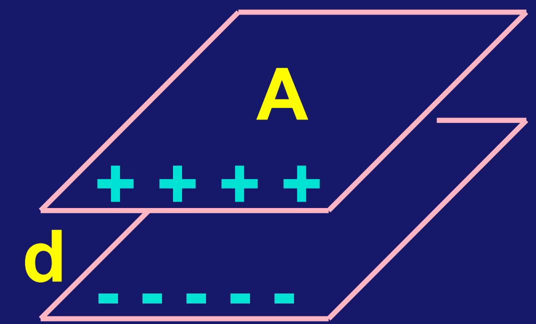
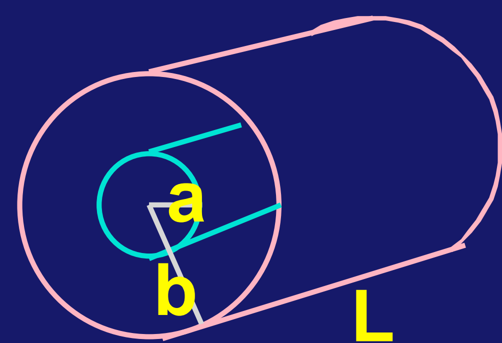
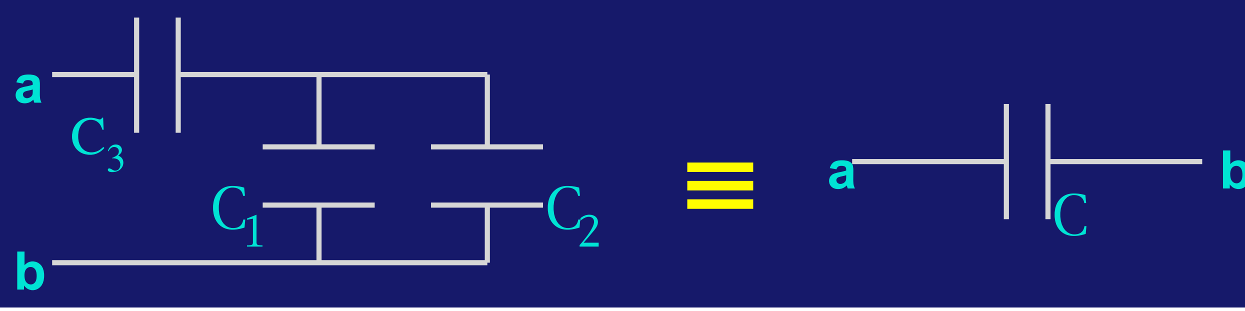
Chapter 24: Capacitance and Dielectrics

$$C \equiv \frac{Q}{V}$$

- Definition of Capacitance
- Example Calculations
  - Parallel Plate Capacitor
  - Cylindrical Capacitor
  - Spherical Capacitor
- Combinations of Capacitors
  - Capacitors in Parallel
  - Capacitors in Series
- Energy in Capacitors
- Dielectrics





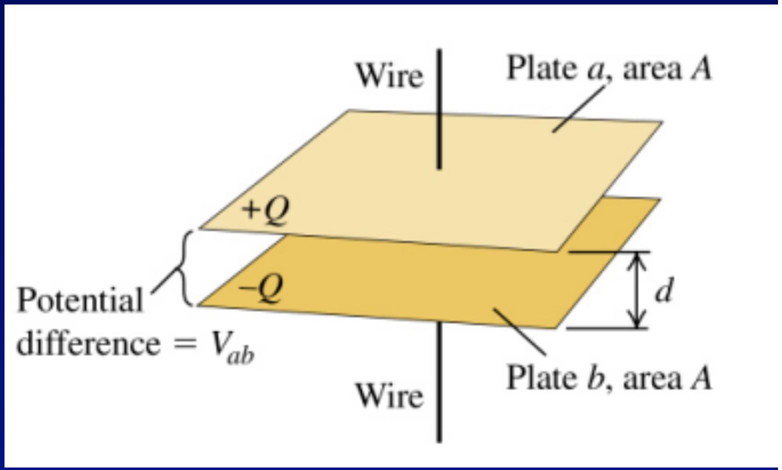


Capacitors

New Topic

Capacitors

- device to store charge and energy
- connect capacitor to battery (V)
  - plates become charged (Q)



charge ∝ potential difference

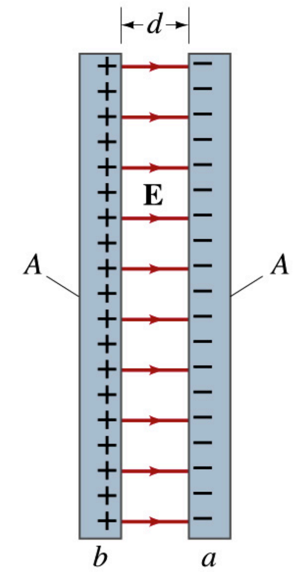
$$Q = C\,V$$

- C is called *capacitance*
  - units: coulomb / volt ≡ Farad
  - larger C ⇒ bigger Q (fixed V) ("capacity" to hold charge)

Example: Parallel Plate (the simplest capacitor)

- The electric field in between is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$



The potential difference is

$$V = Ed = \left(\frac{d}{\varepsilon_0 A}\right) Q$$

€

Thus from $C = \frac{Q}{V}$, we get

$$C = \varepsilon_0 \frac{A}{d}$$

Capacitance only depends on the geometry.

ε_0 = 8.85 x 10^{-12} $C^2/N\text{-}m^2$ (permittivity of free space)

Example: Cylindrical Capacitor

- Determine the capacitance for the cylindrical capacitor shown.

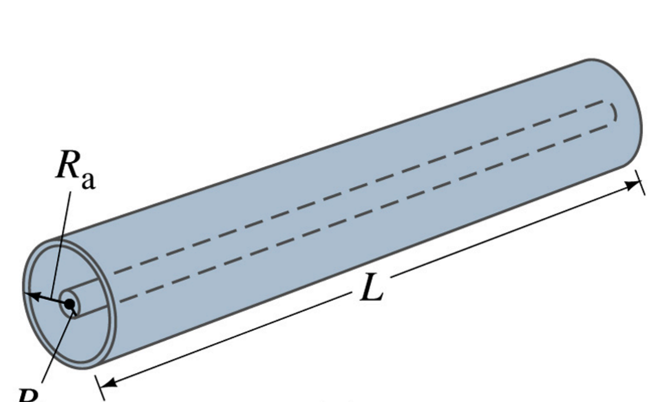
We want to use C=Q/V. What is V?

Outside: E=0 (why?)

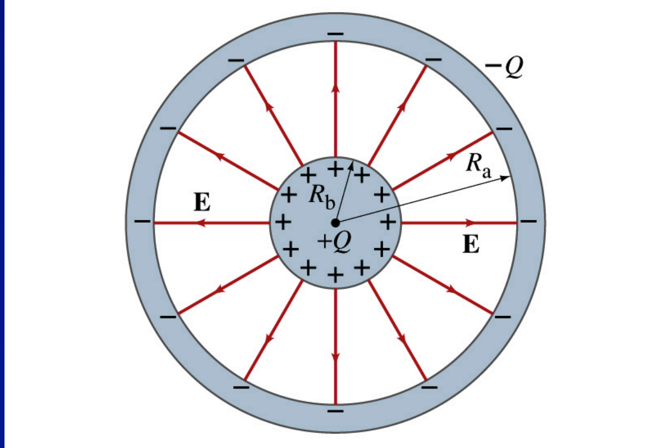
$$\text{In between}: E = \frac{1}{2\pi\varepsilon_0}\frac{Q/L}{r}$$

$$V = \frac{Q/L}{2\pi\varepsilon_0}\ln\frac{R_a}{R_b}$$

$$C = \frac{2\pi\varepsilon_0 L}{\ln(R_a/R_b)}$$



(a)



(b)

Example: Spherical Capacitor

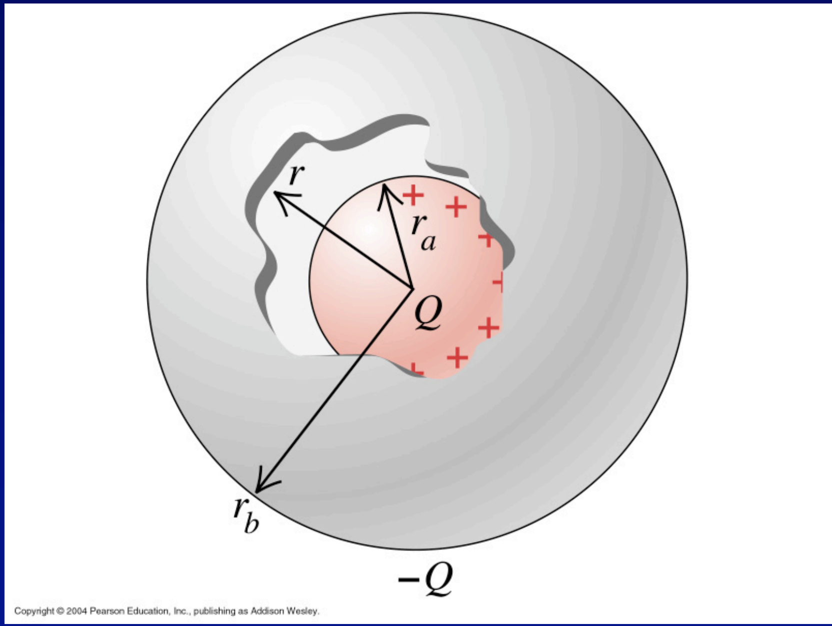
- Determine the capacitance for the spherical capacitor shown.

We want to use C=Q/V. What is V?

Outside: E=0 (why?)

$$\text{In between}: E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

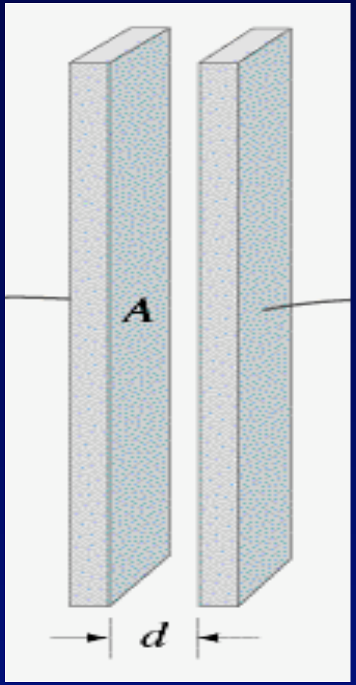


$$C = 4\pi\varepsilon_0 \frac{r_a r_b}{r_a - r_b}$$

Example: How big is 1 Farad?

- choose $d = 1$ mm
  - find the area A for such a capacitor

$$C = \varepsilon_0 \frac{A}{d}$$



$A = C\,(d/\varepsilon_0)$

$= (1\text{ F})\,(0.001\text{ m}) / (8.85\times10^{-12})$

$= 1.1 \times 10^{8}\text{ m}^2$ = 43 sq. miles!

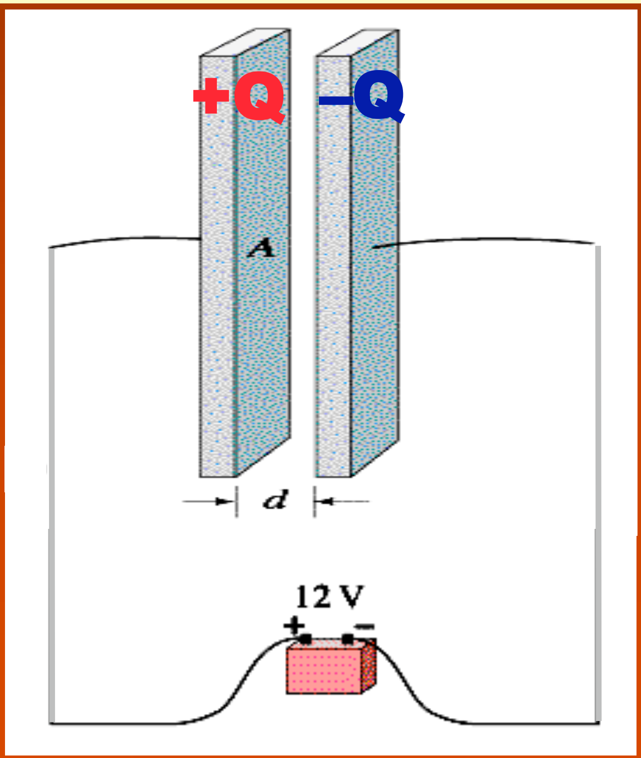
This capacitor is as big as a city!!!

Typical capacitance is in the range of pF to µF.

ConcepTest 24.1 Capacitors

- A parallel-plate capacitor initially has a potential difference of 400 V and is then disconnected from the charging battery. If the plate spacing is now doubled (without changing Q), what is the new value of the voltage?

1) 100 V
2) 200 V
3) 400 V
4) 800 V
5) 1600 V

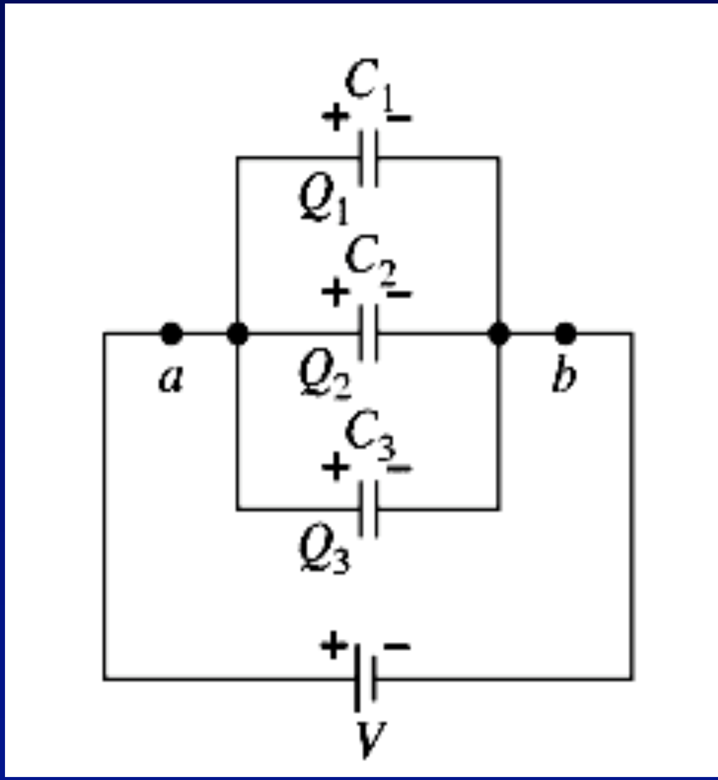


Combination of Capacitors

New Topic

Capacitors in parallel

- Potential difference between points *a* and *b* is the *same* for all 3 capacitors
  - $V_1 = V_2 = V_3 = V$



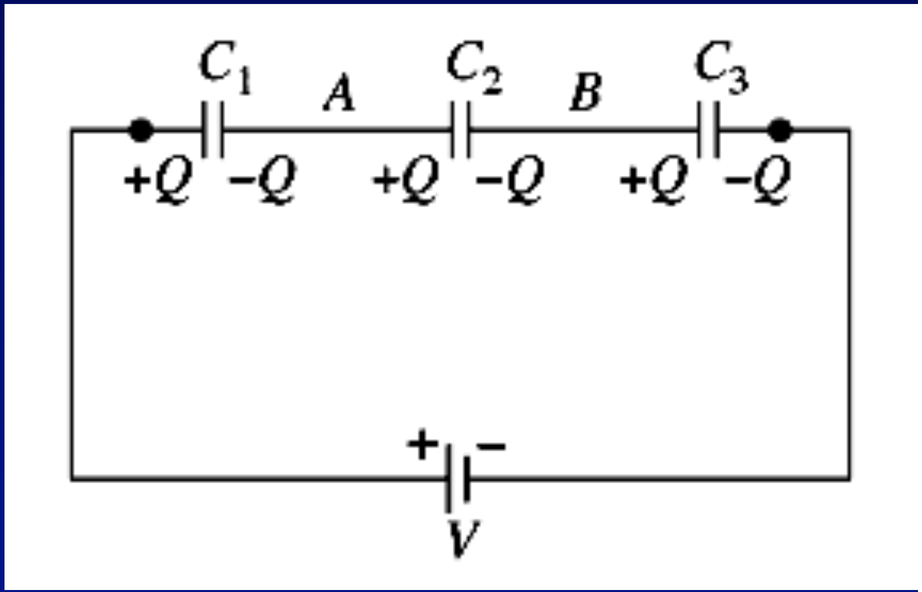
- However, charges add:
  - $Q_1 + Q_2 + Q_3 = Q$
- Since $Q = C V$, we have
  - $C_1V + C_2V + C_3V = CV$

$$C = C_1 + C_2 + C_3$$

C is called an equivalent capacitor.

Capacitors in series

- Each capacitor has to hold the *same charge:*
  - $Q_1 = Q_2 = Q_3 = Q$
- However, voltages add:
  - $V_1 + V_2 + V_3 = V$



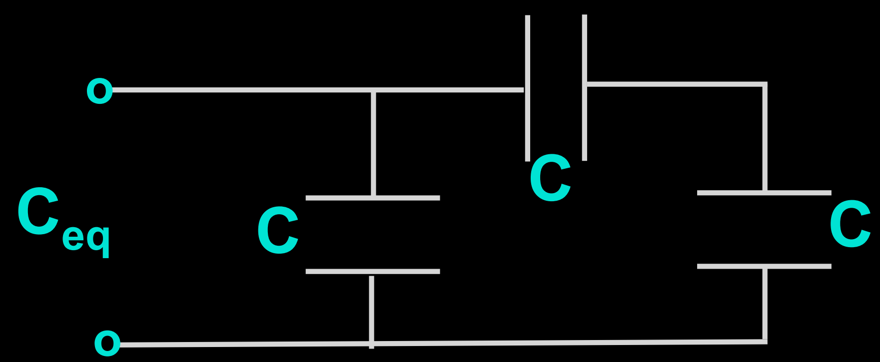
- Since $V = Q/C$, *we have*
  - $Q/C_1 + Q/C_2 + Q/C_3 = Q/C$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

ConcepTest 24.2 Capacitors

- **What is the equivalent capacitance, C_{eq}, of the combination shown?**

(a) $C_{eq} = (3/2)C$
(b) $C_{eq} = (2/3)C$
(c) $C_{eq} = 3C$



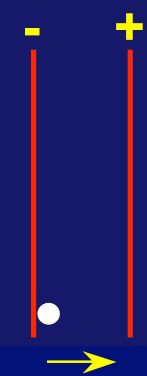
Electric Energy Storage

New Topic

Energy of a Capacitor

- How much energy is stored in a charged capacitor?
  - Calculate the work provided (usually by a battery) to charge a capacitor to +/- Q:

Calculate incremental work dW needed to add charge dq to capacitor at voltage V:



$$dW = dq(V) = dq\left(\frac{q}{C}\right)$$

- The total work W to charge to Q is then given by:

$$W = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\frac{Q^2}{C}$$

$$U = \frac{1}{2}\frac{Q^2}{C}$$

- Since Q=CV, we can write:

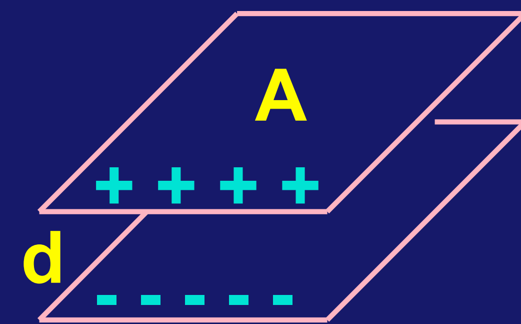
$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Where is the Energy Stored?

- **Claim:** **energy is stored in the Electric Field itself.** **Think of the energy needed to charge the capacitor as being the energy needed to create the field.**

- **To calculate the energy density in the field, first consider the constant field generated by a parallel plate capacitor:**

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2 Ad$$



- **The energy density ($u = U$ / volume) in the field is given by:**

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Units: $\frac{\mathrm{J}}{\mathrm{m}^3}$

Valid for any capacitor.

ConcepTest 24.3 Capacitors

Capacitors 1 and 2 have the same voltage, but capacitor 2 stores twice the charge of capacitor 1. If the energy stored in capacitor 1 is U, the energy stored in capacitor 2 is

(1) U/4

(2) U/2

(3) U

(4) 2U

(5) 4U

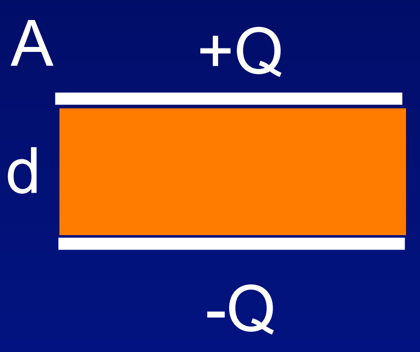
Dielectrics

New Topic

Dielectrics

- Experimental observation: When a piece of material fills the space in a capacitor, the potential difference decreases by a factor: $V=V_0/K$.

Since the charge on the capacitor remains the same, it means, according to $Q=CV$, that the capacitance increases by the same factor: $C=KC_0$.

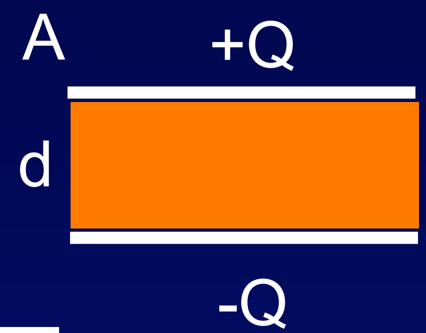


K is called dielectric constant

Material	Dielectric constant K	Dielectric strength (V/m)
Air	1.0006	3×10^6
Paper	3.7	15×10^6
Glass	5	14×10^6
Water	80	
Mica	7	150×10^6
Strontium titanate	300	8×10^6

- Benefits of dielectrics:
  - Increase capacitance ($C=KC_0$)
  - Higher voltage possible
  - Decrease spacing without touching (increase C)
  - Store more energy

Dielectrics



$$\text{permittivity of vacuum}: \varepsilon_0$$

$$\text{permittivity of dielectric}: \quad \varepsilon = K\varepsilon_0$$

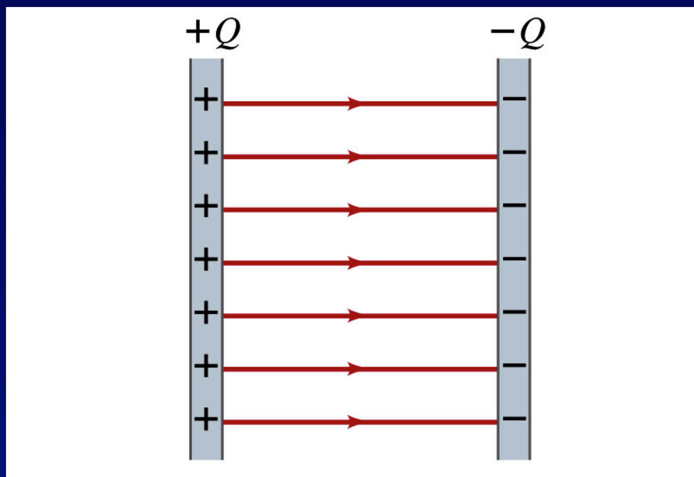
Replace everywhere: $\varepsilon_0 \Rightarrow \varepsilon = K\varepsilon_0$

For example: $C_0 = \varepsilon_0 \dfrac{A}{d} \Rightarrow C = \varepsilon \dfrac{A}{d} = KC_0$

For constant Q, electric field $E_0=\sigma/\varepsilon_0$. With dielectrics: $E=\sigma/\varepsilon = E_0/K$. Then $V=Ed=V_0/K$. Then $U=1/2\ CQ=K\ U_0$.

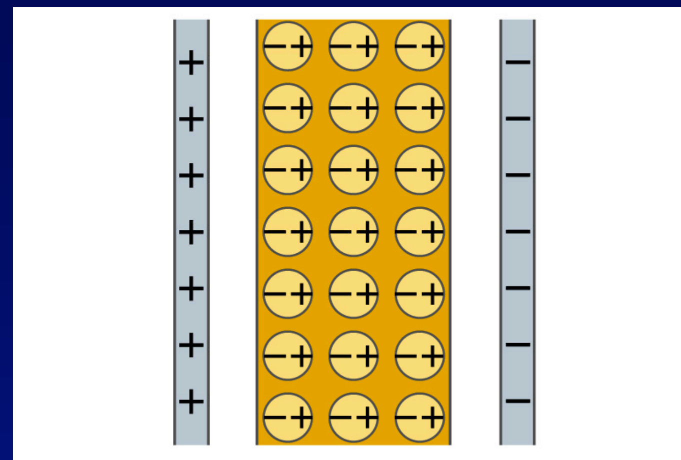
Molecular View of Dielectrics

Fixed charge capacitor

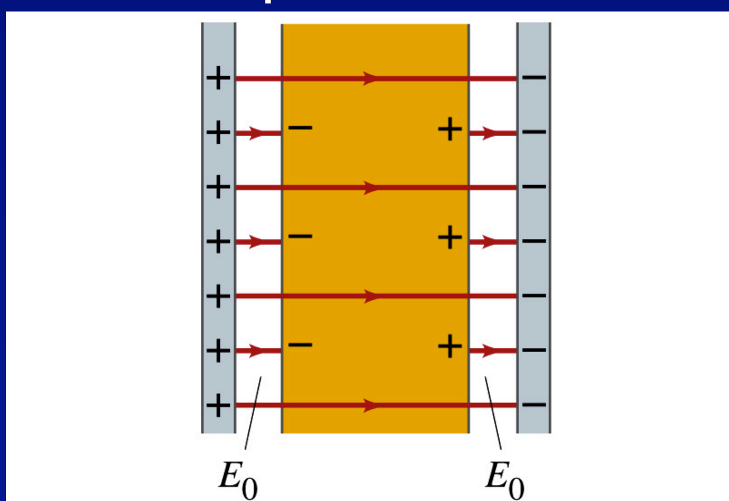


(a)

Polarization of molecules


(b)

Net polarization



(c)

How much induced charge?

$$E = \frac{E_0}{K}$$

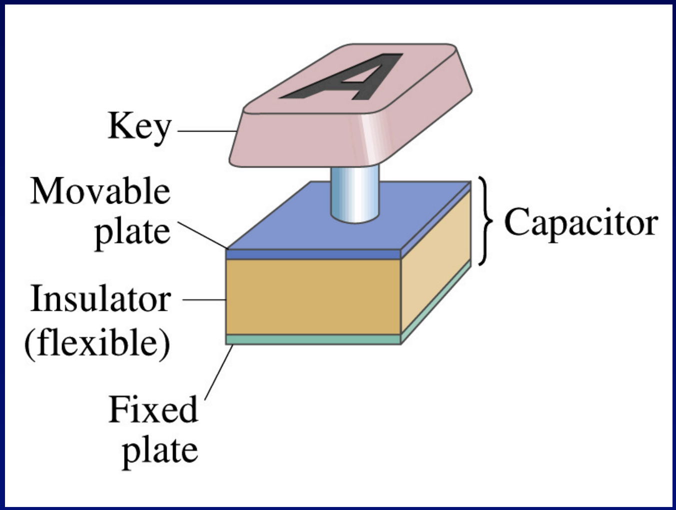
$$E_0 = \frac{\sigma}{\varepsilon_0} \qquad E = \frac{\sigma - \sigma_i}{\varepsilon_0}$$

$$\sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$

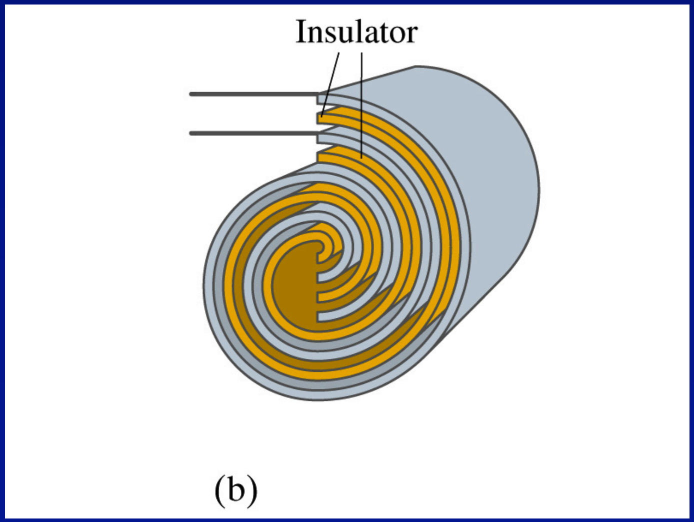
Application of Capacitors

lightening

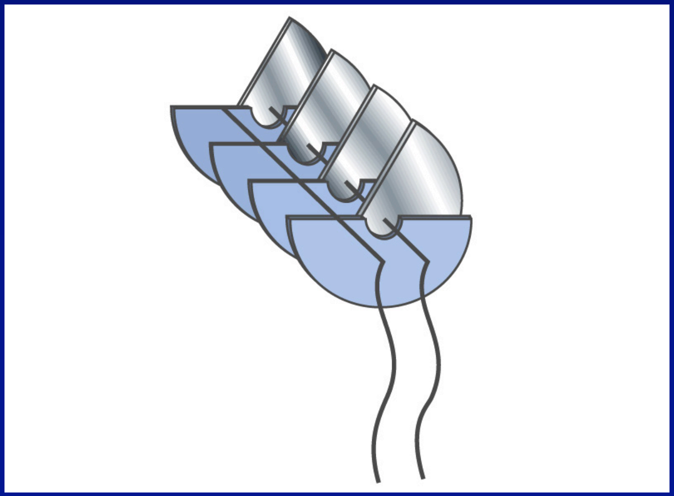
Hitting a key



Rolling it up

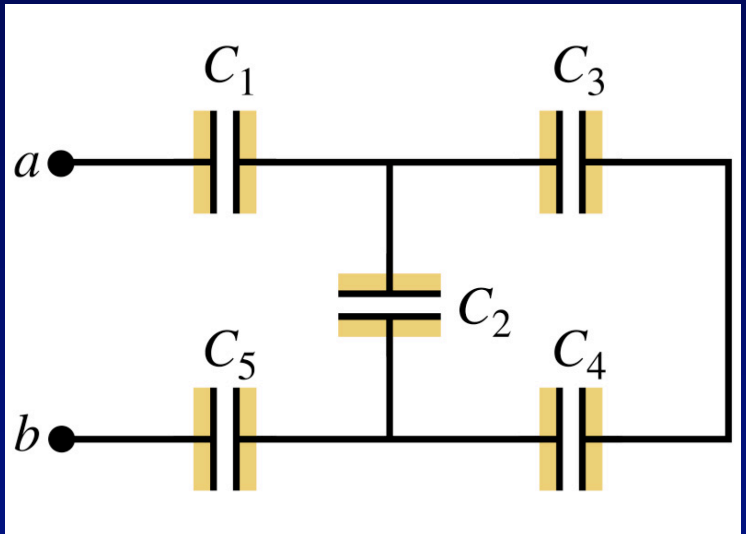


Turning a knob



Problem 24.59: capacitor calculations

- In the figure, $C_1 = C_5 = 8.4\ \mu F$, $C_2 = C_3 = C_4 = 4.2\ \mu F$. The applied potential is $V_{ab} = 220$ V.
  - What is the equivalence capacitance of the network between points a and b?
  - What is the charge on each capacitor?
  - What is the voltage across each capacitor?
  - What's the total energy stored in the system?



What happens if a dielectric of K=100 fills every capacitor?

ConcepTest 24.4 Capacitors

- **A parallel-plate capacitor is connected to a battery so that the voltage is constant.**

If a dielectric is inserted, how does the charge change?

(a) increases
(b) stays the same
(c) decreases