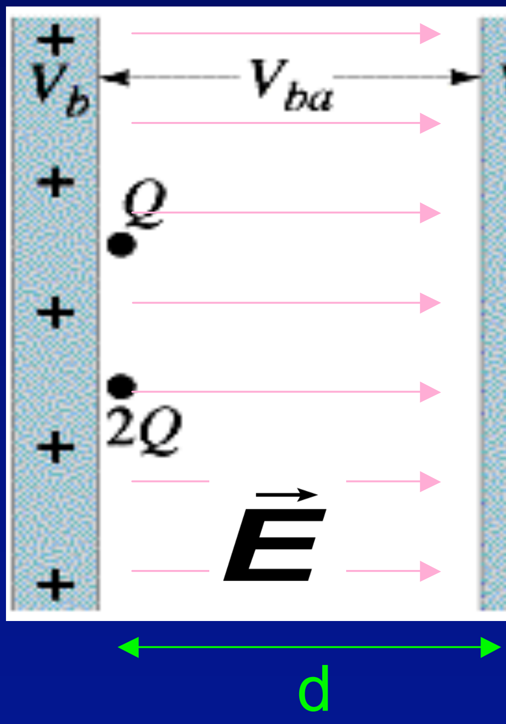
Chapter 23

Electric Potential

- Electric potential energy
- Electric potential
  - Point charges
  - Charge distributions



Electric Potential Energy

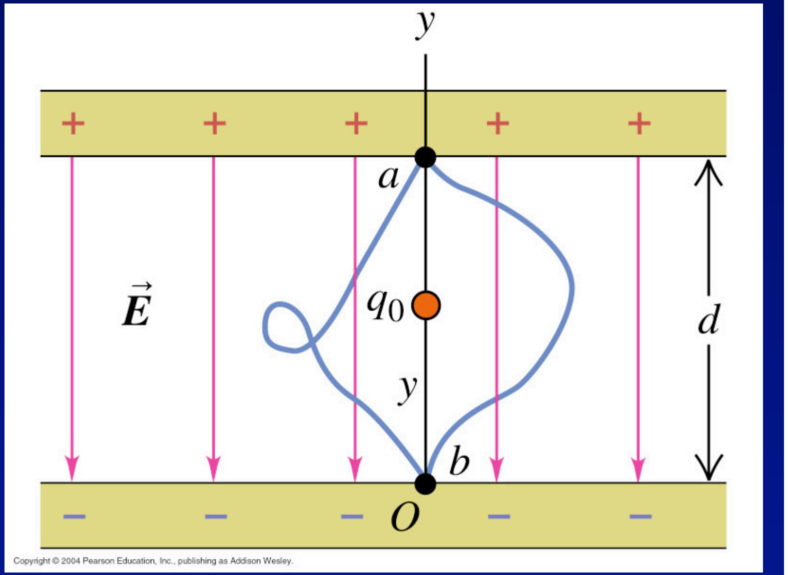
New Topic

Conservative Forces and Potential Energy

- Conservative force
  - Work done by a conservative force does not depend on the path. It only depends on the initial and final points.

- Define potential energy function U via the work done by a conservative force F in the following way:

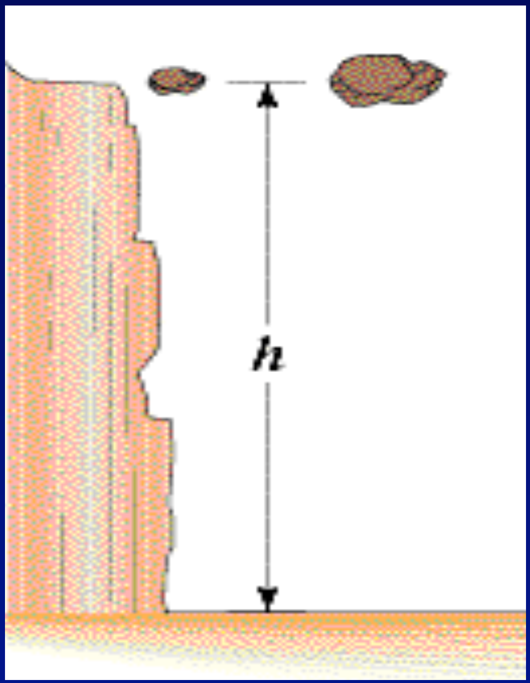
$$W = \int_1^2 \vec{F} \cdot d\vec{l} = U_1 - U_2 = -\Delta U$$



The work done by a conservative force is equal to the decrease in the potential energy function.

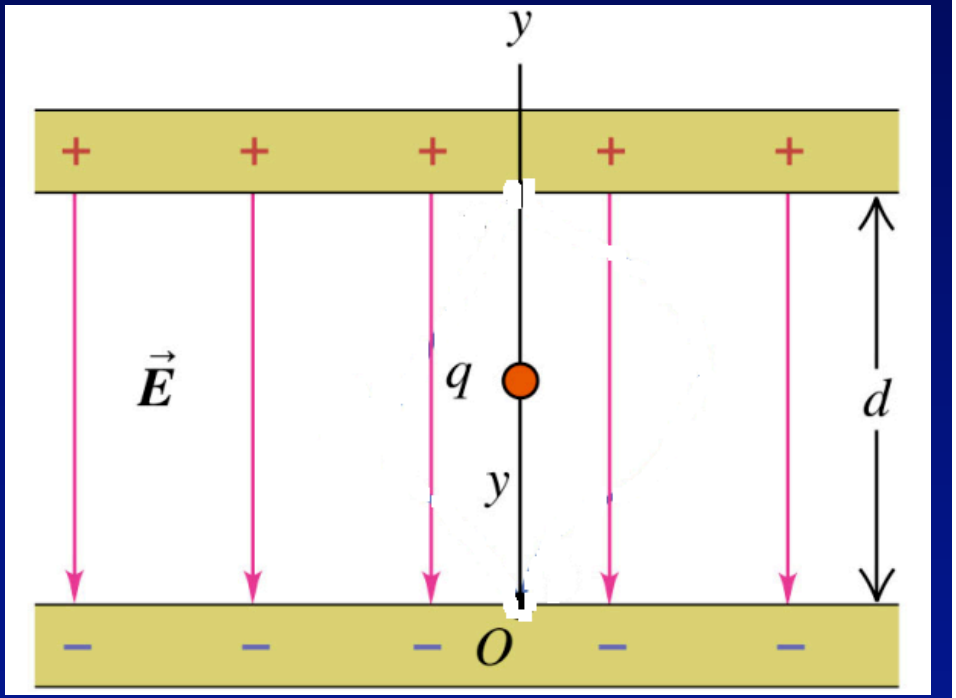
Gravitational and Electric Potential Energy

Gravity



$$\Delta PE_{grav} = mgh$$

Electric



$$\Delta PE_{elec} = qEy$$

(*work = force × displacement*)

Examples of Conservative Forces and Potential Energies

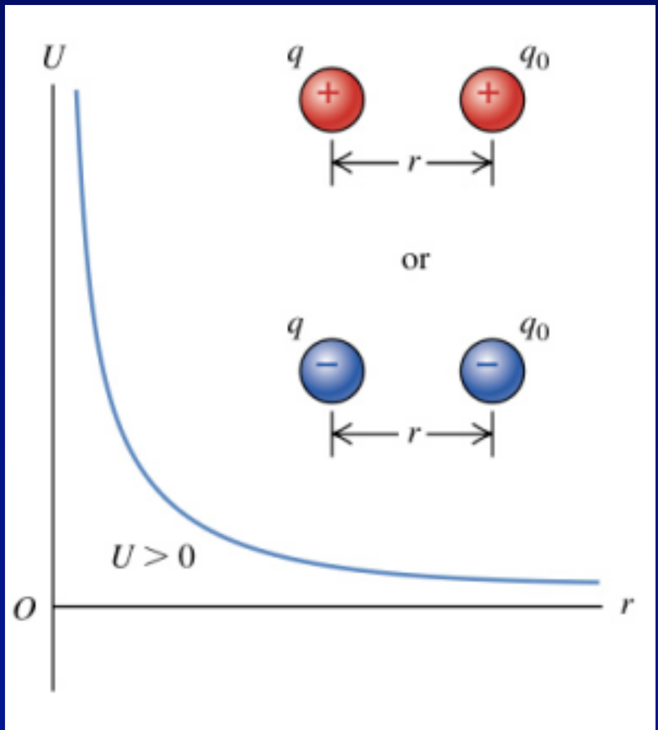
Force F	Work done $W_{1\to 2}$	Change in potential energy $\Delta U = U_2 - U_1$	Potential energy function U
$\vec{F} = -mg\,\hat{j}$	$-mg(y_2 - y_1)$	$mg(y_2 - y_1)$	mgy
$\vec{F} = -G\dfrac{m_1 m_2}{r^2}\hat{r}$	$Gm_1m_2\left\{\dfrac{1}{r_2} - \dfrac{1}{r_1}\right\}$	$-Gm_1m_2\left\{\dfrac{1}{r_2} - \dfrac{1}{r_1}\right\}$	$-\dfrac{Gm_1m_2}{r}$
$\vec{F} = -kx\,\hat{i}$	$-\dfrac{1}{2}k\left\{x_2^2 - x_1^2\right\}$	$\dfrac{1}{2}k\left\{x_2^2 - x_1^2\right\}$	$\dfrac{1}{2}kx^2$
$\vec{F} = q\vec{E}$	$-qE(y_2 - y_1)$	$qE(y_2 - y_1)$	qEy
$\vec{F} = k\dfrac{q_1q_2}{r^2}\,\hat{\mathrm{r}}$	$-kq_1q_2\left(\dfrac{1}{r_2} - \dfrac{1}{r_1}\right)$	$kq_1q_2\left(\dfrac{1}{r_2} - \dfrac{1}{r_1}\right)$	$k\dfrac{q_1q_2}{r}$

Potential energy is a relative concept: only the difference in potential energy is meaningful.

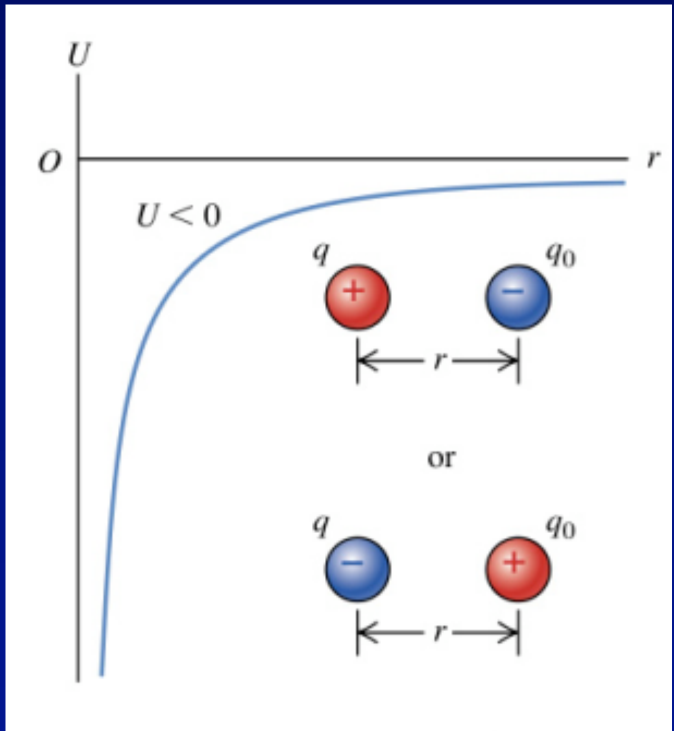
Electric Potential Energy between Two Point Charges

$$U = k\frac{q_1 q_2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

- It depends on the sign of the charges.



(a) q, q_0 have same sign



(b) q, q_0 have opposite signs

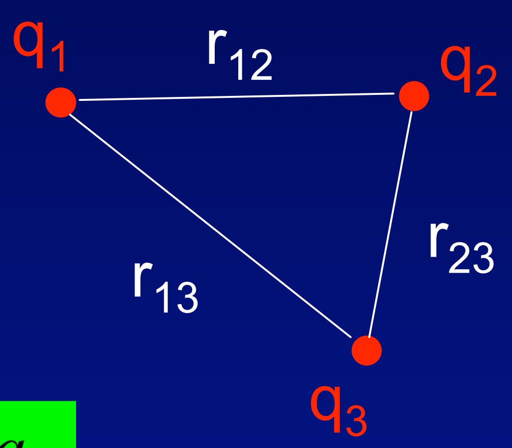
Relative to what point is this potential energy defined?

Electric Potential Energy in a System of Charges

For a pair of charges

$$U = k\frac{q_1 q_2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

For a system of 3 charges



$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

Generalization to N charges

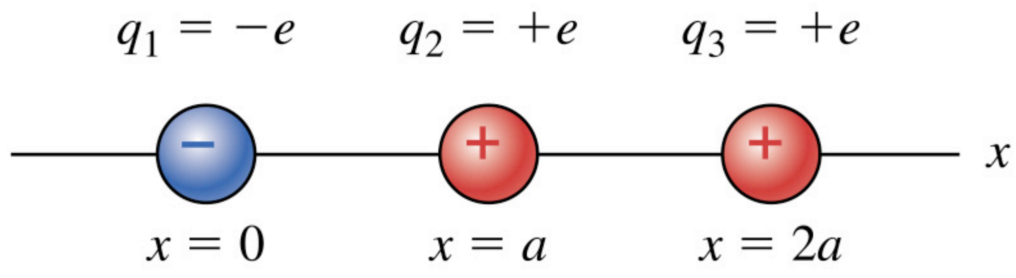
$$U = \frac{1}{4\pi\varepsilon_0}\sum_{i<j}^{N}\frac{q_i q_j}{r_{ij}}$$

Avoid double-counting. Watch for sign of charges.

Physical meaning of U: the total work done by us to assemble the charges from infinity to the present configuration.

Example: 3 charges

- How much work was done to assemble the 3 charges into the current positions?



$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

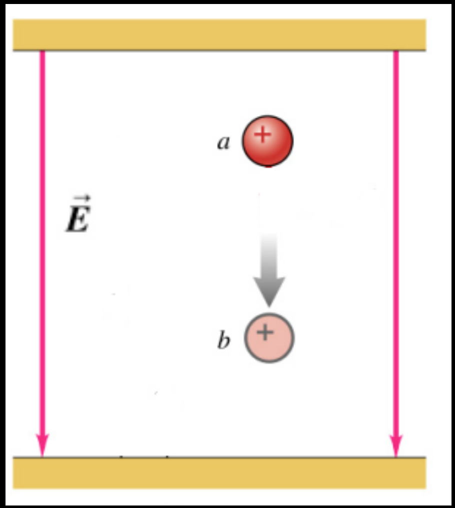
ConcepTest 23.1 Electric potential energy

A positive charge is moved from a to b in a uniform electric field as shown. What happens to the potential energy of the charge?

(a) increase

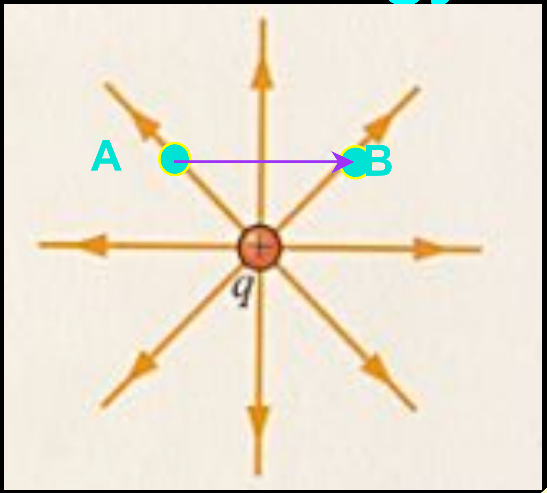
(b) decrease

(c) stays the same



ConcepTest 23.2 Electric potential energy

A negative charge is moved from A to B in the field of a positive charge along the path shown (A and B have equal distance to the charge). What is the net work done by us?



(a) $W_{AB} < 0$
(b) $W_{AB} = 0$
(c) $W_{AB} > 0$

Electric Potential

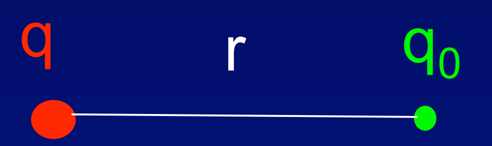
New Topic

The Concept of Electric Potential

- Definition: potential is defined as potential energy per unit charge: $$V = \frac{U}{q_0}$$
- SI unit: volt.   1 V = 1 J/C

For example: the potential energy between two point charges

$$U = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$$



Therefore, the potential of point charge q is

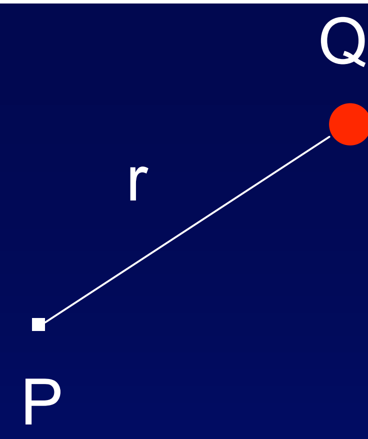
$$V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

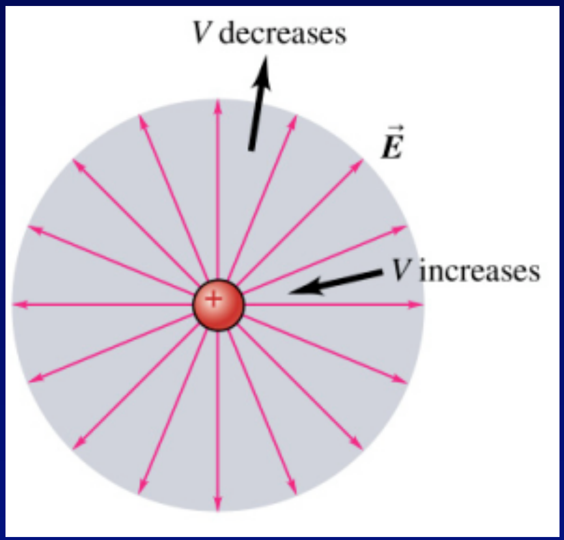
For a system of point charges: $$V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$ (algebraic sum)

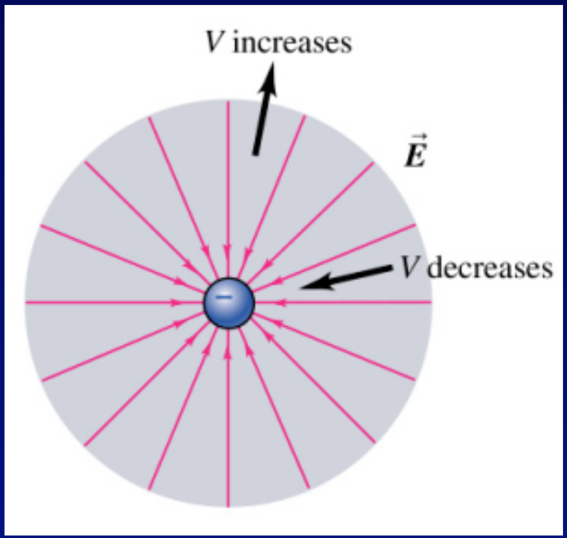
Potential is a scalar. There’s no direction to worry about.
But you do have to watch about the sign of charges.

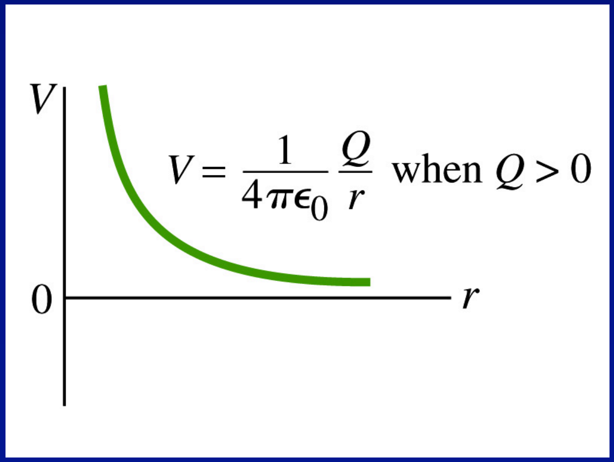
Example: a point charge

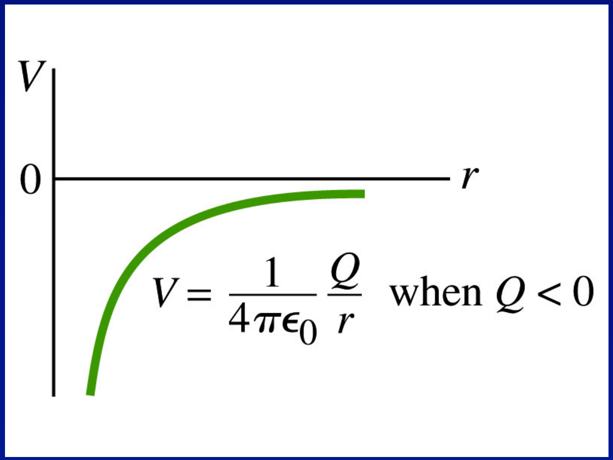
$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$







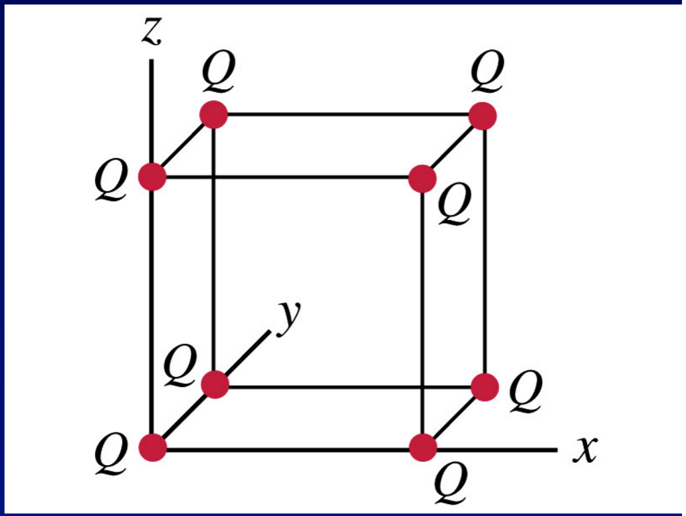




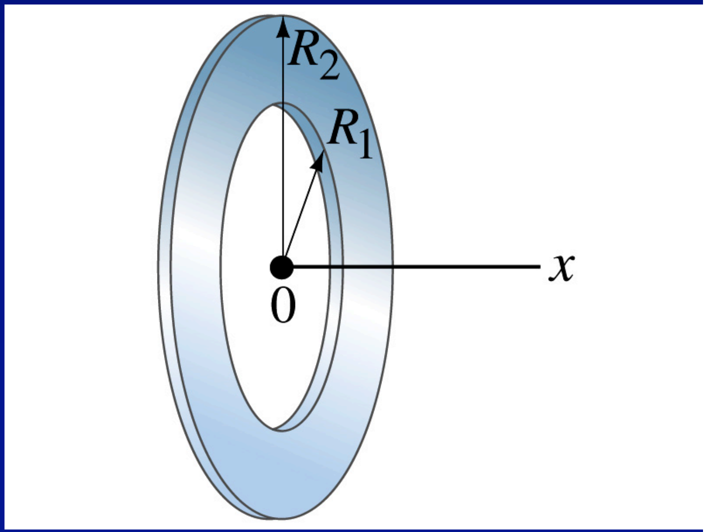
Relative to which point is this potential defined? infinity

Electric Potential
from
Continuous Charge Distributions

$$\text{discrete}: \quad V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{Q_i}{r_i}$$



$$\text{continuous}: V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$



Example 23-11: A ring of charge

- A ring of radius R carries a total charge Q distributed uniformly around it. Determine the electric potential at a point on its axis.

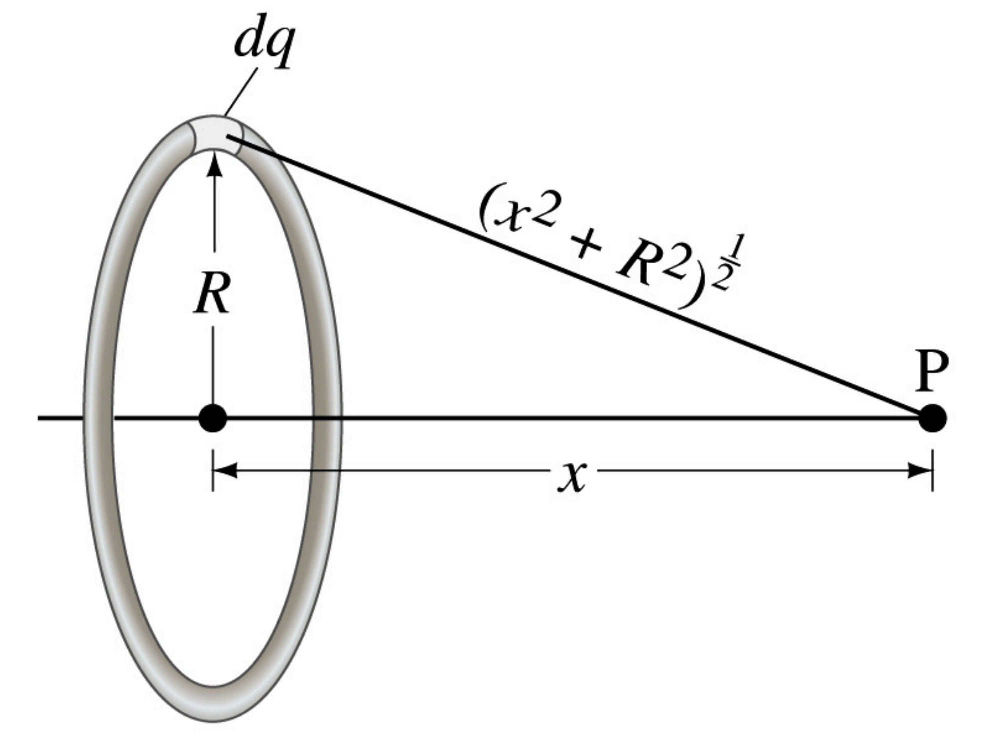
Add up contributions from small segments :

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{1}{r}\int dq = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(x^2+R^2)^{1/2}}$$

Special cases:

1) At $x = 0$, $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R}$

2) At $x >> R$, $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{x}$



ConcepTest 23.3 Electric field and potential

- Four point charges are arranged at the corners of a square. Find the **electric field E** and the **potential V** at the **center of the square**.

(1) $E = 0$, $V = 0$

(2) $E = 0$, $V \neq 0$

(3) $E \neq 0$, $V \neq 0$

(4) $E \neq 0$, $V = 0$

(5) $E = V$ regardless of the value