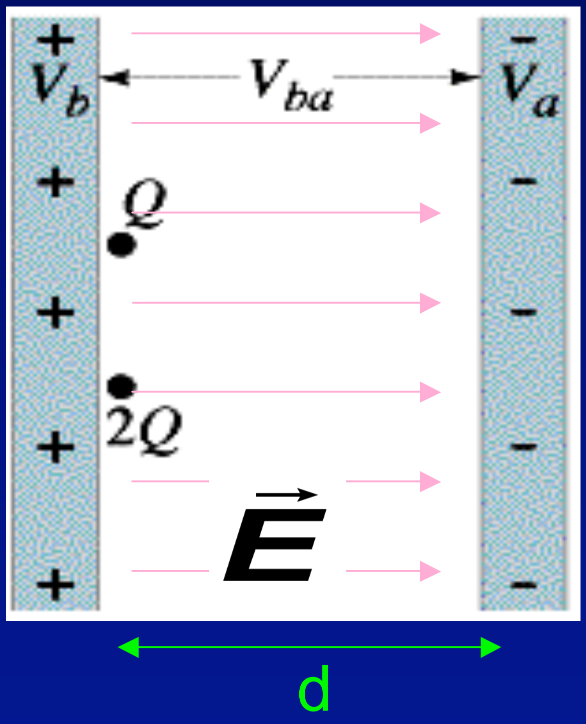
Chapter 23
(part II)

Electric Potential

- Equipotential Surfaces
- Relation between electric field and electric potential
  - How to get V from E?
  - How to get E from V?



Equipotential Surfaces

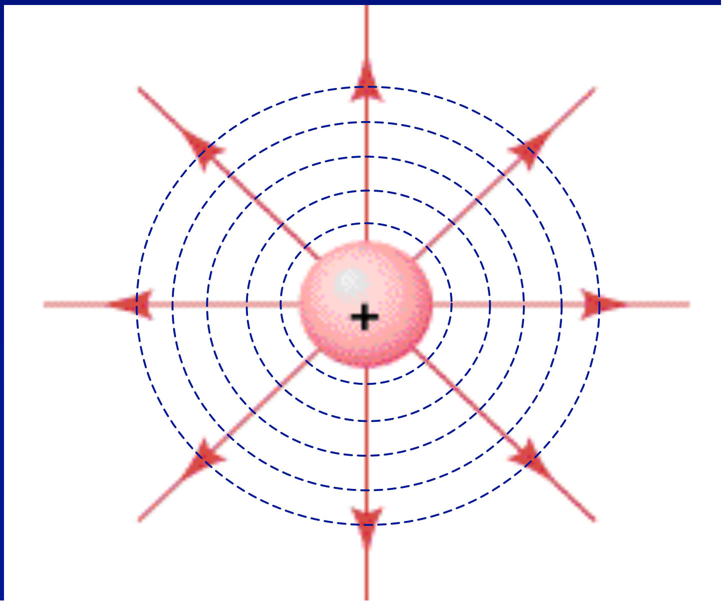
New Topic

Equipotential Surfaces

- contours of **constant** potential
  - all points on the contour have the **same** value of **V**

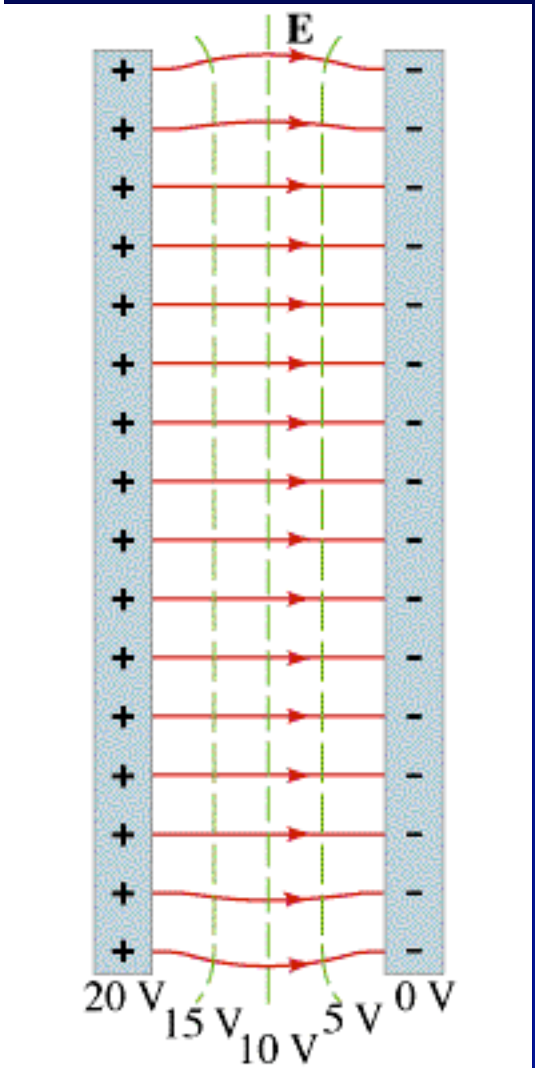
- **no work** is required to move charge along an equipotential surface
  - $W = \Delta PE = q(V_B - V_A) = 0$ if $V_B = V_A$

- $\vec{E} \perp$ equipotential surface

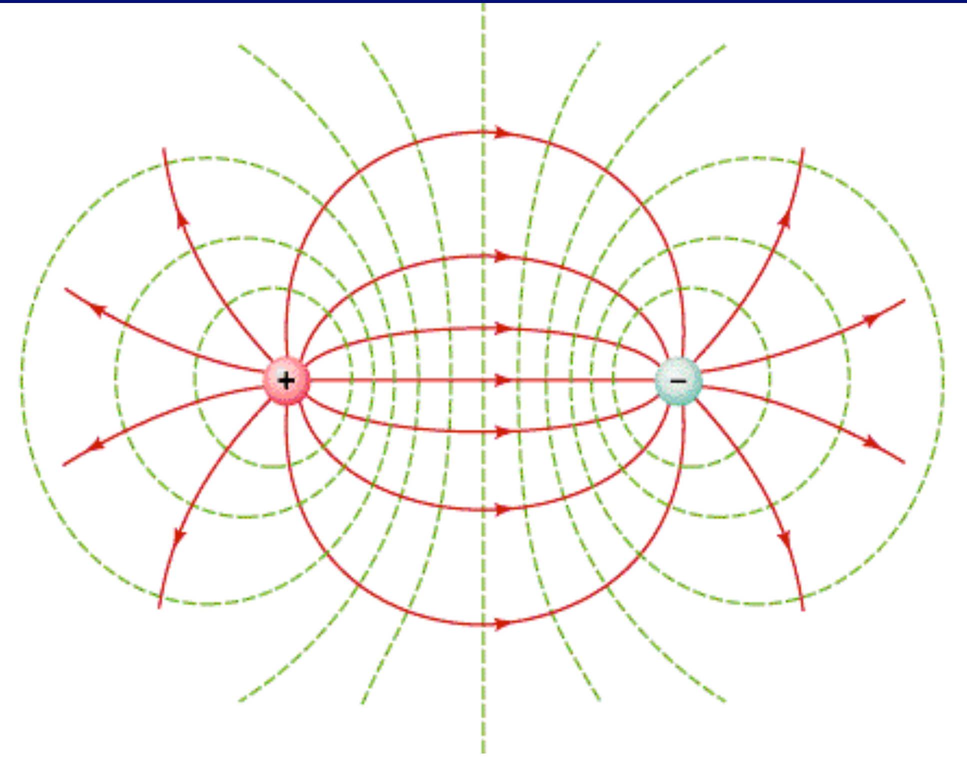


Where E field is <u>constant</u> (same distance between E field lines)

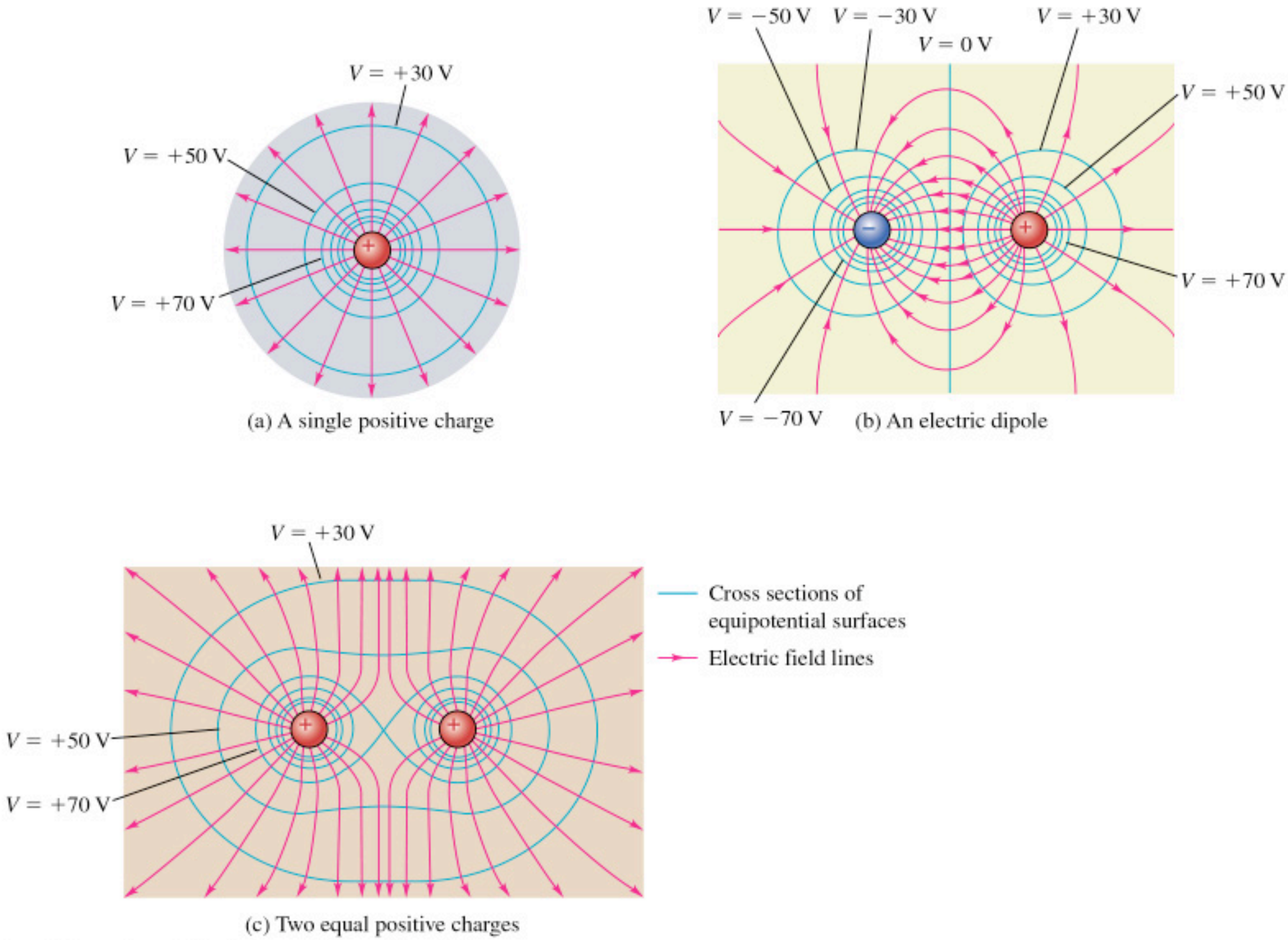
⇒ same distance is between equipotential lines



Where E field is stronger (E field lines closer together) ⇒ equipotential lines are closer together

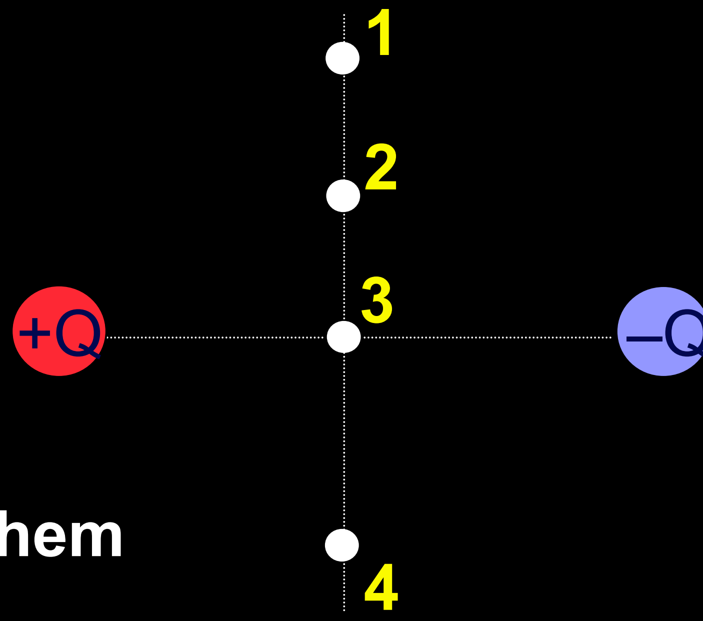


Electric field lines vs. electric potential lines



ConcepTest 23.4
Electric potential

- At which point is the electric potential V = 0?



(1)
(2)
(3)
(4)
(5) all of them

Relation between
Electric Field
and
Electric Potential

New Topic

How to get V from $\vec{E}$?

$$V_b - V_a = \frac{U_b - U_a}{q_0} = -\frac{W_{ab}}{q_0} = -\frac{\int_a^b \vec{F}\cdot d\vec{l}}{q_0} = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

€

Potential difference is equal to the line integration over field over any path that connects the two points.

(In practice, pick the line that makes the integral the easiest to do)

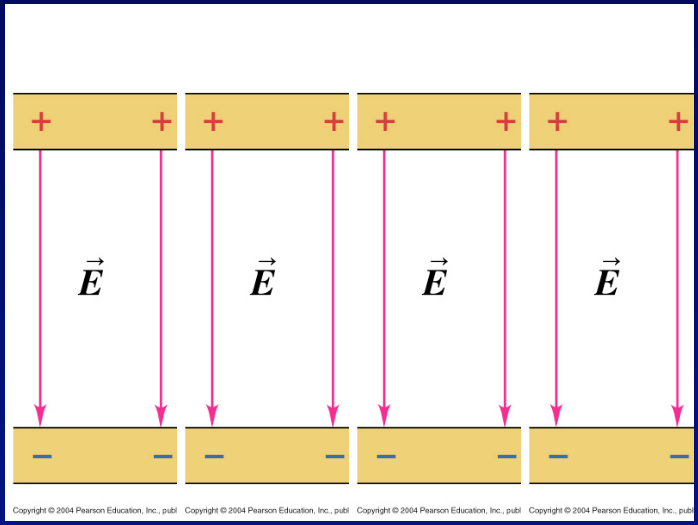
Another unit for electric field: volt / meter

1 V/m = 1 N/C

Example: potential difference between two parallel and oppositely-charged plates

- The field is constant between the plates. So the line integral is easy

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = Ed$$



- The electric field is E = σ/ε_0 so it provides a practical way to measure the surface charge density on the plate

$$\sigma = \frac{\varepsilon_0 (V_a - V_b)}{d}$$

Example: potential of a point charge

- The field is radial, with a magnitude

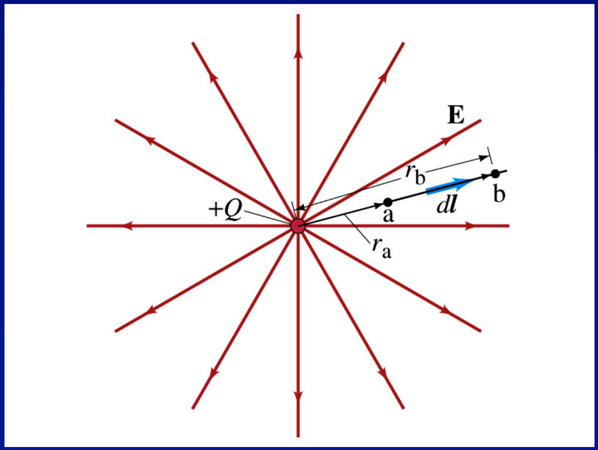
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

- Choose the line along the normal line, then the line integral becomes

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E dr = \int_{r_a}^{r_b} \frac{Q}{4\pi\varepsilon_0}\frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

- Choose reference point: V_b=0 at r_b=infinity.

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$



Example: potential of an infinite line charge

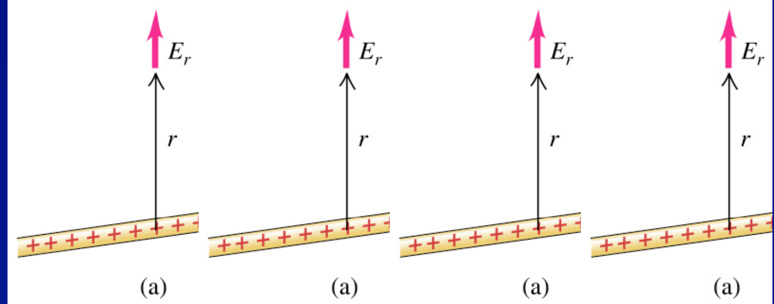
- We know the field is $2k\lambda/r$. So the line integral is easy

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\,dr = \int_{r_a}^{r_b} \frac{\lambda}{2\pi\varepsilon_0}\frac{dr}{r} = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_b}{r_a}$$

$$V_a - V_b = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_b}{r_a}$$

- This is also the potential of an infinite long cylinder.

€



How to get $\vec{E}$ from V ?

- We can obtain the electric field E from the potential V by inverting our previous relation between E and V:

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$$

$$-dV = \vec{E} \cdot d\vec{l} = E_x dx + E_y dy + E_z dz$$

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

- Expressed as a vector, E is the negative gradient of V

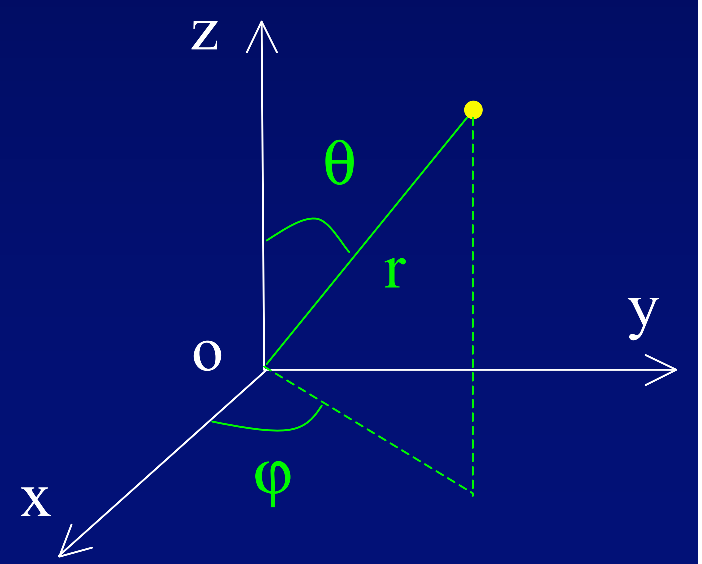
$$\vec{E} = -\vec{\nabla} V$$

Cartesian coordinates:

$$\vec{\nabla} \mathrm{V} = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$$

Spherical coordinates:

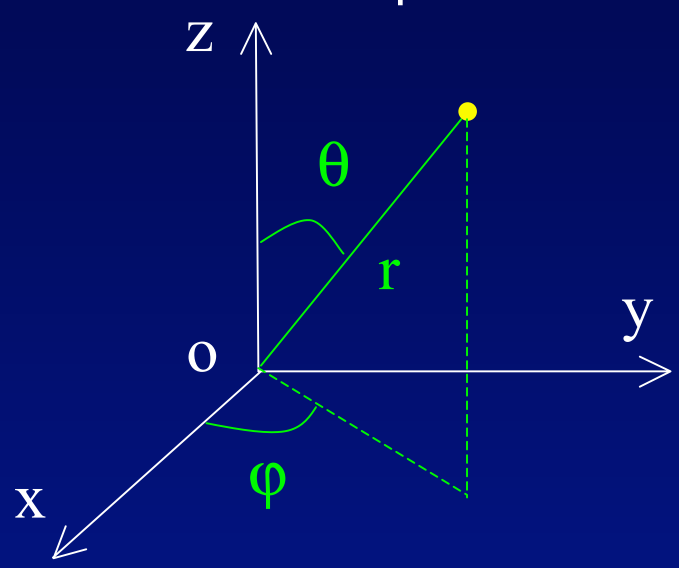
$$\vec{\nabla} \mathrm{V} = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$$



Example: A point charge

- The electric potential depends only on the distance from the point charge:

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

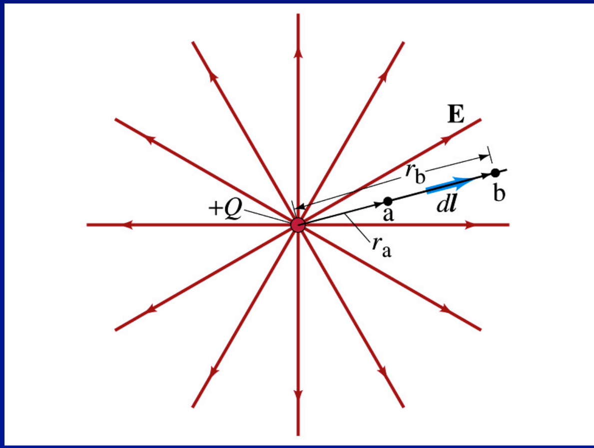


In spherical coordinates (r,θ,φ):

$$E_r = -\frac{\partial V}{\partial r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

No angular dependence, so

$$E_\theta = E_\varphi = 0$$



Example 23-11: A ring of charge

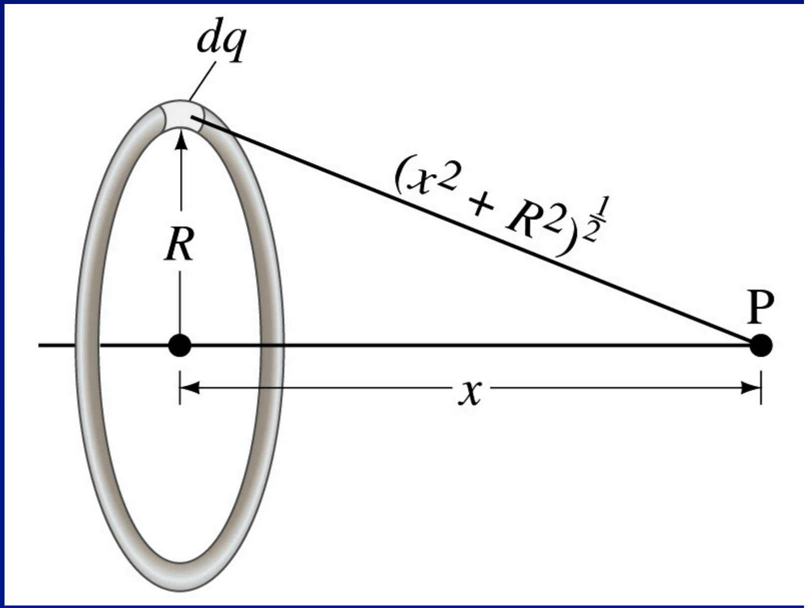
- A ring of radius R carries a total charge Q distributed uniformly around it. The electric potential at a point on its axis is

$$V(x,y,z) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(x^2+R^2)^{1/2}}$$

$$E_x = -\frac{\partial V}{\partial x} = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(x^2+R^2)^{3/2}}$$

$$E_y = 0$$

$$E_z = 0$$

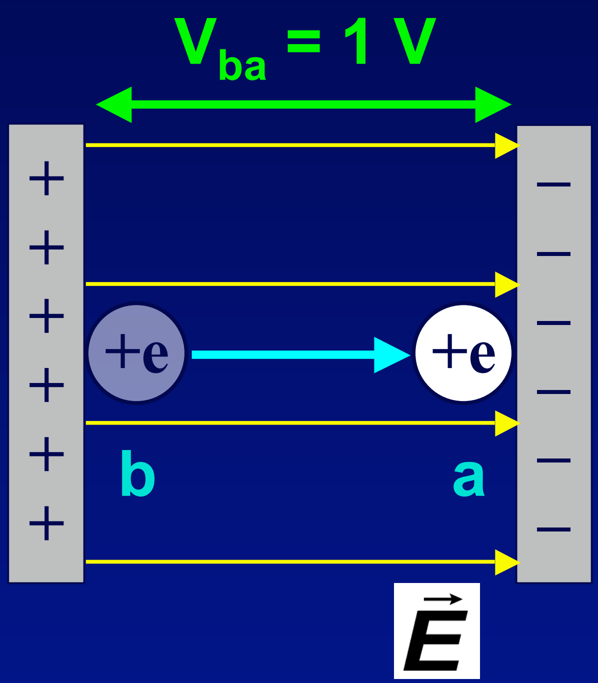


We obtained the same result as that with vector sums of electric fields. But it's much easier here.

The electron volt as an unit for energy

- The kinetic energy acquired by a particle carrying a charge **e**, moving across a potential difference of **1 V.**
  - by conservation of energy:

    KE gained = PE lost



- KE = q V_{ba}
- So 1 eV = 1.6×10^{-19} Joules

For electron, $\frac{1}{2} m_e v^2 = 1\ eV$ gives

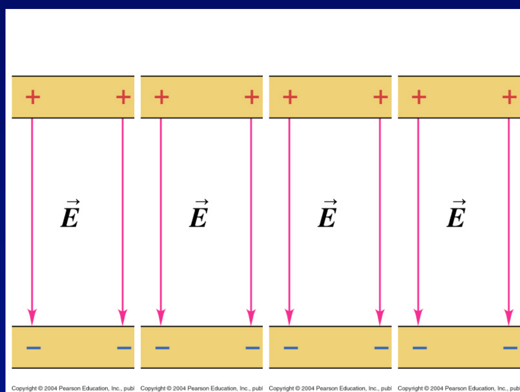
$$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 5.9 \times 10^5 \text{ m/s}$$

For proton, $\frac{1}{2} m_p v^2 = 1\ eV$ gives

$$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}}} = 1.4 \times 10^4 \text{ m/s}$$

Summary of fields:

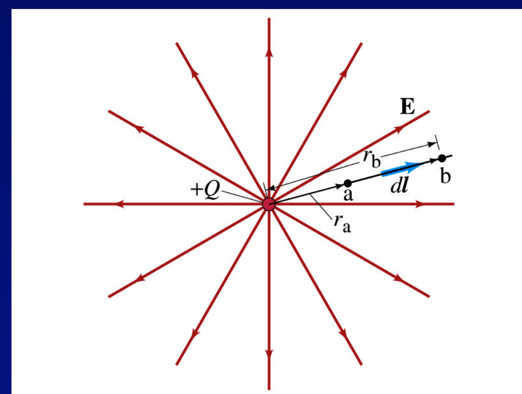
Constant field:



$$\vec{E} = \text{constant}$$

$$V_{ab} = Ed$$

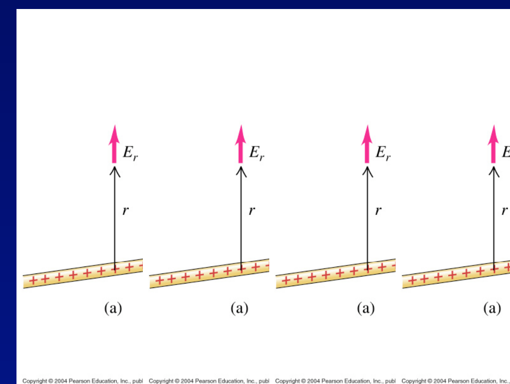
Point charge:



$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r}$$

$$V(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$$

Line charge:

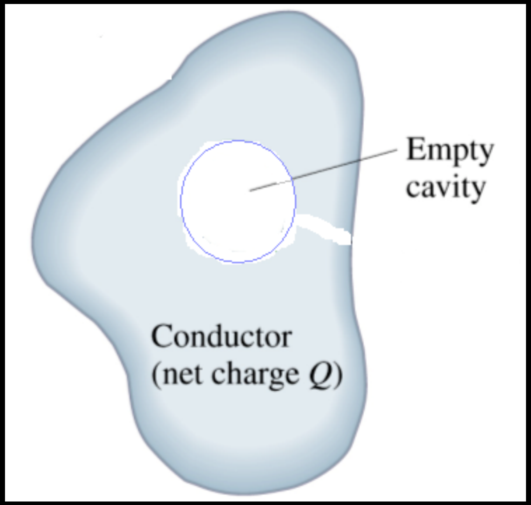


$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0} \frac{1}{r} \hat{r}$$

$$V_{ab} = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{r_b}{r_a}$$

ConcepTest 23.5 Electric potential

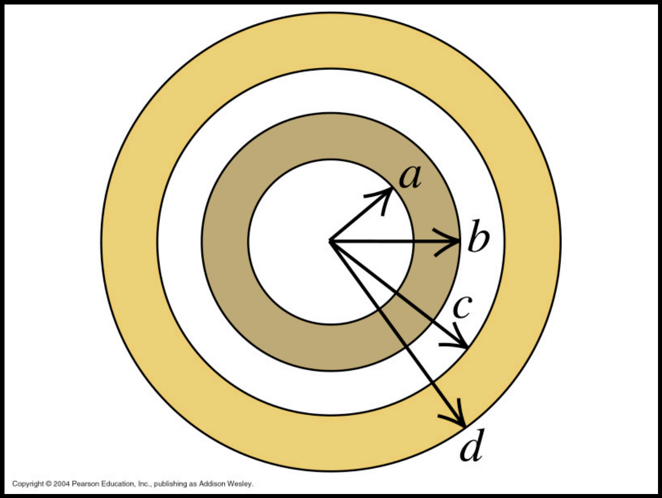
A conductor carries a net charge +Q and has a cavity inside. What is the potential difference between the inner and outer surfaces?



(a) Inner surface has higher potential
(b) Outer surface has higher potential
(c) They have the same potential

ConcepTest 23.6 Electric potential

The inner shell of the two conducting, concentric shells carries charge +q. How does the electric potential compare on the surfaces?



1) $V_a > V_b > V_c > V_d$
2) $V_a < V_b < V_c < V_d$
3) $V_a = V_b > V_c = V_d$
4) $V_a = V_b < V_c = V_d$
5) $V_a = V_b = V_c = V_d$
6) None of these