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Brief Paper

# Design and performance analysis of a direct adaptive controller for nonlinear systems☆

T. Zhang, S.S. Ge*, C.C. Hang

*Department of Electrical Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260, Singapore*

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**Abstract**

In this paper, a direct adaptive controller is developed based on multilayer neural networks (MNNs) for a class of nonlinear systems. The proposed scheme avoids the possible singularity problem of the controller usually met in adaptive control design. The system tracking error is proven to converge to a small neighborhood of zero, while the stability of the closed-loop system is guaranteed. The transient performance of the resulting adaptive system is analytically quantified, and an example is given to show the effectiveness of the scheme. © 1999 Elsevier Science Ltd. All rights reserved.

*Keywords:* Nonlinear systems; Adaptive control; Neural networks; Performance

## 1. Introduction

In recent years, adaptive control of nonlinear systems has been an active research area and many remarkable results have been obtained for a large class of nonlinear systems, which include global state-feedback adaptive schemes (e.g., Sastry & Isidori, 1989; Teel, Kadiyala, Kokotovic & Sastry, 1991; Pomet & Praly, 1992; Kanellakopoulos, Kokotovic & Morse, 1991; Marino & Tomei, 1995; Krstic, Kanellakopoulos & Kokotovic, 1995), and global or semi-global output-feedback adaptive methods (Kanellakopoulos, Kokotovic & Morse, 1992; Marino & Tomei, 1995; Krstic et al., 1995; Jankovic, 1996; Khalil, 1996). In this paper, we are only concerned with the adaptive control problem of nonlinear systems which can be transformed to the following normal form (Isidori, 1989):

$$\begin{aligned} \dot{x}_i &= x_{i+1}, \quad i = 1, \ldots, n-1, \\ \dot{x}_n &= a(x) + b(x)u, \\ y &= x_1, \end{aligned} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$, $u \in R$, $y \in R$ are the state variables, system input and output, respectively; $a(x)$ and $b(x)$ are smooth functions which may not be linearly parameterized. The development of feedback linearization techniques provides a powerful tool for nonlinear system control. Based on these techniques, the most commonly used control structure is $u = [-a(x) + v]/b(x)$ with $v$ being a new control variable. In an effort to solve the problem of unknown nonlinearly parameterized $a(x)$ and $b(x)$, adaptive control schemes based on neural networks (NNs) or fuzzy systems have been studied (Chen & Liu, 1994; Yesidirek & Lewis, 1995; Kosmatopoulos, 1996; Sanner & Slotine, 1992; Spooner & Passino, 1996; Wang, 1994; Ge, Lee & Harris, 1998b). In these control schemes, the nonlinearity $b(x)$ is usually approximated by NNs or fuzzy systems $\hat{b}(x, \hat{W})$ (where $\hat{W}$ denotes the estimated weights). Therefore, additional precautions should be made to avoid possible singularities of the controllers (i.e., $\hat{b}(x, \hat{W}) \neq 0$). To cope with such a problem, Chen and Liu (1994) suggested that the initial values of the NN weights be chosen sufficiently close to the ideal values. Therefore, off-line training phases are needed before the controller is put into operation. Other methods include applying projection algorithm to project $\hat{W}$ inside a feasible set where no singularity problem happens (Wang, 1994; Spooner & Passino, 1996), and modifying the controllers by introducing switching control portions to keep the control magnitudes bounded (Yesidirek & Lewis, 1995;

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* Corresponding author. Tel.: 00-65-874-6821; fax: 00-65-779-1103.
*E-mail address:* elegesz@nus.edu.sg (S.S. Ge)

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Kosmatopoulos, 1996). As an alternative approach, stable direct adaptive controllers are studied for nonlinear systems with the absolute value of the first derivative of $b(x)$ assumed to be bounded by a known function (Sanner & Slotine, 1992; Spooner & Passino, 1996).

Based on Lyapunov synthesis method and neural network approximation, several adaptive controllers have been presented for a general class of unknown non-affine nonlinear systems without causing any control singularity problem (Ge, Hang & Zhang, 1997; 1998a; Zhang, Ge & Hang, 1997, 1998a, b). However, the plants considered are very general, the approaches provided in these works can only guarantee uniformly ultimate boundedness of the closed-loop systems, and several restrictive assumptions are required. Motivated by the fact that better results can be obtained when more properties of the studied systems are exploited, in this paper we restrict our attention to a special class of nonlinear system (1), whose nonlinearity $b(x)$ does not depend on the state $x_n$. By utilizing this nice property, the developed scheme avoids the controller singularity problem completely, and the stability of the resulting adaptive system is guaranteed without the requirement for off-line training. In addition, two performance criteria (the mean-square tracking error bound and the $L_\infty$ tracking error bound) are provided to quantify the control performance of the proposed method. In Section 2, we describe the class of nonlinear systems under study, and propose a desired feedback control (DFC) in the ideal case. Section 3 presents the structure and properties of three-layer neural networks used in the controller. A direct adaptive NN controller and its stability are discussed in Section 4, and performance analysis of the closed-loop system are studied in Section 5. Finally, the effectiveness of the method is illustrated via an example in Section 6 followed by the conclusion in Section 7.

## 2. Problem formulation and desired feedback control

The control objective is to design an adaptive controller for system (1) such that the output $y$ follows the desired trajectory $y_d$. Let $\|\cdot\|$ denote the 2-norm, $\|\cdot\|_F$ denote the Frobenius norm and $|A|_1 = \sum_{i=1}^{m}|a_i|$ with $A = [a_1, a_2, \ldots, a_m]^T \in R^m$.

**Assumption 1.** The sign of $b(x)$ is known and there exist two constants $b_0, b_1 > 0$ such that $b_0 \le |b(x)| \le b_1$, $\forall x \in \Omega \subset R^n$ with compact subset $\Omega$ containing the origin.

**Assumption 2.** $\partial b(x)/\partial x_n = 0$, $\forall x \in \Omega$.

**Remark 2.1.** Assumption 1 implies that the smooth function $b(x)$ is strictly either positive or negative. From now onwards, without losing generality, we shall assume $b(x) > 0$. It is worth noting that Assumption 2 may restrict the range of the applied plants, however, it brings us a nice property

$$\dot{b}(x) = \frac{\mathrm{d}[b(x)]}{\mathrm{d}t} = \frac{\partial b(x)}{\partial x}\dot{x} = \sum_{i=1}^{n-1}\frac{\partial b(x)}{\partial x_i}x_{i+1} \tag{2}$$

which only depends on the states $x$. This property is utilized to design a novel adaptive controller here to avoid the singularly problem discussed in Introduction. In fact, many practical systems, such as pendulum plants (Cannon, 1967; Balestrino, De Maria & Zinober, 1984) and single-link robots with flexible joints (Marino & Tomei, 1995) and many others, can be transformed to system (1) and possess this property.

Define $x_d = [y_d, \dot{y}_d, \ldots, y_d^{(n-1)}]^T$, $\bar{x}_d = [x_d^T, y_d^{(n)}]^T$ and

$$e = x - x_d = [e_1, e_2, \ldots, e_n]^T, \quad e_s = [\Lambda^T \quad 1]e, \tag{3}$$

where $\Lambda = [\lambda_1, \lambda_2, \ldots, \lambda_{n-1}]^T$ is chosen such that the polynomial $s^{n-1} + \lambda_{n-1}s^{n-2} + \cdots + \lambda_1$ is Hurwitz. Therefore, $e(t) \to 0$ as $e_s \to 0$. In addition, the tracking error can also be expressed as $e_1 = H(s)e_s$ with $H(s)$ a proper stable transfer function. Let $v = -y_d^{(n)} + [0 \quad \Lambda^T]e$, the time derivative of $e_s$ may be written as

$$\dot{e}_s = a(x) + b(x)u + v. \tag{4}$$

**Assumption 3.** $\|\bar{x}_d\| \le c$ with known constant $c > 0$.

**Lemma 2.1.** *Consider system* (1) *satisfying Assumptions* 1–3, *there exists a desired feedback control* (*DFC*)

$$u^* = -\frac{1}{b(x)}[a(x) + v] - \left[\frac{1}{\varepsilon} - \frac{\dot{b}(x)}{2b^2(x)}\right]e_s \tag{5}$$

*with constant* $\varepsilon > 0$, *such that* $\lim_{t\to\infty}\|e\| = 0$.

**Proof.** Substituting the DFC controller $u = u^*$ into Eq. (4), we have

$$\dot{e}_s = -b(x)\left[\frac{1}{\varepsilon} - \frac{\dot{b}(x)}{2b^2(x)}\right]e_s. \tag{6}$$

Choosing a positive function $V_s = e_s^2/2b(x)$ and differentiating it along Eq. (6), we obtain

$$\dot{V}_s = \frac{e_s\dot{e}_s}{b(x)} - \frac{\dot{b}(x)}{2b^2(x)}e_s^2 = -\frac{e_s^2}{\varepsilon} \le 0. \tag{7}$$

Since $b_0 \le b(x) \le b_1$ (Assumption 1), it follows from Eq. (7) that $V_s$ is a Lyapunov function. According to the Lyapunov theorem (Narendra & Annaswamy 1989), we have $\lim_{t\to\infty}|e_s| = 0$. Therefore $\lim_{t\to\infty}\|e\| = 0$ holds. □

Since $\dot{b}(x)$ depends on $x$ only (Remark 2.1), the DFC input $u^*$ (5) can be expressed as a function of $x$, $e_s$ and $v$ as follows:

$$u^* = u_1^*(z) - \frac{1}{\varepsilon} e_s, \qquad z = [x^{\mathrm{T}}, e_s, v]^{\mathrm{T}} \in \Omega_z \subset R^{n+2}, \tag{8}$$

where

$$u_1^*(z) = -\frac{1}{b(x)}[a(x) + v] + \frac{\dot{b}(x)}{2b^2(x)} e_s \tag{9}$$

and the compact set $\Omega_z = \{(x, e_s, v) \,|\, x \in \Omega;\ \|\bar{x}_d\| \le c\}$. When the nonlinear functions $a(x)$ and $b(x)$ are unknown, the nonlinearity $u_1^*(z)$ is not available. In the next section, multilayer neural networks are introduced to construct the unknown function $u_1^*$ for approximating the DFC input $u^*$.

## 3. Multilayer neural networks

Consider the following three-layer NNs (Lewis, Yesildirek & Liu, 1996):

$$g(z) = W^{\mathrm{T}} S(V^{\mathrm{T}} \bar{z}), \tag{10}$$

where

$$W = [w_1, w_2, \ldots, w_l]^{\mathrm{T}} \in R^l$$

and

$$V = [v_1, v_2, \ldots, v_l] \in R^{(n+3) \times l}$$

are the first-to-second layer and the second-to-third layer weights, respectively; the input vector $\bar{z} = [z^{\mathrm{T}}, \theta_0]^{\mathrm{T}}$ with constant $\theta_0 > 0$; $S(V^{\mathrm{T}}\bar{z}) = [s(v_1^{\mathrm{T}}\bar{z}), s(v_2^{\mathrm{T}}\bar{z}), \ldots, s(v_{l-1}^{\mathrm{T}}\bar{z}), 1]^{\mathrm{T}}$ where $s(z_a) = 1/(1 + \mathrm{e}^{-\gamma z_a})$ with constant $\gamma > 0$; and the NN node number $l > 1$. It has been proven that neural network (10) satisfies the conditions of the Stone–Weierstrass Theorem and can therefore approximate any continuous function to any desired accuracy over a compact set (Funahashi, 1989; Lewis et al., 1996). Therefore, the smooth function $u_1^*(z)$ can be approximated as

$$u_1^*(z) = W^{*\mathrm{T}} S(V^{*\mathrm{T}} \bar{z}) + \mu_l, \quad \forall z \in \Omega_z, \tag{11}$$

where $\mu_l$ is the NN approximation error.

**Assumption 4.** On the compact set $\Omega_z$, there exist ideal constant weights $W^*$ and $V^*$ such that $\mu_l$ is bounded by $|\mu_l| \le \mu_0$ with constant $\mu_0 > 0$.

In general, the ideal weights $W^*$ and $V^*$ are unknown and need to be estimated in controller design. Let $\hat{W}$ and $\hat{V}$ be the estimates of $W^*$ and $V^*$, respectively, and the weight estimation errors $\tilde{W} = \hat{W} - W^*$ and $\tilde{V} = \hat{V} - V^*$.

**Lemma 3.1.** *For neural network* (10), *the estimation error can be expressed as*

$$\begin{aligned} &\hat{W}^{\mathrm{T}} S(\hat{V}^{\mathrm{T}} \bar{z}) - W^{*\mathrm{T}} S(V^{*\mathrm{T}} \bar{z}) \\ &\quad = \tilde{W}^{\mathrm{T}} (\hat{S} - \hat{S}' \hat{V}^{\mathrm{T}} \bar{z}) + \hat{W}^{\mathrm{T}} \hat{S}' \tilde{V}^{\mathrm{T}} \bar{z} + d_u, \end{aligned} \tag{12}$$

*where*

$$\hat{S} = S(\hat{V}^{\mathrm{T}} \bar{z}),\ \hat{S}' = diag\{\hat{s}'_1, \hat{s}'_2, \ldots, \hat{s}'_l\}$$

*with*

$$\hat{s}'_i = s'(\hat{v}_i^{\mathrm{T}} \bar{z}) = \mathrm{d}[s(z_a)]/\mathrm{d}z_a|_{z_a = \hat{v}_i^{\mathrm{T}} \bar{z}}, \quad i = 1, 2, \ldots, l,$$

*and the residual term $d_u$ is bounded by*

$$|d_u| \le \|V^*\|_F \|\bar{z} \hat{W}^{\mathrm{T}} \hat{S}'\|_F + \|W^*\| \, \|\hat{S}' \hat{V}^{\mathrm{T}} \bar{z}\| + |W^*|_1. \tag{13}$$

**Proof.** See Appendix A.

It should be noticed that if linear NNs are used to approximate the nonlinearity $u_1^*$, e.g., the functions $s(v_i^{\mathrm{T}}\bar{z})$ are chosen as fixed radial basis functions (RBFs), the NN estimation error can be simply expressed as $\hat{W}^{\mathrm{T}} S - W^{*\mathrm{T}} S = \tilde{W}^{\mathrm{T}} S$ with $S$ the basis function vector. The design and analysis of the adaptive NN controller might be simplified significantly. However, the basis functions need to be chosen a priori, and the total number of RBFs will become prohibitively large if the dimension of the input is very high (Sanner & Slotine, 1992). In general, multilayer NNs possess better approximation properties than linear ones (Chen & Liu, 1994; Lewis et al., 1996).

## 4. Controller design and stability analysis

Now, we are ready to present the direct adaptive controller given by

$$u = \hat{W}^{\mathrm{T}} S(\hat{V}^{\mathrm{T}} \bar{z}) - \frac{e_s}{\varepsilon} + u_r \tag{14}$$

with

$$u_r = -\frac{e_s}{\varepsilon} (\|\bar{z} \hat{W}^{\mathrm{T}} \hat{S}'\|_F^2 + \|\hat{S}' \hat{V}^{\mathrm{T}} \bar{z}\|^2), \tag{15}$$

which is introduced for improving the robustness of the controller in the presence of the NN residual term $d_u$. Consider the following NN weight updating algorithms

$$\dot{\hat{W}} = -\Gamma_{\mathrm{w}} [(\hat{S} - \hat{S}' \hat{V}^{\mathrm{T}} \bar{z}) e_s + \delta_{\mathrm{w}} |e_s| \hat{W}], \tag{16}$$

$$\dot{\hat{V}} = -\Gamma_{\mathrm{v}} [\bar{z} \hat{W}^{\mathrm{T}} \hat{S}' e_s + \delta_{\mathrm{v}} |e_s| \hat{V}], \tag{17}$$

where $\Gamma_{\mathrm{w}} = \Gamma_{\mathrm{w}}^{\mathrm{T}} > 0$, $\Gamma_{\mathrm{v}} = \Gamma_{\mathrm{v}}^{\mathrm{T}} > 0$ $\delta_{\mathrm{w}}$ and $\delta_{\mathrm{v}}$ are positive constants. For simplicity, let $\Gamma_{\mathrm{w}} = \gamma_{\mathrm{w}} I$ and the guaranteed boundedness of the NN estimate weights is summarized in the following lemma.

**Lemma 4.1.** *For adaptive algorithms* (16) *and* (17), *there exists a compact set*

$$\Theta_w := \left\{ \hat{W} \,|\, \|\hat{W}\| \le \frac{1.224\sqrt{l}}{\delta_w} \right\} \tag{18}$$

*such that, if* $\hat{W}(0) \in \Theta_w$ *and* $\hat{V}(0)$ *is bounded, then* $\hat{W}(t) \in \Theta_w$ *and* $\hat{V}(t) \in L_\infty$, $\forall t \ge 0$.

**Proof.** See Appendix B.

Substituting Eq. (14) into Eq. (4), we have

$$\dot{e}_s = a(x) + v + b(x)\left[\hat{W}^T S(\hat{V}^T \bar{z}) - \frac{e_s}{\varepsilon} + u_r\right]. \tag{19}$$

Adding and subtracting $b(x)u_1^*$ on the right-hand side of Eq. (19) and noting Eq. (9), we obtain

$$\dot{e}_s = -b(x)\left[\frac{1}{\varepsilon} - \frac{\dot{b}(x)}{2b^2(x)}\right]e_s + b(x)[\hat{W}^T S(\hat{V}^T \bar{z}) + u_r - u_1^*(z)]. \tag{20}$$

Substituting Eq. (11) into the above equation yields

$$\frac{\dot{e}_s}{b(x)} = -\left[\frac{1}{\varepsilon} - \frac{\dot{b}(x)}{2b^2(x)}\right]e_s + \hat{W}^T S(\hat{V}^T \bar{z}) + u_r - W^{*T} S(V^{*T}\bar{z}) - \mu_l, \quad \forall z \in \Omega_z. \tag{21}$$

**Theorem 4.1.** *For system* (1), *controller* (14) *and adaptive laws* (16) *and* (17), *there exist a compact set* $\Omega_0 \subset \Omega$, *and positive constants* $c^*$ *and* $\varepsilon^*$ *such that if* (i) *the initial conditions* $x(0) \in \Omega_0$ *and* $\hat{W}(0) \in \Theta_w$, (ii) *the desired signal satisfying* $\|\bar{x}_d\| \le c^*$, *and the design parameter* $\varepsilon \le \varepsilon^*$, *then all the signals in the closed-loop system are bounded and the states* $x$ *remain in the compact set* $\Omega$ *for all time.*

**Proof.** The proof includes two steps: (i) supposing that $x \in \Omega$, $\forall t \ge 0$ holds, we find the upper bound of the system states, (ii) for the appropriate initial states $x(0)$, reference signal $y_d(t)$ and controller parameters, we prove that the supposition made in (i) is never violated, that is, the states $x$ do remain in the compact set $\Omega$ for all time.

*Step* 1: Suppose $x \in \Omega$, then the NN approximation (11) holds and the upper bounds of the system states can be found. Take the Lyapunov function candidate, $V_s = e_s^2/2b(x)$, its time derivative along Eq. (21) is

$$\dot{V}_s = -\frac{e_s^2}{\varepsilon} + [\hat{W}^T S(\hat{V}^T \bar{z}) + u_r - W^{*T} S(V^{*T}\bar{z}) - \mu_l]e_s.$$

Since $\hat{W}^T S(\hat{V}^T \bar{z}) \le \sqrt{l}\|\hat{W}\| \le 1.224l/\delta_w$ (see Lemma 4.1), $W^{*T} S(V^{*T}\bar{z}) \le \sqrt{l}\|W^*\|$, $u_r e_s \le 0$ and $|\mu_l| \le \mu_0$, we obtain

$$\dot{V}_s \le -\frac{e_s^2}{\varepsilon} + \alpha_0 |e_s| \le -\frac{e_s^2}{2\varepsilon} + \frac{\varepsilon \alpha_0^2}{2} \tag{22}$$

with

$$\alpha_0 = \frac{1.224l}{\delta_w} + \sqrt{l}\|W^*\| + \mu_0 \tag{23}$$

Since $e_s^2 = 2b(x)V_s \ge 2b_0 V_s$, inequality (22) leads to $\dot{V}_s + (b_0/\varepsilon)V_s \le (\varepsilon/2)\alpha_0^2$. Multiplied by $e^{(b_0/\varepsilon)t}$, it can be rewritten as $d(V_s e^{(b_0/\varepsilon)t})/dt \le (\varepsilon/2)\alpha_0^2 e^{(b_0/\varepsilon)t}$. Now, Integrating this inequality over $[0, t]$ and then multiplying it by $e^{-(b_0/\varepsilon)t}$, we obtain

$$V_s(t) \le e^{-(b_0/\varepsilon)t} V_s(0) + \frac{\varepsilon^2 \alpha_0^2}{2b_0}(1 - e^{-(b_0/\varepsilon)t}) \le V_s(0) + \frac{\varepsilon^2 \alpha_0^2}{2b_0}, \quad \forall t \ge 0. \tag{24}$$

It follows from $b_0 \le b(x) \le b_1$ that

$$|e_s(t)| \le \sqrt{\frac{b_1}{b_0}[e_s^2(0) + \varepsilon^2 \alpha_0^2]}, \quad \forall t \ge 0. \tag{25}$$

Let $\zeta = [e_1, e_2, \ldots, e_{n-1}]^T$, then a state representation of $e_s = [\Lambda^T \; 1]e$ may be written as $\dot{\zeta} = A_s \zeta + b_s e_s$ with $A_s$ a stable matrix (since $s^{n-1} + \lambda_{n-1}s^{n-2} + \cdots + \lambda_1$ is Hurwitz) and $b_s = [0, 0, \ldots, 0, 1]^T$. In addition, there exist constants $k_0, \lambda_0 > 0$ such that $\|e^{A_s t}\| \le k_0 e^{-\lambda_0 t}$ (Ioannou & Sun, 1996). The solution for $\zeta$ is $\zeta(t) = e^{A_s t}\zeta(0) + \int_0^t e^{A_s(t-\tau)} b_s e_s \, d\tau$, whose bound can be found as

$$\|\zeta(t)\| \le k_0\|\zeta(0)\| e^{-\lambda_0 t} + k_0 \int_0^t e^{-\lambda_0(t-\tau)} |e_s(\tau)| \, d\tau \le k_0\|\zeta(0)\| + \frac{k_0}{\lambda_0}\sqrt{\frac{b_1}{b_0}[e_s^2(0) + \varepsilon^2 \alpha_0^2]}, \quad \forall t \ge 0. \tag{26}$$

From Eq. (3) and $e = [\zeta^T e_n]^T$, it is shown that $e_n = e_s - \Lambda^T \zeta$. Therefore,

$$\|x\| \le (1 + \|\Lambda\|)\|\zeta\| + |e_s| + \|x_d\| \le R(c, \varepsilon, x(0)), \quad \forall t \ge 0, \tag{27}$$

where

$$R(c, \varepsilon, x(0)) = \bar{k}_0\|\zeta(0)\| + \bar{k}_1\sqrt{b_1/b_0[e_s^2(0) + \varepsilon^2 \alpha_0^2]} + c$$

with $\bar{k}_0 = k_0(1 + \|\Lambda\|)$ and $\bar{k}_1 = 1 + \bar{k}_0/\lambda_0$.

*Step* 2: To complete the proof, we need to show that under the following conditions, the supposition made in *Step 1* holds for all time, i.e., $x \in \Omega, \forall t \ge 0$. Define

$$\Omega_0 = \{x(0) \,|\, \{x \,|\, \|x\| < R(0, 0, x(0))\} \subset \Omega, x(0) \in \Omega\}. \tag{28}$$

For $x(0)\in\Omega_0$, define a positive constant

$$c^* = \sup_{c\in R^+}\{c\,|\,\{x\,|\,\|x\| < R(c,0,x(0))\}\subset\Omega\}.$$

Furthermore, for $x(0)\in\Omega_0$ and $c\le c^*$, a positive constant $\varepsilon^*$ can be found

$$\varepsilon^* = \sup_{\varepsilon\in R^+}\{\varepsilon\,|\,\{x\,|\,\|x\|\le R(c,\varepsilon,x(0))\}\subset\Omega\}.$$

In summary, for all initial conditions $x(0)\in\Omega_0$ and $\hat{W}(0)\in\Theta_{\rm w}$, and the desired signal $\|\bar{x}_{\rm d}\|\le c^*$, if the design parameter $\varepsilon\le\varepsilon^*$, then the system state $x$ stays in $\Omega$ for all time. This completes the proof. □

**Remark 4.1.** It is worth noting that Theorem 4.1 guarantees the boundedness of the closed-loop system in the sense of practical stability (Chen, 1987), i.e., the initial conditions are required to satisfy $x(0)\in\Omega_0$ and $\hat{W}(0)\in\Theta_{\rm w}$, and the states belong to the compact set $\Omega$. This is reasonable because Assumptions 1 and 2 hold on $\Omega$, and the neural network approximation property (11) is only valid on compact set $\Omega_z$. From Eqs. (18) and (28), it is shown that $\Theta_{\rm w}$ and $\Omega_0$ can be calculated explicitly. Therefore, it is not difficult to find the conditions for the initial neural network weights and system states such that a stable closed-loop system can be guaranteed.

## 5. Performance analysis

Theorem 4.1 only ensures the boundedness of the signals in the closed-loop system, no transient performance is revealed. We have the following theorem to discuss this problem.

**Theorem 5.1.** *For the closed-loop system* (1), (14), (16) *and* (17), *if* $x(0)\in\Omega_0$, $\hat{W}(0)\in\Theta_{\rm w}$, $\|\bar{x}_{\rm d}\|\le c^*$, *and* $\varepsilon\le\varepsilon^*$, *then*

(i) *the mean-square tracking error bound is*

$$\frac{1}{t}\int_0^t e_1^2(\tau)\,{\rm d}\tau \le \frac{1}{t}[2\varepsilon c_1 V_1(0) + c_2] + 2\varepsilon^2 c_1\beta_0,\quad \forall t>0, \tag{29}$$

*where* $c_1, c_2 > 0$ *are computable constants, constant* $\beta_0\ge 0$, *and constant* $V_1(0)\ge 0$ *depending on system initial conditions* $x(0)$, $\hat{W}(0)$ *and* $\hat{V}(0)$.

(ii) *the* $L_\infty$ *tracking error bound is*

$$\sup_{t\ge 0}|e_1(t)| \le k_0\|\zeta(0)\| + \frac{k_0}{\lambda_0}\sqrt{\frac{b_1}{b_0}[e_s^2(0)+\varepsilon^2\alpha_0^2]}. \tag{30}$$

**Proof.** Theorem 4.1 guarantees that $x\in\Omega$, $\forall t\ge 0$. Therefore, NN approximation (11) is valid. Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2}\left[\frac{e_s^2}{b(x)} + \tilde{W}^{\rm T}\Gamma_{\rm w}^{-1}\tilde{W} + tr\{\tilde{V}^{\rm T}\Gamma_{\rm v}^{-1}\tilde{V}\}\right]. \tag{31}$$

Differentiating Eq. (31) along Eqs. (16), (17) and (21), and using Eq. (12), we have

$$\dot{V}_1 = -\frac{e_s^2}{\varepsilon} + u_r e_s + (d_u-\mu_l)e_s - \delta_{\rm w}|e_s|\tilde{W}^{\rm T}\hat{W} - \delta_{\rm v}|e_s|tr\{\tilde{V}^{\rm T}\hat{V}\}.$$

Since $2\tilde{W}^{\rm T}\hat{W}\ge\|\tilde{W}\|^2-\|W^*\|^2$, $2tr\{\tilde{V}^{\rm T}\hat{V}\}\ge\|\tilde{V}\|_{\rm F}^2-\|V^*\|_{\rm F}^2$, and noting Eqs. (13) and (15), we obtain

$$\begin{aligned}\dot{V}_1 \le& -\frac{e_s^2}{\varepsilon}(\|\bar{z}\hat{W}^{\rm T}\hat{S}'\|_{\rm F}^2 + \|\hat{S}'\hat{V}^{\rm T}\bar{z}\|^2 + 1)\\ &-\frac{\delta_{\rm w}}{2}|e_s|(\|\tilde{W}\|^2-\|W^*\|^2) - \frac{\delta_{\rm v}}{2}|e_s|(\|\tilde{V}\|_{\rm F}^2-\|V^*\|_{\rm F}^2)\\ &+|e_s|(\|V^*\|_{\rm F}\|\bar{z}\hat{W}^{\rm T}\hat{S}'\|_{\rm F} + \|W^*\|\,\|\hat{S}'\hat{V}^{\rm T}\bar{z}\|\\ &+|W^*|_1+|\mu_l|).\end{aligned} \tag{32}$$

Using the facts that

$$\|V^*\|_{\rm F}\|\bar{z}\hat{W}^{\rm T}\hat{S}'\|_{\rm F}|e_s| \le \frac{e_s^2}{\varepsilon}\|\bar{z}\hat{W}^{\rm T}\hat{S}'\|_{\rm F}^2 + \frac{\varepsilon}{4}\|V^*\|_{\rm F}^2,$$

$$\left(\frac{\delta_{\rm w}}{2}\|W^*\|^2 + \frac{\delta_{\rm v}}{2}\|V^*\|_{\rm F}^2\right)|e_s| \le \frac{e_s^2}{8\varepsilon} + \varepsilon\delta_{\rm w}^2\|W^*\|^4 + \varepsilon\delta_{\rm v}^2\|V^*\|_{\rm F}^4,$$

$$(|W^*|_1+|\mu_l|)|e_s| \le \frac{3}{8\varepsilon}e_s^2 + \varepsilon(|W^*|_1^2+2\mu_l^2),$$

$$\|W^*\|\,\|\hat{S}'\hat{V}^{\rm T}\bar{z}\||e_s| \le \frac{e_s^2}{\varepsilon}\|\hat{S}'\hat{V}^{\rm T}\bar{z}\|^2 + \frac{\varepsilon}{4}\|W^*\|^2,$$

and $|\mu_l|\le\mu_0$, inequality (32) can be written as

$$\dot{V}_1 \le -\frac{e_s^2}{2\varepsilon} + \varepsilon\beta_0 \tag{33}$$

with

$$\beta_0 = \tfrac{1}{4}\|W^*\|^2 + \tfrac{1}{4}\|V^*\|_{\rm F}^2 + |W^*|_1^2 + \delta_{\rm w}^2\|W^*\|^4 + \delta_{\rm v}^2\|V^*\|_{\rm F}^4 + 2\mu_0^2. \tag{34}$$

Now, integrating Eq. (33) over $[0,t]$ leads to

$$\begin{aligned}\int_0^t e_s^2(\tau)\,{\rm d}\tau &\le -\int_0^t 2\varepsilon\dot{V}_1\,{\rm d}\tau + 2\varepsilon^2\beta_0 t\\ &\le 2\varepsilon V_1(0) + 2\varepsilon^2\beta_0 t\end{aligned} \tag{35}$$

with

$$V_1(0) = \frac{1}{2}\left[\frac{e_s^2(0)}{b(x(0))} + \tilde{W}^{\mathrm{T}}(0)\Gamma_{\mathrm{w}}^{-1}\tilde{W}(0) \times + tr\{\tilde{V}^{\mathrm{T}}(0)\Gamma_{\mathrm{v}}^{-1}\tilde{V}(0)\}\right]. \quad (36)$$

Since the tracking error $e_1 = H(s)e_s$, we have

$$\int_0^t e_1^2(\tau)\,\mathrm{d}\tau \le c_1\int_0^t e_s^2(\tau)\,\mathrm{d}\tau + c_2 \le 2\varepsilon c_1 V_1(0) + 2\varepsilon^2 c_1\beta_0 t + c_2 \quad (37)$$

with computable constants $c_1, c_2 > 0$ (Ioannou & Sun, 1996). Dividing Eq. (37) by $t$, we arrive at Eq. (29). Because $|e_1(t)| \le \|\zeta(t)\|$, the $L_\infty$ tracking error bound (30) can be obtained from Eq. (26) directly. □

**Remark 5.1.** Two performance criteria (29) and (30) reveal the transient response of the closed-loop system. It can be seen that large initial errors $e_s(0)$, $\tilde{W}(0)$ and $\tilde{V}(0)$ may lead to a large mean-square tracking error during the initial period of adaptation. However, inequality (29) implies

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t e_1^2(\tau)\,\mathrm{d}\tau \le 2\varepsilon^2 c_1\beta_0,$$

which confirms that the tracking error converges to an $\varepsilon$-neighborhood of origin from the mean-square point of view.

**Remark 5.2.** Theorem 5.1 also provides some methods for improving the control performance. Both performance criteria (29) and (30) can be improved if the initial values $\zeta(0)$ and $e_s(0)$ are set to zeros by appropriately initializing the reference trajectory $x_d(0)$ using the method (Krstic et al., 1995). The NN weight errors $\tilde{W}(0)$ and $\tilde{V}(0)$ are, in general, not possible to be zeros because the ideal weights $W^*$ and $V^*$ are not available. Nevertheless, it follows from Eq. (36) that the constant $V_1(0)$ may be reduced by choosing large adaptation gains $\Gamma_{\mathrm{w}}$ and $\Gamma_{\mathrm{v}}$. Therefore, faster adaptation results in better transient performance.

**Remark 5.3.** In view of Eq. (34), we note that a smaller $\beta_0$ can be obtained by choosing smaller $\delta_{\mathrm{w}}$ and $\delta_{\mathrm{v}}$, which may result in a smaller tracking error. It is worth noticing from Lemma 4.1 that too small $\delta_{\mathrm{w}}$ and $\delta_{\mathrm{v}}$ may not be enough to prevent the NN weight estimates from drifting to very large values in the presence of the NN approximation error or external disturbances. In this case, $\hat{W}$ and $\hat{V}$ might increase to very large values, which result in a variation of a high-gain control. Therefore, in practical applications the parameters $\delta_{\mathrm{w}}$ and $\delta_{\mathrm{v}}$ should be adjusted carefully for achieving suitable transient performance and control action.

## 6. Example study

In this section, the simulation result for an inverted pendulum control is presented to show the design procedure and performance of the proposed controller. The model of the inverted pendulum (Cannon, 1967) can be written in the form of system (1) with

$$a(x) = \frac{g\sin x_1 - mLx_2^2\sin(2x_1)/2(M+m)}{L(\frac{4}{3} - m\cos^2 x_1/(M+m))},$$

$$b(x) = \frac{\cos x_1/(M+m)}{L(\frac{4}{3} - m\cos^2 x_1/(M+m))},$$

where $x_1 = [x_1, x_2]^{\mathrm{T}}$ with $x_1$ the angle displacement of the pendulum from the vertical configuration; $g = 9.8\ \mathrm{m/s^2}$ is the gravity acceleration coefficient; $M$ and $m$ are the mass of the cart and the pole, respectively; $L$ is the half-length of the pole, and $u$ is the applied force control. The true values of the parameters are $M = 1.0$ kg, $m = 0.1$ kg and $L = 0.5$ m. The initial states are $[x_1(0), x_2(0)]^{\mathrm{T}} = [\pi/60, 0]^{\mathrm{T}}$, and the control objective is to make $y = x_1$ track $y_{\mathrm{d}} = (\pi/30)\sin(t)$. If we require that the system states remain in the compact set

$$\Omega = \left\{(x_1, x_2)\,|\,|x_1| \le \frac{\pi}{4},\ |x_2| \le \frac{3\pi}{2}\right\},$$

then it can be checked that Assumptions 1 and 2 are satisfied and $b_0 \le b(x) \le b_1$ with $b_0 = 0.998$ and $b_1 = 1.4634$. In the following design, the controller parameters shall be specified such that $(x_1, x_2) \in \Omega$, $\forall t \ge 0$.

The multilayer neural networks are chosen with the input vector $\bar{z} = [x_1, x_2, e_s, v, \theta_0]^{\mathrm{T}}$, $l = 5$, $\gamma = 100.0$ and $\theta_0 = 0.1$; The parameters of adaptive laws (16) and (17) are taken as $\Gamma_{\mathrm{w}} = diag\{10.0\}$, $\Gamma_{\mathrm{v}} = diag\{20.0\}$, $\delta_{\mathrm{w}} = 0.5$ and $\delta_{\mathrm{v}} = 0.25$. Because the plant is a second order system, we have $\zeta = x_1 - y_{\mathrm{d}}$. If choose $\Lambda = 3.0$, then $k_0$ and $\lambda_0$ in Eq. (26) are $k_0 = 1.0$ and $\lambda_0 = 3.0$. From Eq. (26), the bound of $x_1$ is found as

$$|x_1(t)| \le \|\zeta\| + |y_{\mathrm{d}}| \le |\zeta(0)| + \frac{1}{3.0}\sqrt{\frac{b_1}{b_0}[e_s^2(0) + \varepsilon^2\alpha_0^2]} + |y_{\mathrm{d}}|, \quad \forall t \ge 0. \quad (38)$$

Considering the initial states, it is known that $|\zeta(0)| = \pi/60$, $|e_s(0)| = \pi/20$ and $|y_{\mathrm{d}}| \le \pi/30$. Supposing the MNNs satisfying $\|W^*\| \le 1.0$ and $\mu_l \le 0.1$ on the compact set $\Omega_z$, from Eq. (23) we have $\alpha_0 < 14.58$. It follows

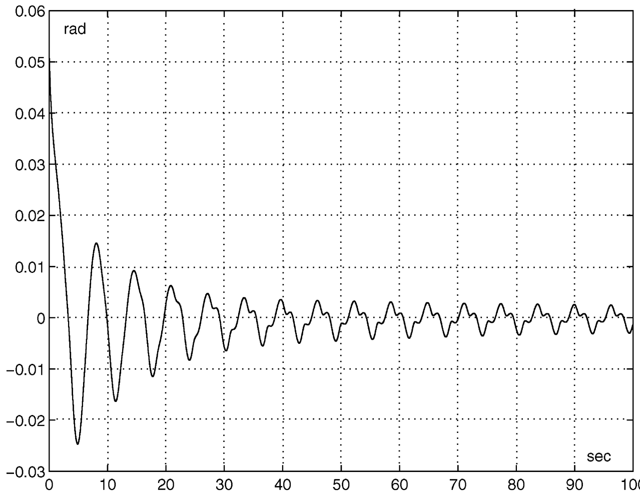


Fig. 1. Tracking error $y - y_d$.

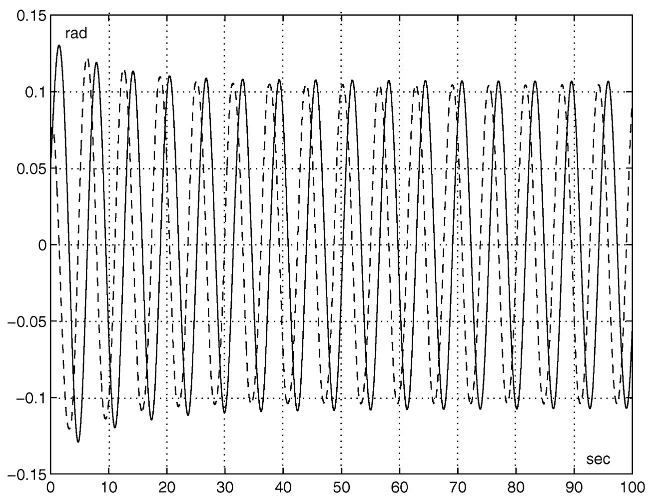


Fig. 2. States $x_1$ ("—") and $x_2$ ("- -").

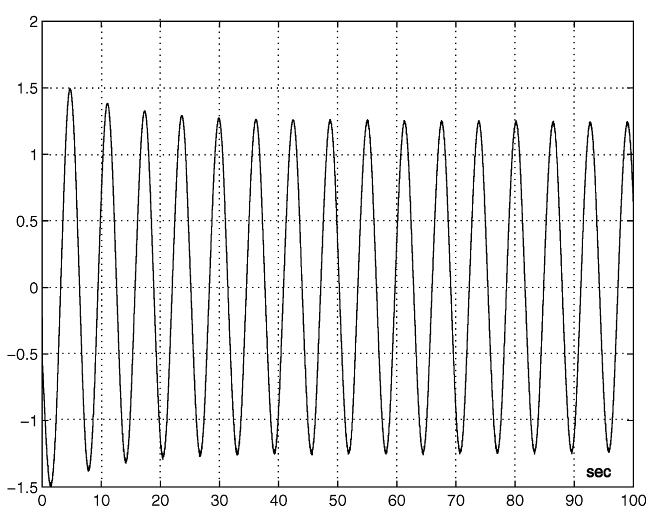


Fig. 3. Control input $u(t)$.

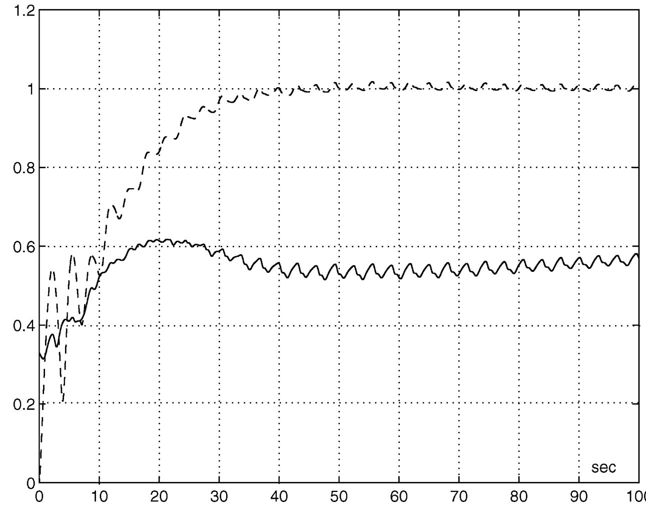


Fig. 4. $\|\hat{W}\|$ ("- -") and $\|\hat{V}\|_F$ ("—").

from Eq. (38) that for all $\varepsilon < \varepsilon^* = 0.106$, the state $|x_1| < \pi/4$, $\forall t \geq 0$. Noticing $x_2 = e_s - 3\zeta + \dot{y}_d$ and $|\dot{y}_d| \leq \pi/30$, and inequalities (25) and (26), it can be checked that $|x_2| < 3\pi/2$, $\forall t \geq 0$ for $\varepsilon < \varepsilon^*$. Therefore, if we choose the design parameter $\varepsilon < \varepsilon^*$, the states $(x_1, x_2)$ do remain in the required set $\Omega$ for all time.

Figs. 1–4 show the simulation result for the proposed NN controller with $\varepsilon = 0.1$, the initial weights $\hat{W}(0) = 0$ and the elements of $\hat{V}(0)$ are taken randomly in the interval $[-0.2, 0.2]$. Although only five NN nodes are taken and no exact model of the pendulum plant is available, through the NN learning phase, it can be seen that the output tracking error shown in Fig. 1 converges to a quite small set after 30 s. Fig. 2 indicates that the system states $x_1$ and $x_2$ are within the compact set $\Omega$ for all time. The response of the control input $u$ is given in Fig. 3 and the boundedness of the NN weight estimates are shown in Fig. 4.

## 7. Conclusion

Based on multilayer neural networks, a direct adaptive control scheme has been studied for a class of nonlinear dynamical systems in this paper. The developed control structure avoids the possible singularity problem in adaptive controller design and the stability of the closed-loop system is guaranteed. Further, the control performance of the scheme has been investigated and some possible methods for improving the system response have been provided.

## Appendix A. Proof of Lemma 3.1

The Taylor series expansion of $S(V^{*T}\bar{z})$ about a given $\hat{V}^T\bar{z}$ can be written as

$$S(V^{*T}\bar{z}) = \hat{S} - \hat{S}'\tilde{V}^T\bar{z} + O(\tilde{V}^T\bar{z})^2, \quad \text{(A.1)}$$

where $O(\tilde{V}^{\mathrm{T}}\bar{z})^2$ denotes the sum of the high-order terms in the Taylor series expansion. Following the similar procedure in the work (Lewis et al., 1996). It can be shown that Eq. (12) holds with $d_u = \tilde{W}^{\mathrm{T}}\hat{S}'V^{*\mathrm{T}}\bar{z} - W^{*\mathrm{T}}O(\tilde{V}^{\mathrm{T}}\bar{z})^2$. It follows from Eq. (12) that $d_u$ can also be expressed as

$$d_u = \hat{W}^{\mathrm{T}}\hat{S} - W^{*\mathrm{T}}S(V^{*\mathrm{T}}\bar{z}) - (\hat{W} - W^*)^{\mathrm{T}}(\hat{S} - \hat{S}'\hat{V}^{\mathrm{T}}\bar{z}) - \hat{W}^{\mathrm{T}}\hat{S}'(\hat{V} - V^*)^{\mathrm{T}}\bar{z}$$

$$= \hat{W}^{\mathrm{T}}\hat{S}'V^{*\mathrm{T}}\bar{z} - W^{*\mathrm{T}}\hat{S}'\hat{V}^{\mathrm{T}}\bar{z} + W^{*\mathrm{T}}[\hat{S} - S(V^{*\mathrm{T}}\bar{z})].$$

Noting that every element of $\hat{S} - S(V^{*\mathrm{T}}\bar{z})$ is bounded by one, we have $W^{*\mathrm{T}}[\hat{S} - S(V^{*\mathrm{T}}\bar{z})] \le |W^*|_1$. Considering $\hat{W}^{\mathrm{T}}\hat{S}'V^{*\mathrm{T}}\bar{z} = tr\{V^{*\mathrm{T}}\bar{z}\hat{W}^{\mathrm{T}}\hat{S}'\} \le \|V^*\|_{\mathrm{F}}\|\bar{z}\hat{W}^{\mathrm{T}}\hat{S}'\|_{\mathrm{F}}$, we conclude that Eq. (13) holds. □

## Appendix B. Proof of Lemma 4.1

Let positive function $V_{\mathrm{w}} = (\gamma_{\mathrm{w}}^{-1}/2)\hat{W}^{\mathrm{T}}\hat{W}$, its time derivative along Eq. (16) is

$$\dot{V}_{\mathrm{w}} \le -|e_s|[\delta_{\mathrm{w}}\|\hat{W}\|^2 - |\hat{W}^{\mathrm{T}}(\hat{S} - \hat{S}'\hat{V}^{\mathrm{T}}\bar{z})|].$$

It can be checked that the elements of $\hat{S}'\hat{V}^{\mathrm{T}}\bar{z}$ are bounded by $|s'_i\hat{v}_i^{\mathrm{T}}\bar{z}| = |z_{ai}\mathrm{e}^{-z_{ai}}/(1 + \mathrm{e}^{-z_{ai}})^2| < 0.224$ with $z_{ai} = \gamma\hat{v}_i^{\mathrm{T}}\bar{z}$. Then the elements $|s(\hat{v}_i^{\mathrm{T}}\bar{z}) - s'_i\hat{v}_i^{\mathrm{T}}\bar{z}|$ of the vector $\hat{S} - \hat{S}'\hat{V}^{\mathrm{T}}\bar{z}$ are not larger than 1.224. It follows that $|\hat{W}^{\mathrm{T}}(\hat{S} - \hat{S}'\hat{V}^{\mathrm{T}}\bar{z})| \le \sum_{i=1}^{l} 1.224|\hat{w}_i| \le 1.224\sqrt{l}\|\hat{W}\|$. Therefore $\dot{V}_{\mathrm{w}} \le -|e_s|\,\|\hat{W}\|(\delta_{\mathrm{w}}\|\hat{W}\| - 1.224\sqrt{l})$, which implies that for the initial condition $\hat{W}(0) \in \Theta_{\mathrm{w}}$, the estimated weight $\hat{W}(t) \in \Theta_{\mathrm{w}}$, $\forall t \ge 0$.

Taking $V_{\mathrm{v}} = \frac{1}{2}tr\{\hat{V}^{\mathrm{T}}\Gamma_{\mathrm{v}}^{-1}\hat{V}\}$ and using the property $\hat{W}^{\mathrm{T}}\hat{S}'\hat{V}^{\mathrm{T}}\bar{z} = tr\{\hat{V}^{\mathrm{T}}\bar{z}\hat{W}^{\mathrm{T}}\hat{S}'\}$, its time derivative along Eq. (17) may be written as

$$\dot{V}_{\mathrm{v}} \le -|e_s|[\delta_{\mathrm{v}}\|\hat{V}\|_{\mathrm{F}}^2 - \|\hat{W}\|\,\|\hat{S}'\hat{V}^{\mathrm{T}}\bar{z}\|].$$

It follows from $\|\hat{S}'\hat{V}^{\mathrm{T}}\bar{z}\| \le 0.224\sqrt{l}$ and $\|\hat{W}\| \le 1.224\sqrt{l}/\delta_{\mathrm{w}}$ that $\dot{V}_{\mathrm{v}} \le -\delta_{\mathrm{v}}|e_s|[\|\hat{V}\|_{\mathrm{F}}^2 - 0.2742l/\delta_{\mathrm{w}}\delta_{\mathrm{v}}]$, which proves that $\dot{V}_{\mathrm{v}} \le 0$ once $\|\hat{V}\|_{\mathrm{F}} \ge 0.2742l/\delta_{\mathrm{w}}\delta_{\mathrm{v}}$. Therefore, $\hat{V}(t) \in L_\infty$ if $\hat{V}(0)$ is bounded. □

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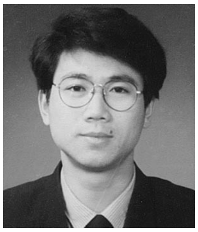
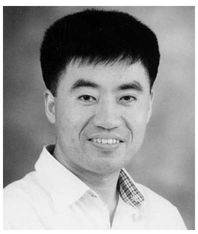


**Tao Zhang** was born in Shenyang, Liaoning Province, People's Republic of China in 1967. He received the B.Eng. and the M.Eng. in the Department of Automatic Control in 1990 and 1993, respectively, both from Northeastern University, People's Republic of China. He is currently a Ph.D. candidate in the Department of Electrical Engineering, the National University of Singapore, and with Seagate Technology International as a research engineer. His technical paper "Adaptive neural network control for strict-feedback nonlinear systems using backstepping design" co-authored with S. S. Ge and C. C. Hang was nominated for the Best Student Paper Award at the 1999 American Control Conference. His research interests include adaptive nonlinear control, robust adaptive control, neural network control, PID auto-tuning and control applications.



**S. S. Ge** received the B.Sc. degree in Control Engineering from Beijing University of Aeronautics and Astronautics, China, in July 1986, and the Ph.D. degree and the Diploma of Imperial College (DIC) in Mechanical/Electrical Engineering from Imperial College of Science, Technology and Medicine, University of London, United Kindom, in January 1993. From May 1992 to June 1993, he was a postdoctoral research associate at Leicester University, United Kindom. He has been with the Department of Electrical Engineering, the National University of Singapore as a lecturer from July 1993 to June 1998, and as a Senior Lecturer from July 1998. He was a visiting staff in Laboratoire de'Automatique de Grenoble, France in 1996 and Department of Electrical and Electronics Engineering, the University of Melbourne, Australia in 1998 and 1999. He has been serving as an associate editor on the Conference Editorial Board of the IEEE Control Systems Society since 1998. His current research interests are Adaptive Control, Neural Networks and Fuzzy Logic, Robot Control, Real-Time Implementation, Genetic Algorithms, Friction Compensation and Sensor Fusion. He has authored and co-authored over 100 international journal and conference papers, one monograph and co-invented one patent.

**C. C. Hang** graduated with a First Class Honours Degree in Electrical Engineering from the University of Singapore in 1970. He received the Ph.D. degree in control engineering from the University of Warwick, England, in 1973. From 1974 to 1977, he worked as a Computer and Systems Technologist in the Shell Eastern Petroleum Company (Singapore) and the Shell International Petroleum Company (The Netherlands). Since 1977, he has been with the National University of Singapore, serving in various positions including being the Vice-Dean of the Faculty of Engineering and Head of the Department of Electrical Engineering. Since October 1994, he has been appointed Deputy Vice-Chancellor. His major area of research is adaptive control in which he has published one book, 170 international journal and conference papers and 4 patents. He was a Visiting Scientist in Yale University in 1983, and in Lund University in 1987 and 1992. Since March 1992, he has been appointed Principal Editor (Adaptive Control) of the Automatica Journal. In January 1998, he was elected a Fellow of IEEE.