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Price Regulation in Property-Liability Insurance: A Contingent-Claims Approach

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ABSTRACT

A discrete-time option-pricing model is used to derive the "fair" rate of return for the property-liability insurance firm. The rationale for the use of this model is that the financial claims of shareholders, policyholders, and tax authorities can be modeled as European options written on the income generated by the insurer's asset portfolio. This portfolio consists mostly of traded financial assets and is therefore relatively easy to value. By setting the value of the shareholders' option equal to the initial surplus, an implicit solution for the fair insurance price may be derived. Unlike previous insurance regulatory models, this approach addresses the ruin probability of the insurer, as well as nonlinear tax effects.

IN RECENT YEARS, THE "fair"-rate-of-return-on-equity criterion has been used in the regulation of property-liability insurance premiums. As with utility regulation, the "fair" rate of return usually is interpreted as that which would prevail under competitive conditions, and in some cases the Sharpe [28]-Lintner [18]-Mossin [21] capital asset pricing model (CAPM) has been used to derive the equilibrium relationship (cf. Hill [15], Fairley [9]). But discontent with this model has led to questioning of its use. In addition to doubt over testability of the CAPM (cf. Roll [23]), this model leaves unexplained some significant pricing anomalies such as the earnings yield and size effects (cf. Reinganum [22]). Applications of the CAPM to insurance regulation have encountered three major problems. First, there are peculiar difficulties in estimating underwriting betas either through the use of market or accounting data (cf. Fairley [9], Hill [15], Cummins and Harrington [5]). Second, the models do not address the effect of insolvency on the return to shareholders despite the attention given to this prospect by regulators and actuaries. Third, the applications either ignore corporate taxation or assume it is proportional over the entire range of corporate income. Tax shields, which are especially important to insurance firms, are known to result in significant nonlinearities in the tax schedule.

In connection with utility regulation, Bower, Bower, and Logue [3] have recently noted the irony that, at the time the CAPM is gaining acceptance by regulators, its preeminent role in the explanation of security returns is being challenged by the arbitrage pricing theory (APT). This paradox obviously applies

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to insurance regulation as well. Does, then, the APT offer a more attractive alternative for insurance regulation? The answer is that it is too early to say. An analytic solution for the fair rate of return on underwriting has been derived by Kraus and Ross [17] in an APT framework, and attempts at the empirical application of the APT to insurance regulation have already been made by Urritia [30]. But, as in the case of the CAPM, doubts have also been raised in the finance literature over the testability of the APT (cf. Shanken [27]; Dhrymes, Friend, and Gultekin [7]; Dybvig and Ross [8]). Furthermore, it is not yet clear that the APT explains the well-known pricing anomalies left unanswered by the CAPM. In applying the APT to insurance regulation, the estimation problems associated with calculating underwriting betas (or factor loadings) remain, due to the inadequacy of market data and the unknown sampling errors that are sure to arise from using accounting data. Moreover, Kraus and Ross's insightful analysis does not address the possibilities of insolvency or tax shield redundancy, nor is it clear how such effects might be encompassed by their model.

We offer an alternative approach to insurance price regulation. The liabilities of the insurer to policyholders, shareholders, and the tax authorities are viewed as contingent claims written on the income generated by the insurer's asset portfolio. If the market value of the asset portfolio is observable, implicit values for these claims may be derived by means of risk-neutral valuation relationships (cf. Brennan [4], Rubinstein [25], Stapleton and Subrahmanyam [29]). Thus, option-pricing techniques may be used to derive the competitive price of the insurance contract as well as the "fair" rates of return on underwriting and on shareholders' equity. The advantages of this approach are that it addresses the possibility of insolvency and the nonlinear tax effects that may arise from the redundancy of tax shields. Although some estimation of risk premiums is required, the menu of possible option-pricing models that can be used permits some choice in the selection of an underlying process for pricing capital assets. In fact, we will limit our presentation to discrete-time models, noting that the advantages of such an approach to valuation are particularly useful in valuing discrete insurance contracts and tax liabilities.

The remainder of the paper is organized in the following manner: In Section I, we provide a generalized single-period valuation model of the claims held by the property-liability insurer's policyholders and shareholders and the tax authorities. In Section II, we offer two special cases of the first section's more general formulation that require restrictions on investor preferences and on the probability distributions that underlie insurance company investment returns and claims costs. In Section III, we present numerical simulations of our model. Section IV concludes.

I. Basic Valuation Relationships for a Property-Liability Insurer

Consider a single-period model of the insurance firm in which investors contribute paid-in equity of S_0 and policyholders pay premiums of P_0. For convenience, premiums will be defined as net of production and marketing expenses. Therefore, the opening cash flow is given as

$$Y_0 = S_0 + P_0. \tag{1}$$

The claims of policyholders and the government are discharged at the end of the period, leaving a residual claim for shareholders. Allowing for investment income at a rate $\tilde{r}_i$, we obtain an expression for terminal cash flow $\tilde{Y}_1$:

$$\tilde{Y}_1 = S_0 + P_0 + (S_0 + kP_0)\tilde{r}_i. \tag{2}$$

The term k is the funds-generating coefficient. This represents an adjustment to compensate for the difference between the period of our model (say one year) and the average delay between receipt of premiums and payment of policyholder claims.[1]

The value $\tilde{Y}_1$ is allocated to various claimholders in a set of payoffs having the characteristics of call options.[2] The payoffs to policyholders, $\tilde{H}_1$, and government, $\tilde{T}_1$, are given in the next two equations:[3]

$$\tilde{H}_1 = \max(\min[\tilde{L}, \tilde{Y}_1], 0) \tag{3}$$

$$\tilde{T}_1 = \max[\tau(\theta(\tilde{Y}_1 - Y_0) + P_0 - \tilde{L}), 0], \tag{4}$$

where the variable $\tilde{L}$ represents the insurer's end-of-period claims costs and τ is the corporate tax rate. The effective tax rate on the insurer's investment income is considerably less than τ in view of the insurer's holding of tax-exempt securities, the somewhat lower capital-gains rate, and the 85-percent shield of dividend income for corporations. The effective tax rate on investment income, therefore, is denoted $\theta\tau$.[4] Since these claims either directly or indirectly involve the valuation of call options, the appropriate expressions for the values of these

[1] Depending upon the type of risk being insured, the time lag between the receipt of the premium and payment of the claim can vary considerably. For example, most casualty insurance lines are characterized by claim delays of less than one year, whereas most liability lines have claim delays of more than one year. Consequently, for every dollar of premiums written, lines of insurance with longer claim delays generate more investable funds than insurance lines with shorter claim delays. Therefore, the "funds-generating coefficient" can be interpreted as the average amount of investable funds per dollar of annual premiums. This type of adjustment is also used in the papers by Hill [15], Fairley [9], Biger and Kahane [1], and Hill and Modigliani [16].

[2] We view shareholders as holding a long position in a call option on the pretax terminal value of the insurer's asset portfolio and a short position in a call option on the taxable income derived from that portfolio. Consequently, policyholders hold a long position in the pretax terminal value of the insurer's asset portfolio and a short position in the call option written on that portfolio, while the government holds a long position in the call option written against the insurer's taxable income. Similar characterizations have been used for modeling nonfinancial firms (e.g., cf. Black and Scholes [2], Galai and Masulis [13], Galai [12], Majd and Myers [19]).

[3] Since our purpose in this section of the paper is to provide as general a formulation of the problem as possible, we initially place explicit lower bounds of zero on both $\tilde{H}_1$ and $\tilde{T}_1$, so as to allow for limited liability. In other words, should a poorly endowed state of nature be revealed at the end of the contracting period, the worst possible outcome for policyholders is one in which they receive no settlement on their claims. Similarly, our restriction on the cash flow associated with the government's tax claim ensures that it does not provide tax rebates to unprofitable firms; viz., at worst, the government does not receive any tax revenues. Subsequently, when we build a model in which we assume that both $\tilde{r}_i$ and $\tilde{L}$ are normally distributed random variables, this lower bound will still need to be observed. However, in the lognormal formulation, this bound will obviously be redundant.

[4] θ is a factor of proportionality defined over the interval [0, 1]. This parameter is functionally related to the composition of the insurer's investment portfolio. For example, if the investment portfolio is comprised of strictly tax-exempt securities, then $\theta = 0$. Conversely, if only fully taxable claims such as corporate bonds and U.S. Treasury securities are chosen, then $\theta = 1$.

claims are given as follows:

$$H_0 = V(\tilde{Y}_1) - C(\tilde{Y}_1; \tilde{L}) \tag{5}$$

$$T_0 = \tau C[\theta(\tilde{Y}_1 - Y_0) + P_0; \tilde{L}], \tag{6}$$

where

$V(\cdot)$ = the valuation operator;
$R_f = 1 + r_f$, where r_f is the riskless interest rate;[5]
$C[A; B]$ = the current market value of a European call option written on an asset with a terminal value of A and exercise price of B.

The market value of the residual claim of the shareholders, V_e, is simply the difference between the market value of the asset portfolio, $V(\tilde{Y}_1)$, and the sum of the values of the policyholders' and government's claims, viz.,

$$\begin{aligned} V_e &= V(\tilde{Y}_1) - [H_0 + T_0] \\ &= C[\tilde{Y}_1; \tilde{L}] - \tau C[\theta(\tilde{Y}_1 - Y_0) + P_0; \tilde{L}] \\ &= C_1 - \tau C_2. \end{aligned} \tag{7}$$

The regulatory problem may now be couched in straightforward terms. Insurance prices must be set such that a "fair" return is delivered to shareholders. This will be achieved if the current market value of the equity claim V_e is equal to the initial equity investment S_0. Noting that $\tilde{Y}_1$ and Y_0 are functions of P_0, we can state the fair rate of return as that implied by a value of P_0^* that satisfies the following equation:

$$\begin{aligned} V_e &= C[\tilde{Y}_1(P_0^*); \tilde{L}] - \tau C[\theta(\tilde{Y}_1(P_0^*) - Y_0(P_0^*)) + P_0^*; \tilde{L}] \\ &= C_1^* - \tau C_2^* \\ &= S_0. \end{aligned} \tag{8}$$

The solution of equation (8) for P_0^* requires the use of an appropriate option-pricing framework, which we present next. Since the payoffs on these call options depend upon the outcomes of the two random variables $\tilde{r}_i$ and $\tilde{L}$, our analysis requires the valuation of options with stochastic exercise prices.

II. Implicit Solutions for the Fair Rate of Return

In this section of the paper, we present two special cases of the previous section's more general formulation. Specifically, we derive pricing relationships based

[5] Since we only consider corporate income taxation, the riskless rate of interest is simply the before-tax rate of interest on riskless bonds (e.g., T-bills). However, in the presence of personal and corporate taxes, it is not entirely clear whether the riskless rate of interest is the before-tax rate of interest on riskless bonds or the certainty-equivalent municipal bond rate. If investors are able to "launder" all of their personal taxes *a la* Miller and Scholes [20], then r_f will continue to be defined as the before-tax rate of interest on riskless bonds. However, if investors are not able to launder taxes on investment income, then the certainty-equivalent municipal bond rate is the appropriate rate. For a lucid discussion of these points, see Hamada and Scholes [14].

upon the discrete-time, risk-neutral-valuation framework pioneered by Rubinstein [25]. In both cases, we make use of Rubinstein's [24] representative investor device; viz., we assume that the conditions for aggregation are met so that securities are priced *as if* all investors have the same characteristics as a representative investor. In addition to the aggregation assumption, we shall assume for the first model that a) the wealth of the representative investor, the rate of return on the insurer's asset portfolio, and the aggregate value of the insurer's claims costs are jointly normally distributed and b) the utility function of the representative investor exhibits constant absolute risk aversion (CARA). The derivation of the second model similarly requires the use of the representative investor device but replaces the distributional and preference assumptions with joint lognormality and constant relative risk aversion (CRRA). Brennan [4] has shown that CARA (CRRA) is a necessary and sufficient condition for pricing *bivariate* contingent claims in discrete time when the price of the underlying asset is normally (lognormally) distributed, while Stapleton and Subrahmanyam [29] have shown that the Brennan results can also be extended to the pricing of *multivariate* contingent claims such as are being considered here.

Although we could have chosen alternative sets of distributional assumptions, we find it convenient to work with joint-normal and lognormal variates for two reasons. The primary advantage of assuming joint normality is that we are able to derive models of insurance pricing that are directly comparable to the existing set of CAPM's and APT-based models. Thus, when we perform our simulation experiments, we are able to determine the relative importance of default risk and tax-shield redundancy in the determination of "fair" insurance prices. As we stated earlier, such effects typically have not been addressed in the CAPM's and APT-based models. Unfortunately, the assumption that investors' utility functions exhibit constant absolute risk aversion is rather restrictive.[6] Furthermore, actual claims-cost distributions are probably better described by skewed probability distributions such as the lognormal than by symmetric probability distributions such as the normal. In view of these considerations, we also offer a model of insurance pricing based upon joint lognormality and constant relative risk aversion.

A. Case 1: Joint Normality and Constant Absolute Risk Aversion

Before we value the two call options described in the previous section, we will first determine the level of premium income P_0 and rate of return on underwriting $E(\tilde{r}_u)$ that would obtain in a competitive, default-free setting in which all tax shields are fully utilized. Under our joint normal and CARA assumptions, we can express the value of equity as the discounted value of the certainty-equivalent

[6] In a study of Federal Reserve data on consumer financial characteristics, Friend and Blume [10] discovered that the percentage of wealth typically invested in risky assets remains virtually unchanged over very different wealth levels. This finding implies that consumers' utility functions exhibit constant relative risk aversion and, consequently, decreasing absolute risk aversion. The same authors [11] also used IRS data to replicate portfolios with power utility functions. As in the study using Fed data, the empirics in the IRS study suggest that the typical investor's utility function exhibits decreasing absolute risk aversion and constant relative risk aversion.

terminal cash flow, viz.,

$$V_e = R_f^{-1} \int_{-\infty}^{\infty} \tilde{Y}_e \hat{f}(\tilde{Y}_e)\, d\tilde{Y}_e$$

$$= R_f^{-1} \hat{E}(\tilde{Y}_e), \tag{9}$$

where

$\tilde{Y}_e$ = random cash flow accruing to shareholders at the end of the period;
$\hat{f}(\tilde{Y}_e)$ = "risk-neutral" normal density function;[7]
$\hat{E}(\tilde{Y}_e)$ = the certainty-equivalent expectation of $\tilde{Y}_e$
$= E(\tilde{Y}_e) - \lambda \operatorname{cov}(\tilde{Y}_e, \tilde{r}_m)$;
λ = the market price of risk
$= [E(\tilde{r}_m) - r_f]/\sigma_m^2$;
$\operatorname{cov}(\cdot)$ = the covariance operator.

The certainty-equivalent expectation of terminal cash flow accruing to shareholders, $\hat{E}(\tilde{Y}_e)$, is given by equation (10):

$$\hat{E}(\tilde{Y}_e) = S_0 + (1 - \theta\tau)\hat{E}(\tilde{r}_i)(S_0 + kP_0) + (1 - \tau)(P_0 - \hat{E}(\tilde{L})), \tag{10}$$

where

$\hat{E}(\tilde{r}_i)$ = the certainty-equivalent expectation of rate of return on the insurer's investment portfolio
$= E(\tilde{r}_i) - \lambda \operatorname{cov}(\tilde{r}_i, \tilde{r}_m) = r_f$;
$\hat{E}(\tilde{L})$ = certainty-equivalent expectation of total claims costs
$= E(\tilde{L}) - \lambda \operatorname{cov}(\tilde{L}, \tilde{r}_m)$.[8]

By substituting the right-hand side of equation (10) into equation (9), setting S_0 equal to V_e, and simplifying, we derive the following analytic expressions for

[7] As shown by Brennan [4] and Stapleton and Subrahmanyam [29], a "risk-neutral" density function is a density function with the location parameter chosen so that the mean of the distribution is its certainty equivalent. In the case of a multivariate risk-neutral density function, the same result holds for the location parameters of the marginal distributions.

[8] In view of the problems associated with accurately estimating $\operatorname{cov}(\tilde{L}, \tilde{r}_m)$, an alternative expression for $\hat{E}(\tilde{L})$ can be derived by assuming that the relationship between $\tilde{L}$ and $\tilde{r}_m$ can be adequately accounted for by the common relationship $\tilde{L}$ has with $\tilde{r}_m$ via its relationship to $\tilde{r}_i$, viz.,

$$\operatorname{cov}(\tilde{L}, \tilde{r}_m) = \frac{\operatorname{cov}(\tilde{L}, \tilde{r}_i)\sigma_m^2}{\operatorname{cov}(\tilde{r}_i, \tilde{r}_m)}$$

$$= \frac{1}{\ } \operatorname{cov}(\tilde{L}, \tilde{r}_i)$$

premium income and the rate of return on underwriting:

$$P_0 = \frac{E(\tilde{L})}{(1 - E(\tilde{r}_u))}, \tag{11}$$

where

$$\begin{aligned} E(\tilde{r}_u) &= [P_0 - E(\tilde{L})]/P_0 \\ &= -\frac{(1 - \theta\tau)}{(1 - \tau)} kr_f + (V_e/P_0)\frac{\theta\tau}{(1 - \tau)} r_f + \lambda \operatorname{cov}(\tilde{r}_u, \tilde{r}_m). \end{aligned} \tag{11a}$$

Hill and Modigliani derive a comparable expression for $E(\tilde{r}_u)$ using the Sharpe-Lintner-Mossin CAPM, and a similar relationship is derived by Fairley.

Next, we value the call options described in equation (7). The value of the first call option, C_1, may be written as the discounted certainty-equivalent expectation of the terminal before-tax value of equity, viz.,

$$\begin{aligned} C_1 &= C[\tilde{Y}_1; \tilde{L}] \\ &= R_f^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \max[(\tilde{Y}_1 - \tilde{L}), 0] \hat{f}(\tilde{Y}_1, \tilde{L})\, d\tilde{Y}_1\, d\tilde{L}, \end{aligned} \tag{12}$$

where $\hat{f}(\tilde{Y}_1, \tilde{L})$ is the bivariate risk-neutral density function governing the realization of the normal variates $\tilde{Y}_1$ and $\tilde{L}$. Examining equation (12), it is obvious that, if the terminal before-tax value of equity is positive, shareholders will own a valuable claim. However, if equity assumes a negative value, shareholders will exercise their "limited-liability option" by declaring bankruptcy.

Next, we simplify equation (12) by defining a normal variate $\tilde{X} = \tilde{Y}_1 - \tilde{L}$, with certainty-equivalent expectation $\hat{E}(\tilde{X}) = \hat{E}(\tilde{Y}_1) - \hat{E}(\tilde{L}) = S_0 + (S_0 + kP_0)r_f + P_0 - \hat{E}(\tilde{L})$, and variance $\sigma_{\tilde{x}}^2 = (S_0 + kP_0)^2\sigma_{\tilde{i}}^2 + \sigma_{\tilde{L}}^2 - 2(S_0 + kP_0)\operatorname{cov}(\tilde{L}, \tilde{r}_i)$. This transformation allows us to rewrite our option value as the solution to

$$C_1 = R_f^{-1} \int_0^{\infty} \tilde{X}\hat{f}(\tilde{X})\, d\tilde{X}. \tag{13}$$

Since $\tilde{X}$ is normally distributed, equation (13) may be rewritten in terms of the standard normal variate $\tilde{z} = (\tilde{X} - \hat{E}(\tilde{X}))/\sigma_x$; hence,

$$C_1 = R_f^{-1}(2\pi)^{-1/2} \int_{-\hat{E}(\tilde{X})/\sigma_x}^{\infty} [\hat{E}(\tilde{X}) + \sigma_x\tilde{z}]e^{-\tilde{z}^2/2}\, d\tilde{z}. \tag{14}$$

Using the properties of the truncated normal distribution and of the standard normal variate, together with the expressions for $\hat{E}(\tilde{X})$ and σ_x, the value of the call can be written in the following form:

$$C_1 = R_f^{-1}(\hat{E}(\tilde{X})N[\hat{E}(\tilde{X})/\sigma_x] + \sigma_x n[\hat{E}(\tilde{X})/\sigma_x]), \tag{15}$$

where

$N[\hat{E}(\tilde{X})/\sigma_x]$ = the standard normal distribution evaluated at $\hat{E}(\tilde{X})/\sigma_x$;[9]
$n[\hat{E}(\tilde{X})/\sigma_x]$ = the standard normal density evaluated at $\hat{E}(\tilde{X})/\sigma_x$.

The value of the second call option, C_2, may be written as the discounted certainty-equivalent expectation of the insurer's terminal taxable income, viz.,

$$C_2 = C[\theta(\tilde{Y}_1 - Y_0) + P_0; \tilde{L}]$$
$$= R_f^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \max[\theta(\tilde{Y}_1 - Y_0) + P_0 - \tilde{L}, 0] \hat{f}(\tilde{Y}_1, \tilde{L})\, d\tilde{Y}_1\, d\tilde{L}. \quad (16)$$

Examining equation (16), it is obvious that, if the terminal value of taxable income is positive, the government will own a valuable claim. However, if taxable income assumes a negative value, shareholders will exercise their "tax-exemption option." Thus, our model allows certain states of nature to arise in which shareholders' claims are less valuable due to the redundancy of tax shields related to the realization of investment losses, underwriting losses, or both.

Next, we simplify equation (16) by defining a normal variate $\tilde{W} = \theta(\tilde{Y}_1 - Y_0) + P_0 - \tilde{L}$, with certainty-equivalent expectation $\hat{E}(\tilde{W}) = \theta(S_0 + kP_0)r_f + P_0 - \hat{E}(\tilde{L})$, and variance $\sigma_w^2 = (S_0 + kP_0)^2\theta^2\sigma_i^2 + \sigma_L^2 - 2(S_0 + kP_0)\theta \operatorname{cov}(\tilde{L}, \tilde{r}_i)$. This transformation allows us to rewrite our option value as the solution to

$$C_2 = R_f^{-1} \int_0^{\infty} \tilde{W}\hat{f}(\tilde{W})\, d\tilde{W}. \quad (17)$$

Using identical analysis to that shown above, the value of the second call option is derived as

$$C_2 = R_f^{-1}(\hat{E}(\tilde{W})N[\hat{E}(\tilde{W})/\sigma_w] + \sigma_w n[\hat{E}(\tilde{W})/\sigma_w]), \quad (18)$$

where

$N[\hat{E}(\tilde{W})/\sigma_w]$ = the standard normal distribution evaluated at $\hat{E}(\tilde{W})/\sigma_w$;[10]
$n[\hat{E}(\tilde{W})/\sigma_w]$ = the standard normal density evaluated at $\hat{E}(\tilde{W})/\sigma_w$.

Substituting the right-hand sides of equations (15) and (18) into equation (7), we obtain an analytic expression for the market value of equity:

$$V_e = R_f^{-1}(\hat{E}(\tilde{X})N[\hat{E}(\tilde{X})/\sigma_x] - \tau\hat{E}(\tilde{W})N[\hat{E}(\tilde{W})/\sigma_w] + \sigma_x n[\hat{E}(\tilde{X})/\sigma_x] - \tau\sigma_w n[\hat{E}(\tilde{W})/\sigma_w]). \quad (19)$$

[9] The term $N[\hat{E}(\tilde{X})/\sigma_x]$ may be interpreted as the pretax certainty-equivalent terminal value of one dollar invested in the firm, provided the firm remains solvent. Because $N[\hat{E}(\tilde{X})/\sigma_x]$ is in effect a "risk-neutral" cumulative distribution function, it understates the solvency probability by the amount of risk-bearing costs borne per dollar of income generated in solvent states of nature, viz., by the difference $N[E(\tilde{X})/\sigma_x] - N[\hat{E}(\tilde{X})/\sigma_x]$.

[10] The term $N[\hat{E}(\tilde{W})/\sigma_w]$ may be interpreted as the certainty-equivalent terminal value of one dollar of taxable income, provided that tax shields are fully utilized. Because $N[\hat{E}(\tilde{W})/\sigma_w]$ is in effect a "risk-neutral" cumulative distribution function, it understates the probability of taxation by the amount of risk-bearing costs borne per dollar of taxable income generated in taxable states of nature, viz., by the difference $N[E(\tilde{W})/\sigma_w] - N[\hat{E}(\tilde{W})/\sigma_w]$.

An implicit solution for the value of P_0^* that satisfies the fair-return criterion implied by equation (8) may be obtained by employing an appropriate algorithm.

B. Case 2: Joint Lognormality and Constant Relative Risk Aversion

Next, we consider the valuation of the options described in equation (7) under the joint lognormal and CRRA assumptions. As in the joint-normal-with-CARA case, we will first determine the level of premium income and rate of return on underwriting that would obtain in a competitive, default-free setting in which all tax shields are fully utilized. As shown by Rubinstein [25], the equilibrium price of the jth risky security that trades in a discrete-time lognormal securities market, V_0^j, must obey the following pricing relationship:

$$V_0^j = R_f^{-1}\hat{E}(\tilde{Y}_1^j) = R_f^{-1}E(\tilde{Y}_1^j)\exp\{-\psi\ \mathrm{cov}[\ln \tilde{R}_j,\ \ln \tilde{R}_m]\}, \tag{20}$$

where

$\tilde{Y}_1^j$ = end-of-period cash flow paid to the holder of security j;
ψ = the representative investor's relative risk aversion parameter $= \dfrac{E(\ln \tilde{R}_m) - \ln R_f}{\mathrm{var}(\ln \tilde{R}_m)} + \dfrac{1}{2}$;
$\tilde{R}_j = 1 + \tilde{r}_j$;
$\tilde{R}_m = 1 + \tilde{r}_m$.

Equation (20) allows us to specify the relationship between the certainty-equivalent expectation of the insurance firm's claims costs, $\hat{E}(\tilde{L})$, and the expected value of claims costs, $E(\tilde{L})$, as follows:

$$\hat{E}(\tilde{L}) = E(\tilde{L})\exp\{-\psi\ \mathrm{cov}[\ln \tilde{L},\ \ln \tilde{R}_m]\}. \tag{21}$$

Next, we incorporate equation (21) into our derivation of an analytic expression for premium income and the competitive rate of return on underwriting in a default-free setting in which all tax shields are fully utilized. By substituting the right-hand side of equation (10) into equation (9), setting S_0 equal to V_e, and simplifying, we derive the following expressions:

$$P_0 = \frac{E(\tilde{L})}{(1 - E(\tilde{r}_u))}, \tag{22}$$

where

$$E(\tilde{r}_u) = 1 - \left(1 + \frac{(1-\theta\tau)}{(1-\tau)}kr_f - (V_e/P_0)\frac{\theta\tau}{(1-\tau)}r_f\right)\exp\{\psi\ \mathrm{cov}[\ln \tilde{L}, \ln \tilde{R}_m]\}. \tag{22a}$$

Next, we value the call options described in equation (7). The value of the first call option, C_1, may be written as

$$C_1 = C[\tilde{Y}_1;\ \tilde{L}] = R_f^{-1}\int_0^\infty\int_0^\infty \max[(\tilde{Y}_1 - \tilde{L}),\ 0]\hat{g}(\tilde{Y}_1, \tilde{L})\ d\tilde{Y}_1\ d\tilde{L}, \tag{23}$$

where $\hat{g}(\tilde{Y}_1, \tilde{L})$ is defined as the bivariate risk-neutral density function governing the realization of the *lognormal* variates $\tilde{Y}_1$ and $\tilde{L}$. By defining a new random variable $\tilde{U} = \tilde{Y}_1 - \tilde{L} + P_0$, we can rewrite equation (23) as

$$C_1 = R_f^{-1} \int_{P_0}^{\infty} (\tilde{U} - P_0)\hat{g}(\tilde{U})\, d\tilde{U}, \tag{24}$$

where $\tilde{U}$ is a lognormal variate with risk-neutral density $\hat{g}(\tilde{U})$.[11]

Changing the random variable $\tilde{U}$ to the standardized normal variate $\tilde{z}$ and simplifying yield

$$C_1 = V_0^U \int_{-d_2}^{\infty} (\sqrt{2\pi})^{-1}\exp\left(-\sigma_u^2 + \tilde{z}\sigma_u - \frac{1}{2}\tilde{z}^2\right) d\tilde{z} - R_f^{-1}P_0 \int_{-d_2}^{\infty} (\sqrt{2\pi})^{-1}\exp\left(-\frac{1}{2}\tilde{z}^2\right) d\tilde{z}, \tag{25}$$

where

V_0^U = the contemporaneous value of the claim $\tilde{U}$
$= V_0^Y - V_0^L + R_f^{-1}P_0 = S_0 + R_f^{-1}P_0(2 + kr_f) - V_0^L$;
$V_0^L = R_f^{-1}\hat{E}(L)$
$= R_f^{-1}E(\tilde{L})\exp\{-\psi \operatorname{cov}[\ln \tilde{L}, \ln \tilde{R}_m]\}$;
$d_1^U = \dfrac{\ln(V_0^U/P_0) + \ln R_f + \sigma_u^2/2}{\sigma_u}$;
$d_2^U = d_1^U - \sigma_u$;
σ_u = the standard deviation of the natural logarithm of $\tilde{U}$
$= [\sigma_y^2 + \sigma_1^2 - 2 \operatorname{cov}(\ln \tilde{Y}, \ln \tilde{L})]^{1/2}$;
σ_y = the standard deviation of the natural logarithm of $\tilde{Y}_1$;
σ_1 = the standard deviation of the natural logarithm of $\tilde{L}$.

Rewriting equation (25) in terms of cumulative standard normal distribution functions yields an expression that is analogous to the familiar Black-Scholes call-option formula:

$$C_1 = V_0^U N(d_1^U) - R_f^{-1}P_0 N(d_2^U), \tag{26}$$

where $N(d_i^U)$ is the standard normal distribution function evaluated at d_i^U.[12]

Next, we consider the valuation of the tax claim, C_2. The value of this option may be written as the discounted certainty-equivalent expectation of the insurer's

[11] The solution procedure used here follows that of Stapleton and Subrahmanyam [29, pp. 223–
241. The caveat that these authors suggest in footnote 19, p. 223, of their article applies here as well;

Table I

Model Parameterization: The Base Case

Initial Equity (S_O)	100.00
Funds-Generating Coefficient (k)	1.00
Standard Deviation of Investment Returns (σ_i)	0.20
Expected Claims Costs ($E(\tilde{L})$)	200.00
Standard Deviation of Claims Costs (σ_L)	50.00
Correlation Between Investment Returns/Claims Costs (ρ_{iL})	0.00
Riskless Rate of Interest (r_f)	0.07
Statutory Tax Rate (τ)	0.46
Tax-Adjustment Parameter (θ)	0.50
Beta of Investment Portfolio (β_i)	0.338
Expected Return on the Market ($E(\tilde{r}_m)$)	0.15
Standard Deviation of Market Return (σ_m)	0.224

terminal taxable income, viz.,

$$C_2 = C[\theta(\tilde{Y}_1 - Y_0) + P_0; \tilde{L}]$$

$$= R_f^{-1} \int_0^\infty \int_0^\infty \max[\theta(\tilde{Y}_1 - Y_0) + P_0 - \tilde{L}, 0]\hat{g}(\tilde{Y}_1, \tilde{L})\, d\tilde{Y}_1\, d\tilde{L}. \quad (27)$$

By defining a new random variable $\tilde{T} = \theta(\tilde{Y}_1 - Y_0) + 2P_0 - \tilde{L}$, we can rewrite equation (27) as

$$C_2 = R_f^{-1} \int_{P_0}^\infty (\tilde{T} - P_0)\hat{g}(\tilde{T})\, d\tilde{T}. \quad (28)$$

Using analysis identical to that shown above, the value of the second call option is derived as[13]

$$C_2 = V_0^T N(d_1^T) - R_f^{-1} P_0 N(d_2^T), \quad (29)$$

where

V_0^T = the contemporaneous value of the claim $\tilde{T}$
$= R_f^{-1}[\theta(S_0 + kP_0)r_f + 2P_0)] - V_0^L$;
$d_1^T = \dfrac{\ln(V_0^T/P_0) + \ln R_f + \sigma_t^2/2}{\sigma_t}$;
$d_2^T = d_1^T - \sigma_t$;
σ_t = the standard deviation of the natural logarithm of $\tilde{T}$
$= [\sigma_{\theta\Delta y}^2 + \sigma_1^2 - 2\,\text{cov}(\ln[\theta(\tilde{Y}_1 - Y_0)], \ln \tilde{L})]^{1/2}$;
$\sigma_{\theta\Delta y}$ = the standard deviation of the natural logarithm of $\theta(\tilde{Y}_1 - Y_0)$.

Substituting the right-hand sides of equations (26) and (29) into equation (7), we obtain an analytic expression for the market value of equity:

$$V_e = V_0^U N(d_1^U) - \tau V_0^T N(d_1^T) - R_f^{-1} P_0(N(d_2^U) - \tau N(d_2^T)). \quad (30)$$

[13] The term $N(d_2^T)$ that appears in equation (29) may be given the same interpretation as the $N[\hat{E}(\tilde{W})/\sigma_w]$ term from equation (18); viz., this term represents the certainty-equivalent terminal value of one dollar of taxable income, provided that tax shields are fully utilized.

Table II

Effects of Variations in Model Parameters upon the Equilibrium Rate of Return on Underwriting

Panel A: Effects of Variations in Initial Equity (S_0)

	CAPM	OPM (Normal)			OPM (Lognormal)		
S_0	$E(\tilde{r}_u)$	$E(\tilde{r}_u)$	P(default)	P(no tax)	$E(\tilde{r}_u)$	P(default)	P(no tax)
25.00	−0.0957	−0.1444	0.3824	0.6135	−0.1249	0.4088	0.6438
50.00	−0.0917	−0.0659	0.1895	0.5060	−0.0636	0.2213	0.5452
75.00	−0.0877	−0.0347	0.0992	0.4558	−0.0355	0.1254	0.4998
100.00	−0.0837	−0.0188	0.0534	0.4271	−0.0199	0.0718	0.4744
150.00	−0.0758	−0.0028	0.0164	0.3941	−0.0034	0.0236	0.4473
200.00	−0.0680	0.0065	0.0055	0.3730	0.0061	0.0077	0.4318

Panel B: Effects of Variations in the Funds Generating Coeffficient (k)

	CAPM	OPM (Normal)			OPM (Lognormal)		
k	$E(\tilde{r}_u)$	$E(\tilde{r}_u)$	P(default)	P(no tax)	$E(\tilde{r}_u)$	P(default)	P(no tax)
0.50	−0.0345	0.0240	0.0264	0.3922	−0.0216	0.0532	0.4310
1.00	−0.0837	−0.0188	0.0534	0.4271	−0.0199	0.0718	0.4744
2.00	−0.1820	−0.1205	0.1192	0.4961	−0.1037	0.1161	0.5526
3.00	−0.2804	−0.2401	0.1766	0.5538	−0.1883	0.1571	0.6154
4.00	−0.3787	−0.3710	0.2204	0.5977	−0.2726	0.1905	0.6653
5.00	−0.4771	−0.5086	0.2534	0.6306	−0.3561	0.2164	0.7060
6.00	−0.5754	−0.6504	0.2786	0.6555	−0.4387	0.2365	0.7403

Panel C: Effects of Variations in the Standard Deviation of Investment Returns (σ_i)

	CAPM	OPM (Normal)			OPM (Lognormal)		
σ_i	$E(\tilde{r}_u)$	$E(\tilde{r}_u)$	P(default)	P(no tax)	$E(\tilde{r}_u)$	P(default)	P(no tax)
0.00	−0.0837	−0.0150	0.0059	0.4094	−0.0196	0.0300	0.4543
0.20	−0.0837	−0.0188	0.0534	0.4271	−0.0199	0.0718	0.4744
0.40	−0.0837	−0.0730	0.1805	0.4985	−0.0364	0.1789	0.5283
0.60	−0.0837	−0.1867	0.2875	0.5771	−0.0718	0.2882	0.5884

Panel D: Effects of Variations in the Standard Deviation of Claims Cost (σ_L)

	CAPM	OPM (Normal)			OPM (Lognormal)		
σ_L	$E(\tilde{r}_u)$	$E(\tilde{r}_u)$	P(default)	P(no tax)	$E(\tilde{r}_u)$	P(default)	P(no tax)
25.00	−0.0837	−0.0336	0.0280	0.4205	−0.0319	0.0169	0.4382
50.00	−0.0837	−0.0188	0.0534	0.4271	−0.0199	0.0718	0.4744
75.00	−0.0837	−0.0099	0.0925	0.4384	−0.0219	0.1555	0.5197
100.00	−0.0837	−0.0103	0.1379	0.4526	−0.0372	0.2381	0.5643
150.00	−0.0837	−0.0411	0.2272	0.4836	−0.0875	0.3677	0.6382
200.00	−0.0837	−0.1140	0.3038	0.5136	−0.1435	0.4563	0.6920

Panel E: Effects of Variations in the Riskless Rate of Interest (r_f)

	CAPM	OPM (Normal)			OPM (Lognormal)		
r_f	$E(\tilde{r}_u)$	$E(\tilde{r}_u)$	P(default)	P(no tax)	$E(\tilde{r}_u)$	P(default)	P(no tax)
0.05	−0.0600	0.0005	0.0490	0.4007	0.0004	0.0692	0.4410
0.07	−0.0837	−0.0188	0.0534	0.4271	−0.0199	0.0718	0.4744
0.09	−0.1071	−0.0381	0.0579	0.4529	−0.0402	0.0743	0.5071
0.11	−0.1304	−0.0574	0.0626	0.4781	−0.0605	0.0767	0.5390
0.13	−0.1534	−0.0767	0.0675	0.5026	−0.0808	0.0789	0.5698

Panel F: Effects of Variations in the Tax Parameter Theta (θ)

	CAPM	OPM (Normal)			OPM (Lognormal)		
θ	$E(\tilde{r}_u)$	$E(\tilde{r}_u)$	P(default)	P(no tax)	$E(\tilde{r}_u)$	P(default)	P(no tax)
0.00	−0.1130	−0.0446	0.0595	0.5678	−0.0470	0.0755	0.5847
0.20	−0.1013	−0.0372	0.0577	0.5115	−0.0389	0.0744	0.5377
0.40	−0.0895	−0.0258	0.0550	0.4538	−0.0270	0.0728	0.4936
0.50	−0.0837	−0.0188	0.0534	0.4271	−0.0199	0.0718	0.4744
0.60	−0.0778	−0.0110	0.0516	0.4026	−0.0122	0.0708	0.4574
0.80	−0.0660	0.0063	0.0478	0.3612	0.0044	0.0687	0.4296
1.00	−0.0543	0.0255	0.0437	0.3294	0.0222	0.0665	0.4088

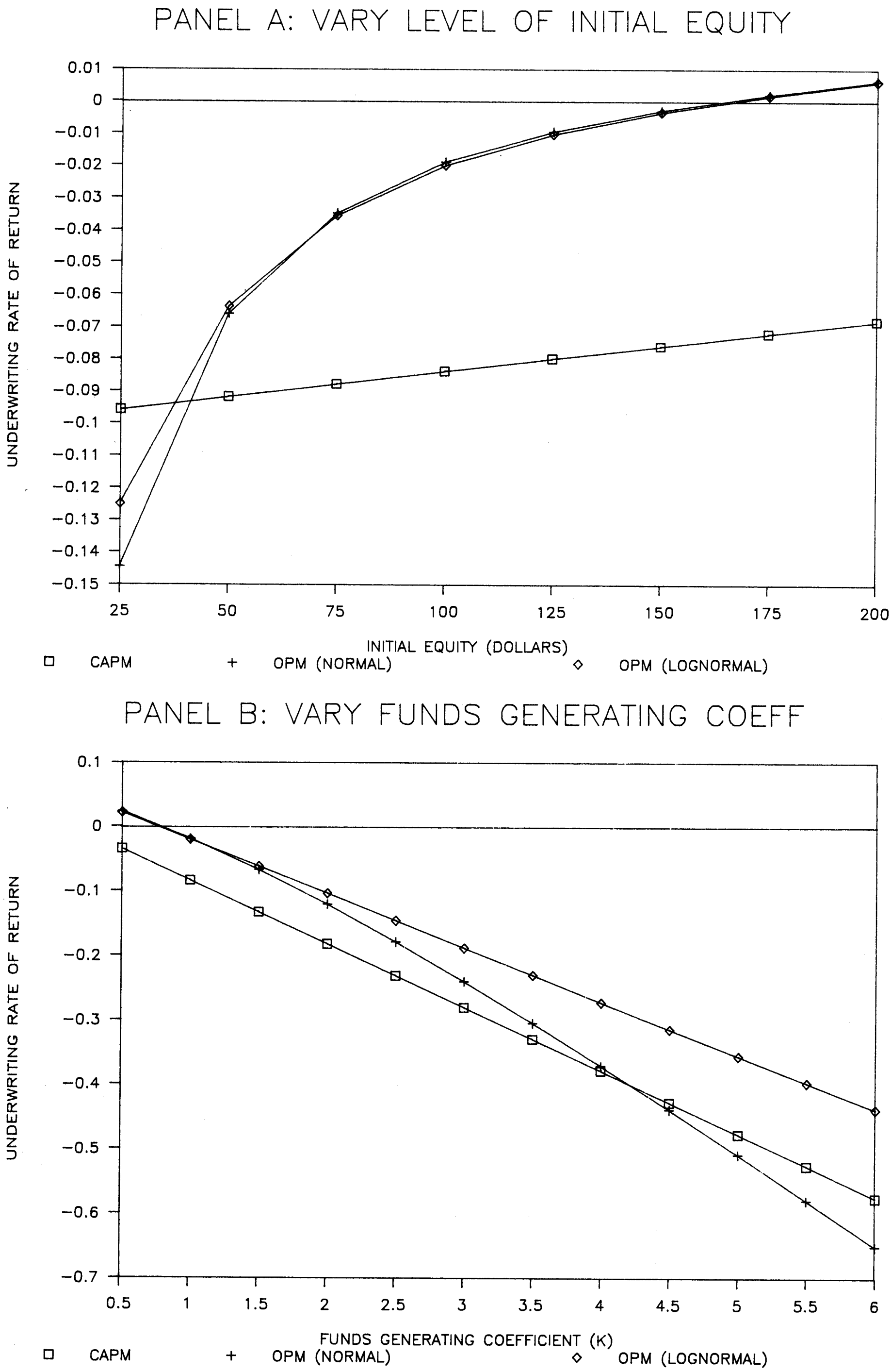
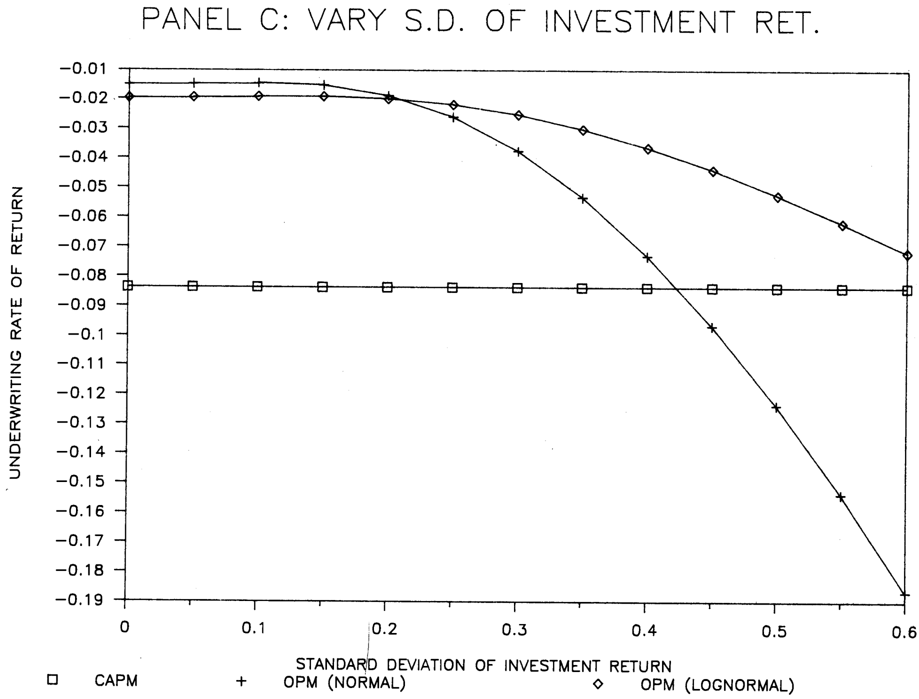


Figure 1. Plots of the Effects of Changes in Model Parameters upon the Equilibrium Rate of Return on Underwriting for a Property-Liability Insurer



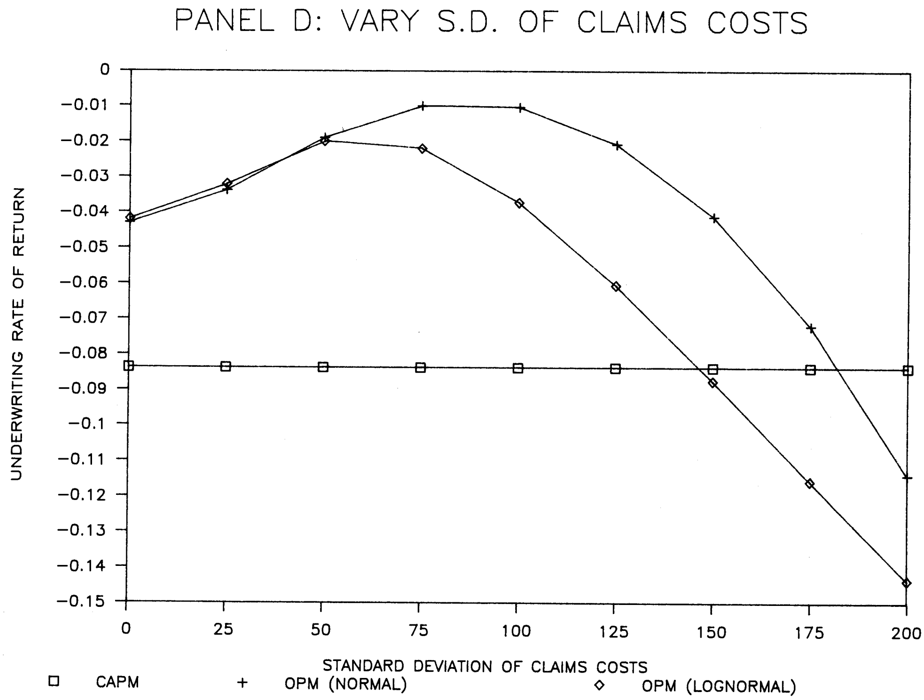
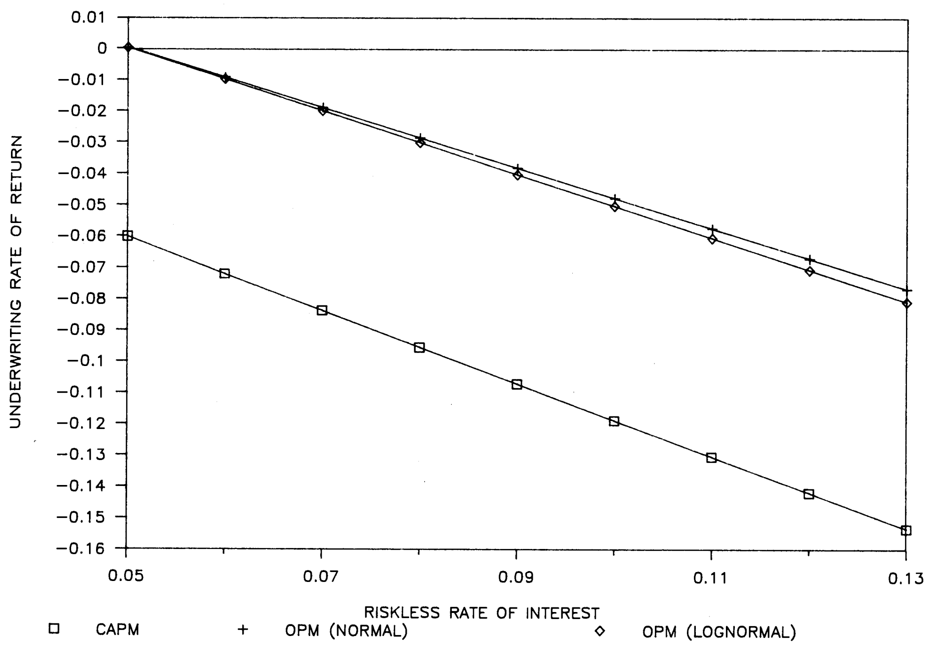


Figure 1 (cont.). Plots of the Effects of Changes in Model Parameters upon the Equilibrium Rate of Return on Underwriting for a Property-Liability Insurer.

PANEL E: VARY RISKLESS RATE OF INTEREST



PANEL F: VARY TAX PARAMETER THETA

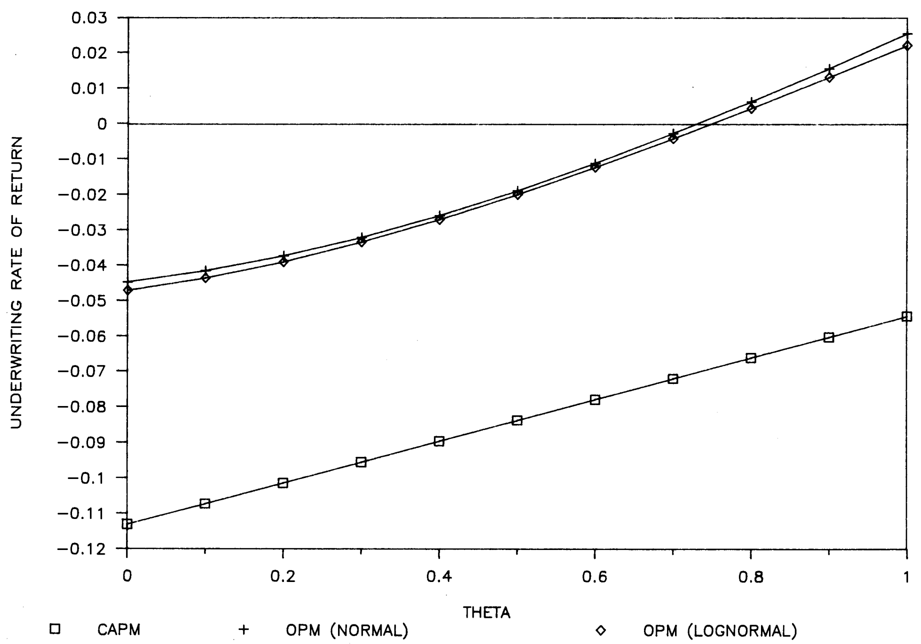


Figure 1 (cont.). Plots of the Effects of Changes in Model Parameters upon the Equilibrium Rate of Return on Underwriting for a Property-Liability Insurer.

An implicit solution for the value of P_0^* that satisfies the fair-return criterion implied by equation (8) may be obtained by employing an appropriate algorithm. Furthermore, the implicit solution P_0^* obtained from equations (19) and (30) may be translated into fair underwriting profit rates $E(\tilde{r}_u^*)$ by the routine solution of

$$E(\tilde{r}_u^*) = \frac{P_0^* - E(\tilde{L})}{P_0^*}. \tag{31}$$

III. A Numerical Illustration

In this section we provide a numerical illustration that provides points of comparison between the alternative option-based models developed here and the regulatory CAPM. The option-based models were solved iteratively from equations (19) and (30), whereas the normal and lognormal CAPM's were solved from equations (11a) and (22a) after substituting for P_0 from equations (11) and (22).[14] The solutions were derived from a set of parameters presented in Table I that are intended as a crude representation of a short-tail (i.e., short settlement period for losses) line of property-liability business. Table II and Figure 1 show the rates of underwriting profit required to deliver a competitive rate of return on equity over different ranges of values for the model parameters. Furthermore, we also show the implied probabilities of insolvency and tax-shield redundancy for the option-based models in Table II.

The points of interest include the following. In general, the option-based models provide higher rates of underwriting profit than the CAPM. The most useful comparison is between the normal CAPM results and those produced under the joint-normal-with-CARA option-pricing model. Since the distributional assumptions are comparable, the differences in fair underwriting profit are explained by the attention paid in our option-pricing model to the probabilities of insolvency and redundant tax shields. Another point of interest is between the insolvency and tax shield redundancy probabilities produced by the two option models. It is well known in the actuarial literature that estimates of the probability mass in the extreme tail of a fitted distribution are highly sensitive to the function form used, a fact borne out by our simulation results. Both normal and lognormal density functions have been used to describe the insurer's aggregate loss distribution (cf. Cummins and Nye [6]), as well as other distributions. Since the probabilities of insolvency and tax-shield redundancy are at issue, some prudent curve fitting should influence the choice of regulatory model.

IV. Conclusion

We have developed a contingent-claim model for estimating the fair rate of return for the property-liability insurance firm. This model offers an alternative regulatory device to the CAPM's and APT-based models. Such an alternative is

[14] Since the results obtained with the lognormal CAPM do not differ materially from the results obtained with the normal CAPM, only the latter model's results are presented here.

considered to be useful in light of the unsettled academic score regarding the appropriate asset-pricing paradigm. Moreover, the proposed option model addresses the ruin probability of the insurer as well as the nonlinear effects of corporate taxation.

Our model specifically applies to the property-liability insurance firm. The features of this particular institution that lend it to an option-pricing application are that its output is a contingent financial claim and that this claim is written on an underlying asset for which a reasonable market value can be provided. With the possible exception of deposit banking, such conditions do not necessarily prevail in other regulated industries.

We will conclude with some qualifying comments on the use of option-based models for insurance price regulation. The first concerns tax nonlinearities. Previous models either ignore taxes (e.g., Kraus and Ross) or assume that the corporate income tax is strictly proportional to corporate income. In effect, the proportional-tax assumption implies that unused tax shields can be sold or carried back and forward at their face value. In contrast, our model makes the opposite assumption that unused tax shields expire worthless. Perhaps a more accurate assumption would be that unused tax shields can be sold or carried back and forward at somewhat less than their face value. Thus, the tax function would still be nonlinear but not as concave as supposed here. While we have not addressed the intermediate approach, it clearly lends itself to the option-modeling techniques developed here.[15] Another problem is that it is not clear from current regulatory practices just how unused tax shields should be treated. This uncertainty gives some advantage to the present model since it is possible to generate a proportional tax as a special case by simply assuming that the probability of tax-shield redundancy is negligible.

A second qualification concerns the application of the regulatory model to individual lines of business. Like most other regulatory models, it is designed to solve the fair return for the firm as though it offered a single line of insurance. The problem in providing simultaneous solutions for multiple lines is that insurers hold common equity and assets and incur common expenses over several lines of business. Thus, the firm, and not the line of business, is valued by the market. In practice, some arbitrary allocations can be made across lines to produce answers, and this indeed is what is usually done. However, the current use of option-pricing models hints at some problems of additivity; e.g., it is well known that an option on a portfolio does not have the same value as a portfolio of options. These issues are not addressed here. Therefore, like previous models, our approach strictly applies only for a single-line insurer.

[15] By using a single-period model, we have implicitly assumed away the possibility of the insurer making use of tax-loss carrybacks and carryforwards (CB-CF), which could be introduced in a multiperiod framework. Although we cannot provide a formal demonstration of the effects of CB-CF provisions on insurance pricing, the effects can nevertheless be inferred from our model. Since tax-shield redundancy effectively increases the burden of the corporate tax on the insurer, this burden will be passed on to policyholders via higher insurance prices and underwriting rates of return, *ceteris paribus*. However, since the effect of CB-CF provisions is to reduce this tax burden, their existence implies lower insurance prices and underwriting rates of return, *ceteris paribus*. The interested reader is referred to the recent paper by Majd and Myers [19], which numerically simulates the valuation effects of CB-CF provisions in a contingent-claims setting.

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