Chapter 21

Electric Charge and Electric Field

- Electric field and force
- Electric Fields
  - Some calculations

Electric Field and Force

New Topic

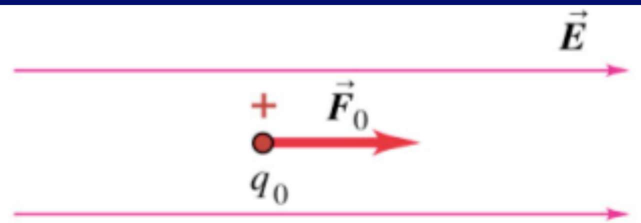
Motion of a charged particle in a given electric field

Once the field is given, the force can be easily calculated by

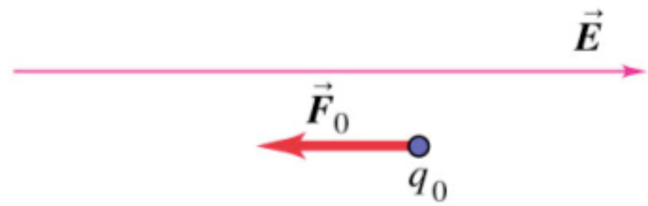
$$\vec{F} = q\vec{E}$$

Once the force is known, the motion can be solved by

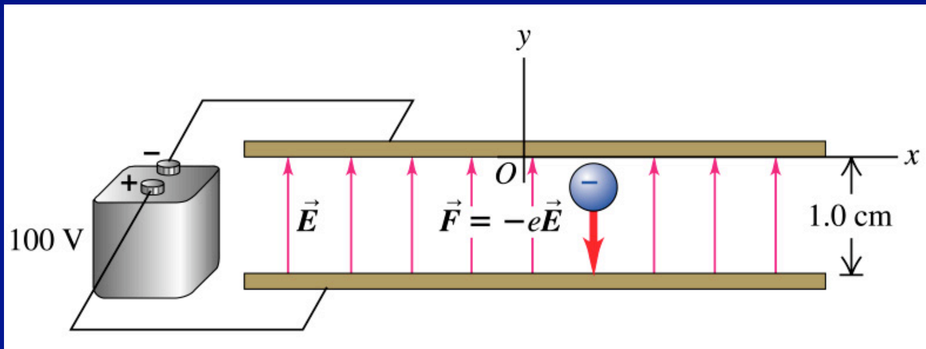
$$\vec{F} = m\vec{a}$$

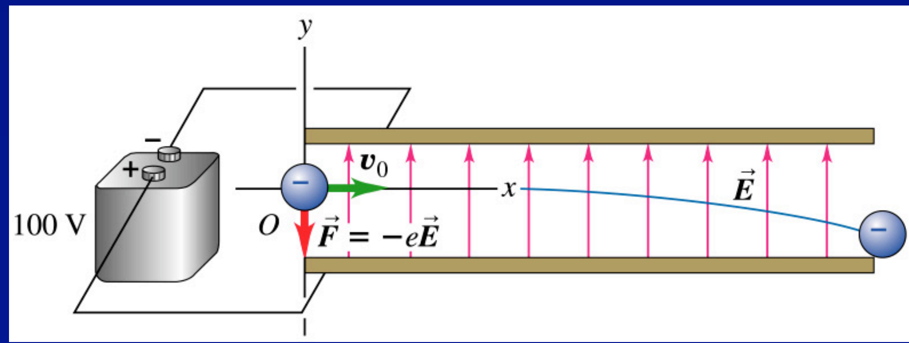


(a) Positive charge q_0 placed in an electric field: force on q_0 is in same direction as $\vec{E}$



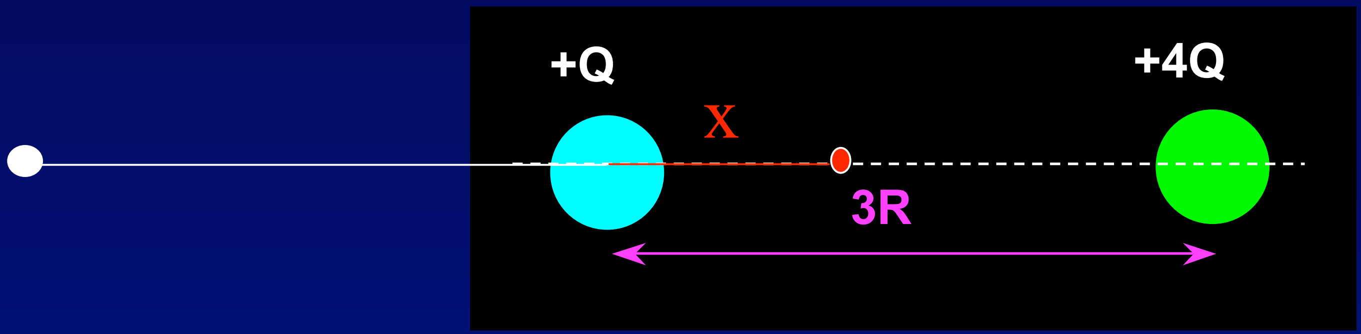
(b) Negative charge q_0 placed in an electric field: force on q_0 is in opposite direction from $\vec{E}$





Example:

Is there a point along the line of the two charges where the electric field vanishes? If yes, find the location.



- Assume the point is in between and x away from the +Q charge. Also assume to the right is positive.

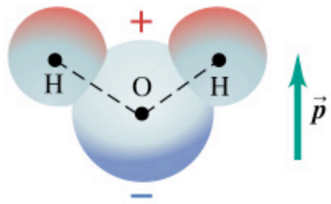
$$E = k\frac{Q}{x^2} - k\frac{4Q}{(3R-x)^2} = 0$$

$$x = R, \text{ or } -3R$$

Discard X=-3R because it's unphysical.

What makes water special?

- The large electric dipole moment of water makes it an excellent solvent.



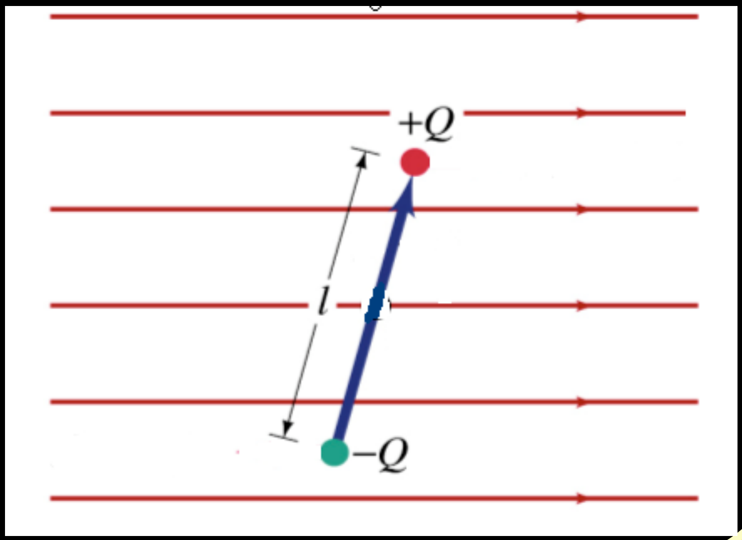
(a)

(b)

ConcepTest 21.6 (Post) Electric Dipole

- Imagine an electric dipole placed in an uniform electric field. What will it experience?

(1) A net force and a net torque
(2) Zero net force and a net torque
(3) Zero net force and zero net torque
(4) A net force and zero net torque



Electric field of continuous charge distributions

New Topic

How to calculate Electric Fields from Continuous Charge Distributions ?

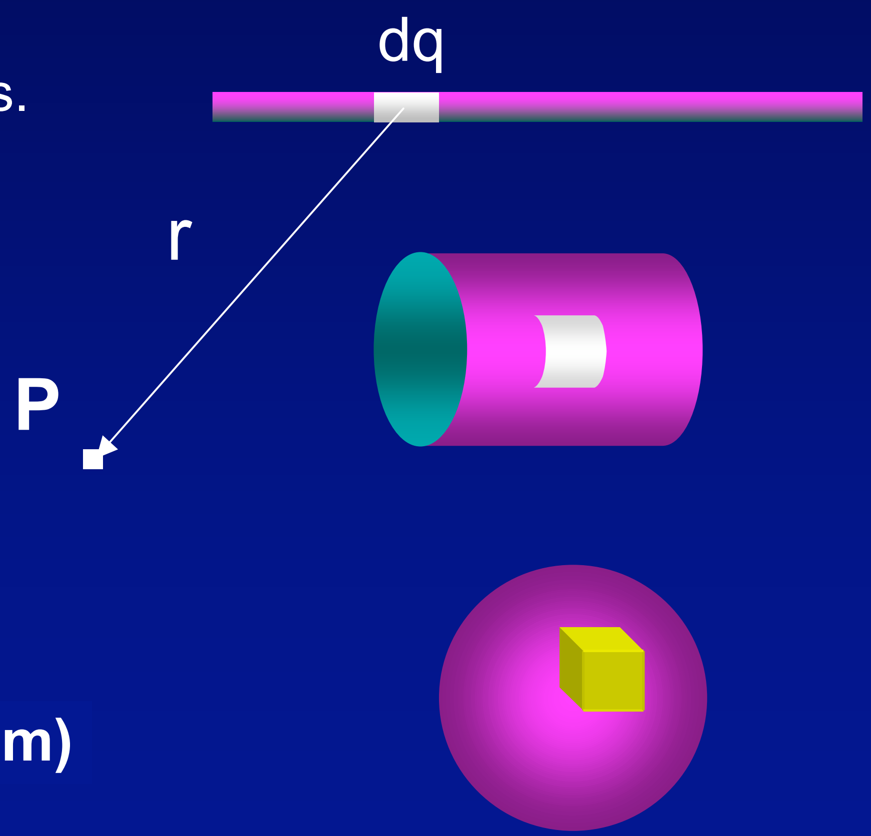
- Divide into small chunks
  - Treat each chunk as a point charge
- Superposition principles of electric fields.
  - Add up contribution from all chunks
  - Sum becomes integral

$$dE = k\frac{dq}{r^2}$$

$$\sum_i \vec{E}_i \rightarrow \int d\vec{E}$$

(vector sum)

What's electric field at point P due to the charge distribution?



Charge Densities

- How do we represent the charge "Q" on an extended object?

total charge $\mathbf{Q}$ $\longrightarrow$ **small pieces of charge** $\mathbf{dq}$

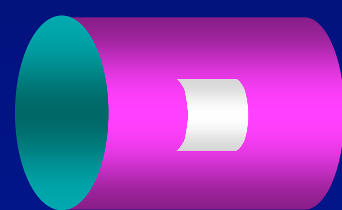
- Line of charge:

  λ = charge per unit length

$$\boxed{dq = \lambda \, dx}$$

- Surface of charge:

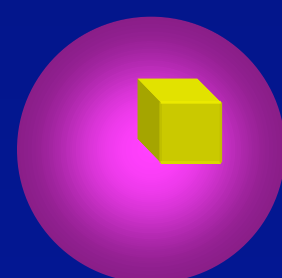
  σ = charge per unit area



$$\boxed{dq = \sigma \, dA}$$

- Volume of charge:

  ρ = charge per unit volume



$$\boxed{dq = \rho \, dV}$$

Example: A ring of charge

- A ring of radius a carries a total charge Q distributed uniformly around it. Determine the electric field at a point on its axis.

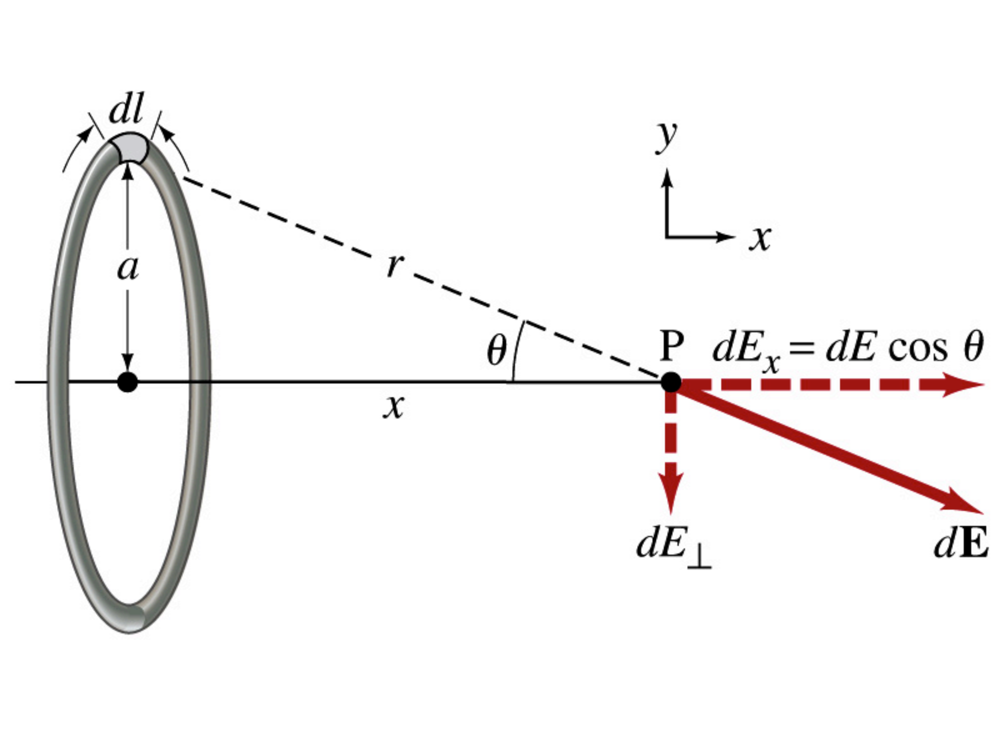
Set up coordinate system:

$$\text{charge density } \lambda = \frac{Q}{2\pi a}$$

$$dq = \lambda\, dl$$

$$dE = k\frac{\lambda\, dl}{r^2}$$

By symmetry: $E_y = 0$
So E is along x-axis.



Example: A ring of charge (continued)

$$E = \int dE_x = \int dE\cos\theta = k\lambda \int \frac{dl}{r^2}\cos\theta$$

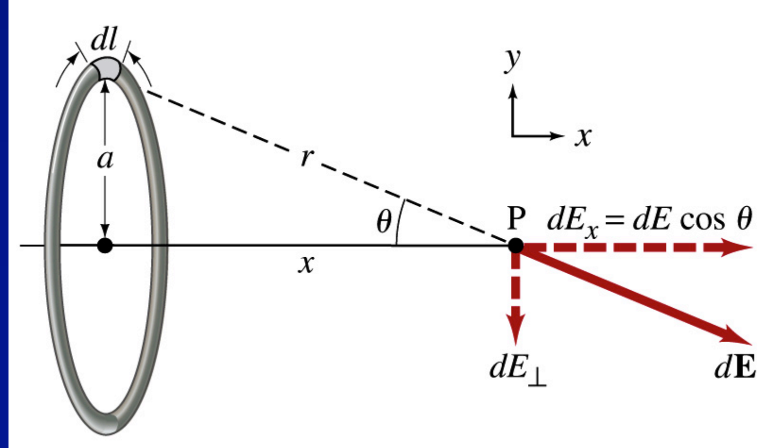
Use $\cos\theta = x/r$, and $r^2 = x^2 + a^2$

$$E = \frac{k\lambda x}{(x^2+a^2)^{3/2}} \int_0^{2\pi a} dl = \frac{kQx}{(x^2+a^2)^{3/2}}$$

Special cases:

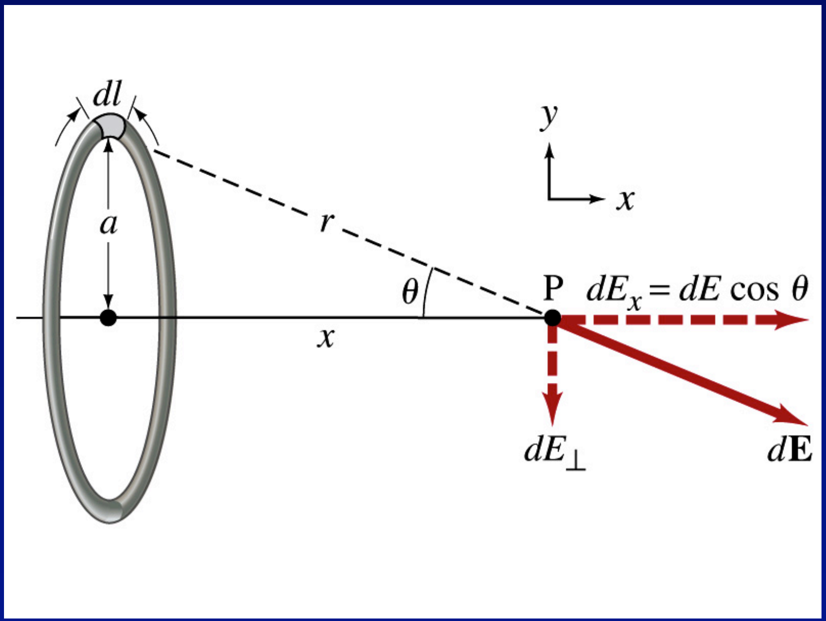
1) At $x = 0$, $E = 0$

2) At $x >> a$, $E = k\dfrac{Q}{x^2}$



Problem-solving techniques:
calculating electric fields from continuous charge distributions

- Break up the charge into small pieces, then add up the fields from each piece. (use charge density $dQ=\lambda\ dl$)
- Use symmetry to simplify the problem.
- Check special cases where you know or expect the answer. If it doesn't check out, the result would be wrong entirely.



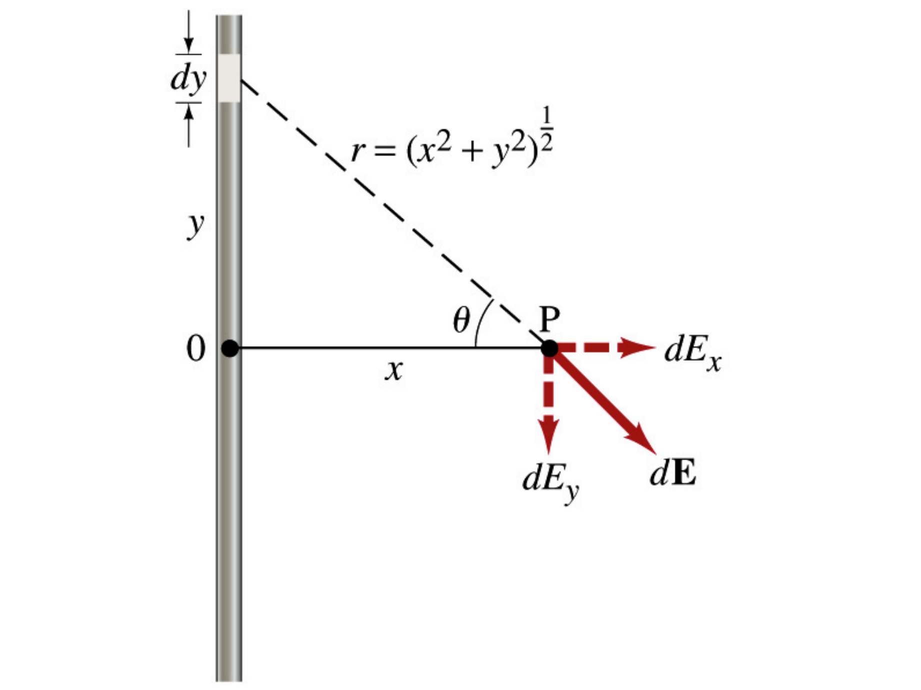
Example: Long line of charge

- Determine the electric field at any point a distance x away from a disk of uniformly distributed charge of density λ.

Set up coordinate system

$$dq = \lambda\, dy$$

$$dE = k\frac{\lambda\, dy}{r^2} \quad \text{(vector)}$$



By symmetry: E_y=0. So E is along x.

$$E = \int dE_x = \int dE \cos\theta = k\lambda \int \frac{dy}{r^2}\cos\theta$$

$$\text{Use } \cos\theta = x/r, \quad \text{and} \quad r^2 = x^2 + y^2$$

$$E = k\lambda x \int_{-\infty}^{+\infty} \frac{dy}{(x^2+y^2)^{3/2}} = k\lambda x\left(\frac{2}{x^2}\right) = \frac{2k\lambda}{x}$$

Example: Uniformly charged disk

- Determine the electric field at any point a distance z away from a long line of uniformly distributed charge of density σ.

Set up coordinate system:

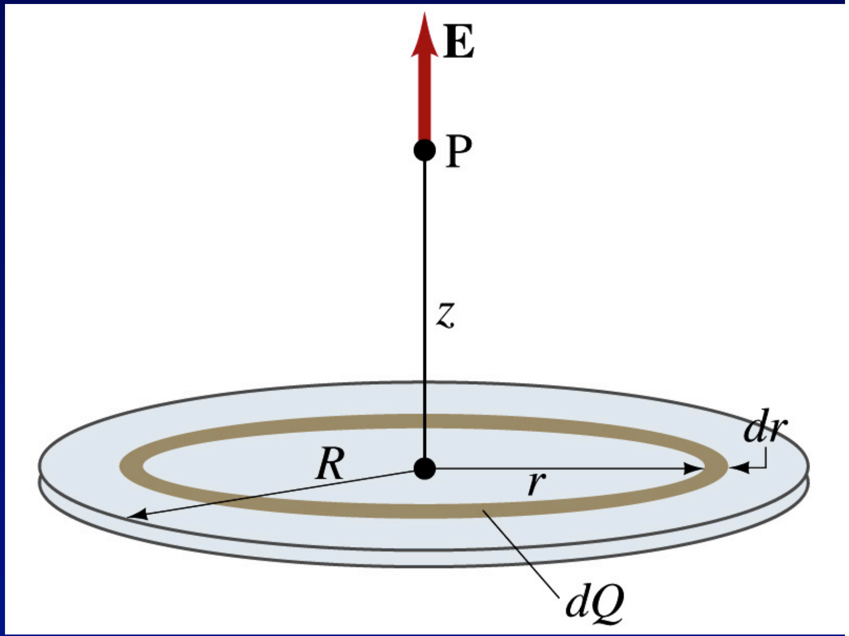
Divide into small rings:

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{z\,dq}{(z^2+r^2)^{3/2}} \quad \text{(along z)}$$

$$dq = \sigma(2\pi r\,dr)$$

$$E = \frac{z\sigma}{2\varepsilon_0}\int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}} = \frac{z\sigma}{2\varepsilon_0}\left[\frac{-1}{(z^2+r^2)^{1/2}}\right]_0^R$$

$$E = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{z}{(z^2+R^2)^{1/2}}\right]$$



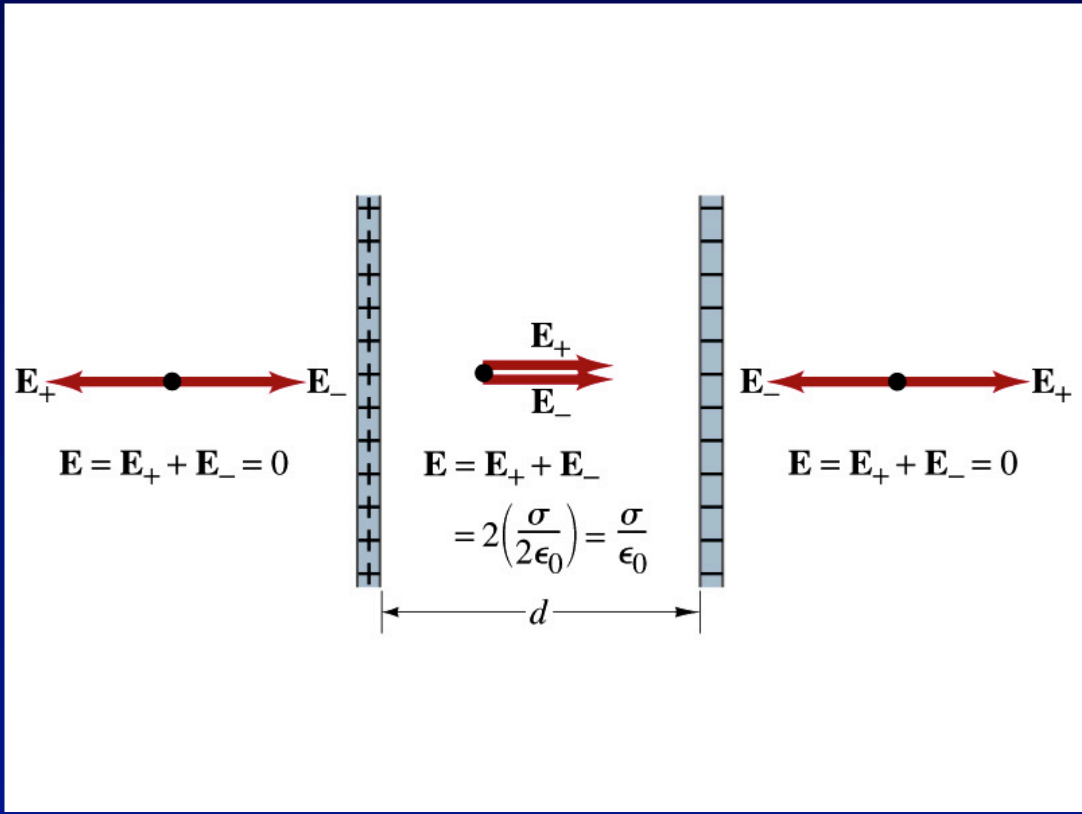
infinite plane (z << R):

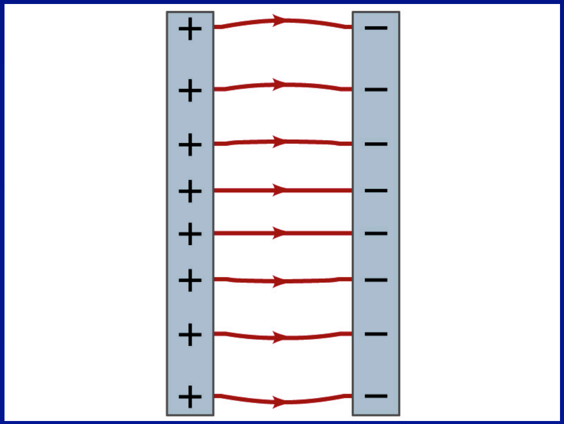
$$E = \frac{\sigma}{2\varepsilon_0}$$

Example: Two parallel plates (one positive, one negative)

$$\text{inside}: E = \frac{\sigma}{\varepsilon_0}$$

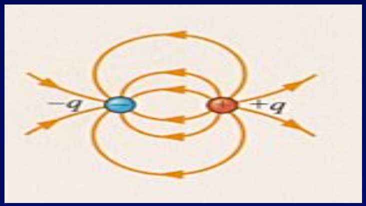
$$\text{outside}: E = 0$$





Review: Some Electric Field Distributions

Dipole



$\sim 1 / r^3$

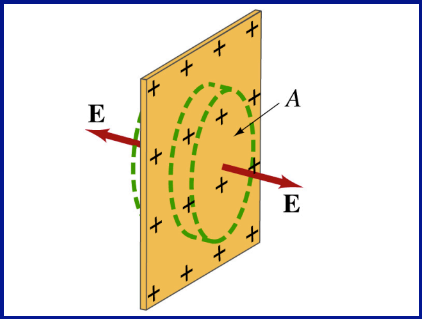
Point Charge

$\sim 1 / r^2$

Infinite
Line of Charge

$\sim 1 / r$

Infinite
Plane of Charge



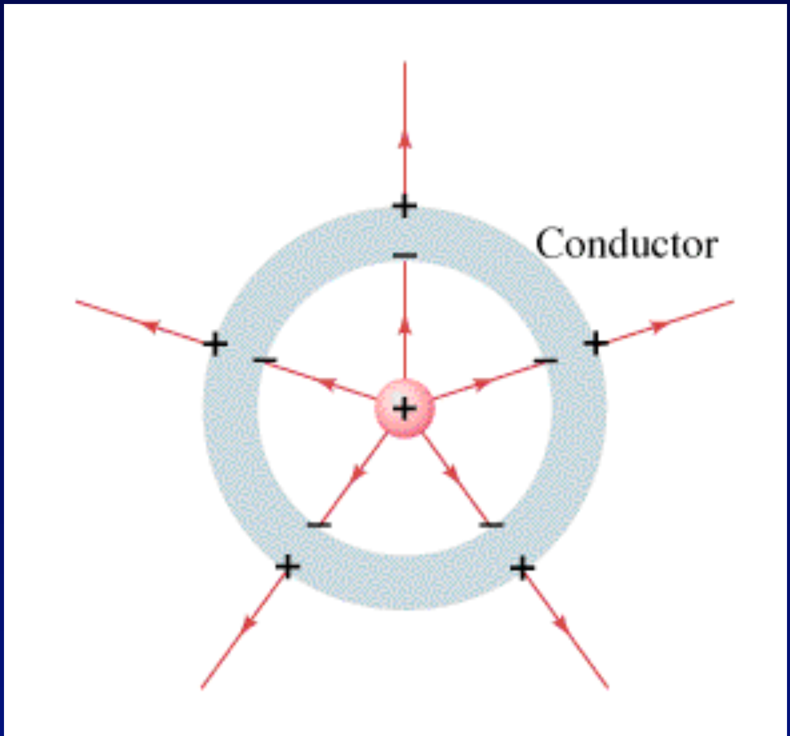
$\sim$ constant

Conductors in Electrostatic Equilibrium

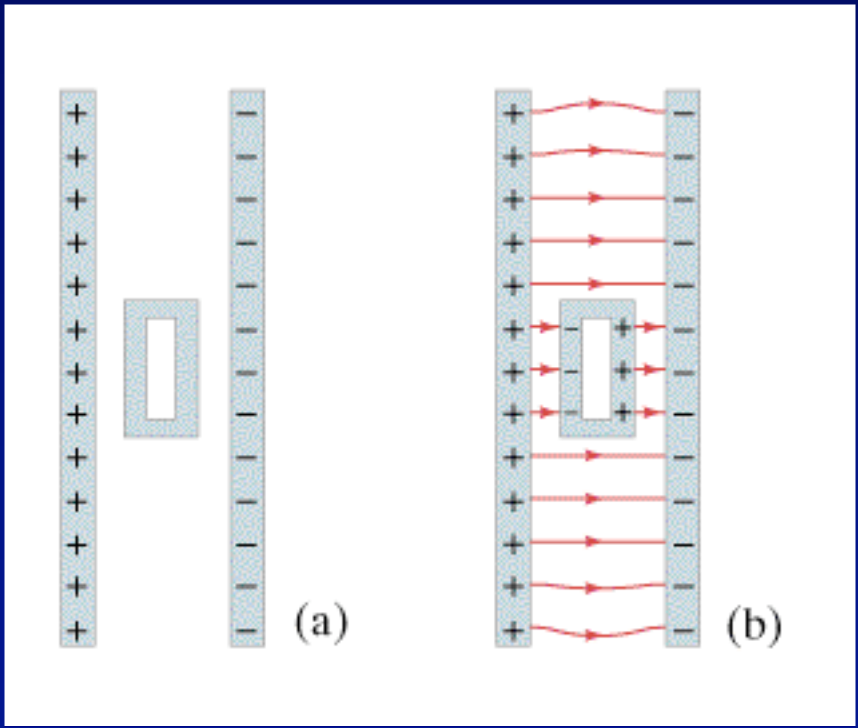
charges can move freely

no motion of charges

- Electric field is **zero** everywhere inside the conductor
  - if $E \neq 0$, then charges would move -- no equilibrium!
- Excess charge on isolated conductor resides only on the **surface** of the conductor
  - mutual repulsion pushes the charges apart
- Electric field is **perpendicular** to the surface of a conductor
  - if a parallel component existed, charges would move!!
- For irregular shaped conductors, more charge accumulates near **sharp points**, i.e. the field strength is greater there



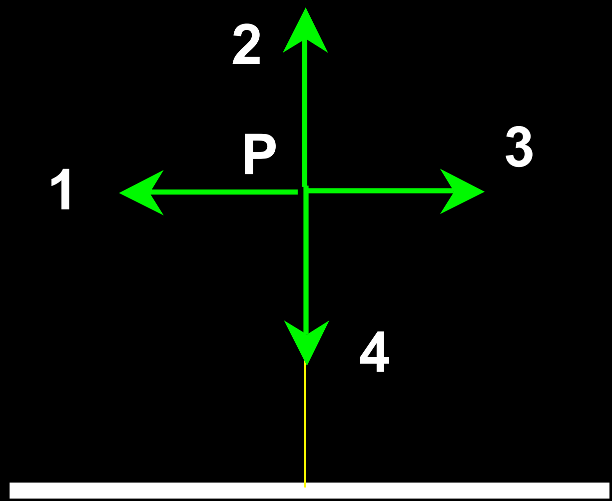
Electric field inside a conductor is zero after charges are settled !



Conductors are good shields.

ConcepTest 21.7 (Post) line charge

- Which arrow best represents the electric field at a point on the middle line from a positive line charge?



(A) 1 (B) 2 (C) 3 (D) 4

(E) none of these