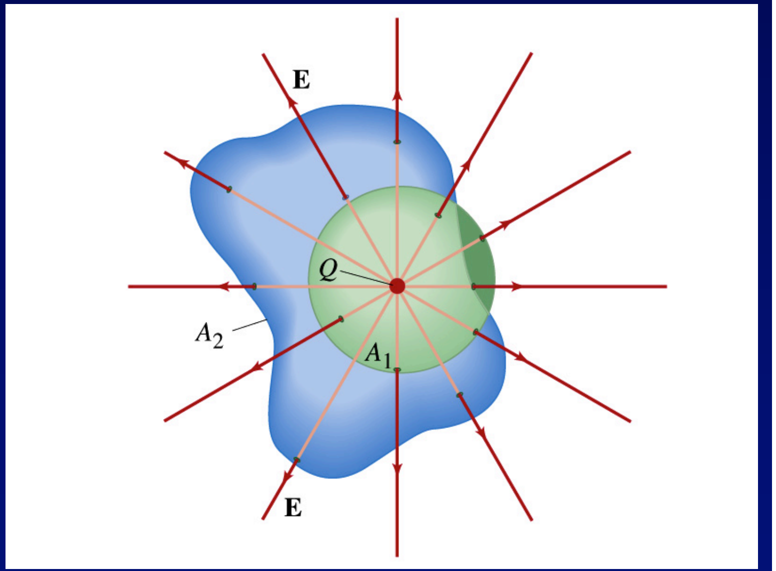
Chapter 22
Gauss's Law

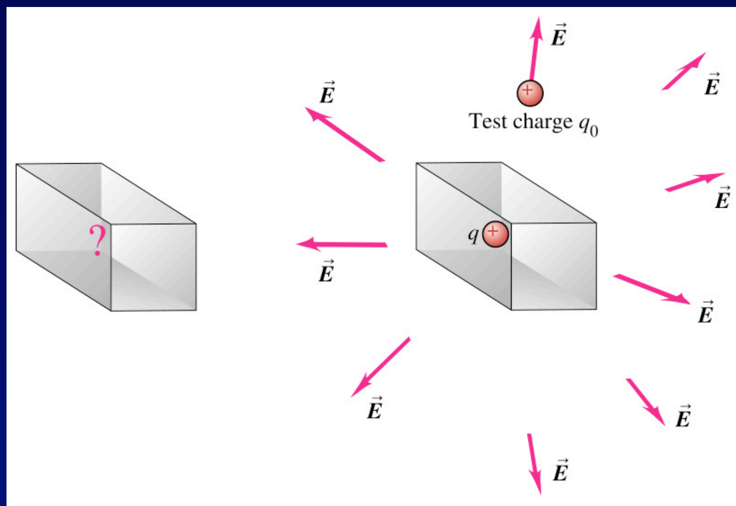
- **Electric Flux**
- **Gauss's Law: Definition**
- **Applications of Gauss's Law**
  - Uniform Charged Sphere
  - Infinite Line of Charge
  - Infinite Sheet of Charge
  - Two infinite sheets of charge



Electric Flux

New Topic

The Concept of Electric Flux



(a) (b)

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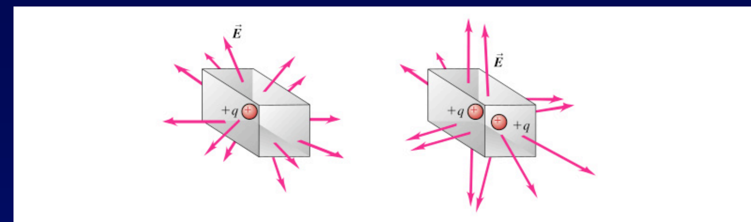

(a) Positive charge inside box, outward flux

(b) Positive charge inside box, outward flux

(c) Negative charge inside box, inward flux

(d) Negative charge inside box, inward flux

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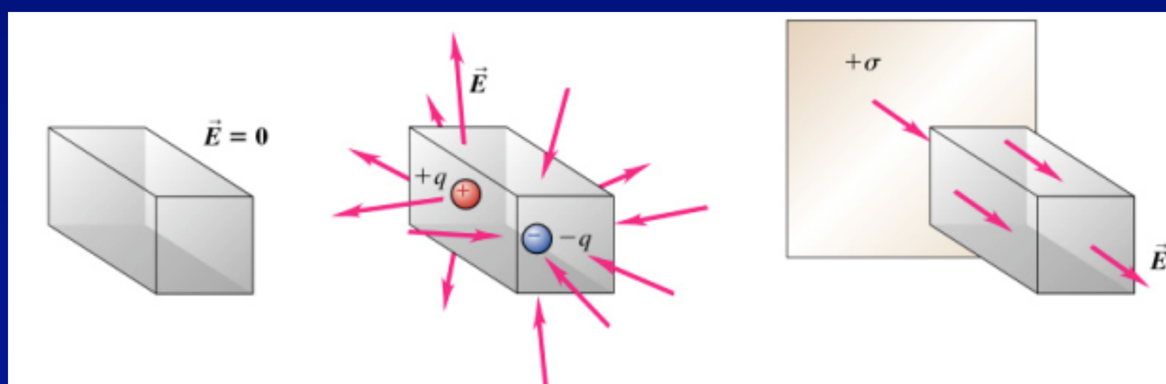


(a) No charge inside box, zero flux

(b) Zero *net* charge inside box, inward flux cancels outward flux

(c) No charge inside box, inward flux cancels outward flux

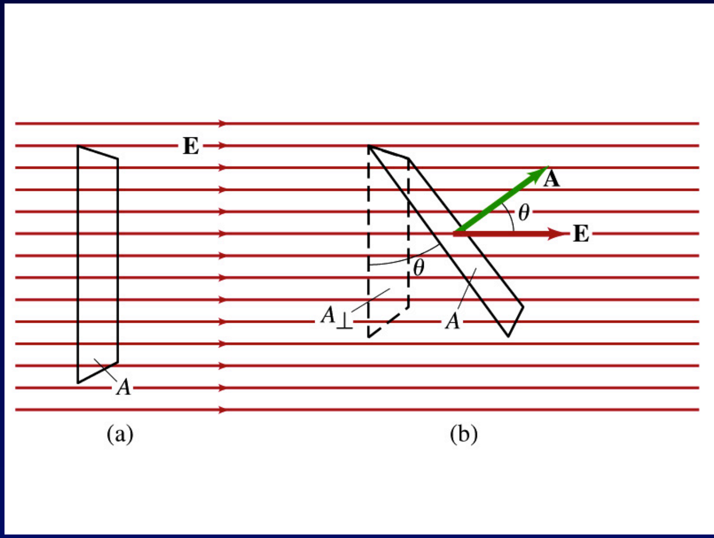
Electric Flux

For uniform field, if E and A are perpendicular, define

$$\Phi_E = EA$$

If E and A are NOT perpendicular, define scalar product

$$\Phi_E = EA\cos\theta = \vec{E}\cdot\vec{A}$$

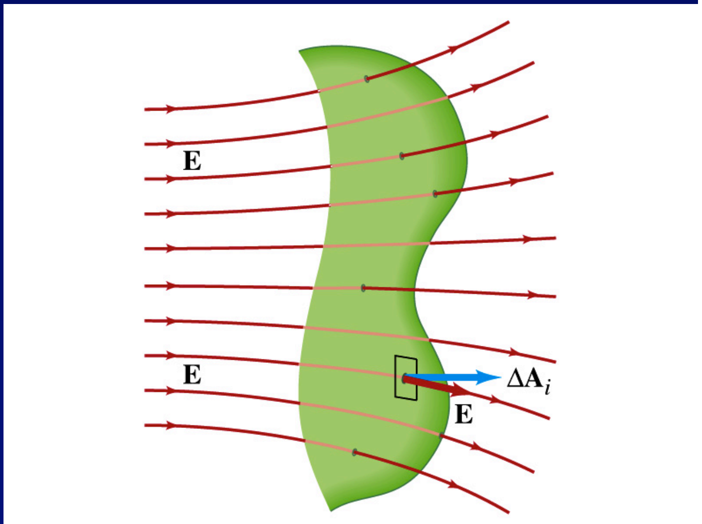


General (surface integral)

$$\Phi_E = \int \vec{E}\cdot d\vec{A}$$

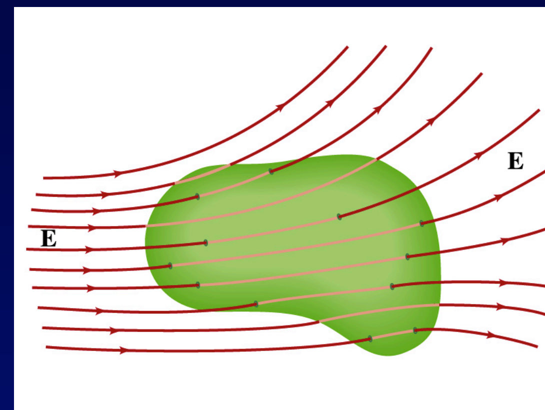
Unit of electric flux: N . m^2 / C

- Electric flux is proportional to the number of field lines passing through the area.

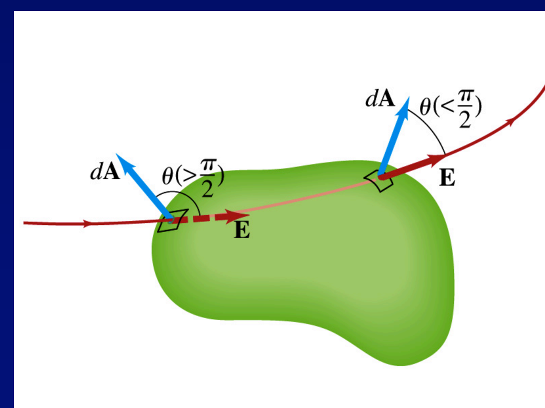


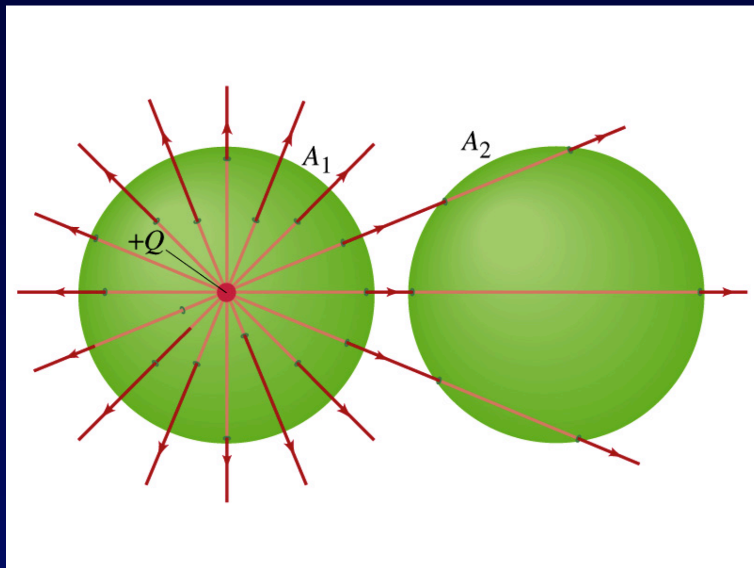
Electric Flux: closed surface

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$



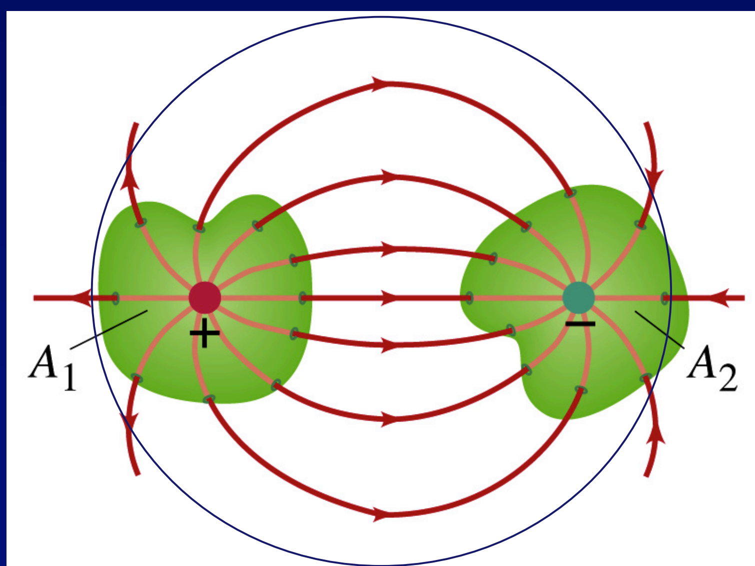
- **Flux entering is negative**
- **Flux leaving is positive**





The net flux through A_1 is positive.

The net flux through A_2 is zero.



The net flux through A_1 is positive.

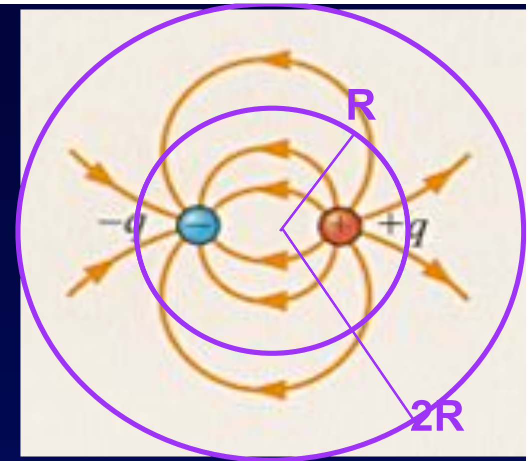
The net flux through A_2 is negative.

What about the net flux through the big sphere surface?

Zero!

Electric Flux for a dipole

$\Phi_R = \Phi_{2R} = 0$



- To directly calculate the flux through these surfaces by $\Phi_E = \oint \vec{E} \cdot d\vec{A}$ is a challenge.

- Make an intelligent "guess". Look at the inner circle.
- The lines going out are symmetric to the lines coming back in.
- This could mean the "net flux" is zero and it does mean that!

- It's not so easy in the outer one, because we didn't extend the lines…
- Imagine if we did. The same would happen. The net flux is zero.

- There is an easier way. Gauss' Law states the net flux is related to the NET enclosed charge. The <u>NET charge is ZERO</u> in both cases.
- *But, what is Gauss' Law ?*

ConcepTest 22.1 Electric Flux

Imagine a cube of side a positioned in a region of constant electric field as shown. Which of the statements about the electric flux through the surface of the cube is true?

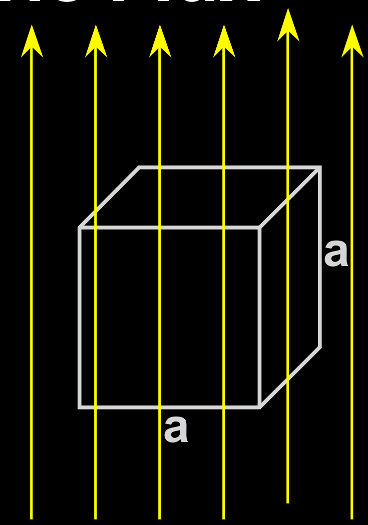
(1) $\Phi_E = 0$

(2) $\Phi_E = 2Ea^2$

(3) $\Phi_E = -2Ea^2$

(4) $\Phi_E = 6Ea^2$

(5) $\Phi_E = -6Ea^2$



Gauss's Law

New Topic

Gauss' Law

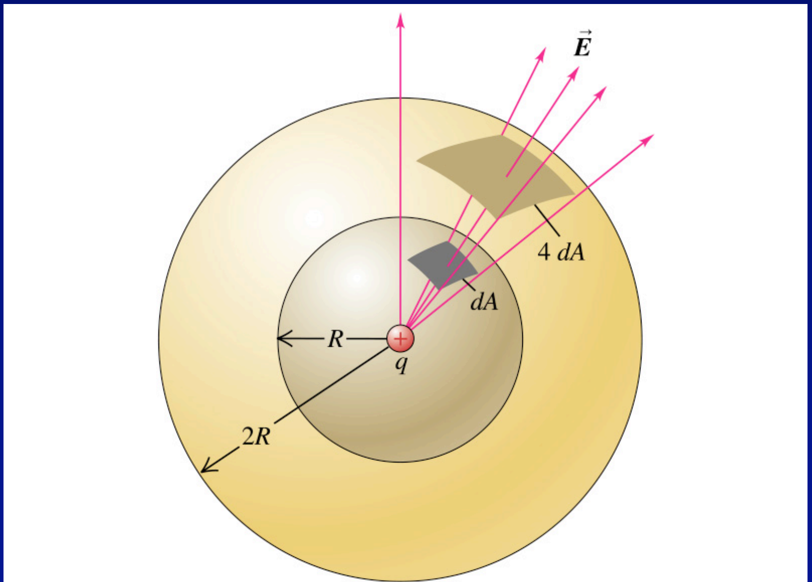
The total electric flux through a closed surface is equal to the total charge enclosed by that surface, divided by ε_0.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = Q_{enclosed} / \varepsilon_0$$

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N.m^2$ is called permittivity of free space

The k in Coulomb's law $F = kq_1q_2/r^2$ is related to ε_0 by

$$k = \frac{1}{4\pi\varepsilon_0}$$



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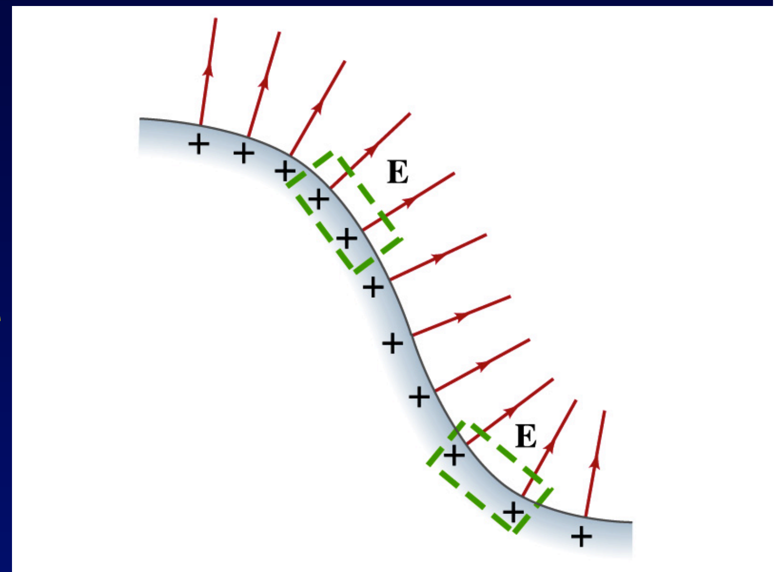
Electric Field near surface of Conductor

- Choose small cylindrical Gaussian box as shown:
  - One end just outside
  - One end just inside
  - The barrel is normal to the surface
- Flux at the end inside is zero
- Flux on the barrel is zero



$$\oint \vec{E} \cdot d\vec{A} = EA = Q/\varepsilon_0 = \sigma A/\varepsilon_0$$

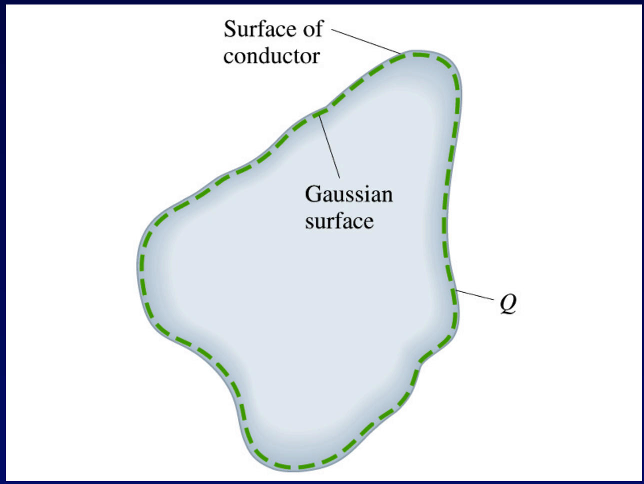
$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Valid near the surface of conductor of any shape

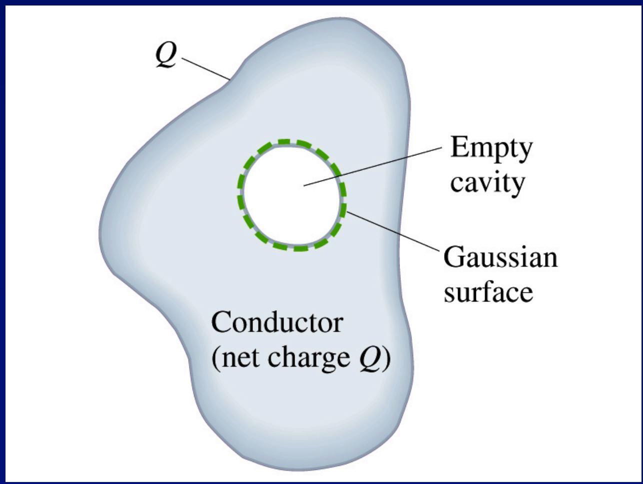
Question: why not $E = \dfrac{\sigma}{2\varepsilon_0}$?

Charges on a Conductor
(the view from Gauss's Law)

- **The E must be zero inside the conductor even if it carries a net charge.**



- **Any net charge on a conductor must all reside on its outside surface.**

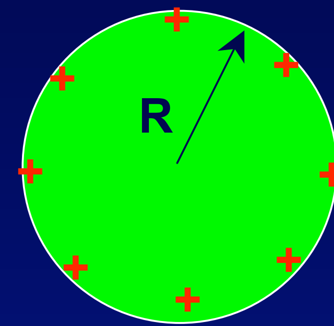


Example: Field of a charged conducting sphere

- We place a positive charge q on a solid conducting sphere with radius R. Find the electric field at any point inside or outside the sphere.

This problem has spherical symmetry. For a spherical surface of any radius, the surface integral becomes

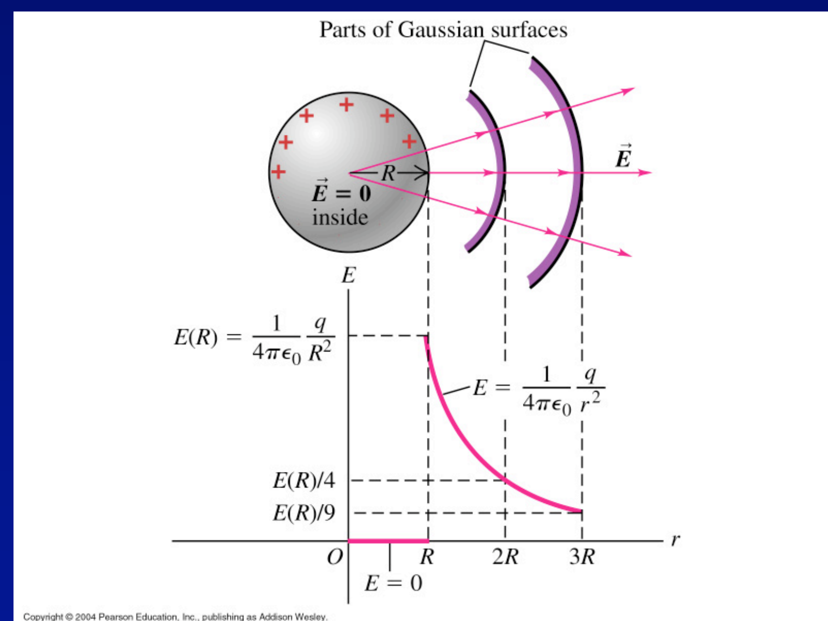
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = 4\pi r^2 E$$



Choose the sphere to be inside the conductor (r<R). Since there is no charge enclosed: $4\pi r^2 E=0$. So E=0 inside.

Choose the sphere to be outside the conductor (r>R). Since the entire charge is enclosed: $4\pi r^2 E=q/\varepsilon_0$. So

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = k\frac{q}{r^2}$$

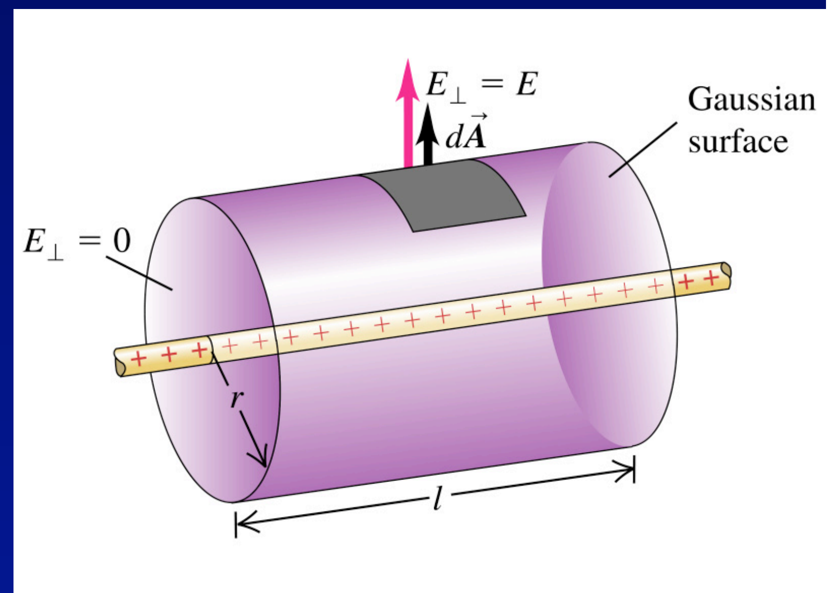


Example: Field of a long line charge

- Electric charge is distributed along a infinitely long, thin wire. The charge per unit length is l. (assumed positive). Find the electric field around the wire.

This problem has cylindrical symmetry. The E field must be perpendicular to the line.

Choose the Gaussian surface to be a cylinder of radius r and length l (a soda can), with the line as the axis.



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The flux through the two ends is zero.

The flux through the side is (2πr l) E.

The total charge enclosed is q=λl.

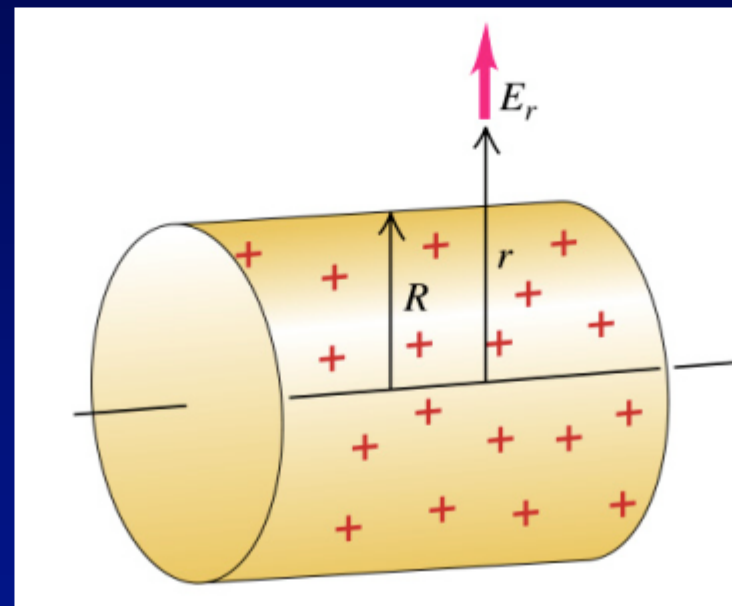
So by Gauss's law (2πr l) E= λl/ ε_0. Or

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r} = \frac{2k\lambda}{r}$$

Example: Field of a line tube charge

- Electric charge is distributed along a infinitely long, thin cylinder. The charge per unit length is l (assumed positive). Find the electric field around the cylinder.

This problem has cylindrical symmetry. The E field must be perpendicular to the line.

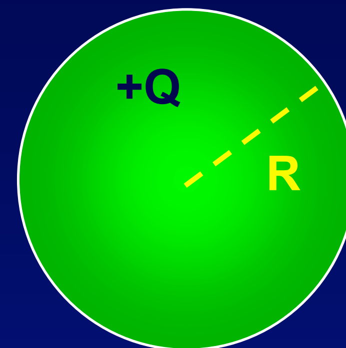


Inside: E=0

Outside
$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r} = \frac{2k\lambda}{r}$$

Example: Field of a uniformly charged sphere

- Positive electric charge Q is distributed uniformly throughout the volume of an insulating sphere with radius R. Find the E field both inside and outside of the sphere.



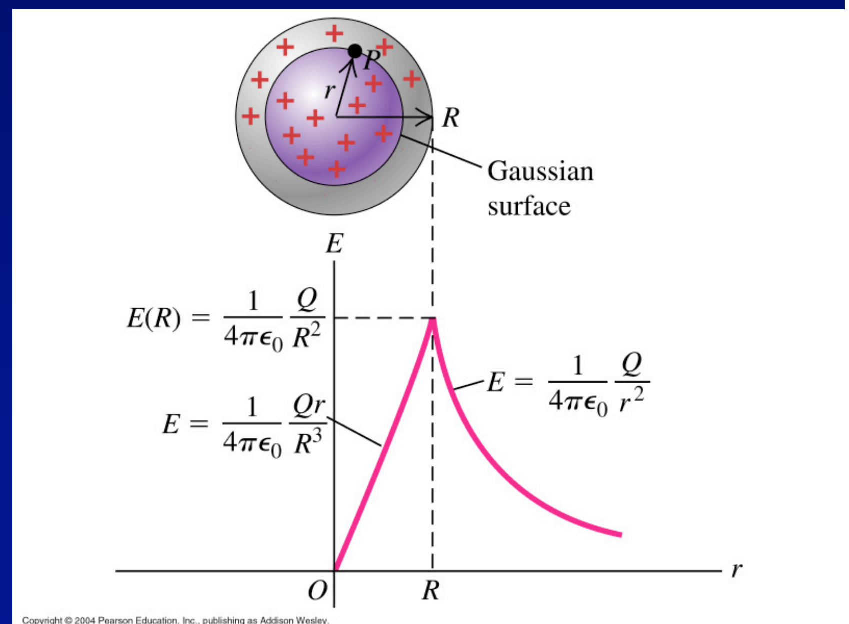
This problem has spherical symmetry. The E field must be radially outward. Choose Gaussian surface as a sphere.

Inside: $$4\pi r^2 E = \frac{q}{\varepsilon_0}$$

$$q = \left(\frac{Q}{\frac{4}{3}\pi R^3} \right) \left(\frac{4}{3}\pi r^3 \right) = Q\frac{r^3}{R^3}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}$$

Outside like a point charge: $$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$



Example: Concentric Spherical Shells

- If the inner shell has total charge +q, find the E field everywhere.

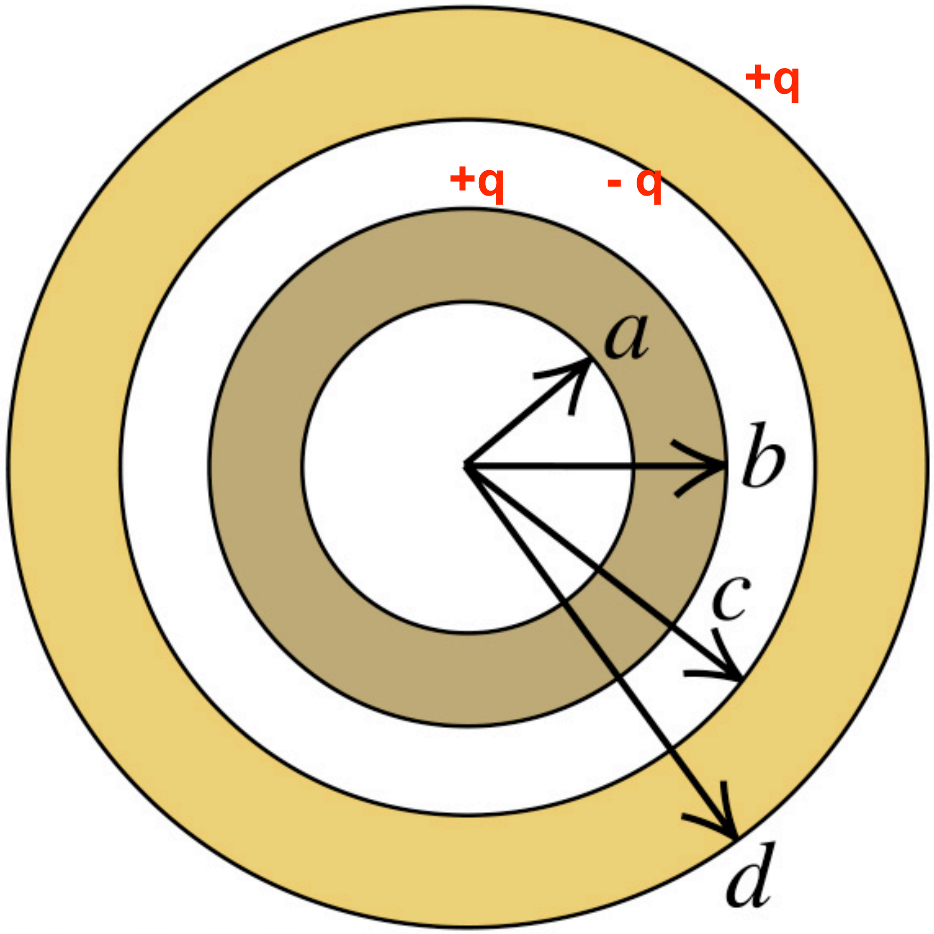
r<a, E=0

a<r<b, E=0

b<r<c, $E=kq/r^2$

c<r<d, E=0

r>d, $E=kq/r^2$



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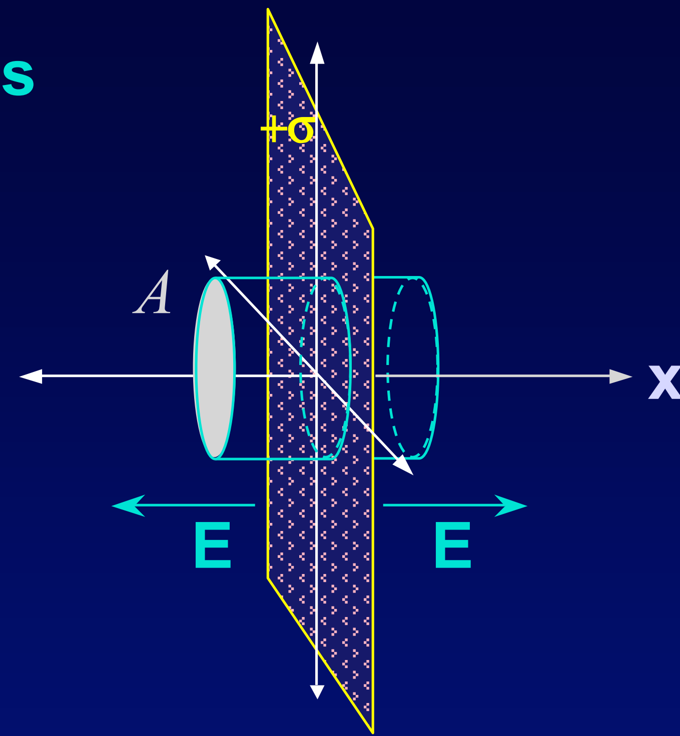
Example : Infinite sheet of charge

- **Symmetry: direction of E is normal to the sheet**



- Therefore, CHOOSE Gaussian surface to be a cylinder whose axis is aligned with the x-axis.

- **Apply Gauss' Law:**
  - On the barrel, flux is zero.
  - On the ends, $\oint \vec{E} \bullet d\vec{S} = 2AE$
  - The charge enclosed = σA

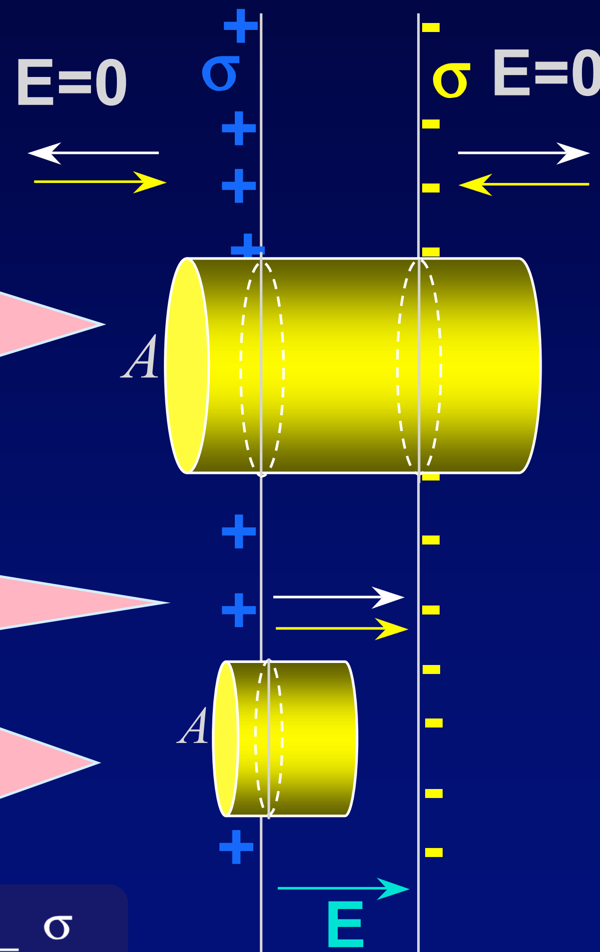
Therefore, Gauss' Law $\Longrightarrow$ $2EA = \sigma A / \varepsilon_0$ $\Rightarrow$ $\boxed{E = \dfrac{\sigma}{2\varepsilon_0}}$

Conclusion: An infinite plane sheet of charge creates a CONSTANT electric field .

Example: Two Infinite Sheets

(into screen)

- **Field outside the sheets must be zero. Two ways to see:**
  - **Superposition**
  - **Gaussian surface encloses zero charge**
- **Field inside sheets is NOT zero:**
  - **Superposition**
  - **Gaussian surface encloses non-zero charge**



$$Q = \sigma A$$

$$\oint \vec{E} \bullet d\vec{S} = \cancelto{0}{AE_{outside}} + AE_{inside}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

Statements about Gauss's Law (true or false?)

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = Q_{enclosed} / \varepsilon_0$$

- If the electric flux through a closed surface is zero, the electric field must be zero at all points on the surface. **(false)**
- The electric field in Gauss's law is only due to the charge enclosed in the surface. **(false)**
- If the electric field is zero at all points on the surface, there must be no net charge within the surface. **(true)**
- If a surface encloses zero net charge, the electric field must be zero at all points on the surface. **(false)**