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A direct adaptive controller for dynamic systems with a class of nonlinear parameterizations[1]

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Abstract

In this note, the adaptive control problem is considered for a class of nonlinearly parametrized systems. By introducing a novel kind of Lyapunov functions, a direct adaptive controller is developed for achieving asymptotic tracking control. The transient performance of the resulting closed-loop system can be guaranteed by suitably choosing the Lyapunov function to construct the controller. The effectiveness of the proposed scheme is illustrated with two examples. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Nonlinear system; Adaptive control; Lyapunov stability; Nonlinear parameterization; Transient performance

1. Introduction

Adaptive control of nonlinear systems has been an active research area and many good theoretical results have been obtained in the literature (Sastry and Isidori 1989; Kanellakopoulos et al. 1991; Krstic et al. 1995; Johansen and Ioannou, 1996; Marino and Tomei, 1995) and the references therein. Most available adaptive controllers deal with control problem of systems with nonlinearities being linear in the unknown parameters. In practice, however, nonlinear parameterization is very common in many physical plants. Adaptive control for nonlinearly parametrized systems is an interesting and challenging problem in control community. Marino and Tomei (1993) studied the global output feedback control for systems containing nonlinear parameterizations, which is designed using high-gain adaptation and applicable to set-point regulation problem. For a class of first-order nonlinearly parametrized models similar to those arising in fermentation processes, Boskovic (1995) provided an interesting adaptive control scheme with three unknown parameters (two of them do not enter linearly). The key points of this design method lie in the appropriate parameterization of the plant and the suitable choice of a Lyapunov function with a cubic term for developing the stable adaptive controller.

In this paper, we deal with the state-feedback adaptive tracking control problem for nonlinear systems with a class of nonlinear parameterizations. A novel kind of Lyapunov functions is developed to construct a Lyapunov-based controller and parameter updating law. It is shown that the globally asymptotic tracking is achieved with guaranteed control performance. This paper is organized as follows. Section 2 shows the control problem and the definition of weighted control Lyapunov function (WCLF). Section 3 presents the direct adaptive controller and stability analysis of the closed-loop system. Tracking performance of the adaptive system is discussed in Section 4. Two examples are given in Section 5 to show the effectiveness of the controller proposed. Section 6 contains the conclusion.

2. Problem statement

Let us consider the nonlinear systems given in the following form

$$\dot{x}_i = x_{i+1}, \quad i = 1, 2, \ldots, n-1$$
$$\dot{x}_n = \frac{1}{\beta(x)}[f(x) + g(x)u], \tag{1}$$
$$y = x_1,$$

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where $x = [x_1 \ x_2 \ \cdots \ x_n]^T \in \mathbb{R}^n$, $u \in \mathbb{R}$, $y \in \mathbb{R}$ are the state variables, system input and output, respectively; $g(x)$ is a known continuous function; functions $f(x), \beta(x) \in C^1$ can be expressed as

$$f(x) = \theta^T w_f(x) + f_0(x), \qquad \beta(x) = \theta^T w_\beta(x) + \beta_0(x) \tag{2}$$

where $\theta \in \mathbb{R}^p$ is a vector of unknown constant parameters, $w_f(x) \in \mathbb{R}^p$ and $w_\beta(x) \in \mathbb{R}^p$ are known regressor vectors, functions $f_0(x), \beta_0(x) \in C^1$ are known. The control objective is to find a controller u such that output y follows a given reference signal y_d.

Clearly, the unknown parameter vector θ enters into system (1) nonlinearly. Many practical systems, such as pendulum plants (Cannon 1967, Balestrino et al. 1984) and fermentation processes (Boskovic, 1995), can be described by system (1) and possess such a kind of nonlinear parameterizations. In this paper, the following assumption is made.

Assumption 1. *$g(x)/\beta(x) \neq 0, \forall x \in \mathbb{R}^n$ and its sign is known.*

The above assumption implies that the continuous function $g(x)/\beta(x)$ is strictly either positive or negative. From now onward, without losing generality, we shall assume $g(x) > 0$ and $\beta(x) > 0$ for all $x \in \mathbb{R}^n$. Define vectors x_d, ε and a filtered error ε_s as

$$x_d = [y_d \ \dot{y}_d \ \cdots \ y_d^{(n-1)}]^T, \ \varepsilon = x - x_d = [\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n]^T,$$

$$\varepsilon_s = \left(\frac{d}{dt} + \lambda\right)^{n-1} \varepsilon_1 = [\Lambda^T \ 1]\varepsilon, \tag{3}$$

where constant $\lambda > 0$ and

$$\Lambda = [\lambda^{n-1} \ (n-1)\lambda^{n-2} \ \cdots \ (n-1)\lambda]^T.$$

Remark 2.1. It has been shown in the reference (Slotine and Li, 1991) that definition (2) has the following properties: (i) the equation $\varepsilon_s = 0$ defines a time-varying hyperplan in $\mathbb{R}^n$ on which the tracking error ε_1 converges to zero asymptotically, (ii) if the magnitude of ε_s is bounded, the error vector $\varepsilon(t)$ is also bounded, and (iii) a state representation of Eq. (3) can be expressed as $\dot{\zeta} = A_s\zeta + b_s\varepsilon_s$ with $\zeta = [\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_{n-1}]^T$, $n \geq 2$, A_s a stable matrix depending on λ and $b_s = [0 \ 0 \ \cdots \ 0 \ 1]^T$.

From Eqs. (1) and (3), the time derivative of ε_s can be written as

$$\dot{\varepsilon}_s = \frac{1}{\beta(x)}[f(x) + g(x)u] + v, \tag{4}$$

where $v = -y_d^{(n)} + [0 \ \Lambda^T]\varepsilon$. Let $\beta_\alpha(x) = \beta(x)\alpha(x)$ with the smooth function $\alpha(x)$: $\mathbb{R}^n \to \mathbb{R}_+$ to be specified later. It is can be seen from Eq. (3) that $x_n = \varepsilon_s + y_d^{(n-1)} - [\Lambda^T \ 0]\varepsilon$. For the ease of discussion, we shall denote $\beta_\alpha(\bar{x}, \varepsilon_s + v_1) = \beta_\alpha(x)$ with $\bar{x} = [x_1 \ x_2 \ \cdots \ x_{n-1}]^T$ and $v_1 = y_d^{(n-1)} - [\Lambda^T \ 0]\varepsilon$.

Definition 2.1. *For a bounded reference vector x_d, a scalar function*

$$V_\varepsilon = \int_0^{\varepsilon_s} \sigma\beta_\alpha(\bar{x}, \sigma + v_1)\,d\sigma \tag{5}$$

is called a weighted control Lyapunov function (WCLF) for system (1), if there exist a smooth function $\alpha(x)$ and a control input u such that V_ε satisfies:

1. V_ε is positive definite in the filtered error ε_s,
2. V_ε is radially unbounded with respect to ε_s, i.e., $V_\varepsilon \to \infty$ as $|\varepsilon_s| \to \infty$, and
3. $\dot{V}_\varepsilon < 0, \forall \varepsilon_s \neq 0$

In addition, $\alpha(x)$ is called a weighting function (WF).

3. Adaptive controller design

In this section, we first show that for system (1) satisfying Assumption 1, there indeed exists a WF $\alpha(x)$ and a control input u such that V_ε defined in Eq. (5) is a WCLF. Then, we construct an adaptive controller using this WCLF for achieving asymptotic tracking control.

As $\beta(x) > 0$ is linear in the unknown constant parameters, a smooth function $\alpha(x)$ can be found such that V_ε satisfies conditions 1 and 2 in Definition 2.1. For example, if $\beta(x) = \exp(-x_n)(\theta_1 + x_n^2)$ with constant $\theta_1 > 0$, then we may choose $\alpha(x) = \exp(x_n)$ which leads to

$$V_\varepsilon = \int_0^{\varepsilon_s} \sigma[\theta_1 + (\sigma + v_1)^2]\,d\sigma$$

$$= \frac{\varepsilon_s^2}{4}[(\varepsilon_s + \tfrac{4}{3}v_1)^2 + \tfrac{2}{9}v_1^2 + 2\theta_1].$$

Clearly, the above function is positive definite and radially unbounded with respect to ε_s. Taking the time derivative of V_ε given in Eq. (5), we have

$$\dot{V}_\varepsilon = \beta_\alpha(x)\varepsilon_s\dot{\varepsilon}_s + \int_0^{\varepsilon_s} \sigma\left[\frac{\partial\beta_\alpha(\bar{x}, \sigma + v_1)}{\partial\bar{x}}\dot{\bar{x}} + \frac{\partial\beta_\alpha(\bar{x}, \sigma + v_1)}{\partial v_1}\dot{v}_1\right]d\sigma. \tag{6}$$

Because $\partial\beta_\alpha(\bar{x},\sigma+v_1)/\partial v_1 = \partial\beta_\alpha(\bar{x},\sigma+v_1)/\partial\sigma$ and $v = -\dot{v}_1$, it follows that

$$\int_0^{\varepsilon_s}\sigma\frac{\partial\beta_\alpha(\bar{x},\sigma+v_1)}{\partial v_1}\dot{v}_1\mathrm{d}\sigma$$

$$= -v\int_0^{\varepsilon_s}\sigma\frac{\partial\beta_\alpha(\bar{x},\sigma+v_1)}{\partial\sigma}\mathrm{d}\sigma$$

$$= -v\left[\sigma\beta_\alpha(\bar{x},\sigma+v_1)\Big|_0^{\varepsilon_s} - \int_0^{\varepsilon_s}\beta_\alpha(\bar{x},\sigma+v_1)\mathrm{d}\sigma\right]$$

$$= -v\varepsilon_s\beta_\alpha(x) + v\int_0^{\varepsilon_s}\beta_\alpha(\bar{x},\sigma+v_1)\mathrm{d}\sigma.$$

Substituting the above equation into Eq. (6) and using Eq. (4), we obtain

$$\dot{V}_\varepsilon = \frac{\beta_\alpha(x)}{\beta(x)}[f(x)+g(x)u]\varepsilon_s + \int_0^{\varepsilon_s}\left[\sigma\frac{\partial\beta_\alpha(\bar{x},\sigma+v_1)}{\partial\bar{x}}\dot{\bar{x}} + v\beta_\alpha(\bar{x},\sigma+v_1)\right]\mathrm{d}\sigma.$$

Noting the expressions in Eq. (2), we have

$$\dot{V}_\varepsilon = \varepsilon_s\alpha(x)[\theta^{\mathrm{T}}w(z) + g(x)u + h(z)], \tag{7}$$

where

$$w(z) = w_f(x) + \frac{1}{\varepsilon_s\alpha(x)} \times \int_0^{\varepsilon_s}\left[\sigma\frac{\partial\bar{w}_\beta(\bar{x},\sigma+v_1)}{\partial\bar{x}}\dot{\bar{x}} + v\bar{w}_\beta(\bar{x},\sigma+v_1)\right]\mathrm{d}\sigma, \tag{8}$$

$$h(z) = f_0(x) + \frac{1}{\varepsilon_s\alpha(x)} \times \int_0^{\varepsilon_s}\left[\sigma\frac{\partial\bar{\beta}_0(\bar{x},\sigma+v_1)}{\partial\bar{x}}\dot{\bar{x}} + v\bar{\beta}_0(\bar{x},\sigma+v_1)\right]\mathrm{d}\sigma, \tag{9}$$

$z = [x^{\mathrm{T}}\ x_d^T\ y_d^{(n)}]^{\mathrm{T}} \in \mathbb{R}^{2n+1}$, $\bar{w}_\beta(\bar{x},\sigma+v_1) = w_\beta(\bar{x},\sigma+v_1)\alpha(\bar{x},\sigma+v_1) \in \mathbb{R}^p$ and $\bar{\beta}_0(\bar{x},\sigma+v_1) = \beta_0(\bar{x},\sigma+v_1)\alpha(\bar{x},\sigma+v_1) \in \mathbb{R}$. It can be checked that

$$\lim_{\varepsilon_s\to 0} w(z) = w_f(x) + \frac{v\bar{w}_\beta(\bar{x},v_1)}{\alpha(x)},$$

$$\lim_{\varepsilon_s\to 0} h(z) = f_0(x) + \frac{v\bar{\beta}_0(\bar{x},v_1)}{\alpha(x)}.$$

Hence, both $w(z)$ and $h(z)$ are well defined. If the parameter vector θ is available, a possible controller is $u^* = g^{-1}(x)[-k(\varepsilon_s/\alpha(x)) - \theta^{\mathrm{T}}w(z) - h(z)]$ with design parameter $k > 0$. For this controller, Eq. (7) becomes $\dot{V}_\varepsilon = -k\varepsilon_s^2 < 0, \forall\varepsilon_s \neq 0$. According to Definition 2.1, we conclude that V_ε is a WCLF and $\varepsilon_s \to 0$ as $t\to\infty$. In the case of unknown parameter θ, we employ its *certainty-equivalence* controller as

$$u = \frac{1}{g(x)}\left[-\frac{k\varepsilon_s}{\alpha(x)} - \hat{\theta}^{\mathrm{T}}w(z) - h(z)\right], \tag{10}$$

where $\hat{\theta}$ is the estimate of θ. Define a parameter estimate error $\tilde{\theta} = \hat{\theta} - \theta$ and substitute Eq. (10) into Eq. (7), we obtain

$$\dot{V}_\varepsilon = -k\varepsilon_s^2 - \tilde{\theta}^{\mathrm{T}}w(z)\alpha(x)\varepsilon_s. \tag{11}$$

The system stability is not clear at this stage because the last term in Eq. (11) is indefinite and contains unknown $\tilde{\theta}$. To remove such an uncertainty, parameter adaptive tuning is introduced for $\hat{\theta}$. For constructing an adaptive law, we augment V_ε as follows

$$V = V_\varepsilon + \tfrac{1}{2}(\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}) \tag{12}$$

with gain matrix $\Gamma = \Gamma^{\mathrm{T}} > 0$. The time derivative of V along Eq. (11) is

$$\dot{V} = -k\varepsilon_s^2 + \tilde{\theta}^{\mathrm{T}}[-w(z)\alpha(x)\varepsilon_s + \Gamma^{-1}\dot{\hat{\theta}}]. \tag{13}$$

To eliminate $\tilde{\theta}$ from $\dot{V}$, the adaptive law can be chosen as

$$\dot{\hat{\theta}} = \Gamma w(z)\alpha(x)\varepsilon_s \tag{14}$$

which leads to

$$\dot{V} = -k\varepsilon_s^2 \leq 0. \tag{15}$$

Since function $\beta_\alpha(x) \in C^1$, Eq. (5) shows that V_ε is a C^1 function of x and x_d. This guarantees that $V_\varepsilon(0) \in L_\infty$ for any bounded initial values $x(0)$ and $x_d(0)$. Integrating Eq. (15), we have $\int_0^\infty k\varepsilon_s^2(\tau)\mathrm{d}\tau \leq V(0) < \infty$ and $0 \leq V(t) \leq V(0)$. This implies that $\varepsilon_s \in L_2 \cap L_\infty$ and $\hat{\theta}(t)$ is bounded. Consequently, u and $\dot{\varepsilon}_s$ are also bounded. Since $\varepsilon_s \in L_2\cap L_\infty$ and $\dot{\varepsilon}_s \in L_\infty$, we conclude $\lim_{t\to\infty}\varepsilon_s = 0$ by Barbalat's lemma (Popov, 1973). It follows from Remark 2.1 that $x \in L_\infty$ and the tracking error converges to zero asymptotically. The above result is summarized in the following Theorem.

Theorem 3.1. *For system* (1) *satisfying Assumption* 1, *controller* (10) *with adaptive law* (14) *guarantees the boundedness of all the signals in the closed-loop system and the globally asymptotic tracking, i.e.,* $\lim_{t\to\infty} y(t) = y_d(t)$.

4. Performance analysis

As shown in the preceding section, a key step in the design procedure is the choice of WF $\alpha(x)$ and WCLF V_ε. It should be pointed out that for a given system, different WF can be found to construct different WCLF. Therefore, the resulting controller is not unique and the control performance also varies with the choice of WCLFs. This brings the designer some degrees of freedom in controller design. In the following, we show that for controller (10) with a suitably chosen WF $\alpha(x)$, transient performance of the closed-loop system can be guaranteed.

Theorem 4.1. *For the closed-loop adaptive system* (1), (10) *and* (14), *if WF $\alpha(x)$ is chosen such that $\beta_\alpha(x) \le c_0$ with c_0 a positive constant, then*

(i) *L_2 transient bound of the filtered error*

$$\int_0^\infty \varepsilon_s^2(\tau)\,d\tau \le \frac{1}{2k}[c_0\varepsilon_s^2(0) + \tilde{\theta}^T(0)\Gamma^{-1}\tilde{\theta}(0)], \tag{16}$$

(ii) *for the systems with $n \ge 2$, the L_∞ tracking error bound*

$$\|\zeta(t)\| \le k_0\|\zeta(0)\|e^{-\lambda_0 t} + \frac{k_0}{2\sqrt{k\lambda_0}}\sqrt{c_0\varepsilon_s^2(0) + \tilde{\theta}^T(0)\Gamma^{-1}\tilde{\theta}(0)} \tag{17}$$

with computable constants $k_0, \lambda_0 > 0$ which depend on the design parameter λ.

Proof. (i) If $\alpha(x)$ is chosen such that $0 < \beta_\alpha(x) \le c_0$, then

$$V_\varepsilon = \int_0^{\varepsilon_s} \sigma\beta_\alpha(\bar{x}, \sigma + v_1)\,d\sigma \le c_0\int_0^{\varepsilon_s}\sigma\,d\sigma = \frac{c_0}{2}\varepsilon_s^2. \tag{18}$$

Integrating Eq. (15) over $[0, t]$ and applying Eq. (18), we obtain

$$\int_0^t k\varepsilon_s^2(\tau)\,d\tau \le -\int_0^t \dot{V}\,d\tau = V(0) - V(t) \le \frac{c_0}{2}\varepsilon_s^2(0) + \frac{1}{2}\tilde{\theta}^T(0)\Gamma^{-1}\tilde{\theta}(0), \quad \forall t \ge 0 \tag{19}$$

from which L_2 bound (16) can be concluded.

(ii) For the systems with order $n \ge 2$, Remark 2.1 shows that $\dot{\zeta} = A_s\zeta + b_s\varepsilon_s$ with stable matrix A_s. It is not difficult to find two constants $k_0, \lambda_0 > 0$ which depend on the design parameter λ such that $\|e^{A_s t}\| \le k_0 e^{-\lambda_0 t}$ (Ioannou and Sun, 1996). The solution for ζ can be written as

$$\zeta(t) = e^{A_s t}\zeta(0) + \int_0^t e^{A_s(t-\tau)}b_s\varepsilon_s(\tau)\,d\tau.$$

Therefore

$$\|\zeta(t)\| \le k_0\|\zeta(0)\|e^{-\lambda_0 t} + k_0\int_0^t e^{-\lambda_0(t-\tau)}|\varepsilon_s(\tau)|\,d\tau. \tag{20}$$

Applying the following Schwartz inequality (Ioannou and Sun, 1996)

$$\int_0^t |a(\tau)b(\tau)|\,d\tau \le \left(\int_0^t a^2(\tau)\,d\tau\right)^{1/2}\left(\int_0^t b^2(\tau)\,d\tau\right)^{1/2}, \tag{21}$$

we have

$$\|\zeta(t)\| \le k_0\|\zeta(0)\|e^{-\lambda_0 t} + k_0\left(\int_0^t e^{-2\lambda_0(t-\tau)}\,d\tau\right)^{1/2}\left(\int_0^t \varepsilon_s^2(\tau)\,d\tau\right)^{1/2} \le k_0\|\zeta(0)\|e^{-\lambda_0 t} + \frac{k_0}{\sqrt{2\lambda_0}}\left(\int_0^t \varepsilon_s^2(\tau)\,d\tau\right)^{1/2}.$$

Using Eq. (19), the inequality (17) follows. □

Remark 4.1. The L_∞ bound of the error vector ζ in Theorem 4.1 is obtained for high-order systems ($n \ge 2$). For a first-order system, to get an explicit bound of the tracking error, an additional condition $\beta_\alpha(x) \ge c_1$ is needed for the choice of $\alpha(x)$. In this case ($\varepsilon_s = \varepsilon_1$ for $n = 1$)

$$V_\varepsilon = \int_0^{\varepsilon_1}\sigma\beta_\alpha(\bar{x}, \sigma + v_1)\,d\sigma \ge c_1\int_0^{\varepsilon_1}\sigma\,d\sigma = \frac{c_1}{2}\varepsilon_1^2. \tag{22}$$

Noticing $V_\varepsilon(t) \le V(t) \le V(0)$, we have $\varepsilon_1^2(t) \le 2V(0)/c_1$. From $V_\varepsilon(0) \le c_0\varepsilon_1^2(0)/2$, the L_∞ tracking bound for the first-order system can be found

$$|\varepsilon_1(t)| \le \sqrt{\frac{c_0}{c_1}\varepsilon_1^2(0) + \frac{1}{c_1}\tilde{\theta}^T(0)\Gamma^{-1}\tilde{\theta}(0)}. \tag{23}$$

Remark 4.2. Theorem 4.1 shows that different choices of WF $\alpha(x)$ may produce different control performance. As $\beta_\alpha(x) = [\theta^T w_\beta(x) + \beta_0(x)]\alpha(x)$ with known functions $w_\beta(x)$ and $\beta_0(x)$, it is not difficult to design a WF $\alpha(x)$ to make $0 < c_1 \le \beta_\alpha(x) \le c_0$. For example, if $\beta(x) = \exp(-x_n^2)(\theta_1 + \theta_2 x_n^2)$ with constant parameters θ_1, $\theta_2 > 0$, then one may take $\alpha(x) = \exp(x_n^2)/(1 + x_n^2)$ which leads to

$$\min[\theta_1, \theta_2] \le \beta_\alpha(x) = \frac{\theta_1 + \theta_2 x_n^2}{1 + x_n^2} \le \max[\theta_1, \theta_2].$$

Remark 4.3. From a practical point of view, Assumption 1 holds on whole space might be a strong restriction for many physical plants. If Assumption 1 holds only on a compact subset $\Omega \subset \mathbb{R}^n$, the proposed approach is still applicable if the controller parameters are designed appropriately. The reason is that by suitably choosing the design parameters, upper bounds of the states (derived from Eqs. (17) and (23)) are adjustable by the designer, and subsequently can be guaranteed within the given compact set Ω in which Assumption 1 is satisfied for all time. The second example given in Section 5 illustrates such an application.

5. Case study

Example 5.1. To show the controller design procedure and validate the effectiveness of the developed scheme, we consider a second-order system

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = \frac{x_1^2 + u}{\exp(-x_1^2)(\theta_1 + \theta_2 x_2^2)} \tag{24}$$

with unknown parameters $\theta_1, \theta_2 > 0$. The objective is to control the output $y = x_1$ to follow the reference $y_d(t) = \sin(0.5t)$. Plant (24) can be expressed in the form of system (1) with $f(x) = x_1^2$, $g(x) = 1$ and $\beta(x) = \exp(-x_1^2)(\theta_1 + \theta_2 x_2^2)$. Comparing with Eq. (2), we have $f_0(x) = x_1^2$, $\theta = [\theta_1 \ \theta_2]^{\mathrm{T}}$ and $w_\beta(x) = [\exp(-x_1^2) \ \exp(-x_1^2) x_2^2]^{\mathrm{T}}$. In view of Remark 4.2, we choose the WF $\alpha(x) = \exp(x_1^2)/(1 + x_2^2)$. It follows from Eqs. (8) and (9), that

$$w(z) = \frac{v}{\varepsilon_s \alpha(x)} \left[\int_0^{\varepsilon_s} \frac{1}{1 + (\sigma + v_1)^2} \mathrm{d}\sigma \int_0^{\varepsilon_s} \frac{(\sigma + v_1)^2}{1 + (\sigma + v_1)^{\ 2}} \mathrm{d}\sigma \right]^{\mathrm{T}}$$

$$= \frac{v}{\varepsilon_s \alpha(x)} [\tan^{-1} x_2 - \tan^{-1} v_1 \ \varepsilon_s - \tan^{-1} x_2 + \tan^{-1} v_1]^{\mathrm{T}}$$

with $v_1 = \dot{y}_d - \lambda(x_1 - y_d)$, $v = -\ddot{y}_d + \lambda(x_2 - \dot{y}_d)$, and $h(z) = x_1^2$. Then, Eqs. (10) and (14) suggest the following controller

$$u = -k(1 + x_2^2)\exp(-x_1^2)\varepsilon_s - \hat{\theta}^{\mathrm{T}} w(z) - x_1^2$$

with adaptive laws

$$\dot{\hat{\theta}}_1 = \gamma_1 v(\tan^{-1} x_2 - \tan^{-1} v_1),$$

$$\dot{\hat{\theta}}_2 = \gamma_2 v(\varepsilon_s - \tan^{-1} x_2 + \tan^{-1} v_1).$$

In the simulation, the true values of the system parameters are $[\theta_1 \ \theta_2]^{\mathrm{T}} = [2.0 \ 0.5]^{\mathrm{T}}$ and the initial condition is $[x_1(0) \ x_2(0)]^{\mathrm{T}} = [0.5 \ 0.0]^{\mathrm{T}}$. The parameters of the adaptive controller are $\lambda = 1.0$, $k = 1.0$, $\gamma_1 = 10.0$, $\gamma_2 = 25.0$ and $[\hat{\theta}_1(0) \ \hat{\theta}_2(0)]^{\mathrm{T}} = [0.0 \ 0.0]^{\mathrm{T}}$. The simulation result given in Fig. 1a indicates that the output tracking error converges to zero asymptotically. The responses of the estimated parameters and control input are shown in Fig. 1b and c, respectively.

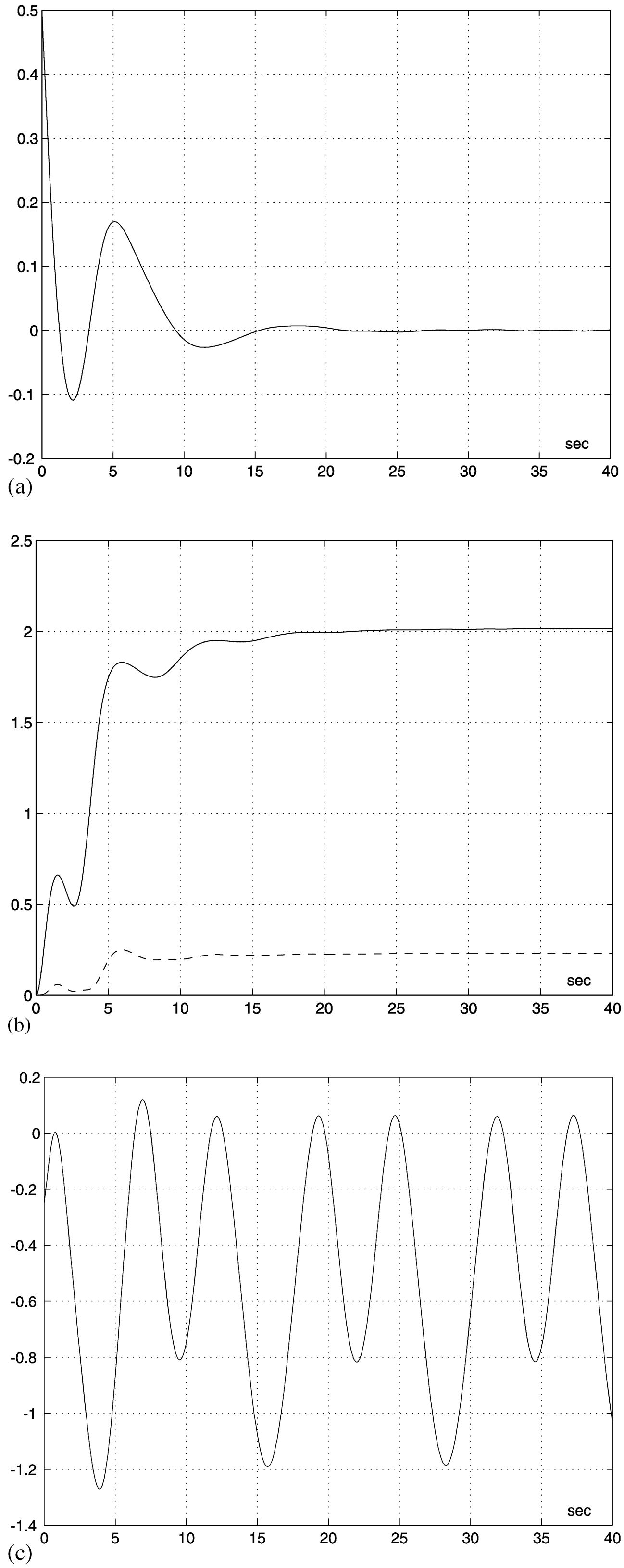

Fig. 1. Responses of the adaptive system in Example 5.1. (a) Tracking error $y - y_d$ (b) $\hat{\theta}_1$ ("—") and $\hat{\theta}_2$ ("- -") (c) Control input $u(t)$.

Example 5.2. In this example, we apply the proposed approach to an inverted pendulum plant (Cannon, 1967) described by

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = \frac{g \sin x_1 - \dfrac{mLx_2^2 \sin x_1 \cos x_1}{M + m}}{L\left(\dfrac{4}{3} - \dfrac{m \cos^2 x_1}{M + m}\right)} + \frac{\dfrac{\cos x_1}{M + m}}{L\left(\dfrac{4}{3} - \dfrac{m \cos^2 x_1}{M + m}\right)} u,$$

$$y = x_1, \tag{25}$$

where x_1 and x_2 are the angular displacement and velocity of the pendulum, respectively; $g = 9.8\ \mathrm{m/s^2}$ is the gravity acceleration coefficient; M and m are the masses of the cart and the pole, respectively; L is the half-length of the pole, and u is the applied force control. The true values of the plant are $M = 1.0$ kg, $m = 0.2$ kg and $L = 0.5$ m, initial states are $[x_1(0)\ \ x_2(0)]^{\mathrm{T}} = [0\ \ 0]^{\mathrm{T}}$, and reference signal is $y_d(t) = \pi/6\sin(t)$. Let

$$\theta = \begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\end{bmatrix} = \begin{bmatrix} M+m \\ -mL \\ \frac{4}{3}L(M+m)\end{bmatrix}, \qquad w_f(x) = \begin{bmatrix} g\sin x_1 \\ x_2^2 \sin x_1 \cos x_1 \\ 0\end{bmatrix},$$

$$w_\beta(x) = \begin{bmatrix} 0 \\ \cos^2 x_1 \\ 1\end{bmatrix}.$$

The plant (25) can be written in the form of system (1) with $f(x) = \theta^{\mathrm{T}} w_f(x)$, $g(x) = \cos x_1$ and $\beta(x) = \theta^{\mathrm{T}} w_\beta(x)$. Although the pendulum plant (25) does not satisfy Assumption 1 for $x \in \mathbb{R}^2$, it can be checked that $g(x)/\beta(x) \neq 0$ for all $|x_1| < \pi/2$. In order to apply the proposed method, the design parameters should be specified such that $|x_1| < \pi/2$ holds for all time as discussed in Remark 4.3. Take WF $\alpha(x) = 1$, it follows from Eqs. (8) and (9) that

$$w(z) = \begin{bmatrix} g\sin x_1 \\ v_1 x_2 \sin x_1 \cos x_1 + v\cos^2 x_1 \\ v\end{bmatrix}, \qquad h(z) = 0.$$

According to Eqs. (10) and (14), the adaptive controller can be chosen as

$$u = \frac{1}{\cos x_1}[-k\varepsilon_s - \hat{\theta}^{\mathrm{T}} w(z)], \quad \text{with } \dot{\hat{\theta}} = \Gamma w(z)\varepsilon_s.$$

In the simulation, controller parameters are set as $\lambda = 1.0$ and $\Gamma = \mathrm{diag}\{0.2\}$, and initial condition $\hat{\theta}(0) = 0.0$. To avoid possible controller singularity when $|x_1| = \pi/2$, the design parameter k is chosen as follows. Suppose that very conservative bounds of plant parameters M, m and L are known as $M \le 1.5$ kg, $m \le 0.3$ kg and $L \le 0.75$ m. It can be shown that $\beta(x) = L[\frac{4}{3}(M+m) - m\cos^2 x_1] \le 1.8$ and $\|\tilde{\theta}(0)\|^2 \le 6.5306$. According to Eq. (17) in Theorem 4.1, the upper bound of x_1 can be obtained

$$|x_1(t)| \le |y_d| + |\varepsilon_1(0)| + \frac{1}{2\sqrt{k}}\sqrt{1.8\varepsilon_s^2(0) + \frac{1}{0.2}\|\tilde{\theta}(0)\|^2}, \quad \forall t \ge 0. \quad (26)$$

Since $y_d = \pi/6\sin(t)$ and the initial states $[x_1(0)\ \ x_2(0)]^{\mathrm{T}} = [0\ \ 0]^{\mathrm{T}}$, we know that $|y_d| \le \frac{\pi}{6}$, $\varepsilon_1(0) = 0$ and $|\varepsilon_s(0)| = \pi/6$. It can be calculated from Eq. (26) that if the gain $k > 7.6$, then $|x_1| < \pi/2$ can be guaranteed. In the simulation test, we let $k = 10.0$. Fig. 2a shows that

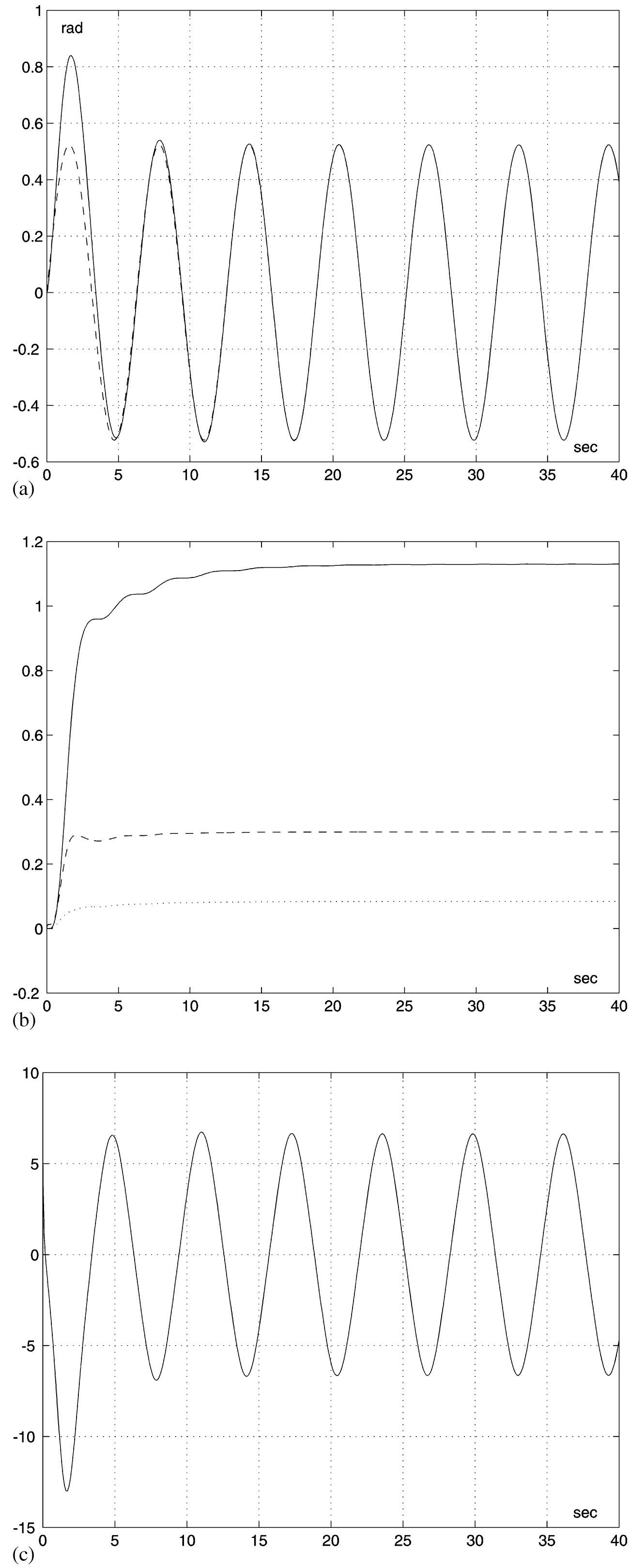

Fig. 2. Responses of the adaptive system in Example 5.2. (a) Output y ("—") follows y_d ("- -") (b) $\hat{\theta}_1$ ("—"), $\hat{\theta}_2$ ("⋯") and $\hat{\theta}_3$ ("- -") (c) Control input $u(t)$.

although the tracking error is large during the initial 5 s due to the inadequate initial parameter $\hat{\theta}(0) = 0.0$, the transient bound of $|x_1|$ is smaller than $\pi/2$. As the parameters are adaptively tuned on-line, the output $y(t)$ tracks the reference $y_d(t)$ asymptotically. The boundedness of the estimated parameters and control signal are also presented in Fig. 2b and c, respectively.

6. Conclusion

We have presented a direct adaptive controller for a class of dynamic systems with nonlinear parameterization. The main feature of the paper is the construction of the weighted control Lyapunov function, which can be used to remove the nonlinear parameterization for adaptive controller design. Global stability and asymptotic convergence of tracking error have been obtained and the control performance of the resulting adaptive system has been investigated.

References

Balestrino, A., De Maria, G., & Zinober, A. S. I. (1984). Nonlinear adaptive model-following control. *Automatica*, *20*, 559–568.

Boskovic, J. D. (1995). Stable adaptive control of a class of first-order nonlinearly parameterized plants. *IEEE Trans. Automat. Control*, *AC-40*, 347–350.

Cannon, R. H. (1967). *Dynamics of physical systems*, New York: McGraw-Hill.

Ioannou, P. A., & Sun, J. (1996). *Robust adaptive control.* Englewood Cliffs, NJ: Prentice-Hall.

Johansen, T. A., & Ioannou, P. A. (1996). Robust adaptive control of minimum phase non-linear systems. *Int. J. Adaptive Control Signal Process*, *10*, 61–78.

Kanellakopoulos, I., Kokotovic, P. V., & Morse, A. S. (1991). Systematic design of adaptive controller for feedback linearizable systems. *IEEE Trans. Automat. Control*, *AC-36*, 1241–1253.

Krstic, M., Kanellakopoulos, I., & Kokotovic, P. V. (1995). *Nonlinear and adaptive control design.* New York: Wiley.

Marino, R., & Tomei, P. (1993). Global adaptive output-feedback control of nonlinear systems, part II: Nonlinear parameterization. *IEEE Trans. Automat. Control*, *AC-38*, 17–48.

Marino, R., & Tomei, P. (1995). *Nonlinear adaptive design*: *Geometric*, *adaptive*, *and robust*. London: Printice-Hall.

Popov, M. V. (1973). *Hyperstability of control systems.* New York, NY: Springer.

Sastry, S. S., & Isidori, A. (1989). Adaptive control of linearizable systems. *IEEE Trans. Automat. Control*, *AC-34*, 1123–1131.

Slotine, J. E., & Li, W. (1991). *Applied nonlinear control.* Englewood Cliff, NJ: Prentice-Hall.