \frac{k_0^2\xi_1^2}{2}.$$

Choose the virtual control α_1 and the tuning function τ_1 as

$$\alpha_1 = k_1\xi_1 + (n-1)\left(1 - \frac{c_0}{k_0}\right)z_1 + \frac{\psi_1^{\mathrm{T}}\hat{\theta}}{k_0} + \frac{k_0\xi_1}{2}, \quad k_1 > 0 \tag{41}$$

$$\tau_1 = \Gamma\xi_1(\psi_1 + k_0\zeta_2), \quad \Gamma = \Gamma^{\mathrm{T}} > 0. \tag{42}$$

The time derivative of V_1 is given by

$$\dot{V}_1 \leqslant -\frac{\tilde{e}^{\mathrm{T}}\tilde{e}}{2} - k_0 k_1\xi_1^2 - k_0\xi_1\xi_2 + \tilde{\theta}^{\mathrm{T}}\Gamma^{-1}(\dot{\tilde{\theta}} + \tau_1) \tag{43}$$

where the coupling term $k_0\xi_1\xi_2$ will be cancelled at the next step.

Step i $(2 \leqslant i \leqslant n)$: Consider the Lyapunov function candidate $V_i = V_{i-1} + \frac{1}{2}\xi_i^2$. Again, by completing the squares as in Step 1, there exist a smooth nonnegative function $p_i(x_0, z_1, \ldots, \hat{z}_i, \hat{\theta})$ and a smooth function vector $g_i(x_0, z_1, \ldots, \hat{z}_i, \hat{\theta})$ such that

$$\xi_i\frac{\partial\alpha_{i-1}}{\partial z_1}k_0 e_2 + \xi_i l_i e_1 + \xi_i\sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{z}_j}l_j e_1 \leqslant \frac{\tilde{e}^{\mathrm{T}}\tilde{e}}{2^i} + \xi_i^2 p_i - \xi_i g_i^{\mathrm{T}}\tilde{\theta}.$$

Let $h_1 = 0$, and define

$$w_i = -\left(g_i + \frac{\partial\alpha_{i-1}}{\partial z_1}\psi_1\right), \tag{44}$$

$$\eta_i = \zeta_i^{\mathrm{T}} - \sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{z}_j}\zeta_j^{\mathrm{T}} + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}, \tag{45}$$

$$h_i = h_{i-1} + \xi_i\eta_i \tag{46}$$

which are computable for $2 \leqslant i \leqslant n$.

By choosing the virtual control α_i, the tuning function τ_i, $2 \leqslant i \leqslant n-1$, as

$$\begin{aligned} \alpha_i = {} & -\xi_{i-1} + k_i\xi_i + (n-i)\left(1 - \frac{c_0}{k_0}\right)\hat{z}_i + \frac{\psi_i^{\mathrm{T}}\hat{\theta}}{k_0} + \frac{\partial\alpha_{i-1}}{\partial x_0} \\ & \times(1 - c_0/k_0)x_0 + \frac{\partial\alpha_{i-1}}{\partial z_1}\left[\hat{z}_2 - (n-1)\left(1 - \frac{c_0}{k_0}\right)\xi_1\right] \\ & - \frac{\partial\alpha_{i-1}}{\partial z_1}\frac{\psi_1^{\mathrm{T}}\hat{\theta}}{k_0} + \frac{\xi_i p_i}{k_0} + \sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{z}_j}\left[\hat{z}_{j+1} - \frac{\psi_j^{\mathrm{T}}\hat{\theta}}{k_0}\right. \\ & \left. - (n-j)\left(1 - \frac{c_0}{k_0}\right)\hat{z}_j\right] - \frac{h_{i-1}\Gamma w_i}{k_0} - \frac{\eta_i\tau_i}{k_0}, \end{aligned} \tag{47}$$

$$\tau_i = \tau_{i-1} + \Gamma\xi_i w_i \tag{48}$$

and the actual control u_1, adaptation law for $\hat{\theta}$ as

$$\begin{aligned} u_1 = {} & k_0\xi_{n-1} - k_n\xi_n - \psi_n^{\mathrm{T}}\hat{\theta} + \frac{\partial\alpha_{n-1}}{\partial x_0}(c_0 - k_0)x_0 \\ & + \frac{\partial\alpha_{n-1}}{\partial z_1}[-k_0\hat{z}_2 + (n-1)(k_0 - c_0)\xi_1] - \xi_n p_n \\ & + \frac{\partial\alpha_{n-1}}{\partial z_1}\psi_1^{\mathrm{T}}\hat{\theta} + \sum_{j=2}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{z}_j}[(n-j)(k_0 - c_0)\hat{z}_j \\ & - k_0\hat{z}_{j+1} + \psi_j^{\mathrm{T}}\hat{\theta}] + h_{n-1}\Gamma w_n + \eta_n\tau_n \end{aligned} \tag{49}$$

$$\dot{\hat{\theta}} = \tau_n \tag{50}$$

then, at the last step, we can show that

$$\dot{V}_n \leqslant -\frac{\tilde{e}^{\mathrm{T}}\tilde{e}}{2^n} - \sum_{j=1}^{n-1}k_0 k_j\xi_j^2 - k_n\xi_n^2. \tag{51}$$

It is of interest to note that $u_1(x_0,\xi,\hat\theta)$ and $\alpha_i(x_0,\bar{\xi}_i,\hat\theta)$ are smooth functions and satisfy $u_1(x_0,0,\hat\theta)=0$, and $\alpha_i(x_0,0,\hat\theta)=0$, $\forall x_0,\hat\theta\in R\times R^l$.

We have thus far completed the output feedback controller design procedure for $x_0(0)\neq 0$. For $x_0(0)=0$, the switching control strategies in Section 3.2 can be applied and the following result for the output feedback stabilization is obtained.

Theorem 2. *Under Assumption 2, if the static output feedback law (24) and the dynamic output feedback law (49) are applied to (1) with adaptation law (50) along with the switching control strategy, uncertain system (1) is globally regulated at the origin. Furthermore, the estimated parameters are bounded.*

Proof. Choose the Lyapunov function $V=V_0+\tilde e^{\mathrm T}P\tilde e+\sum_{i=1}^{n}\frac12\xi_i^2+\frac12\tilde\theta^{\mathrm T}\Gamma^{-1}\tilde\theta$. Its time derivative is given by $\dot V\leqslant-(k_0-c_0)x_0^2-\sum_{j=1}^{n-1}k_0k_j\xi_j^2-k_n^2\xi_n^2-\frac{\tilde e^{\mathrm T}\tilde e}{2^n}$, which means $E=(x_0,\xi_1,\ldots,\xi_n,\tilde e)\in L_\infty^{2n+1}$ and $\tilde\theta\in L_\infty^l$. Hence, we can conclude that ψ is bounded. Since θ is constant, we know that $\hat\theta$ is bounded. From (29), since $(A-LC)$ is a stable matrix and ψ is bounded, we conclude that ζ is bounded.

From LaSalle's Invariant Theorem, it further concludes that $(E,\hat\theta)$ converges to the largest invariant set M contained in the set where $\dot V=0$, which implies that $E(t)\to 0$ as $t\to\infty$. From Lemma 2, $\psi\to 0$ as $t\to\infty$. Hence, it can be conclude that $\zeta\to 0$ as $t\to\infty$ from (29). From the definition of $\tilde e$, since $\tilde e,\zeta\to 0$ as $t\to\infty$ and $\tilde\theta$ is bounded, $e\to 0$ as $t\to\infty$.

From the design procedure, we can see that $\alpha_i, 1\leqslant i<n-1$, and u_1 are smooth functions of their arguments and for all $\hat\theta\in R^l$, $\alpha_i(0,\ldots,0,\hat\theta)=0$, and $u_1(0,\ldots,0,\hat\theta)=0$. These properties along with $E(t)\to 0$ as $t\to\infty$ imply that $(x_0(t),z_1(t),\hat z_2(t),\ldots,\hat z_n(t))\to 0$ as $t\to\infty$. Since we already have $e\to 0$ as $t\to\infty$, which implies that $(x_0(t),z(t))\to 0$ as $t\to\infty$. Accordingly, we conclude that $(x_0(t),x(t))\to 0$ as $t\to\infty$.

Remark 4. From linear system theory, since (A,C) is observable and (A,b) is controllable, it is natural to use a traditional full order linear observer to estimate the unmeasured states, i.e. $\dot{\hat z}=A\hat z+bu_1+L(y-C\hat z)$. From $e=z-\hat z$, the observer error dynamics would become

$$\dot e=(A-LC)e+\psi^{\mathrm T}\theta. \tag{52}$$

However, because of the presence of uncertainty θ, it is hard to judge the stability of observer (52). Furthermore, because e is unmeasured, it is impossible to use backstepping technique to find any one control input to stabilize the observation error.

To overcome such difficulties, adaptive observer (28) was used, then the observation error dynamics becomes a linear and stable one (32). Hence, the appearance of unknown regressor matrix has been avoided, and the adaptive backstepping control design can be continued.

Remark 5. It is interesting to note that for the third order system, $n=2$, the condition of not containing any unknown parameter on the x_0-subsystem can be relaxed to allow the existence of uncertainty because of the corresponding regressor matrix does not contain any unmeasured state. In fact, in this case, there is only one state, x_2 needs to be estimated.

Let us consider the system

$$\begin{aligned}\dot x_0&=d_0u_0+c_0x_0,\\ \dot x_i&=u_0x_{i+1}+\phi_i^{\mathrm T}(u_0,x_0,\bar x_i)\theta\quad 1\leqslant i<n,\ n\geqslant 2,\\ \dot x_n&=u_1+\phi_n^{\mathrm T}(u_0,x_0,x)\theta,\end{aligned} \tag{53}$$

where d_0 is unknown but lower bounded by $d_0\geqslant d_1>0$, constant c_0 is known, and ϕ_i satisfy Assumption 2.

Using the knowledge of d_1 and c_0, we know that control

$$u_0=-k_0x_0,\quad k_0>\frac{c_0}{d_1} \tag{54}$$

guarantees $x_0\to 0$ as $t\to\infty$. Through the transformation $z_i=x_i/x_0^{n-i}$, $1\leqslant i\leqslant n$, the x-subsystem in (53) can be similarly transformed into

$$\dot z=Az+bu_1+\varphi^{\mathrm T}(x_0,x)\vartheta \tag{55}$$

where

$$A=\begin{bmatrix}-(n-1)c_0 & -k_0 & 0 & \cdots & 0\\ 0 & -(n-2)c_0 & -k_0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & -k_0\\ 0 & 0 & 0 & \cdots & 0\end{bmatrix},$$

$$b=\begin{bmatrix}0\\0\\ \vdots\\0\\1\end{bmatrix}$$

$$\varphi^{\mathrm T}=\begin{bmatrix}\dfrac{\phi_1^{\mathrm T}(-k_0x_0,x_0,x_1)}{x_0^{n-1}} & (n-1)k_0z_1\\ \dfrac{\phi_2^{\mathrm T}(-k_0x_0,x_0,\bar x_2)}{x_0^{n-2}} & (n-2)k_0z_2\\ \vdots & \vdots\\ \phi_n^{\mathrm T} & 0\end{bmatrix},\quad \vartheta=\begin{bmatrix}\theta\\ d_0\end{bmatrix}.$$

Due to the introduction of unknown d_0, the regressor matrix φ has to be augmented as well. From the expression of φ, we can clearly see the presence of unavailable states $z_2,\ldots,z_{n-1}$ which prohibits output feedback controller design.

However, for the third order system, φ is simply given by

$$\varphi^{\mathrm{T}} = \begin{bmatrix} \phi_1^{\mathrm{T}} & k_0 z_1 \\ \phi_2^{\mathrm{T}} & 0 \end{bmatrix} \tag{56}$$

which does not contain the unknown state x_2 or z_2 at all. As the φ in (56) is known, the proposed control design procedure can be applied directly.

When $c_0 = 0$, the lower bound condition for the x_0-subsystem can be further relaxed to known sign. For example, it is relaxed from condition $d_0 \geqslant d_1 > 0$ to $d_0 > 0$ as the control for u_0 can be simply chosen as $u_0 = -k_0 x_0$ with $k_0 > 0$.

5. Simulation results

In this section, the bilinear model of a mobile robot with small angle measurement error (Morin, Pomet, & Samson, 1998) is simulated as it is a practical system, which is described by

$$\dot{x}_l = \left(1 - \frac{\varepsilon^2}{2}\right) v, \qquad \dot{y}_l = \theta_l v + \varepsilon v, \qquad \dot{\theta}_l = w. \tag{57}$$

By using the transformation $x_0 = x_l$, $x_1 = y_l$, $x_2 = \theta_l + \varepsilon$, $u_0 = v$, $u_1 = w$, it becomes

$$\begin{aligned} \dot{x}_0 &= \left(1 - \frac{\varepsilon^2}{2}\right) u_0 \\ \dot{x}_1 &= x_2 u_0 \\ \dot{x}_2 &= u_1 \end{aligned} \tag{58}$$

which is in the form as discussed in Remark 5.

Due to space limitation, state feedback control is omitted. Assuming that x_2 is unmeasured, the controller design procedure in Section 4 is applied to regulate the states of the transformed system (58) to $(0, 0, -\varepsilon)$. For simplicity, it is assumed that $(1 - \varepsilon^2/2) > 0$ and $x_0(0) \neq 0$. Hence, for any $k_0 > 0$, the control law (24) can be applied. After performing the state scaling (7), we have $\theta = 1 - \varepsilon^2/2$ and

$$A = \begin{bmatrix} 0 & -k_0 \\ 0 & 0 \end{bmatrix}, \qquad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$\psi^{\mathrm{T}} = \begin{bmatrix} k_0 z_1 \\ 0 \end{bmatrix}, \qquad C = [1\ 0].$$

Then the adaptive observer, control law for u_1 and the update law are given by (28), (49) and (50), respectively.

In the simulation, the design parameters are chosen as $k_0 = k_1 = k_2 = 1$, $l_1 = 10$ and $l_2 = -25$ to place both poles of $(A - LC)$ at -5. The system parameter is chosen as $\varepsilon = 0.1$ and the initial conditions are $(x_l(0), y_l(0), \theta_l(0)) = (1, 1, 1)$ and $\hat{\theta}(0) = 0$.

As can be seen from Fig. 1, all signals $(x_l, y_l, \theta_l + \varepsilon)$ globally converge to zero. Fig. 2 shows the control inputs.

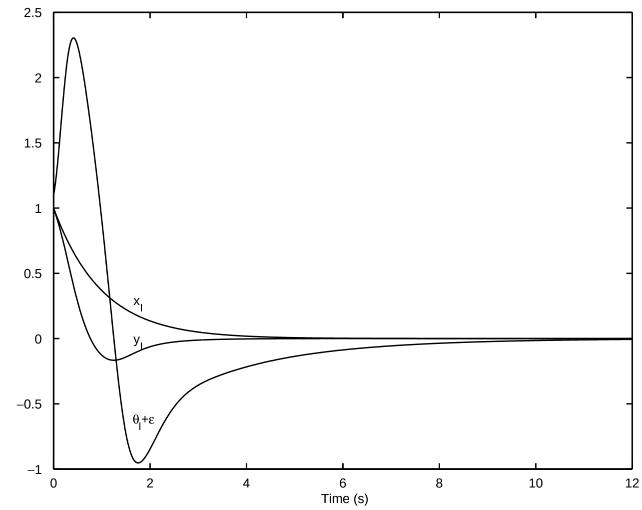

Fig. 1. System states.

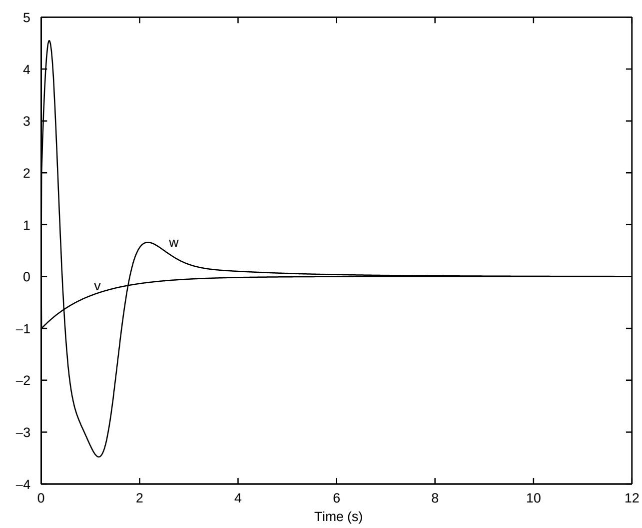

Fig. 2. Control inputs.

Comparing Fig. 3 with Fig. 2 in Jiang (2000), it can be seen that the control efforts using the controller derived in the paper are relatively mild in contrast to the robust controller presented in Jiang (2000). Fig. 3 shows that the estimation of the unknown constant is bounded but does not approach to its true value.

6. Conclusion

In this paper, constructive adaptive state feedback control has been presented for stabilizing a class of uncertain nonholonomic chained systems without imposing any restriction on the system order and the growth of the drift nonlinearities. A new switching control strategy has been proposed. Observer-based output feedback control has been proposed when only partial system states are measurable. An adaptive

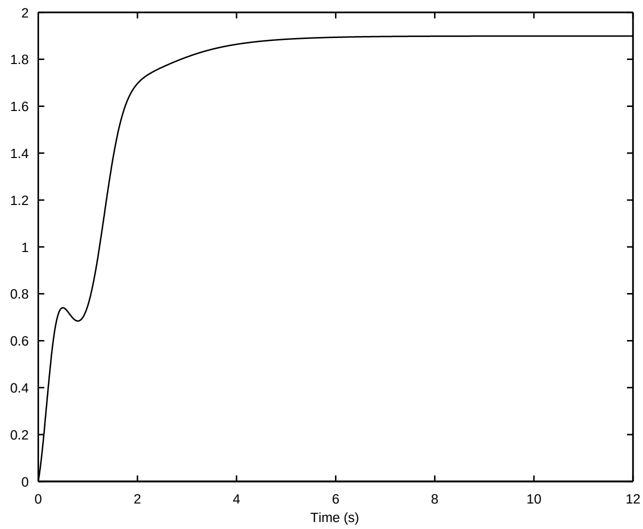

Fig. 3. $\hat{\theta}$.

observer has been constructed to handle the technical problem due to the presence of unavailable states in the regressor matrix. All the system states have been proven to globally converge to the origin, while the estimated parameters have maintained bounded. Simulation results have shown the effectiveness of our proposed control approach.

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References

Arnold, V. I. (1987). *Geometrical methods in the theory of ordinary differential equations*. Berlin: Springer.

Astolfi, A. (1996). Discontinuous control of nonholonomic systems. *Systems and Control Letters*, *27*, 37–45.

Astolfi, A., & Schaufeberger, W. (1996). State and output feedback stabilization of multiple chained systems with discontinuous control. In *Proceedings of the 35th IEEE conference on decision & control*, Kobe, Japan (pp. 1443–1447).

Brockett, R. W. (1983). Asymptotic stability and feedback stabilization. In *Differential geometry control theory* (pp. 181–208). Basel: Birkhauser.

Canudas de Wit, C., Berghuis, H., & Nijmeijer, H. (1994). Practical stabilization of nonlinear systems in chained form. In *Proceedings of the 33rd IEEE conference on decision & control*, Lake Buena Vista, FL, USA (pp. 3475–3480).

Colbaugh, R., Barany, R., & Glass, K. (1996). Adaptive control of nonholonomic mechanical systems. In *Proceedings of the 35th IEEE conference on decision & control*, Kobe, Japan (pp. 1428–1434).

Dixon, W. E., Dawson, D. M., Zergeroglu, E., & Behal, A. (2001). Nonlinear control of wheeled mobile robots. In *Lecture Notes in Control and Information Sciences*, Vol. 262. London: Springer.

Do, KahDuc., & Pan, J. (2002). Adaptive global stabilization of nonholonomic systems with strong nonlinear drifts. *Systems & Control Letters*, *46*, 195–205.

Fierro, R., & Lewis, F. L. (1995). Control of a nonholonomic mobile robot: Backstepping kinematics into dynamics. In *Proceedings of the 34th IEEE conference on decision & control*, New Orleans, LA, USA (pp. 1722–1727).

Fliess, M., Levine, J., Martin, P., & Rouchon, P. (1995). Flatness and defect of nonlinear systems: Introductory theory and examples. *International Journal of Control*, *61*, 1327–1361.

Ge, S. S., Sun, A., Lee, T. H., & Spong, M. W. (2001a). Feedback linearization and stabilization of second-order nonholonomic chained systems. *International Journal of Control*, *74*(14), 1383–1392.

Ge, S. S., Wang, J., Lee, T. H., & Zhou, G. Y. (2001b). Adaptive robust stabilization of dynamic nonholonomic chained systems. *Journal of Robotic Systems*, *18*(3), 119–133.

Hespanha, J. P., Liberzon, S., & Morse, A. S. (1999). Towards the supervisory control of uncertain nonholonomic systems. In *Proceedings of the 1999 American control conference*, San Diego, CA, USA (pp. 3520–3524).

Huo, W., & Ge, S. S. (2001). Exponential stabilization of nonholonomic systems: An eni approach. *International Journal of Control*, *74*(15), 1492–1500.

Jiang, Z. P. (2000). Robust exponential regulation of nonholonomic systems with uncertainties. *Automatica*, *36*(2), 189–209.

Jiang, Z. P., & Nijmeijer, H. (1999). A recursive technique for tracking control of nonholonomic systems in chained form. *IEEE Transactions on Automatic Control*, *44*(2), 265–279.

Kolmanovsky, I., & McClamroch, N. H. (1995). Developments in nonholonomic control problems. *IEEE Control System Magazine*, *15*(6), 20–36.

Krstic, M., Kanellakopoulos, I., & Kokotovic, P. V. (1995). *Nonlinear and adaptive control design*. New York: Wiley.

M'Closkey, R. T., & Murray, R. M. (1992). Convergence rate for nonholonomic systems in power form. In *Proceedings of the American control conference*, Chicago, USA (pp. 2489–2493).

Morin, P., Pomet, J. B., & Samson, C. (1998). Developments in time-varying feedback stabilization of nonlinear systems. In *Preprints of nonlinear control systems design symposium (NOLCOS'98)*, Enschede (pp. 587–594).

Murray, R. M. (1993). Control of nonholonomic systems using chained form. *Fields Institute of Communications*, *1*, 219–245.

Murray, R. M., & Sastry, S. S. (1993). Nonholonomic motion planning: Steering using sinusoids. *IEEE Transactions on Automatic Control*, *38*, 700–716.

Samson, C. (1993). Time-varying feedback stabilization of a nonholonomic wheeled mobile robot. *International Journal of Robotics Research*, *12*, 55–66.

Sontag, E. D. (1989). Smooth stabilization implies coprime factorization. *IEEE Transactions on Automatic Control*, *34*(4), 435–443.

Sordalen, O. J., & Canudas de Wit, C. (1995). Exponential stabilization of nonholonomic chained systems. *IEEE Transactions on Automatic Control*, *40*(1), 35–49.

Sun, Z., Ge, S. S., Huo, W., & Lee, T. H. (2001). Stabilization of nonholonomic chained systems via nonregular feedback linearization. *System & Control Letters*, *44*, 279–289.

Walsh, G. C., & Bushnell, L. G. (1995). Stabilization of multiple input chained form control systems. *System & Control Letters*, *25*, 227–234.

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