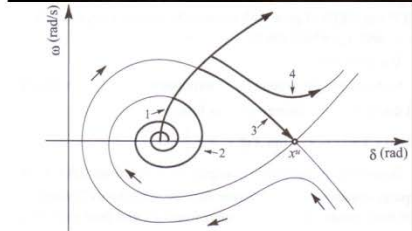
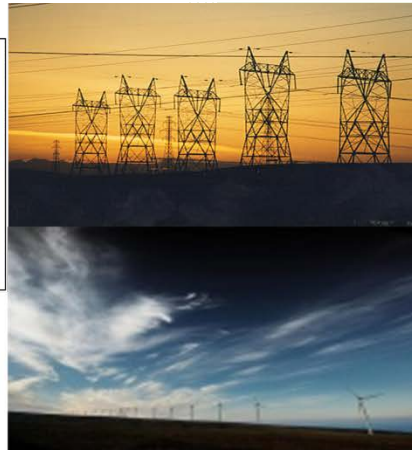
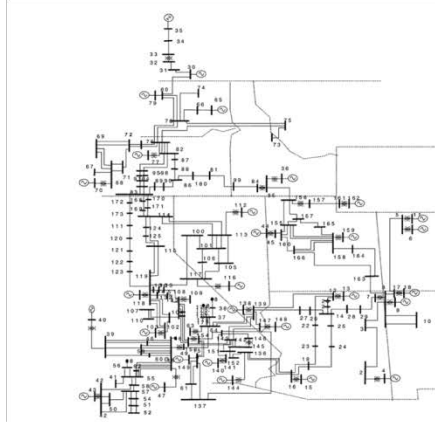


# 2015

## Risk and Resiliency of the Electric Power Grid: A Division of Mathematical Sciences Workshop

$\min_{p, \alpha} \mathbb{E}[c(p)]$	Min Cost
s.t. $\sum_{i \in G} \alpha_i = 1, \alpha \geq 0$	Freq. regulation model
$B\delta = \alpha, \delta_n = 0$	
$\sum_{i \in G} \bar{p}_i + \sum_{i \in W} \mu_i = \sum_{i \in D} d_i$	Avg. power balance
$\bar{T}_{ij} = y_{ij}(\bar{\theta}_i - \bar{\theta}_j),$	Line flows
$B\bar{\theta} = \bar{p} + \mu - d, \bar{\theta}_n = 0$	DC power flow
$s_{ij}^2 \geq y_{ij}^2 \sum_{k \in W} \sigma_k^2 (B_{ik}^+ - B_{jk}^+ - \delta_i + \delta_j)^2$	Auxiliary constraint
$ \bar{T}_{ij}  + s_{ij} \phi^{-1}(1 - \epsilon_{ij}) \leq \epsilon_{ij}^{\max}$	Chance constraint



1: Fault-on trajectory,  
2: Stable case after fault clearing,  
3: Critical case, 4: Unstable case,  
x\*: Unstable Equilibrium Point (UEP)

$$P_i = V_i^t \sum_{k=1}^{n-1} (G_{ik} V_k^t - B_{ik} V_k^{im}) + V_i^{im} \sum_{k=1}^{n-1} (B_{ik} V_k^t + G_{ik} V_k^{im}) \quad i=1, \dots, n$$

$$Q_i = -V_i^t \sum_{k=1}^{n-1} (B_{ik} V_k^t + G_{ik} V_k^{im}) + V_i^{im} \sum_{k=1}^{n-1} (G_{ik} V_k^t - B_{ik} V_k^{im}) \quad i=r+1, \dots, n$$

$$|V_i^t|^2 = |V_i^{im}|^2 \quad i=1, \dots, r$$

Emmanuel Taylor  
DOE OE

Barry Lee  
SMU

7/20/2015

# 1. Introduction

The electric power grid is often considered to be one of the most complex machinery designed by mankind. Its societal, national, and economic significance is acknowledged every time there is a major blackout. Thus, insuring a reliable and economical production of electric power, and insuring a reliable transmission of electric power have been major goals of power grid engineers. The reliability of the production and transmission is often assessed through computer simulations using mathematical methods. These simulations, together with the mathematical techniques behind them, are invaluable to mitigating risks in the power grid and insuring the grid's resiliency. Unfortunately, even for the traditional power grid, computational and analytical techniques to optimally and efficiently tackle many of the challenges in the power grid are often lacking. Development of these techniques is welcome by the power grid community. Moreover, insight into the fundamental and intrinsic properties of the power grid that can expose the mathematical and statistical structures of the grid is welcome. Such development and insight will be necessary for modeling and analyzing the emergent grid, which will present new computational challenges and have more risks and less resilience than the traditional grid.

Indeed, the emergent power grid continues to fundamentally change with the incorporation of new technologies and new policies. For example, the shift to renewable energy sources and the deregulation of electric markets are changing the dynamics, the actual mathematical dynamics, of the power grid system. The emergent grid also presents new opportunities and challenges in developing a more resilient and safer network. For example, the enormous amount of data generated by new measurement devices can be analyzed to determine the state of the power grid system, which in turn, can be used to mitigate risks that can propagate through large regions of the grid in seconds; and the cyber-based infrastructure of the grid presents new challenges in the design of a secure system. Given the national security and national interest of the power grid, the importance of designing a stable, resilient system is being recognized even outside the engineering and science communities. However, this design will require new technical tools and methodologies. Developing these new tools and methodologies will require innovative application of mathematics and statistics techniques, and more importantly, will require mathematics and statistics research of the power grid for understanding the grid's intrinsic nature.

This power grid research must be an interdisciplinary effort consisting of power grid engineers, mathematicians, and statisticians. Unfortunately the mathematics and statistics communities have not been as actively involved in this research field as in other fields of science and engineering. A major barrier to more mathematics and statistics involvement is the complexity of the power grid itself, which leads to a steep learning curve. Thus, to germinate more interest in these communities, this workshop brought together power grid engineers, mathematicians, and statisticians to discuss the mathematics and statistics challenges in the power grid. Topics included stability analysis, uncertainty quantification, model reduction, anomaly detection, data analytics, and computational methods for the grid. Further, several talks on the mathematics of networks were presented, to investigate the applicability of state-of-the-art network results to the power grid.

The workshop involved representatives from academia and the Department of Energy (DOE). Power grid experts from DOE shared their knowledge, experience, and visions on the current and emergent power grid. This insured that the challenges of the actual power grid were presented to the mathematics and

statistics communities. Talks were also presented by a mathematician and a statistician that have investigated the power grid. Talks from this mixture of disciplines helped bridged the language barrier between the different communities.

The goal of this workshop was to generate interest in the power grid in mathematicians and statisticians. This was achieved, as reflected by the good attendance and increased interest in the electric grid. We envision that the interest generated in the mathematics and statistics participants will lead to recruitment of math/stats students to research the risk and resiliency of the electric power grid. Such recruitment may be natural since the power grid is rich in mathematical topics that mathematicians have historically investigated for other science and engineering applications.

This workshop was funded by NSF, Division of Mathematical Sciences, grant DMS-1550666.

## 2. Presentations

Seven presentations were given in this workshop, several describing the power grid challenges from an engineering perspective and several from a purely mathematics perspective (networks). Abstracts of these presentations, and summaries connecting the power grid challenges to the mathematical research are described in the following.

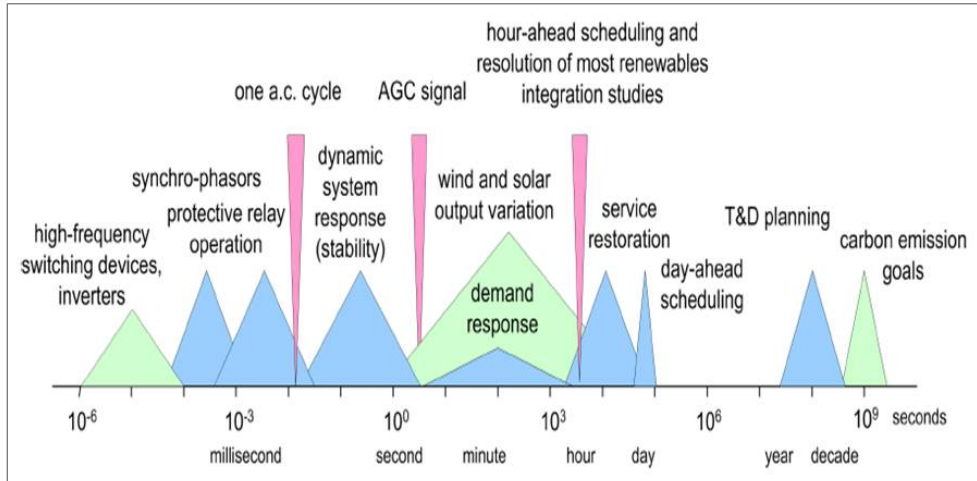
1. Power Grid Risk and Resiliency, presenter Emmanuel Taylor (DOE Office of Electricity)

There are many factors affecting the need for mathematics research specific to power systems. The portfolio of generating technologies is shifting throughout the US. The dynamics of electrical loads are changing. Reliability is more difficult to guarantee, and new threats are emerging that have the potential to disrupt the grid. New strategies and methodologies will be needed to inform the operation of the grid, planning the grid of the future, and ensuring its safety and reliability. Underpinning these new strategies will be newfound approaches to mathematics, developed specifically for grid applications.

Summary:

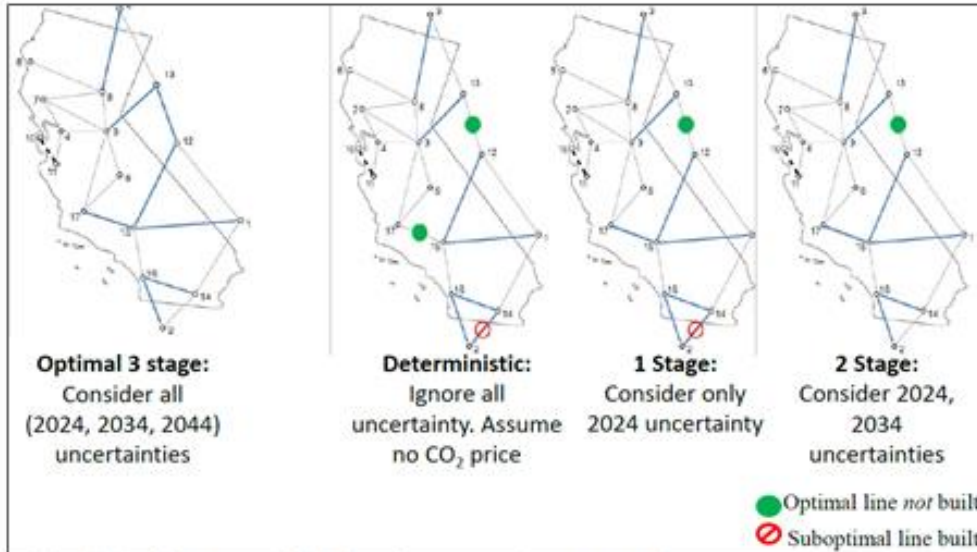
This presentation provided the attendees with an overview of the structure of the power grid, its operations, its challenges, and its mathematical needs. This presentation also highlighted current programs through which the DOE sponsors research in mathematics.

**Planning for the grid happens on the scale of decades.**



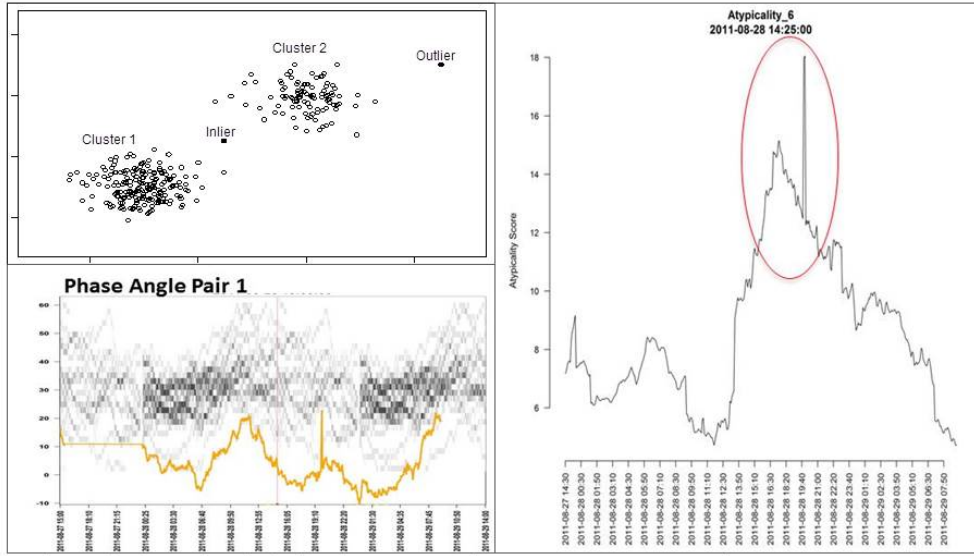
A. Von Meier; "Integration of Renewable Generation in California"; DOI: 10.1109/EPQU.2011.6128888

**Uncertainties have a real effect on the system of today, and tomorrow.**



Hobbs, Benjamin F.; "What Investments Should We Make Now?"; CBETS Program Review; August 5, 2004; JShaw, NY

Multivariate statistical techniques used to identify atypical grid events.



Amidan, Brett; "Baselining Studies and Analysis", "DOE/OE Transmission Reliability Program Review, June 3-4, 2013; Washington, DC

1

2. Challenges to Electricity Markets, presenter Richard P. O’Neill (Federal Energy Regulatory Commission)

The limitations in the long-term storage of electricity present a challenge in the economical generation and distribution of electricity. Because of these limitations, electricity markets play a major role in this production and distribution: a smooth, relatively uninterrupted and near-optimal rate of production is possible if surplus energy can be sold quickly. To achieve this near-optimality, large-scale optimization problems must be solved. In particular, non-convex mixed integer programs (MIP), with constraints that are based on the physical limits (e.g., thermal limits) of the states in the power grid, arise. We will describe these MIP formulations and summarize the economical savings that they have achieved.

Summary:

Mixed integer programming has successfully advanced the electricity markets. New MIP approaches, with better convergence properties, is an active area of mathematical research, and may provide further cost savings in the generation and transmission of electricity. Integrating other components of the power grid (e.g., AC optimal power flow and stability) with the MIP will lead to new mathematical formulations and algorithms. Utilizing high-performance parallel computers also will require development of new algorithms.

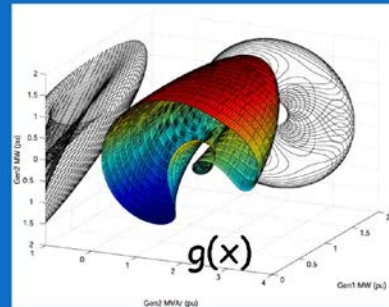
# From real time dispatch to investment planning



Mixed Integer Nonconvex Program

maximize  $b(x,z)-c(x,z)$   
subject to  $g(x,z) \leq 0$   
 $Ax-bz \leq 0$   
 $l \leq x \leq u,$   
 $z \in \{0,1\}$

$c(x), g(x)$  may be non-convex



I didn't know what I would find there

8

# Mixed Integer Program

maximize  $cx$   
subject to  $Ax = b,$   
 $l \leq x \leq u,$   
some  $x \in \{0,1\}$

And though the holes  
were rather small  
They had to count  
them all

Better modeling for

Start-up and shutdown  
Transmission switching  
Investment decisions

It was twenty  
years ago today

solution times improved by  $> 10^7$  in last 30 years  
10 years becomes 10 minutes

9

## Rectangular ACOPF-IV formulation.

Network-wide objective function:  $\text{Min } c(P, Q, I, V)$  (50)

Network-wide constraint:  $I = YV$  (51)

Bus-specific constraints:

$P = V^r \cdot I^r + V^j \cdot I^j \leq P^{\max}$  (54)       $P^{\min} \leq P = V^r \cdot I^r + V^j \cdot I^j$  (55)

$Q = V^j \cdot I^r - V^r \cdot I^j \leq Q^{\max}$  (56)       $Q^{\min} \leq Q = V^j \cdot I^r - V^r \cdot I^j$  (57)

$V^r \cdot V^r + V^j \cdot V^j \leq (V^{\max})^2$  (58)       $(V^{\min})^2 \leq V^r \cdot V^r + V^j \cdot V^j$  (59)


$(i_{nmk})^2 \leq (i_{k}^{\max})^2$  for all k (60)

$[\theta_{nm}^{\min} \leq \arctan(v_n^j/v_n^r) - \arctan(v_m^j/v_m^r) \leq \theta_{nm}^{\max}]$  (61)


$V^r \geq 0$  (62)

July 14, 2015

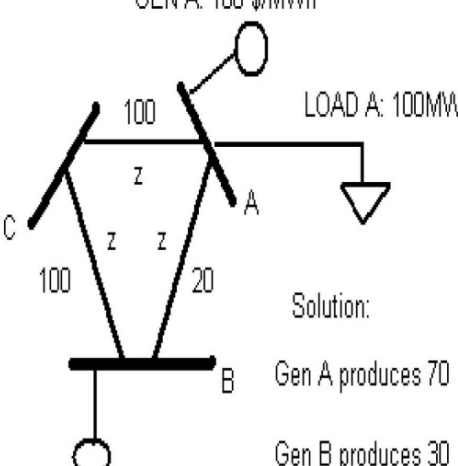
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# Optimal Topology



GEN A: 100 \$/MWh



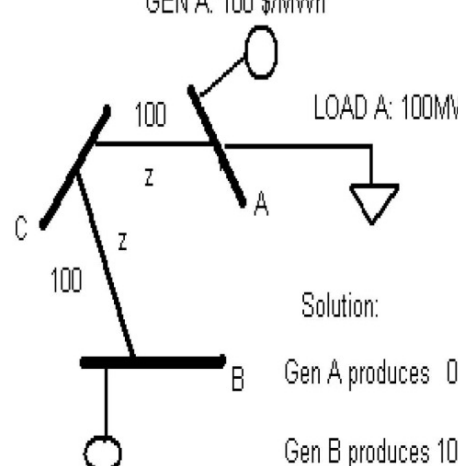
Solution:

Gen A produces 70  
Gen B produces 30

GEN B: 50 \$/MWh

Cost: 8500

GEN A: 100 \$/MWh



Solution:

Gen A produces 0  
Gen B produces 100

GEN B: 50 \$/MWh

Cost: 5000

3. The Mathematics of the Electric Power Grid, presenter Barry Lee (Southern Methodist University)

The electric power grid is modeled by a system of nonlinear differential-algebraic equations (DAEs) defined on a complex graph network. With the incorporation of new technologies and policies, the emergent power grid system is changing drastically when compared to the traditional system of a few decades ago. Correspondingly, the mathematical formulations and properties of the emergent system will undergo major changes. However, the core mathematics and algorithms for modeling and solving the traditional system will persist for the future power grid. Thus, in this talk, we present the mathematics of some of the central challenges of the traditional power grid, and describe how the mathematics must be modified for the emergent grid. These challenges include stability of the grid (transient, small-signal, and voltage), multiple time-scale features of the DAEs, and the assimilation of measured data from the system.

Summary:

Mathematics and statistics arise in many of the core components of power grid modeling. Specifically, mathematics and statistics research in the emergent power grid will be a necessity. Decentralization and incorporation of renewable energies will change the intrinsic structure of the grid and will lead to new mathematical challenges. These challenges include analysis of asynchronous procedures in multi-component settings, uncertainty quantification for highly varying electricity generation and loads, multiple time-scales analysis for AC/DC converters needed to connect local renewable sources, etc. Mathematics areas that will be relevant to the power grid are nonlinear dynamics, network analysis, uncertainty quantification, algorithmic development, and numerical analysis. However, given the immense engineering details that must be considered in understanding the problem, this research must be performed collaboratively with power grid engineers.

## The Mathematics of the Power Grid

**Assume that the network consists of m generators and n buses, with**  
**m generator buses**  
**(n-m) load buses.**

**Then the network can be described as an undirected graph  $\mathcal{G}(n, E)$  where**  
**E is the set of edges.**

**The most general and basic mathematical description of the network is**

$$\begin{cases} \dot{x} = f(x, y, \alpha) \\ 0 = g(x, y, \alpha) \end{cases}$$

← **generators**

**differential & algebraic variables,**  
**and system parameters**

← **Kirchhoff's law (power flow equations):  $I = YV$  and  $P = I^*V$**

**For example, the simplest/lowest-order approximation of the generator dynamics**

**Swing equations:**  $M_i \ddot{\theta}_i = -D_i \dot{\theta}_i + P_{m_i} - P_{e_i} = -D_i \dot{\theta}_i + P_{m_i} - \sum_{j=1}^n a_{ij} \sin(\theta_i - \theta_j)$   $i \in$  generator buses

$\dot{\theta}_i, \theta_i, M_i, D_i, P_{m_i}$  **rotor speed, rotor angle, inertial constant, damping coeff, mechanical power**  
 $a_{ij} = |V_i||V_j| \text{Im}(Y_{ij})$  **voltage and nodal admittance matrix**



## Incorporating dynamics into the voltage stability problem: bifurcation techniques

Voltage stability can be examined considering the full DAE system. Voltage collapse is the emergence of sudden changes in the system arising from smooth parameter variations. Mathematically, this is a bifurcation in a nonlinear dynamical system.

$$\begin{cases} \dot{x} = f(x, y, \alpha) \\ 0 = g(x, y, \alpha) \end{cases}$$

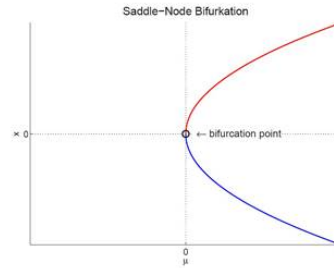
At the equilibrium pt.  $(\bar{x}, \bar{y})$ , we have

$$\begin{cases} 0 = f(\bar{x}, \bar{y}, \alpha) \\ 0 = g(\bar{x}, \bar{y}, \alpha) \end{cases}$$

and a saddle-node bifurcation (SNB) occurs when the Jacobian of

$$\begin{cases} 0 = f(\bar{x}, \bar{y}, \alpha) \\ 0 = g(\bar{x}, \bar{y}, \alpha) \end{cases}$$

is singular.



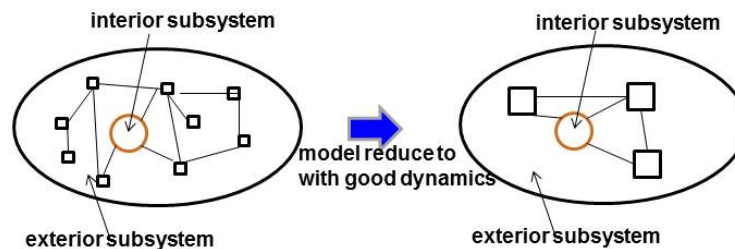
Again, to determine regions of voltage collapse, find the  $\alpha$ 's where the Jacobian is singular, or find the SNB's of the DAE.

## Emergent Complex System and New Challenges

Decentralization of the network due to deregulation and new technologies will change the dynamics of the power grid → more risks and lower resiliency. Decentralization will occur in the macro and microscale.

Mathematically,

1. Leads to multi-component nonlinear system (complex stability)
2. Leads to an asynchronous system with local subsystems processed and analyzed with weak coupling to the rest of the system
3. Multiscale in time- e.g., local DC systems connected to AC systems
4. More sources of uncertainty especially for renewable sources
5. Complex model reduction



## Conclusion

1. **The electric power grid is a complex system**
  - Power grid engineers have done a remarkable job laying out and tackling the math
  - But there are many open problems, both for the traditional power grid and especially for the emergent power grid, which mathematicians can help
    - ✓ Dynamical systems
    - ✓ Network analysis
    - ✓ Uncertainty quantification
    - ✓ Numerical analysis
    - ✓ Algorithmic development
2. **There is a huge language barrier between the different disciplines. The most beneficial interdisciplinary collaboration will require mathematicians to really dive into the power grid problem (math as a tool for solving these problems)**
3. **Many of the fundamental problems have already been examined by for different applications, but at a much smaller scale. Math research must look at the large scale, which implies algorithmic and computational research.**

4. Integrated Smart Grid Analytics for Anomaly Detection, presenter George Michailidis (University of Florida)

The overarching objective of the smart grid, is to integrate two-way communication technologies across power generation, transmission and distribution to deliver electricity efficiently, securely and cost-effectively. However, real-time messaging exposes the entire grid to security threats ranging from attacks that disable information exchange between smart meters and data fusion centers to spurious payload content that would lead to incorrect assessment of actual demand. Such malicious activities can compromise grid stability and efficiency. We discuss a framework for detection of false data injection attacks in smart grids. We present a measurement-based situation awareness framework that combines evidence from sensors at home-area level, and aims to infer anomalies that signify a coordinated, well-orchestrated attack on residential smart meters at increasing spatial scales. By leveraging multi-view sensor readings, we present a Bayesian based monitoring approach that quickly detects power shifts to anomalous regimes. We evaluate the proposed algorithms on real power consumption measurements.

### Summary:

With the cyber-based infrastructure of the modern power grid, anomaly detection will be a relatively new area in power grid research. In this talk, a statistical approach that analyzes the data collected from the hierarchical components of the smart grid allows the construction of a predictive model for the energy consumption was presented. This model can be used to forecast the error of the model with the actual energy consumption and, in turn, be used to detect anomalies in the data for the energy consumption. This is a practical approach for detecting anomalies. Using this statistical

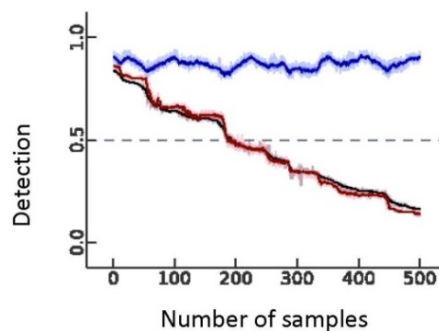
approach, together with mathematical graph-theoretical techniques (reachability and observability) for the power grid equations, can lead to strong anomaly detection in the emergent grid.

## Manipulation of the predictive model by the attacker

- Examine two possibilities for a **targeted attack**
  - Attacker manipulates inputs
  - Attacker manipulates model structure (e.g. by changing the sign of the forecasting error)
- (Very) preliminary results
  - Depending on the intensity of the attack, performance can deteriorate in terms of detection rates from 50% to all the way down to 20%
- Defense strategy under consideration → **randomized prediction/classification**
  - Build a library of predictive models and select one at random to predict over a short time interval
  - Performance depends on the similarity of the model used compared to the one under attack

## Manipulation of the predictive model by the attacker (ctd)

- Performance deterioration by manipulating the model structure



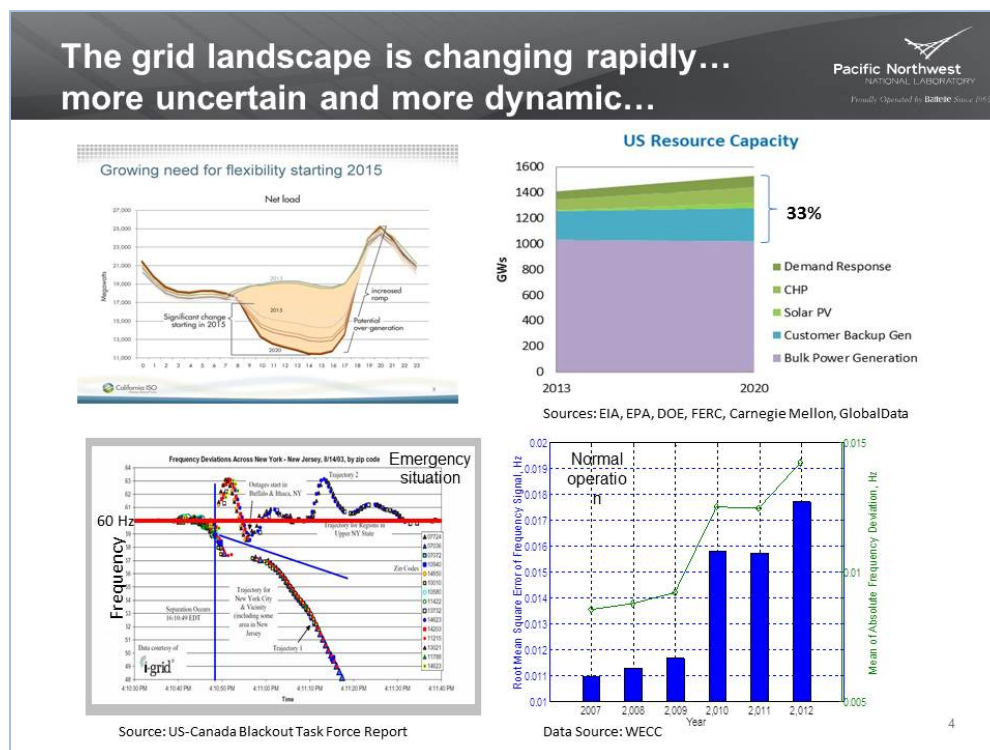
- Preliminary finding: the “dissimilarity” of the candidate predictive models impacts the performance of the randomized strategy

5. Capturing Dynamics and Understanding Stochastics in Emerging Power Grids, presenter Henry Huang (Pacific Northwest National Laboratory)

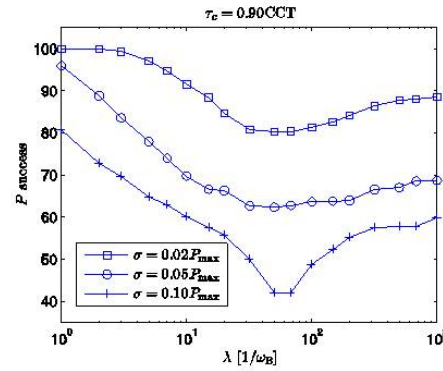
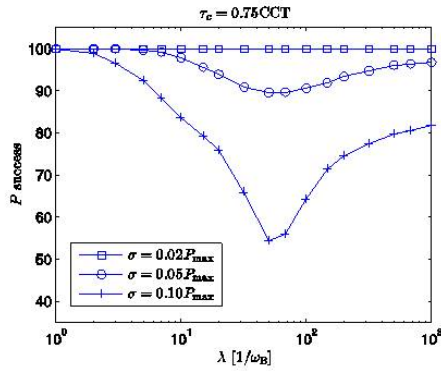
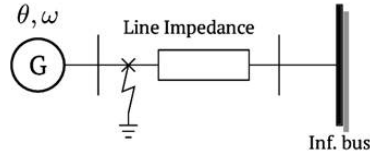
The power grid is evolving with significant penetration of renewable energy generation, smart loads, electric vehicles, and other emerging technologies. Such a new mix of generation and consumption results in emerging stochastic behaviors and dynamics. To capture such dynamics and understand the stochastics, new mathematics is required in the areas of uncertainty quantification and propagation, data assimilation, and stability. This talk will characterize the mathematical attributes of the emerging power grid, identify mathematical challenges, and present some initial mathematical development for new power grid applications.

Summary:

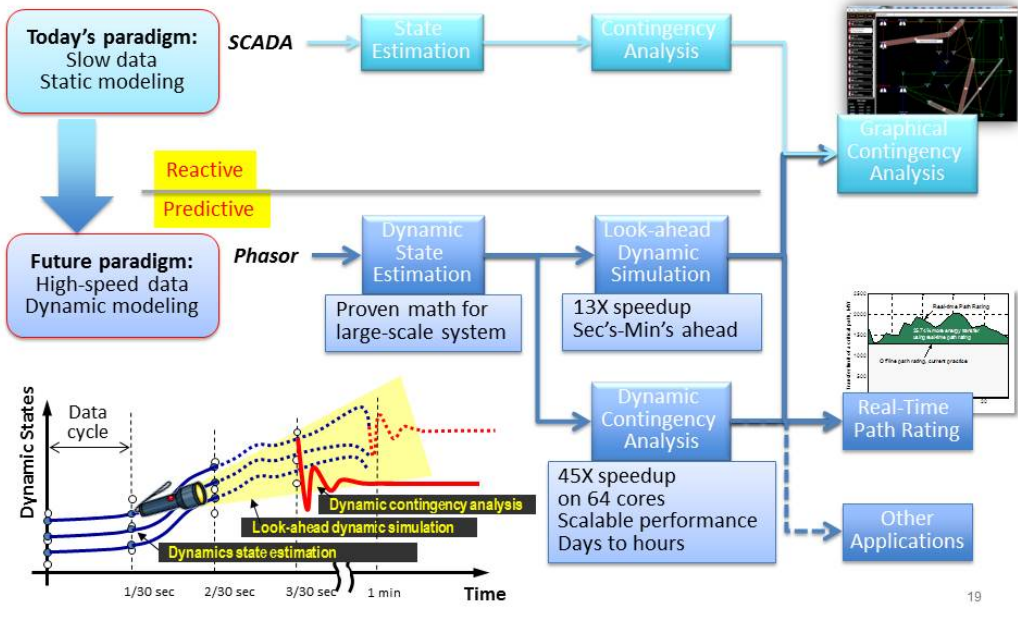
Like in most science and engineering applications, incorporating stochastic aspects into the power grid models will lead to a better understanding of the physics. But this will lead to new mathematical methods and computational challenges. For the power grid, given the large sources of uncertainties and the scale of the network, uncertainty quantification (UQ) of the grid will require new UQ approaches. Preliminary results using a probability density function (PDF) approach on a very small scale problem show that the uncertainty can lead to noticeable changes in the stability of the system. Moreover, these results show that the uncertainties are colored rather than white. For realistic power grids, this PDF approach will not be applicable because of the scale of these grids. New UQ methods are needed. Further, methods must be developed for the assimilation of data with non-Gaussian noise.

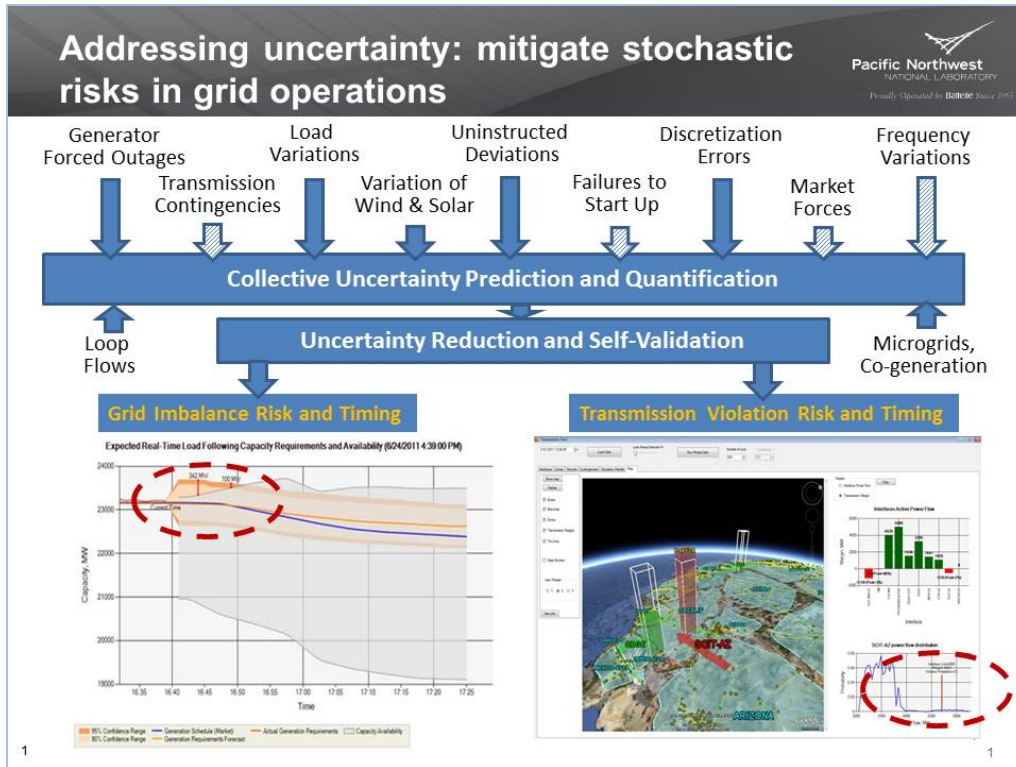


# Uncertainties could push transient stability over the boundary



# Capturing dynamics: enable grid operations from reactive to predictive





6. Distribution Level Synchronous Measurement and Its Applications, presenter Yilu Liu (University of Tennessee Knoxville, Oakridge National Laboratory)

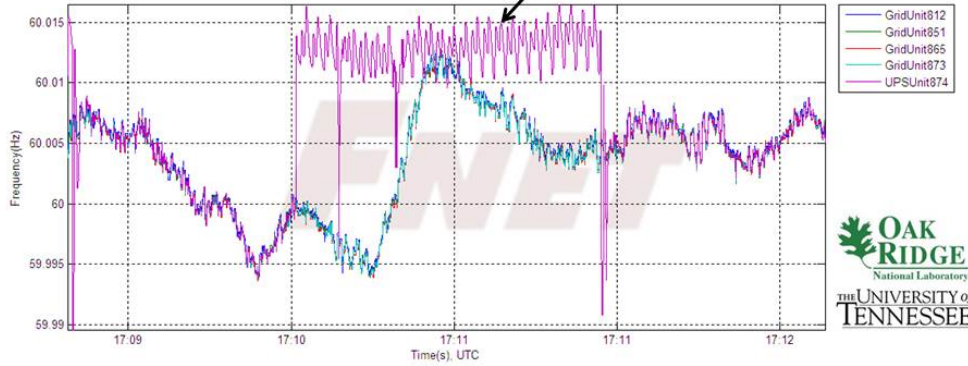
With the development and deployment of new measurement devices, an enormous amount of data is being collected on the dynamics of the power grid. This data has the potential for use in the design of early detection procedures for mitigating effects of catastrophic events. However, to achieve this, big data analytic methods are needed; specifically, big data methods that can mine sets quickly enough for practical implementation of early warning procedures, i.e., data mined at time-scales faster than the propagation of the effects of a disturbance. In this talk, we will present results for our Frequency Disturbance Recorder (FDR) measurement device. The data collected from these devices deployed in different locations in North America show that frequency disturbances propagate at extremely fast rates and over large regions of the power grid interconnect. Fast analysis of the collected data is a major power grid challenge for the mathematics and statistics communities.

Summary:

New measurement devices have the potential to help mitigate the risks and improve the resiliency of the emergent power grid. However, this potential can be realized only if the large quantity of collected data can be efficiently analyzed. Because of the time-scale of the propagation of these disturbances, this big data challenge will require new mathematical and statistical algorithms with real-time capabilities.

# Off Grid Detection for Hospitals and Data Centers

Grid → UPS → Grid Detected → On UPS

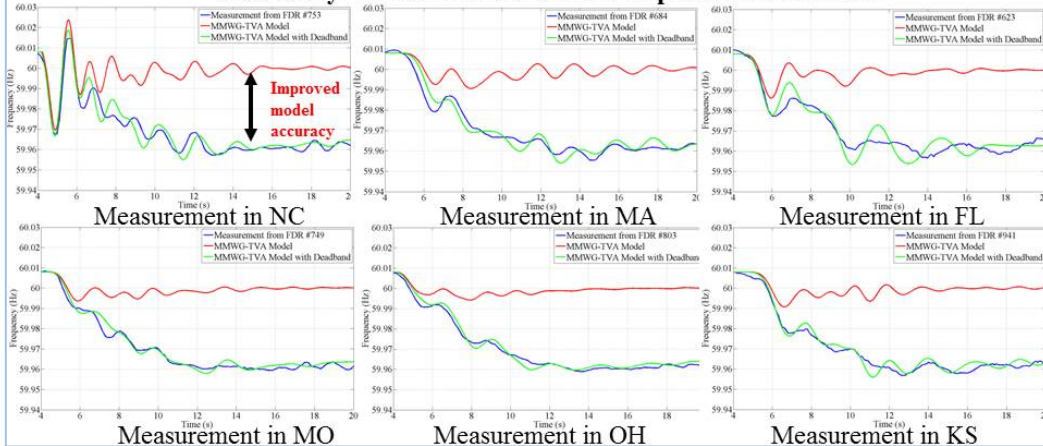


Central alarm system in operation

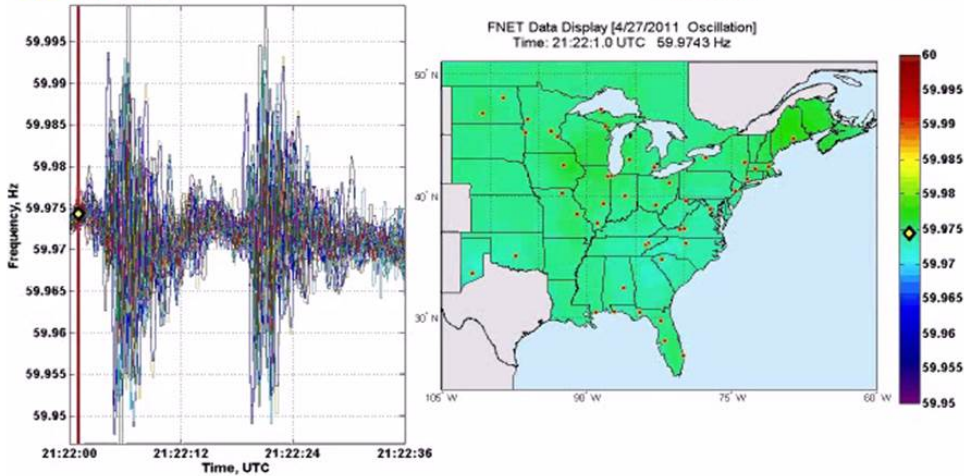
## Synchrophasor-based Dynamic Model Validation on the Eastern Interconnection with Governor Deadband

- Synchrophasor measurement collected by FNET/GridEye is used to calibrate the simulated frequency response.
- Governor deadband is adjusted by measurements to reflect the actual system performance.

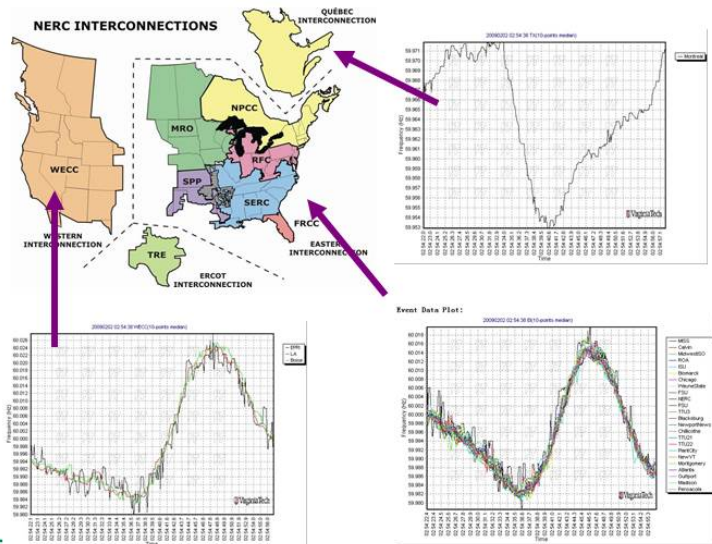
### Case Study: a 1100 MW Generation Trip in North Carolina



# Frequency Disturbance Propagation



# Super Bowl Frequency swings during commercials





7. A Differential Equation Approach for Information Propagation on Networks, presenter Haomin Zhou (Georgia Technical University)

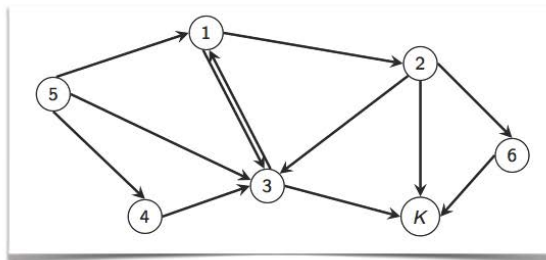
As information, such as viral signals or infectious diseases, spreads on various kinds of real-world networks, we are interested in estimating and predicting their influences which, for example, are usually defined as the expected number of individuals involved in the spread, up to a target time. Methods characterizing continuous-time propagation adaptively model the dynamics of such spreads. However, they tend to encounter significant computational challenges in influence prediction which is a fundamental problem in many applications. In this presentation, we introduce a new analysis framework, based on the discrete Fokker-Planck equations, and associated computational methods to tackle this problem. It gives rise to a class of novel algorithms that work effectively for networks with unknown propagation models or parameters, or even unobserved network structure. Numerical tests are carried out on several synthetic and real-world networks to show the promising performance of the proposed method.

Summary:

This technique for modeling the propagation of information through networks with incomplete knowledge of the dynamics may be used to predict the propagation of disturbances in the power grid network. In particular, this probability formulation of the information diffusion may be used to predict the spread of cascading blackouts. Using the Fokker-Planck formulation will permit fast computation of this spread in large-scale power grid networks.

## Dynamics on Networks

There are **complex random dynamics** on **large networks** (directed graph):



- ❖ **Node:** person, webpage, computer client...
- ❖ **Edge:** contact, hyperlink, connection...
- ❖ **Weight:** strength of relationship between nodes.
- ❖ **Dynamics:** infection, repost, computer virus...
- ❖ Also known as **diffusion networks**.

## Problem, Challenges, and Our Aim

### Problem:

We want to **predict the influence**, defined as the expected number of individuals involved in the propagation:

$$\sigma(t) = \sum_{k=1}^N k p_k(t)$$

$p_k(t)$  is the probability of exact  $k$  infected nodes.

### Challenges:

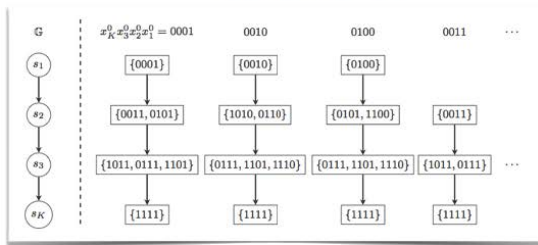
- **Incomplete network** structure: nodes, edges, weights.
- **Unknown dynamics** for propagation mechanism, often impossible to model.
- **Limited observations** (partial data, short time) are available.

### Our Aim:

We want to find **efficient approaches** to extract information hidden in the **big network data** and predict future influence.

## Our Strategy

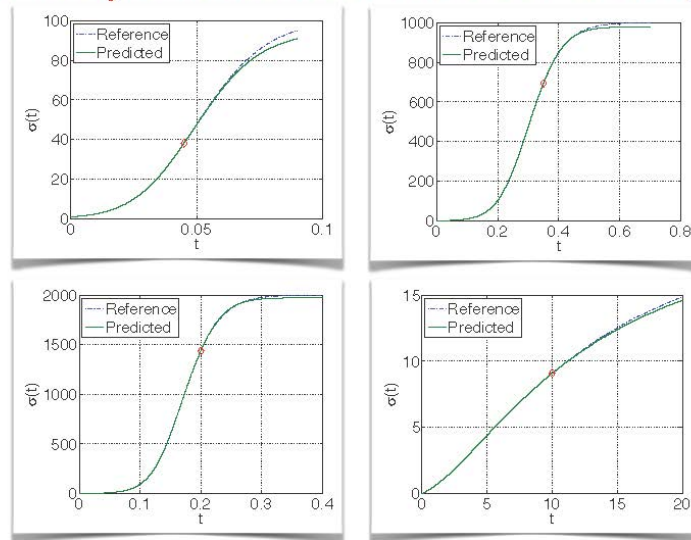
- We work on an aggregate graph:



A linked list (1-D lattice) with node  $k$  aggregating all nodes with exact  $k$  infectious node in the state graph.

- We use Fokker-Planck equation on graph,
  - ✦ Information propagation on network
  - ✦ Stochastic process on state graph
  - ✦ Fokker-Planck equation defined on state graph
- We employ the well developed methods in conservation laws and optimal transport theory to carry out calculations.

## Four synthetic networks for testing



The influence prediction results from our algorithm.

### 3. Conclusion

The electric power grid is a complex mathematical system. In fact, some components of the emergent grid (for example, faster than real-time analysis of enormous amounts of collected data) have yet to be mathematically formulated. Collaboration between power grid engineers and mathematicians/statisticians will be beneficial for the design of low-risk, highly resilient systems. Areas of mathematics and statistics that will be beneficial are dynamical systems, graph network analysis, uncertainty quantification, statistical big-data analysis, statistical anomaly detection, algorithmic development, and numerical analysis. Because of the complexity of the power grid, this collaboration will require researchers to be acquainted with power grid engineering. This will insure not only that the developed methods are indeed applicable to the actual power grid problem, but also will give a correct mathematical perspective of the many sub-problems of the full power grid problem. Such perspective will help develop more efficient algorithms that exploit the intrinsic structures of the emergent grid.

The workshop accomplished its goal of exposing the mathematics and statistics community to the needs of the future power grid. This workshop represented the input and collaboration of government agencies, academia, and research institutions. This workshop helped to identify barriers that have prevented statisticians and mathematicians from effectively collaborating with power engineers. Language barriers must be overcome, and the complexity of the grid must be effectively communicated, to enable the collaborations required to insure the safe and reliable operation of the future US electric grid.