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Brief paper

An ISS-modular approach for adaptive neural control of pure-feedback systems☆

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Abstract

Controlling non-affine non-linear systems is a challenging problem in control theory. In this paper, we consider adaptive neural control of a completely non-affine pure-feedback system using radial basis function (RBF) neural networks (NN). An ISS-modular approach is presented by combining adaptive neural design with the backstepping method, input-to-state stability (ISS) analysis and the small-gain theorem. The difficulty in controlling the non-affine pure-feedback system is overcome by achieving the so-called "ISS-modularity" of the controller-estimator. Specifically, a neural controller is designed to achieve ISS for the state error subsystem with respect to the neural weight estimation errors, and a neural weight estimator is designed to achieve ISS for the weight estimation subsystem with respect to the system state errors. The stability of the entire closed-loop system is guaranteed by the small-gain theorem. The ISS-modular approach provides an effective way for controlling non-affine non-linear systems. Simulation studies are included to demonstrate the effectiveness of the proposed approach.

Keywords: Adaptive neural control; Pure-feedback systems; Non-affine systems; Input-to-state stability; Small-gain theorem

1. Introduction

In non-linear control design, the backstepping design method (Krstic, Kanellakopoulos, & Kokotovic, 1995) has been successful for special classes of non-linear systems with its constructive Lyapunov design procedures. A great deal of progress has been achieved for the control of strict-feedback systems with unknown parameters (Kokotovic & Arcak, 2001; Krstic et al., 1995) and with unknown non-linearities (Choi & Farrell, 2001; Ge, Hang, Lee, & Zhang, 2001; Ge & Wang, 2002a; Kwan & Lewis, 2000; Lewis, Jagannathan, & Yeildirek, 1999; Polycarpou & Mears, 1998; Zhang, Peng, & Jiang, 2000). Nevertheless, it is noticed that relatively fewer results have been obtained for the class of pure-feedback systems, which is given in a general form as (Krstic et al., 1995)

$$\dot{x}_i = f_i(\bar{x}_i, x_{i+1}), \quad i = 1, \ldots, n-1,$$

$$\dot{x}_n = f_n(\bar{x}_n, u),$$

$$y = x_1, \tag{1}$$

where $\bar{x}_i = [x_1, \ldots, x_i]^{\mathrm{T}} \in R^i$, $i = 1, \ldots, n$, $u \in R$, $y \in R$ are state variables, system input and output, respectively; $f_i(\cdot)$ $(i = 1, \ldots, n)$ are smooth non-linear functions. The pure-feedback system (1) represents a class of lower-triangular non-linear systems which has a more representative form than the strict-feedback systems. In practice, there are many systems falling into this category featured with a cascade and non-affine structure, such as biochemical process (Krstic et al., 1995), Duffing oscillator (Dong, Chen, & Chen, 1997), aircraft flight control system (Hunt & Meyer, 1997), mechanical systems (Ferrara & Giacomini, 2000), etc. A more recent example of

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practical pure-feedback systems is a simplified dynamic model for a reduced scale autonomous helicopter (Mahony & Lozano, 2000).

It can be seen that pure-feedback system (1) has no affine appearance of the variables to be used as virtual controls, and of the actual control u itself. The cascade and non-affine properties make it quite difficult to find the explicit virtual controls and the actual control to stabilize the pure-feedback systems using backstepping design (Krstic et al., 1995). In the literature of pure-feedback system control, parametric pure-feedback systems were mainly considered (Ferrara & Giacomini, 2000; Kanellakopoulos, Kokotovic, & Morse, 1991; Krstic et al., 1995; Seto, Annaswamy, & Baillieul, 1994). Recently, by combining the backstepping methodology with adaptive neural design, several special cases of pure-feedback systems, which are affine in control u, were investigated (Ge & Wang, 2002b; Wang & Huang, 2002). However, the problem of controlling the completely non-affine pure-feedback system (1) remains unsolved in the literature. The main difficulty for adaptive neural control of pure-feedback system (1) lies in that, when neural networks are used to approximate some desired virtual controls α_i^* and desired practical control u^* in the backstepping design, as done for lower-triangular systems (Ge & Wang, 2002a, 2002b; Kwan & Lewis, 2000; Wang & Huang, 2002; Zhang et al., 2000), it will generally involve the NN approximation of a function of u and $\dot{u}$. As the NN approximation is one part of control u, this will lead to a circular construction of the practical controller. In Ge and Wang (2002b), Wang and Huang (2002), the circularity problem was avoided because much simpler pure-feedback systems were investigated.

In this paper, we consider adaptive neural control of the completely non-affine pure-feedback system (1). To overcome the aforementioned difficulty, we employ the input-to-state stability (ISS) analysis (Sontag, 1989; Sontag & Wang, 1996) and the small gain theorem (Jiang, Teel, & Praly, 1994) rather than constructing an overall Lyapunov function for the entire closed-loop. It is observed that in the adaptive neural control approaches (e.g., Ge & Wang, 2002a, 2002b; Kwan & Lewis, 2000; Zhang et al., 2000), the resulting closed-loop system commonly consists of two interconnected subsystems: the state error subsystem and the weight estimation subsystem. The interconnected structure motivates us to solve this problem using the celebrated small-gain theorem, especially the ISS-type small-gain theorem (Jiang et al., 1994), which will be shown useful to achieve the main results of this paper. By combining adaptive neural design with the ISS-type small-gain theorem, we present an ISS-modular approach for non-affine pure-feedback system control. The adaptive neural control approach is designed to achieve a significant level of "ISS-modularity" of the controller-estimator pair, i.e., to stabilize the interconnected state error subsystem and the weight estimation subsystem, any ISS neural controller can be combined with any ISS neural weight estimator, provided that the small-gain condition of the interconnected subsystems is satisfied. The neural controller is to achieve ISS with respect to the NN weight estimation errors. The neural weight estimator, in turn, will be designed to achieve ISS with respect to the system state errors. The stability of the entire closed-loop system will be guaranteed by using the small-gain theorem. By achieving the ISS-modularity of the interconnected control module and estimation module, the difficulty in controlling non-affine pure-feedback system (1) is separated into two relatively easier ones: the input-to-state stability analyses of the two subsystems, and the derivation of the entire closed-loop stability by using the small-gain theorem. The employment of ISS analysis and the small gain theorem avoids the construction of an overall Lyapunov function for the entire system, and subsequently overcomes the aforementioned circular controller construction problem.

The ISS-modular approach is inspired by the modular design in Krstic et al. (1995), which was developed for parametric strict-feedback systems. Compared with existing results for affine-in-control pure-feedback systems (Ge & Wang, 2002b; Wang & Huang, 2002), this paper presents yet another method in controlling non-affine pure-feedback system (1) with less restrictive assumptions. The ISS-modular approach provides a simple and effective way for adaptive neural control of uncertain non-linear systems. The proposed adaptive neural controller can also be directly applied to the uncertain strict-feedback systems. Since there are many practical systems falling into the category of non-linear strict-feedback and pure-feedback forms, the proposed scheme will find a wide variety of industrial applications.

The rest of the paper is organized as follows: the problem formulation as well as some preliminary results are presented in Section 2. Section 3 presents the ISS-modular approach for adaptive neural control of uncertain pure-feedback system (1). Simulation results performed on an illustrative example are included in Section 4 to demonstrate the effectiveness of the approach. Section 5 contains the conclusions.

Terminology: a continuous function $\alpha : R_+ \to R_+$ is said to belong to class $\mathscr{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class $\mathscr{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$. A continuous function $\beta : R_+ \times R_+ \to R_+$ is said to belong to class $\mathscr{KL}$ if, for each fixed s, the mapping $\beta(r, s)$ belongs to class $\mathscr{K}$ with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.

2. Problem formulation and preliminaries

2.1. Problem formulation

For the control of pure-feedback system (1), define

$$g_i(\bar{x}_i, x_{i+1}) = \frac{\partial f_i(\bar{x}_i, x_{i+1})}{\partial x_{i+1}}, \quad i = 1, \ldots, n-1, \tag{2}$$

$$g_n(\bar{x}_n, u) = \frac{\partial f_n(\bar{x}_n, u)}{\partial u}. \tag{3}$$

For simplicity of presentation, denote $x_{n+1} = u$.

Assumption 1. The signs of $g_i(\cdot, \cdot)$, $i = 1, \ldots, n$ are known, and there exist constants $0 < \underline{g}_i \leqslant \bar{g}_i < \infty$ such that (i) $|g_i(\bar{x}_i, x_{i+1})| > \underline{g}_i$ $(i = 1, \ldots, n)$, $\forall (\bar{x}_i, x_{i+1}) \in R^i \times R$; and

(ii) $|g_i(\bar{x}_i, x_{i+1})| \leqslant \bar{g}_i$ $(i=1, \ldots, n)$, $\forall(\bar{x}_i, x_{i+1}) \in \Omega_{\bar{x}_{i+1}}$ where $\Omega_{\bar{x}_{i+1}} \subset R^{i+1}$ is a compact set.

Assumption 1 implies that partial derivatives g_i $(i=1, \ldots, n)$ are strictly either positive or negative. Without losing generality, it is assumed that $g_i > \underline{g}_i > 0$.

Remark 1. In Assumption 1, although $g_i(\cdot)$ $(i = 1, \ldots, n)$ appears to be similar with the affine terms in a strict-feedback system (Krstic et al., 1995), a major difference lies in that $g_i(\cdot)$ is a function of $\bar{x}_{i+1}$, and thus, it is still a non-affine term in character.

The control objective is to design a direct adaptive neural controller for system (1) such that (i) all the signals in the closed-loop system remain uniformly ultimately bounded, and (ii) the output y follows a desired trajectory y_d generated from the following smooth, bounded reference model:

$$\dot{x}_{di} = f_{di}(x_d), \quad 1 \leqslant i \leqslant n,$$

$$y_d = x_{d1}, \tag{4}$$

where $x_d = [x_{d1}, x_{d2}, \ldots, x_{dn}]^{\mathrm{T}} \in R^n$ are the states, $y_d \in R$ is the system output, $f_{di}(\cdot), i = 1, 2, \ldots, n$ are known smooth non-linear functions. Assume that the states of the reference model remain bounded, i.e., $x_d \in \Omega_d$, $\forall t \geqslant 0$.

2.2. Gaussian RBF networks

In this paper, the following RBF NN (see, e.g., Haykin, 1999) is used to approximate the continuous function $h(Z) : R^q \to R$,

$$h_{nn}(Z) = W^{\mathrm{T}} S(Z), \tag{5}$$

where the input vector $Z \in \Omega \subset R^q$, weight vector $W = [w_1, w_2, \ldots, w_l]^{\mathrm{T}} \in R^l$, the NN node number $l > 1$; and $S(Z) = [s_1(Z), \ldots, s_l(Z)]^{\mathrm{T}}$, with $s_i(Z)$ being chosen as the commonly used Gaussian functions, which have the form

$$s_i(Z) = \exp\left[\frac{-(Z-\mu_i)^{\mathrm{T}}(Z-\mu_i)}{\eta^2}\right], \quad i = 1, 2, \ldots, l, \tag{6}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{iq}]^{\mathrm{T}}$ is the center of the receptive field and η is the width of the Gaussian function.

It has been proven that network (5) can approximate any continuous function over a compact set $\Omega_Z \subset R^q$ to arbitrary any accuracy as

$$h(Z) = W^{*^{\mathrm{T}}} S(Z) + \varepsilon(Z), \quad \forall Z \in \Omega_Z, \tag{7}$$

where W^* is ideal constant weights, and $\varepsilon(Z)$ is the approximation error ($\varepsilon(Z)$ is denoted as ε to simplify the notation).

Assumption 2. There exist ideal constant weights W^* such that $|\varepsilon| \leqslant \varepsilon^*$ with constant $\varepsilon^* > 0$ for all $Z \in \Omega_Z$. Moreover, W^* is bounded by $\|W^*\| \leqslant \overline{W}^*$ on the compact set Ω_Z.

It is clear that W^* is usually unknown and need to be estimated in function approximation. Let $\widehat{W}$ be the estimates of W^*, and the weight estimation error be $\widetilde{W} = \widehat{W} - W^*$.

For Gaussian RBF networks, the following lemma provides an upper bound on the 2-norm of vector $S(Z)$, which is essential in proving of our main result.

Lemma 1 (Kurdila, Narcowich, & Ward, 1995). *Consider the Gaussian RBF networks* (5) (6). *Let* $\rho := \frac{1}{2}\min_{i \neq j}\|\mu_i - \mu_j\|$, *and let q be the dimension of input Z, and η be the width of Gaussian function* (*as in* (6)). *Then we may take an upper bound of* $\|S(Z)\|$ *as*

$$\|S(Z)\| \leqslant \sum_{k=0}^{\infty} 3q(k+2)^{q-1} \mathrm{e}^{-2\rho^2 k^2/\eta^2} := s^*. \tag{8}$$

Remark 2. It can be easily proven that the sum $\sum_{k=0}^{\infty} 3q(k+2)^{q-1}\mathrm{e}^{-2\rho^2k^2/\eta^2}$ has a limited value s^*, since the infinite series $\{3q(k+2)^{q-1}\mathrm{e}^{-2\rho^2k^2/\eta^2}\}$ $(k=0, \ldots, \infty)$ is convergent by the Ratio Test Theorem (Apostol, 1963). Note also that this limited value s^* is independent of Z (the NN inputs) and l (the dimension of neural weights W).

3. Adaptive neural control design

In this section, we develop an ISS-modular approach to overcome the circularity problem (as mentioned in the Introduction). In Section 3.1, an adaptive neural controller is designed to achieve ISS with respect to the NN weight estimation errors. In Section 3.2, a neural weight estimator is designed to achieve ISS with respect to the system state errors. The stability of the entire closed-loop system will be guaranteed by using the small-gain theorem in Section 3.3.

3.1. ISS neural controller design

To achieve the ISS-modularity of the neural control module, in this subsection, we develop an ISS neural controller by combining backstepping design with the Implicit Function Theorem, as shown in Ge and Wang (2002b, Lemma 1). At each recursive step i, a desired feedback control α_i^* is firstly shown to exist, which possesses some desired properties. Then, a stabilizing function α_i $(u = \alpha_n)$ is designed, where a Gaussian RBF network is employed to approximate the desired feedback control α_i^* $(i = 1, \ldots, n)$. With the stabilizing functions α_i $(i=1, \ldots, n)$, the state error subsystem is obtained (as seen from the following steps):

Step 1: Define $z_1 = x_1 - x_{d1}$. Its derivative is

$$\dot{z}_1 = f_1(x_1, x_2) - \dot{x}_{d1}. \tag{9}$$

From Assumption 1, we know that $\partial f_1(x_1, x_2)/\partial x_2 > \underline{g}_1 > 0$ for all $(x_1, x_2) \in R^2$. Define $v_1 = -\dot{x}_{d1} = -f_{d1}(x_d)$. It is clear that v_1 is a function of x_d. Considering the fact that $\partial v_1/\partial x_2 = 0$, we have $\partial[f_1(x_1, x_2) + v_1]/\partial x_2 > \underline{g}_1 > 0$. By viewing x_2 as a virtual control input, for every value of x_1 and v_1, there exists a smooth ideal control input $x_2 = \alpha_1^*(x_1, v_1)$ such that $f_1(x_1, \alpha_1^*) + v_1 = 0$.

Using the Mean Value Theorem (Apostol, 1963), there exists λ_1 $(0<\lambda_1<1)$ such that

$$f_1(x_1,x_2)=f_1(x_1,\alpha_1^*)+g_{\lambda_1}(x_2-\alpha_1^*), \tag{10}$$

where $g_{\lambda_1}:=g_1(x_1,x_{\lambda_1})$, $x_{\lambda_1}=\lambda_1 x_2+(1-\lambda_1)\alpha_1^*$. Note that Assumption 1 on $g_1(x_1,x_2)$ is still valid for g_{λ_1}.

Combining (9)–(10) yields

$$\begin{aligned}\dot z_1&=f_1(x_1,x_2)+v_1\\&=g_{\lambda_1}(x_2-\alpha_1^*).\end{aligned}$$

By employing an RBF neural network $W_1^{\mathrm T}S_1(Z_1)$ to approximate $\alpha_1^*(x_1,v_1)$, where

$$Z_1=[x_1,\dot x_{d1}]^{\mathrm T}\in\Omega_1\subset R^2, \tag{11}$$

α_1^* can be expressed as $\alpha_1^*=W_1^{*\mathrm T}S_1(Z_1)+\varepsilon_1, \forall Z_1\in\Omega_1\subset R^2$ where $|\varepsilon_1|\leqslant\varepsilon_1^*$ is the approximation error with constant $\varepsilon_1^*>0$ over Ω_1. Let $\widehat W_1$ be the estimate of W_1^*, $\widetilde W_1=\widehat W_1-W_1^*$. Define $z_2=x_2-\alpha_1$ and let

$$\alpha_1=-c_1z_1+\widehat W_1^{\mathrm T}S_1(Z_1), \tag{12}$$

where c_1 is a positive constant to be specified later. Then, the dynamics of z_1 is governed by

$$\begin{aligned}\dot z_1&=g_{\lambda_1}(z_2+\alpha_1-\alpha_1^*)\\&=g_{\lambda_1}[z_2-c_1z_1+\widetilde W_1^{\mathrm T}S_1(Z_1)-\varepsilon_1].\end{aligned} \tag{13}$$

Step i $(2\leqslant i\leqslant n)$: Define $z_i=x_i-\alpha_{i-1}$. The derivative of z_i is

$$\dot z_i=f_i(\bar x_i,x_{i+1})-\dot\alpha_{i-1},\quad 2\leqslant i\leqslant n, \tag{14}$$

where $x_{n+1}:=u$ as denoted before. From Assumption 1, we know that $\partial f_i(\bar x_i,x_{i+1})/\partial x_{i+1}>\underline{g}_i>0$ for all $\bar x_{i+1}\in R^{i+1}$ $(2\leqslant i\leqslant n)$. Let $v_i=-\dot\alpha_{i-1}$ $(2\leqslant i\leqslant n)$. It is seen from Eq. (12) that $v_2=-\dot\alpha_1$ is not a function of x_3, thus we have $\partial(-v_2)/\partial x_3=0$. Subsequently, it can be seen from the recursive design procedure that $\partial(-v_i)/\partial x_{i+1}=0$ $(3\leqslant i\leqslant n)$. Then, we have $\partial[f_i(\bar x_i,x_{i+1})+v_i]/(\partial x_{i+1})>\underline{g}_i>0, 2\leqslant i\leqslant n$. For every value of $\bar x_i$ and v_i, there exists a smooth ideal control input $x_{i+1}=\alpha_i^*(\bar x_i,v_i)$ $(u^*=\alpha_n^*(\bar x_n,v_n)$ for $i=n)$ such that $f_i(\bar x_i,\alpha_i^*)+v_i=0,\ 2\leqslant i\leqslant n$. Using the Mean Value Theorem, there exists λ_i $(0<\lambda_i<1)$ such that

$$f_i(\bar x_i,x_{i+1})=f_i(\bar x_i,\alpha_i^*)+g_{\lambda_i}(x_{i+1}-\alpha_i^*),\quad 2\leqslant i\leqslant n, \tag{15}$$

where $g_{\lambda_i}:=g_i(\bar x_i,x_{\lambda_{i+1}})$ with $x_{\lambda_{i+1}}=\lambda_i x_{i+1}+(1-\lambda_i)\alpha_i^*$. Note that Assumption 1 on $g_i(\bar x_i,x_{i+1})$ is still valid for g_{λ_i}. Combining (14)–(15) yields

$$\dot z_i=g_{\lambda_i}(x_{i+1}-\alpha_i^*).$$

Since α_{i-1} is a function of $\bar x_{i-1}$, x_d and $\widehat W_1,\ldots,\widehat W_{i-1}$, $\dot\alpha_{i-1}$ is given by $\dot\alpha_{i-1}=\sum_{k=1}^{i-1}(\partial\alpha_{i-1}/\partial x_k)f_k(\bar x_{k+1})+\phi_{i-1}$, where $\phi_{i-1}=\sum_{k=1}^{i-1}(\partial\alpha_{i-1}/\partial x_d)\dot x_d+\sum_{k=1}^{i-1}(\partial\alpha_{i-1}/\partial\widehat W_k)\dot{\widehat W}_k$ is computable ($\dot{\widehat W}_k$ will be given in the next subsection).

By employing an RBF neural network $W_i^{\mathrm T}S_i(Z_i)$ to approximate $\alpha_i^*(\bar x_i,v_i)$, where

$$Z_i=\left[\bar x_i,\frac{\partial\alpha_{i-1}}{\partial x_1},\ldots,\frac{\partial\alpha_{i-1}}{\partial x_{i-1}},\phi_{i-1}\right]^{\mathrm T}\in\Omega_i\subset R^{2i}, \tag{16}$$

α_i^* $(u^*=\alpha_n^*)$ can be expressed as $\alpha_i^*=W_i^{*\mathrm T}S_i(Z_i)+\varepsilon_i$. Define $z_{i+1}=x_{i+1}-\alpha_i$ $(2\leqslant i\leqslant n-1)$ and let

$$\alpha_i=-z_{i-1}-c_iz_i+\widehat W_i^{\mathrm T}S_i(Z_i),\quad 2\leqslant i\leqslant n-1 \tag{17}$$

and

$$u=-z_{n-1}-c_nz_n+\widehat W_n^{\mathrm T}S_n(Z_n), \tag{18}$$

where c_i $(2\leqslant i\leqslant n)$ is a positive constant to be specified later. Then, Eq. (14) becomes

$$\begin{aligned}\dot z_i&=g_{\lambda_i}[z_{i+1}-z_{i-1}-c_iz_i+\widetilde W_i^{\mathrm T}S_i(Z_i)-\varepsilon_i],\\&\quad i=2,\ldots,n-1,\end{aligned} \tag{19}$$

$$\dot z_n=g_{\lambda_n}[-z_{n-1}-c_nz_n+\widetilde W_n^{\mathrm T}S_n(Z_n)-\varepsilon_n]. \tag{20}$$

Combining Eqs. (13), (19) and (20), we arrive at the state error subsystem

$$\begin{aligned}\dot z_1&=g_{\lambda_1}[z_2-c_1z_1+\widetilde W_1^{\mathrm T}S_1(Z_1)-\varepsilon_1],\\ \dot z_i&=g_{\lambda_i}[z_{i+1}-z_{i-1}-c_iz_i+\widetilde W_i^{\mathrm T}S_i(Z_i)-\varepsilon_i],\\&\quad i=2,\ldots,n-1,\\ \dot z_n&=g_{\lambda_n}[-z_{n-1}-c_nz_n+\widetilde W_n^{\mathrm T}S_n(Z_n)-\varepsilon_n].\end{aligned} \tag{21}$$

In the following, we prove that the state error subsystem (21) is ISS with respect to the NN weight estimation errors $\widetilde W$ and the NN approximation errors ε.

Lemma 2. *The state error subsystem* (21), *viewed as a system with states $z=[z_1,\ldots,z_n]^{\mathrm T}$, and inputs $\widetilde W=[\widetilde W_1^{\mathrm T},\ldots,\widetilde W_n^{\mathrm T}]^{\mathrm T}$ and $\varepsilon=[\varepsilon_1,\ldots,\varepsilon_n]^{\mathrm T}$, is input-to-state stable.*

Proof. Consider the ISS-Lyapunov function candidate $V_z=\frac12\|z\|^2=\frac12\sum_{i=1}^n z_i^2$. Its derivative along (21) is

$$\begin{aligned}\dot V_z=&-\sum_{i=1}^n c_ig_{\lambda_i}z_i^2+\sum_{i=1}^{n-1}(g_{\lambda_i}-g_{\lambda_{i+1}})z_iz_{i+1}\\&+\sum_{i=1}^n g_{\lambda_i}\widetilde W_i^{\mathrm T}S_i(Z_i)z_i-\sum_{i=1}^n g_{\lambda_i}z_i\varepsilon_i\\ \leqslant&-\sum_{i=1}^n c_ig_{\lambda_i}z_i^2+\sum_{i=1}^{n-1}|g_{\lambda_i}-g_{\lambda_{i+1}}|\frac{(z_i^2+z_{i+1}^2)}{2}\\&+\sum_{i=1}^n g_{\lambda_i}\|\widetilde W_i\|\|S_i(Z_i)\||z_i|-\sum_{i=1}^n g_{\lambda_i}z_i\varepsilon_i\\ \leqslant&\left(-\sum_{i=1}^n c_{i0}g_{\lambda_i}z_i^2+\sum_{i=1}^{n-1}|g_{\lambda_i}-g_{\lambda_{i+1}}|\frac{(z_i^2+z_{i+1}^2)}{2}\right)\\&-\sum_{i=1}^n c_{i1}g_{\lambda_i}z_i^2+\sum_{i=1}^n g_{\lambda_i}(-c_{i2}z_i^2+s_i^*\|\widetilde W_i\||z_i|)\\&+\sum_{i=1}^n g_{\lambda_i}(-c_{i3}z_i^2-z_i\varepsilon_i),\end{aligned}$$

where $c_i = c_{i_0} + c_{i_1} + c_{i_2} + c_{i_3}$, with c_{i_j} $(i = 1, \dots, n,\ j = 0, \dots, 3) > 0$. By completion of squares, the following inequalities hold:

$$-c_{i_2} g_{\lambda_i} z_i^2 + g_{\lambda_i} s_i^* \|\widetilde{W}_i\| |z_i| \leqslant \frac{g_{\lambda_i} s_i^{*2} \|\widetilde{W}_i\|^2}{4c_{i_2}},$$

$$-c_{i_3} g_{\lambda_i} z_i^2 - g_{\lambda_i} z_i \varepsilon_i \leqslant \frac{g_{\lambda_i} \varepsilon_i^2}{4c_{i_3}}.$$

Since

$$\begin{aligned}\left(-\sum_{i=1}^{n} c_{i0} g_{\lambda_i} z_i^2 + \sum_{i=1}^{n-1} |g_{\lambda_i} - g_{\lambda_{i+1}}| \frac{(z_i^2 + z_{i+1}^2)}{2}\right) &\leqslant -\sum_{i=1}^{n} c_{i0} g_{\lambda_i} z_i^2 + \frac{|g_{\lambda_1} - g_{\lambda_2}|}{2} z_1^2 + \frac{|g_{\lambda_{n-1}} - g_{\lambda_n}|}{2} z_n^2 \\ &\quad + \sum_{i=2}^{n-1} \left(\frac{|g_{\lambda_{i-1}} - g_{\lambda_i}|}{2} + \frac{|g_{\lambda_i} - g_{\lambda_{i+1}}|}{2} \right) z_i^2\end{aligned}$$

and

$$|g_{\lambda_i} - g_{\lambda_{i+1}}| \leqslant \max(\bar{g}_i, \bar{g}_{i+1}) - \min(\underline{g}_i, \underline{g}_{i+1}) := g_{\lambda_{i+1}}^{\lambda_i}.$$

Choosing c_{i0} such that $c_{1_0} \geqslant g_{\lambda_2}^{\lambda_1}/2\underline{g}_1$, $c_{i_0} \geqslant (g_{\lambda_i}^{\lambda_{i-1}} + g_{\lambda_{i+1}}^{\lambda_i})/2\underline{g}_i (i = 2, \dots, n-1)$, $c_{n_0} \geqslant g_{\lambda_n}^{\lambda_{n-1}}/2\underline{g}_n$, the derivative of V_z satisfies

$$\dot{V}_z < -\sum_{i=1}^{n} c_{i_1} g_{\lambda_i} z_i^2 + \sum_{i=1}^{n} \frac{g_{\lambda_i} s_i^{*2} \|\widetilde{W}_i\|^2}{4c_{i_2}} + \sum_{i=1}^{n} \frac{g_{\lambda_i} \varepsilon_i^2}{4c_{i3}}.$$

Denote $s^* := \max_{1 \leqslant i \leqslant n} s_i^*$, and

$$c_{*_1} = \min_{1 \leqslant i \leqslant n} c_{i_1}, \quad c_{*_2} = \min_{1 \leqslant i \leqslant n} c_{i_2}, \quad c_{*_3} = \min_{1 \leqslant i \leqslant n} c_{i_3},$$

we have

$$\begin{aligned}\dot{V}_z &\leqslant g_{\lambda_i} \left(-c_{*_1} \|z\|^2 + \frac{s^{*2} \|\widetilde{W}\|^2}{4c_{*_2}} + \frac{\|\varepsilon\|^2}{4c_{*_3}} \right) \\ &\leqslant g_{\lambda_i} \left[-\frac{c_{*_1}}{2} \|z\|^2 - \left(\frac{c_{*_1} \|z\|^2}{2} - \frac{s^{*2} \|\widetilde{W}\|^2}{4c_{*_2}} - \frac{\|\varepsilon\|^2}{4c_{*_3}} \right) \right].\end{aligned}$$

Since

$$\begin{aligned}\|z\| &> \frac{s^*}{\sqrt{2c_{*_1} c_{*_2}}} \|\widetilde{W}\| + \frac{1}{\sqrt{2c_{*_1} c_{*_3}}} \|\varepsilon\| \\ &> \sqrt{\frac{s^{*2} \|\widetilde{W}\|^2}{2c_{*_1} c_{*_2}} + \frac{\|\varepsilon\|^2}{2c_{*_1} c_{*_3}}},\end{aligned}$$

implies $\dot{V}_z < -\alpha_z(\|z\|)$, where $\alpha_z(r) = (c_{*_1} \underline{g}_*/2) r^2$ with $\underline{g}_* := \min_{1 \leqslant i \leqslant n} \underline{g}_i > 0$. Thus, according to (Christofides & Teel, 1996, Definition 1), the state error subsystem (21) is input-to-state stable, with gain functions

$$\gamma_1^{\widetilde{W}}(r) = \frac{s^*}{\sqrt{2c_{*_1} c_{*_2}}} r, \tag{22}$$

$$\gamma_1^{\varepsilon}(r) = \frac{1}{\sqrt{2c_{*_1} c_{*_3}}} r. \quad \square \tag{23}$$

3.2. ISS neural weights estimator

In this subsection, a Lyapunov-based neural weights estimator is presented to achieve the ISS-modularity of the estimation module, i.e., to make the neural weight estimation subsystem input-to-state stable with respect to the system state errors z. Concerning parameter estimator design, roughly two approaches are available in the literature: the Lyapunov function based design, and the one based on optimization techniques (e.g., gradient or least-square algorithms). In the modular approaches proposed in Krstic et al. (1995), optimization based algorithms were employed, where observers were constructed and the boundedness of the parameter errors was guaranteed. The complete controller-estimator separation was implemented. However, it is in fact not necessary to achieve boundedness when we consider modularity in the sense of ISS. Moreover, the optimization based results are complicated for implementation since observers have to be developed in the design.

The following Lyapunov-based neural weight estimator (with σ-modification) has been used in the literature of adaptive NN control (Ge & Wang, 2002a, 2002b; Kwan & Lewis, 2000; Polycarpou & Mears, 1998)

$$\dot{\widetilde{W}}_i = \dot{\widehat{W}}_i = \Gamma_i [S_i(Z_i) z_i - \sigma_i \widehat{W}_i], \quad i = 1, \dots, n, \tag{24}$$

where $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$, and $\sigma_i > 0$, $i = 1, \dots, n$ are positive constant design parameters. Define $W^* = [W_1^{*\mathrm{T}}, \dots, W_n^{*\mathrm{T}}]^{\mathrm{T}}$, and note that $\widehat{W}_i = \widetilde{W}_i + W_i^*$. System (24) can be rewritten as

$$\dot{\widetilde{W}} = \Gamma [S(Z) z - \Upsilon \widetilde{W} - \Upsilon W^*], \tag{25}$$

where $S(Z) = \mathrm{diag}\{S_1(Z_1), \dots, S_n(Z_n)\}$, $\Gamma = \mathrm{diag}\{\Gamma_1, \dots, \Gamma_n\}$, and $\Upsilon = \mathrm{diag}\{\sigma_1 I, \dots, \sigma_n I\}$.

Compared with the optimization-based estimator, the Lyapunov-based neural weight estimator is much simpler. It is used in our ISS-modular neural control approach because it can achieve input-to-state stability with respect to z and W^*.

Lemma 3. *The neural weight estimation subsystem* (25), *viewed as a system with state* $\widetilde{W}$, *inputs* z *and* W^* (*the ideal NN weights*), *is input-to-state stable with respect to* z *and* W^*.

Proof. For the $\widetilde{W}$-subsystem (25), consider the ISS-Lyapunov function candidate $V_{\widetilde{W}} = \frac{1}{2} \|\widetilde{W}\|^2$. Its derivative along the trajectories of (25) is

$$\begin{aligned}\dot{V}_{\widetilde{W}} &= \widetilde{W}^{\mathrm{T}} \dot{\widetilde{W}} = \widetilde{W}^{\mathrm{T}} \Gamma [S(Z) z - \Upsilon \widetilde{W} - \Upsilon W^*] \\ &= \widetilde{W}^{\mathrm{T}} \Gamma [-(1-\theta) \Upsilon \widetilde{W} - \theta \Upsilon \widetilde{W} + S(Z) z - \Upsilon W^*] \\ &\leqslant -(1-\theta) \lambda_{\min}(\Upsilon) \|\Gamma\| \|\widetilde{W}\|^2,\end{aligned}$$

for all $\|\widetilde{W}\| \geqslant s^*\|z\|/\theta\lambda_{\min}(\Upsilon) + \|W^*\|/\theta$, where $0<\theta<1$ is a constant. Therefore, system (25) is ISS with respect to inputs (z, W^*), with gain functions

$$\gamma_2^z(r) = \frac{s^*}{\theta\lambda_{\min}(\Upsilon)} r, \tag{26}$$

$$\gamma_2^{W^*}(r) = \frac{1}{\theta} r. \quad \square \tag{27}$$

Remark 3. The Lyapunov-based estimator might be one of the simplest types that can achieve ISS; however, it is certainly not the only one. The ISS neural weight estimator can also use any other kind of estimators, as long as the input-to-state stability of the estimation subsystem can be achieved.

3.3. Stability of closed-loop system by small-gain theorem

The research on small-gain theorem has a long history. Most of the classical work on the small-gain theorem applies to norm-based (linear) gains (Vidyasagar, 1993). Recently, the small-gain theorem in terms of non-linear gain functions was established by Hill and Mareels (Hill, 1991; Mareels & Hill, 1992) within the input-output context. Expressed in the ISS framework, Jiang et al. (1994) extended further the monotone stability result in Mareels and Hill (1992) and established an ISS-type small-gain theorem. These results are very important in the analysis and control of non-linear systems.

Traditionally, the small-gain theorems are used to verify stability of the closed-loop systems consisted of the plant, as well as the controllers (auxiliary systems) connected to the plant. The controller thus designed is usually referred to as "SG-controller". In the present work, the small-gain theorem is used in a different way: to achieve ISS-modularity of the interconnected control module and estimation module, and thus to guarantee stability and performance for the closed-loop system.

It has been shown in Sections 3.1 and 3.2 that when applying the controller (18) and the NN weight estimator (24) to the plant (1), the closed-loop system is described by two interconnected subsystems: the state error subsystem (21) and the weight estimation subsystem (25). The following theorem states the main result of this paper.

Theorem 1. *Consider the closed-loop system consisting of plant* (1), *reference model* (4), *controller* (18) *and NN weight estimator* (24). *Then, for bounded initial conditions, all signals in the closed-loop system remain bounded, and the output tracking error* $y(t)-y_d(t)$ *converges to a neighborhood around zero.*

Proof. From Lemmas 2 and 3, the ISS property has been established for the state error subsystem (21) and the weight estimation subsystem (25), respectively. Thus, there exist class $\mathscr{KL}$ function β_z and $\beta_{\widetilde{W}}$, and class $\mathscr{K}$ functions $\gamma_1^{\widetilde{W}}$, γ_1^{ε}, γ_2^z and $\gamma_2^{W^*}$, such that

$$\|z(\cdot)\|_\infty \leqslant \max\{\beta_z(\|z(0)\|), \gamma_1^{\widetilde{W}}(\|\widetilde{W}\|_\infty), \gamma_1^{\varepsilon}(\|\varepsilon\|_\infty)\}, \tag{28}$$

$$\|\widetilde{W}(\cdot)\|_\infty \leqslant \max\{\beta_{\widetilde{W}}(\|\widetilde{W}(0)\|), \gamma_2^z(\|z\|_\infty), \gamma_2^{W^*}(\|W^*\|_\infty)\}. \tag{29}$$

According to the small-gain theorem (Jiang et al., 1994), by checking the following condition:

$$\gamma_2^z(\gamma_1^{\widetilde{W}}(r)) < r, \tag{30}$$

we have $(s^*/\theta\lambda_{\min}(\Upsilon))(s^*/\sqrt{2c_{*1}c_{*2}})r < r$, i.e., $c_{*1} > s^{*2}/\sqrt{2}\theta\lambda_{\min}(\Upsilon)$ where we choose $c_{*1} = c_{*2}$.

Then, for bounded initial conditions $z(0)$ and $\widetilde{W}(0)$, we have

$$\|z(\cdot)\|_\infty \leqslant \max\{\beta_z(\|z(0)\|), \gamma_1^{\widetilde{W}}\beta_{\widetilde{W}}(\|\widetilde{W}(0)\|), \gamma_1^{\varepsilon}(\|\varepsilon\|_\infty), \gamma_1^{\widetilde{W}}\gamma_2^{W^*}(\|W^*\|_\infty)\} \tag{31}$$

and

$$\|\widetilde{W}(\cdot)\|_\infty \leqslant \max\{\beta_{\widetilde{W}}(\|\widetilde{W}(0)\|), \gamma_2^z\beta_z(\|z(0)\|), \gamma_2^z\gamma_1^{\varepsilon}(\|\varepsilon\|_\infty), \gamma_2^{W^*}(\|W^*\|_\infty)\}, \tag{32}$$

which means that the closed-loop system is (locally) input-to-state stable (see (Isidori, 1999)) with respect to ε and W^*. Since $\|\varepsilon\| < \varepsilon^*$, and both ε^* and W^* are assumed to be constants, the boundedness of $(z, \widetilde{W})$, and consequently, the boundedness of x, $\widehat{W}$ and the control signal u can be established. Thus, all the signals in the closed-loop remain bounded. Note that the boundedness of $(x, \widehat{W})$ implies that there exist compact sets Ω_i, $1 \leqslant i \leqslant n$, which can be constructed to be sufficiently large such that all the NN inputs $Z_i(t)$ stay within Ω_i for all $t \geqslant 0$.

In particular, the response of z satisfies

$$\lim_{t\to\infty} \sup \|z(t)\| \leqslant \max\{\gamma_1^{\varepsilon}(\|\varepsilon\|), \gamma_1^{\widetilde{W}}\gamma_2^{W^*}(\|W^*\|)\} \leqslant \max\left\{\frac{1}{\sqrt{2c_{*1}c_{*3}}}\varepsilon^*, \frac{s^*}{\sqrt{2}c_{*1}\theta}\overline{W^*}\right\},$$

which means that $\|z(t)\|$ (and consequently $z_1 = x_1 - x_{d1} = y - y_d$) will converge to a small neighborhood of zero by choosing c_i large enough. $\square$

Remark 4. It is seen from Eqs. (31) (32) that bounded initial conditions can lead to the boundedness of all signals in the closed-loop system, which implies the existence of compact sets Ω_i $(i = 1, \dots, n)$ such that $Z_i(t) \in \Omega_i$ $\forall t \geqslant 0$. With the employed neural networks being constructed on large enough approximation regions, a stable adaptive neural controller can be developed such that bounded initial conditions guarantee the boundedness of all the signals in the closed-loop system.

In adaptive neural control, the problem of how to determine the NN approximation region a priori is still open (Choi & Farrell, 2001). From our point of view, the difficulty mainly lies in that, (i) NN approximation (i.e., parameterization) is only valid within a compact set; (ii) while the upper bound on the NN approximation error, i.e., ε^*, might be given by a constructive procedure for RBF networks as in (12), the upper bound on the ideal NN weights, i.e., W^*, which actually represents some kind of detailed information on the non-linearity within the entire approximation region Ω_i, is generally unavailable in practical neural control design. These two points make adaptive neural control more challenging compared with conventional adaptive control, in which either the parametric representation is valid globally, or the upper bounds of unknown parameters are available. This controversial problem is worthwhile for further investigation.

4. An example

To verify the effectiveness of the proposed approach, the developed adaptive NN controller is applied to the following non-linear system:

$$\begin{aligned}\dot{x}_1 &= x_1 + x_2 + \frac{x_2^3}{5},\\ \dot{x}_2 &= x_1 x_2 + u + \frac{u^3}{7},\\ y &= x_1,\end{aligned} \tag{33}$$

which is in the non-affine pure-feedback form (1).

The reference model is taken as the famous van der Pol oscillator (see, e.g., Vidyasagar, 1993)

$$\begin{aligned}\dot{x}_{d1} &= x_{d2},\\ \dot{x}_{d2} &= -x_{d1} + \beta(1 - x_{d1}^2)x_{d2},\\ y_d &= x_{d1},\end{aligned} \tag{34}$$

which yields a limit cycle trajectory when $\beta > 0$ ($\beta = 0.2$ in this simulation), for initial states starting from points other than (0, 0).

The control objective is to design controller for system (33) such that (i) all the signals in the closed-loop system remain bounded, and (ii) the output of system (33) follows the desired reference trajectory y_d generated from the van der Pol oscillator.

As system (33) is of second order, the adaptive NN controller is chosen according to (18) as follows:

$$u = -z_1 - c_2 z_2 - \widehat{W}_2^{\mathrm{T}} S_2(Z_2), \tag{35}$$

where $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$ and $Z_2 = [x_1, x_2, \partial\alpha_1/\partial x_1, \phi_1]^{\mathrm{T}}$ with

$$\begin{aligned}\alpha_1 &= -c_1 z_1 - \widehat{W}_1^{\mathrm{T}} S_1(Z_1), \quad Z_1 = [x_1, \dot{x}_{d1}]^{\mathrm{T}},\\ \phi_1 &= \frac{\partial\alpha_1}{\partial x_{d1}}\dot{x}_{d1} + \frac{\partial\alpha_1}{\partial x_{d2}}\dot{x}_{d2} + \frac{\partial\alpha_1}{\partial \widehat{W}_1}\dot{\widehat{W}}_1,\end{aligned} \tag{36}$$

and NN weights $\widehat{W}_1$ and $\widehat{W}_2$ are updated by (24).

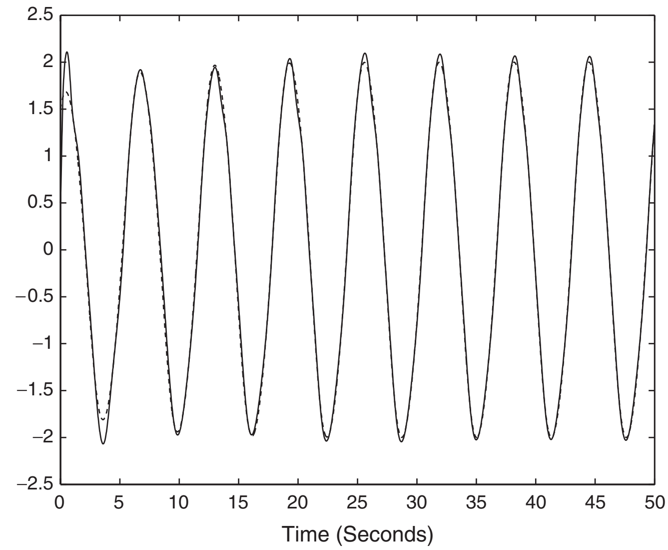

Fig. 1. Output tracking performance (y—solid line and y_d–dashed line).

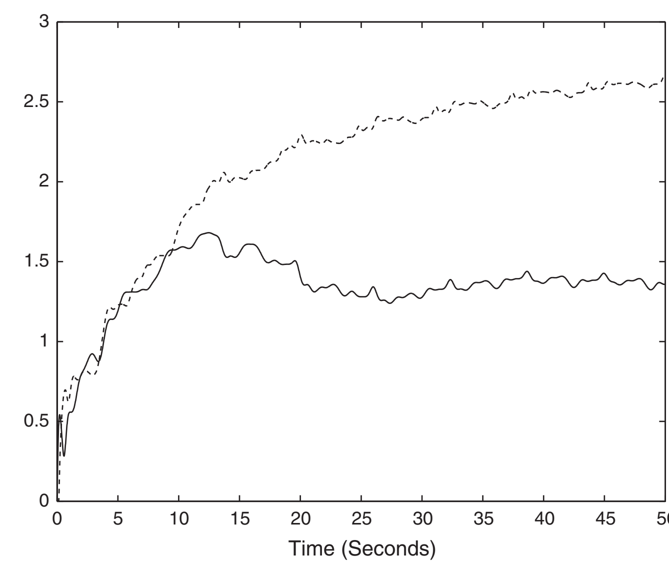

Fig. 2. L_2 norms of the NN weights: $\widehat{W}_1$ (solid line) and $\widehat{W}_2$ (dashed line).

Neural networks $\widehat{W}_1^{\mathrm{T}} S_1(Z_1)$ contains 25 nodes (i.e., $l_1 = 25$), with centers μ_l ($l = 1, \ldots, l_1$) evenly spaced in $[-4, 4] \times [-4, 4]$, and widths $\eta_l = 2$ ($l = 1, \ldots, l_1$). Neural networks $\widehat{W}_2^{\mathrm{T}} S_2(Z_2)$ contains 135 nodes (i.e., $l_2 = 135$), with centers μ_l ($l = 1, \ldots, l_2$) evenly spaced in $[-4, 4] \times [-4, 4] \times [-4, 0] \times [-6, 6]$, and widths $\eta_l = 2$ ($l = 1, \ldots, l_2$). The design parameters of the above controller are $c_1 = 3.0$, $c_2 = 5.0$, $\Gamma_1 = \Gamma_2 = \mathrm{diag}\{2.0, 2.0\}$, $\sigma_1 = \sigma_2 = 0.2$. The initial weights $\widehat{W}_1(0) = 0$, $\widehat{W}_2(0) = 0$. The initial conditions $[x_1(0), x_2(0)]^{\mathrm{T}} = [0.5, 1.8]^{\mathrm{T}}$ and $[x_{d1}(0), x_{d2}(0)]^{\mathrm{T}} = [1.5, 0.8]^{\mathrm{T}}$.

Figs. 1–3 show the simulation results of applying controller (35) to system (33) for tracking reference signal y_d. From Fig. 1, we can see that fairly good tracking performance is

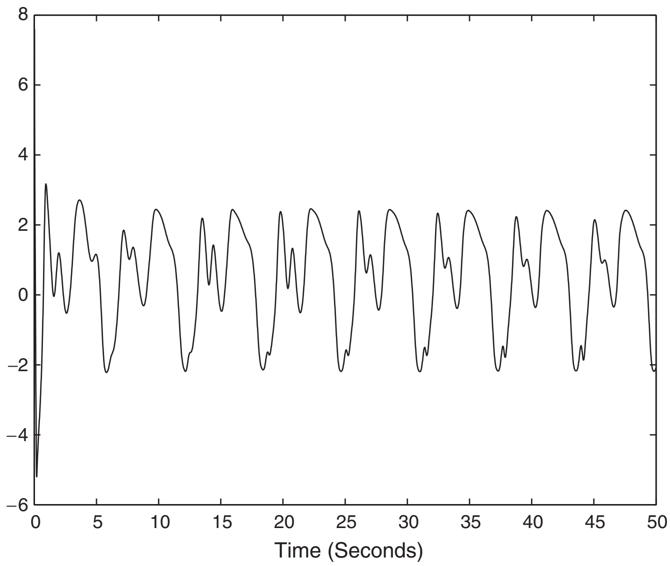

Fig. 3. Boundedness of the control u.

obtained. The boundedness of NN weights $\widehat{W}_1$, $\widehat{W}_2$ and control signal u are shown in Figs. 2 and 3, respectively.

5. Conclusion

An "ISS-modular" approach for adaptive neural control of the non-affine pure-feedback system was presented. By achieving the ISS-modularity of the interconnected control module and estimation module, the difficult problem of non-affine pure-feedback system control was resolved by combining adaptive neural design with the backstepping method, ISS analysis and the small-gain theorem. The employment of ISS analysis and the small gain theorem avoids the construction of an overall Lyapunov function for the closed-loop system, and subsequently overcomes the circular design problem in NN control of pure-feedback systems. The ISS-modular approach was finally shown by example to provide a simple and effective way for stabilizing non-affine non-linear systems using neural networks.

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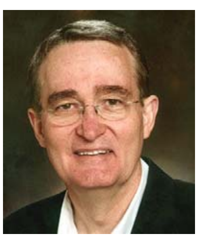

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