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Project STAIR is a federally-funded research project that supports middle-school mathematics teachers in implementing data-based individualization (DBI). STAIR coaches work with teachers to help support students who experience difficulty with math to develop algebra readiness skills needed to be successful in high school and beyond. Project STAIR is supported by the Office of Special Education Programs (OSEP) under grant H326M170006. The project is housed at the University of Missouri, Southern Methodist University, and the University of Texas at Austin.

Authors of this guide:
Samantha E. Bos
Rachel Juergensen
Tiffini Pruitt-Britton
Sarah R. Powell

PROJECT STAIR TEAM MEMBERS:

Erica S. Lembke
Leanne Ketterlin-Geller
Sarah R. Powell
Tessa L. Arsenault
Samantha E. Bos
Taylor Cox
Stacy Hirt
Stephanie Hopkins

Jiyung Hwang
Rachel Juergensen
Erica N. Mason
Alain Mota
Yaacov Petscher
Tiffini Pruitt-Britton
Elizabeth Thomas
In this guide, we provide suggestions for implementing evidence-based mathematics (math) instruction. We suggest you design an instructional platform and intensify the platform based on student needs.

**WHAT IS EVIDENCE-BASED MATHEMATICS INSTRUCTION?**

Evidence-based instruction is instruction based on the results of several high-quality research studies that find a practice to effectively improve students' math skills and understandings across a variety of settings.

In this guide, we provide an introduction to key evidence-based math practices. Additionally, we provide a brief description of the research-base that supports each practice, and we provide additional resources if you are interested in learning more.
# Project STAIR

Supporting Teaching of Algebra: Individual Readiness

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*NOTE. INTERACTIVE HYPERLINKS ARE EMBEDDED THROUGHOUT THE DOCUMENT. THIS DOCUMENT MAY BE REPRODUCED OR DISTRIBUTED FOR NONPROFIT PROFESSIONAL LEARNING EDUCATION PURPOSES. DISCLAIMER: NOT ALL EXAMPLES ARE LISTED IN DOCUMENT. RESOURCE MAY NOT BE REPRODUCED FOR MONETARY CHARGE.*
Math Instructional Practices

We highlight 7 evidence-based math practices.

EXPLICIT INSTRUCTION
Explicit instruction includes proper modeling, practice, and support of math learning. Teachers should be purposeful in their explanations and interventions to ensure student participation and reflection.

MULTIPLE REPRESENTATIONS
The use of multiple representations, including concrete, pictorial, and abstract representations allows students to have multiple access points in understanding the math concepts. There is no hierarchy between the three forms of representation, and it is important that students at all grade levels have access to these multiple forms of learning.

MATH LANGUAGE
Math language includes the use of precise and concise formal math language. It is important that students hear, see, write, and practice speaking using the mathematically correct term, even at a young age. Teachers can explicitly teach math language and integrate it into their practice.

WORD-PROBLEM INSTRUCTION
Many students benefit from explicit word problem instruction, including attack strategies to break down the word problem as well as schema instruction. Learning the different schemas, or structures, of word problems helps student identify strategies to solve each problem.

MNEMONICS
Mnemonics includes devices or strategies to help students remember math concepts or the steps in a math procedure. Mnemonics can be a powerful tool to help students learn multi-step strategies or complex multi-component procedures.

GRAPHIC ORGANIZERS
Graphic organizers are visual, or graphic, displays of math vocabulary, concepts, procedures, or relationships between larger math topics. Graphic organizers are an effective tool to help students organize their thoughts and implement complex math procedures.

FLUENCY BUILDING ACTIVITIES
To help students master math facts and math essentials, it is important they are provided frequent, brief opportunities to develop their fluency. Students can play games, use manipulatives, or practice their facts in a variety structured ways.
Utilize Explicit Instruction

DEFINITION

Teachers should be explicit in the modeling and practicing of math. In the figure below, interventions should be divided between modeling and practice. Modeling includes a step-by-step explanation and clarity in the language used during explanations. Practice includes practice with the teacher (i.e., guided practice) as well as independent practice. During modeling and practice, teachers should use supports, such as asking a variety of question types, eliciting student responses and providing immediate corrective feedback while maintaining a brisk pace.

WHAT DOES THE RESEARCH SAY?

Explicit and systematic instruction is essential when providing math intervention (Fuchs et al., 2021). In reviews of math intervention, explicit instruction is often noted in almost all interventions (Jitendra et al., 2018; Stevens et al., 2018).

Researchers have used explicit instruction to teach a variety of math content, including operations (Milton et al., 2019; Parker et al., 2019), fractions (Bouck et al., 2017; Dyson et al., 2020), word-problem solving (Bottge et al., 2014; Jitendra & Star, 2012), algebra (Bouck et al., 2019; Bryant et al., 2020), and math writing (Hughes & Lee, 2020).
Utilize Explicit Instruction

EXAMPLE: MODELING

MODELING

Clear Explanation
Planned Examples

MODELING

State the goal and its importance.
Model steps.
Use precise math language.

“To solve 26 plus 79, we first decide about the operation. Do we add, subtract, multiply or divide?”
“The plus sign tells us to add. So, we'll add 26 plus 79. We'll use the partial sums strategy. First, we add 20 plus 70. What’s 20 plus 70?”
... "Finally, we add the partial sums: 90 and 15. 90 plus 15 is 105.”

MODELING

Clear Explanation
Planned Examples

Use examples.
Also use non-examples.

24 ÷ 6  28 ÷ 7  35 \( \div \) 5
32 ÷ 8  42 ÷ 7  25 − 5

EXAMPLE: PRACTICE

PRACTICE

Guided Practice
Independent Practice

Teacher and student practice together.
"Let’s work on this problem together. First..."

Student practices with teacher support.
"Now you'll practice a problem on your own. Don’t forget to use your attack strategy."
Utilize Explicit Instruction

**EXAMPLE: SUPPORTS**

**SUPPORTS**

- Asking the right questions
  - Ask low-level and high-level questions.
  - “What is 7 times 9?”
  - “How would you solve this problem?”

- Eliciting frequent responses
  - Vary responses, such as: classwide, individual, partner, write on paper, write on whiteboard, thumbs up, etc.
  - “Turn and discuss the formula for perimeter with your partner.”
  - “Write 2 + 6 on your whiteboard.”

- Providing immediate specific feedback
  - Provide affirmative and correct feedback.
  - “Let’s look at that again. Tell me how you added in the hundreds column.”

- Maintaining a brisk pace
  - Be planned and organized.
  - “Grab your bag of manipulatives as you enter the classroom.”

**ADDITIONAL RESOURCES**

- National Center on Intensive Intervention Course on Explicit Instruction
- Vanderbilt IRIS Center Module on Explicit Instruction
- Explicit Instruction Homepage
## Utilize Explicit Instruction

### PROCEDURES

| Clear Explanation with Precise Language | When modeling, start with a clear explanation about why the math is important. For example, say, “Today, we are learning about division. This is important because sometimes you have to share objects or things with your friends.” This will help students start to make the connection between math and real life. Precise language includes the formal language of math. For example, say “numerator” instead of “top number” or “product” instead of “answer.” |
| Model Steps | While modeling, model the steps to solve a problem. Involve your students in the process by asking questions and giving them opportunities to respond. Modeling should feel like a dialogue between you and your students. |
| Planned Examples and Non-Examples | Plan the examples you choose for your interventions. For example, when modeling a division problem, think about the different ways to show division and how you want to represent the problems. Include non-examples to help students understand when to apply the strategy you have modeled. |
| Guided Practice | You and the students practice problems together. Provide scaffolding so students can start to see how they can solve these problems on their own. |
| Independent Practice | Students practice independently while you provide feedback. Ensure that students are able to complete problems on their own before they begin independent practice. |
| Supports: Questions | During both modeling and practice, ask a mix of low-level questions (to check for understanding) and high-level questions (to learn what students understand about different concepts and procedures). |
| Supports: Responses | During both modeling and practice, provide opportunities for students to respond. Typically students should respond at least every 30 to 60 seconds during modeling and guided practice. |
| Supports: Feedback | During both modeling and practice, provide affirmative feedback. Make the affirmative feedback specific to math. Provide corrective feedback when necessary. |
| Supports: Brisk Pace | During both modeling and practice, maintain a brisk pace by being prepared, organized, and ready to teach in order to maximize time spent learning. |
Teachers should use multiple representations to help students understand different math concepts and procedures. Multiple representations include concrete representations. These are hands-on tools students can use to show math in different ways. Multiple representations also includes pictorial representations. These might be pictures, graphs, or organizers printed or drawn on a two-dimensional surface. Pictorial representations could also include virtual manipulatives accessed via technology. Finally, multiple representations includes abstract representations of math presented with numerals, symbols, and words.

When students use multiple representations, their math performance increases (Bouck & Park, 2018). This is true for hands-on tools (Namkung & Bricko, 2021), virtual manipulatives (Bouck et al., 2018), and graphic organizers. Multiple representations have been used to increase student understanding of operations (Bennett & Rule, 2005), fractions (Bouck et al., 2020), word-problem solving (Xin et al., 2020), geometry (Strickland & Maccini, 2012), and algebra (Scheuermann et al., 2009).
Concrete representations include three-dimensional objects or virtual manipulatives. Shown below are examples of manipulatives, but many manipulatives can be used to teach a variety of math topics.

<table>
<thead>
<tr>
<th>Operations</th>
<th>Whole Numbers</th>
<th>Fractions</th>
<th>Algebra</th>
<th>Geometry</th>
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</tr>
</tbody>
</table>
**Multiple Representations**

Visual representations include two-dimensional objects and virtual manipulatives. Shown below are examples of manipulatives, but many manipulatives can be used to teach multiple math topics.

<table>
<thead>
<tr>
<th>Operations</th>
<th>Whole Numbers</th>
<th>Fractions</th>
<th>Algebra</th>
<th>Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Operations Image" /></td>
<td><img src="image2" alt="Whole Numbers Image" /></td>
<td><img src="image3" alt="Fractions Image" /></td>
<td><img src="image4" alt="Algebra Image" /></td>
<td><img src="image5" alt="Geometry Image" /></td>
</tr>
</tbody>
</table>
Abstract representations include symbols and numerals. It is beneficial to always have the abstract representation present when working with additional representations as well.

2 + 8 = 10

34 = 3 tens and 4 ones

x – 6 = 8

4,179 + 569

Additional Resources

National Council of Teachers of Mathematics: Multiple Representations

Vanderbilt IRIS Center Module on Visual Representations

Virtual Manipulatives Library
Math Language

DEFINITION

Teachers should be sure to use precise and concise math language. Math terms should be explicitly taught. Precise math language includes the formal math terms that accurately portray a math concept or procedure. For example, teachers should say **numerator**, not **top number**. When teachers use concise math language, they use short but accurate and student-friendly definitions when they introduce and discuss math concepts.

WHAT DOES THE RESEARCH SAY?

Research suggests that teachers and students should have multiple opportunities to access and use precise and concise math language (Powell et al., 2019). Teachers who use informal language or casual terms such as **box** instead of **cube** are setting students up for later failure and confusion when they are assessed using the formal language or move to another classroom in which a different term is used. Instead, it is important that teachers use the mathematically correct and exact term for the concepts they are teaching. Teachers should explicitly teach math terms just as they would teach math concepts to students.

In addition to explicitly teaching math terms, teachers can also use Frayer models, math journals, word walls with student-friendly definitions and images, and vocabulary practice sheets (depicted here starting at the top and moving counter-clockwise).
“The denominator is the number of equal parts that make the whole.”

“A difference problem is two amounts compared for the difference.”

“To show the fraction, look at the denominator. A denominator of 5 means I need to break the whole into 5 equal parts.”

“To use partial sums, add the hundreds, then add the tens, then add the ones. Finally, add all the partial sums.”
Word-Problem Instruction

DEFINITION

Many students require explicit instruction in solving word problems. It is important that teachers do not focus on teaching key words (Karp et al., 2019), but instead teach students how to use an attack strategy for working through word problems (Powell & Fuchs, 2018). High-quality attack strategies require the students to read the problem, create a plan for solving the problem, solve the problem, and check the answer. Attack strategies should be flexible to address any word problem.

WHAT DOES THE RESEARCH SAY?

In addition to teaching students to use attack strategies, it is also important to introduce schemas (Jitendra et al., 2015). A schema refers to the structure of the word problem. There are three primary additive schemas: Total, Difference, and Change problems. There are also three multiplicative schemas: Equal Groups, Comparison, and Ratio/Proportion problems.

- **Total**: Parts put together into a **total**
- **Difference**: Greater and lesser amounts compared for a difference
- **Change**: An amount that increases or decreases
- **Equal Groups**: Groups multiplied by an equal number in each group
- **Comparison**: Set compared a number of times
- **Ratios/Proportions**: Description of relationships among quantities
Word-Problem Instruction

EXAMPLE: SCHEMAS

Total
Max baked 40 cookies and 75 brownies. How many baked goods did Max bake?

Difference
The Brazos River is 840 miles. The Red River is 1,360 miles. How much longer is the Red River?

Change
There were 23 passengers on the bus. Then, 13 more passengers boarded the bus. How many passengers are on the bus now?

Equal Groups
Mark has 2 boxes of crayons. There are 24 crayons in each box. How many crayons does Mark have?

Comparison
Jill picked 6 apples. Meg picked 2 times as many apples as Jill. How many apples did Meg pick?

Ratios/Proportions
There are 176 slices of bread in 8 loaves. If there are the same number of slices in each loaf, how many slices of bread are in 5 loaves?
**DEFINITION**

Mnemonics are a device or strategy designed to help students remember a strategy, concept, or series of steps. Many mnemonics include a series of letters in which each letter represents a step or component of an intervention (e.g., SOLVE: Study the problem, Organize the facts, Line up a plan, Verify your plan with action, Examine your results). Mnemonics can be a helpful tool for students with limited working memory or as students are first learning the steps in a complex intervention.

**WHAT DOES THE RESEARCH SAY?**

Mnemonic devices or mnemonic strategies are evidence-based components that have been shown to aid students in mastering math concepts (Cuenca-Carlino et al., 2016; Freeman-Green et al., 2015). Mnemonics can used to help students remember the components in a multi-step process, including advanced algebraic procedures. Mnemonics are also often found as part of word problem instruction. Mnemonics can used to remember an attack strategy or a cognitive or meta-cognitive strategy.

Matt bought 1 orange and 3 apples for a total of $2.25. The orange cost $0.60. The apples each cost the same amount. What amount did Matt pay to buy each apple?

\[
\begin{align*}
\text{P1} + \text{P2} &= \text{T} \\
0.60 + ? &= 2.25 \\
? &= \$1.65 \\
\text{for} \quad \text{apples} \\
? &= \$0.55 \\
\text{per apple}
\end{align*}
\]
**EXAMPLES: MNEMONICS**

**RIDE**
- Read the problem.
- Identify the relevant information.
- Determine the operation and unit for the answer.
- Enter the correct numbers and calculate, then check the answer.

**SOLVE**
- Study the problem.
- Organize the facts.
- Line up the plan.
- Verify the plan with computation.
- Examine the answer.

**UPS CHECK**
- Understand: Read the problem.
- Plan: Choose a strategy.
- Solve: Show all your work.
- Check: Explain & justify your answer.

**RIDGES**
- Read the problem.
- I know statement.
- Draw a picture.
- Goal statement.
- Equation development.
- Solve the equation.

**RICE**
- Read and Record the problem.
- Illustrate your thinking.
- Compute.
- Explain your thinking.

**ADDITIONAL RESOURCES**
- IRIS Center Modules
- LD Online Resources
Graphic Organizers

**DEFINITION**

Graphic organizers are visual or graphic representations of procedures or relationships within a math concept. Similar to mnemonics, graphic organizers can be a useful tool in helping students to remember specific procedures. Alternatively, graphic organizers can be used to help students better understand the relationship between math concepts or learn math vocabulary. Depending on the purpose of the graphic organizer, students can either be presented with a prepared graphic organizer or students may benefit from drawing their own graphic organizers.

**WHAT DOES THE RESEARCH SAY?**

Graphic organizers have been shown to be an effective tool to help students learn new material, practice multi-step procedures, and make connections between math concepts for a deeper conceptual understanding (Shin & Bryant, 2017; van Garderen, 2007). Graphic organizers are often used as part of multi-component interventions, and can be tied with mnemonics and explicit instruction to help students master math concepts.

<table>
<thead>
<tr>
<th>Define:</th>
<th>Characteristics:</th>
</tr>
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<tbody>
<tr>
<td>Example:</td>
<td>Non-example:</td>
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</table>
Graphic Organizers

EXAMPLES: GRAPHIC ORGANIZERS

COMPARISON SCHEMA

TEN FRAME

MATH LANGUAGE OR MATH CONCEPT

VENN DIAGRAM

2, 4, 6, 10

1, 5, 7, 21

Triangle

Square

Circle

Ellipse

ADDITIONAL RESOURCES

National Center on Intensive Intervention Resources

The Access Center Graphic Organizers

Understood Graphic Organizers
WHAT DOES THE RESEARCH SAY?

Fluency building activities provide opportunities for students to master math facts and other necessary math knowledge. When students are fluent in math facts, they can spend more energy solving problems or tackling complex problems. In addition to math facts, as students mature, it is recommended that they become fluent in determining equivalent and benchmark fractions, determining common denominators, and adding, subtracting, multiplying, or dividing positive and negative integer patterns.

DEFINITION

It is recommended that all teachers include brief fluency building activities everyday in math class (Burns et al., 2010). Fluency practice is important for students with limited working memory or students who experience difficulty with math. When students are fluent in essential math facts and math understandings, it frees up cognitive processing power which can then be devoted to more complex math. Students who struggle with math often benefit from explicit instruction in learning math facts and strategies.
EXAMPLES: FLUENCY BUILDING ACTIVITIES

- **Roll the Dice**
  - __ + __ = ___
  - __ + __ = ___
  - __ - __ = ___
  - __ - __ = ___

- **Dominoes**
  - __ + __ = ___
  - __ + __ = ___
  - __ - __ = ___
  - __ - __ = ___

- **Cards**
  - __ + __ = ___
  - __ + __ = ___
  - __ - __ = ___
  - __ - __ = ___

**File Folder**

<table>
<thead>
<tr>
<th>6 + 3 =</th>
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<th>8 + 5 =</th>
<th>9 + 6 =</th>
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ADDITIONAL RESOURCES

Combining Explicit Instruction and Mastery Practice to Build Fact Fluency Article
Multicomponent Interventions

**KEY CONSIDERATIONS**

Many math interventions include many, if not all, of the components discussed in this guide. Therefore, this makes it difficult to determine which component(s) are responsible for student learning. Instead, researchers can only evaluate the impact of the overall intervention. Therefore, when designing an intervention, it is important to consider which components align with the goals of the intervention. For example, if designing a word problem solving intervention, mnemonics, graphic organizers and attack strategies all support student learning and align well with each other.
References


References


