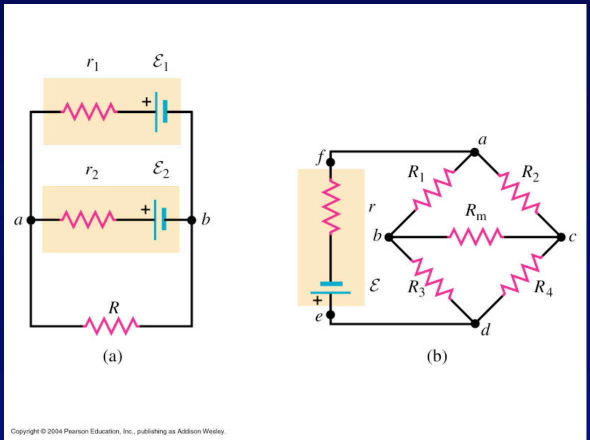
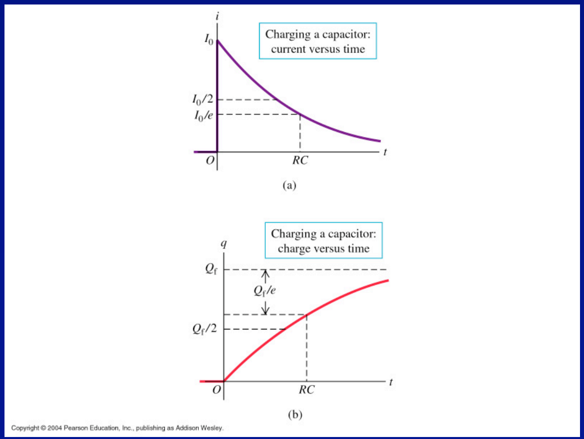
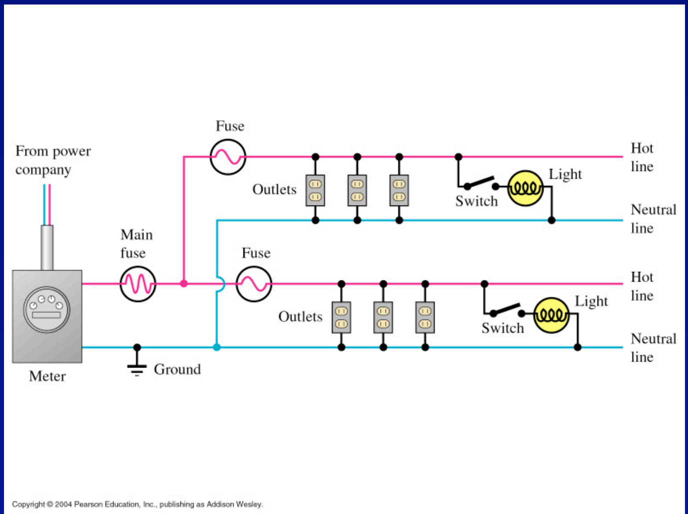
Chapter 26: Direct-Current Circuits

- Batteries in series and parallel
- Resistors in series and parallel
- Kirchhoff's Rules
- Electrical measuring instruments

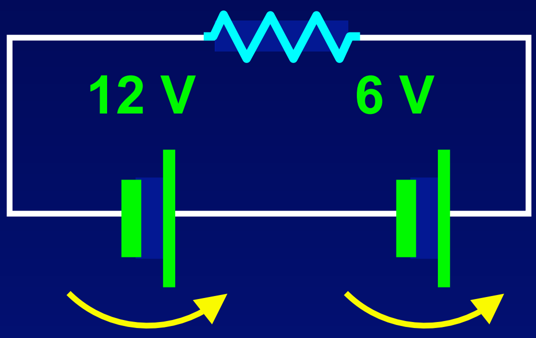




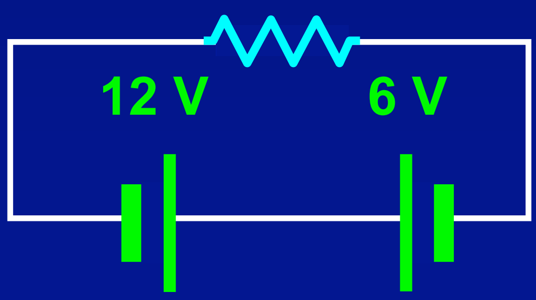


Batteries in series

- Batteries in **series**:
  - first battery does work on charge
  - second battery does more work
    - » voltages add (18 V across R)



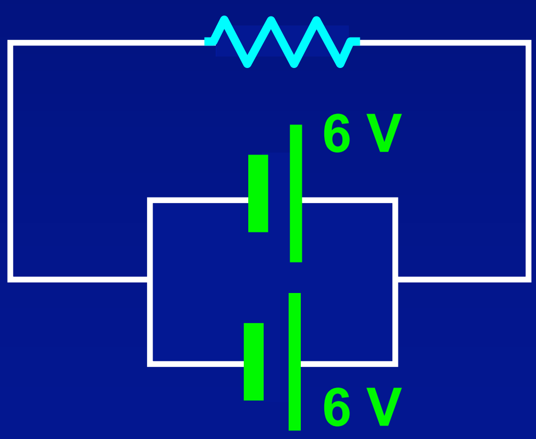
- Batteries in **series**:
  - first battery does work on charge
  - charge does work on second battery
    - » voltages subtract (6 V across R)
    - » second battery is being charged by the first one



Batteries in parallel

- Batteries in parallel:
  - each charge only goes through one of the batteries
    - » voltage is the same (6 V across R)
  - but each battery does less work (since only some of the charge goes through it)
    - » batteries last longer
    - » Can be used to recharge

(see Examples 26.4 and 26.5).

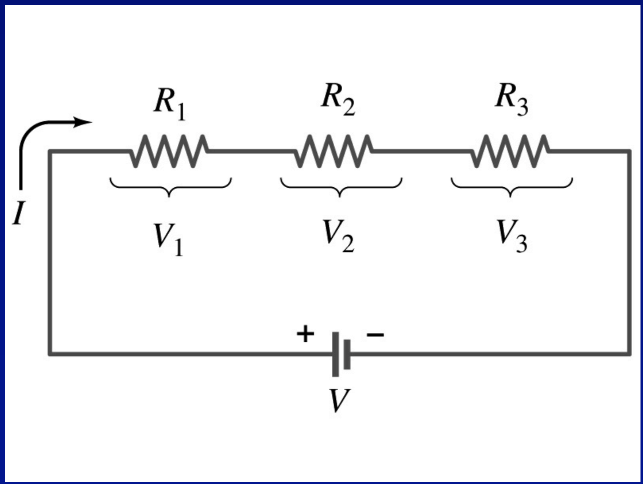


Resistors in series

- Same charge has to flow through all the resistors

  same current: $I_1 = I_2 = I_3 = I$

- Total work done by battery must equal sum of energy lost as charge moves through resistors
  - voltages add: $V = V_1 + V_2 + V_3$



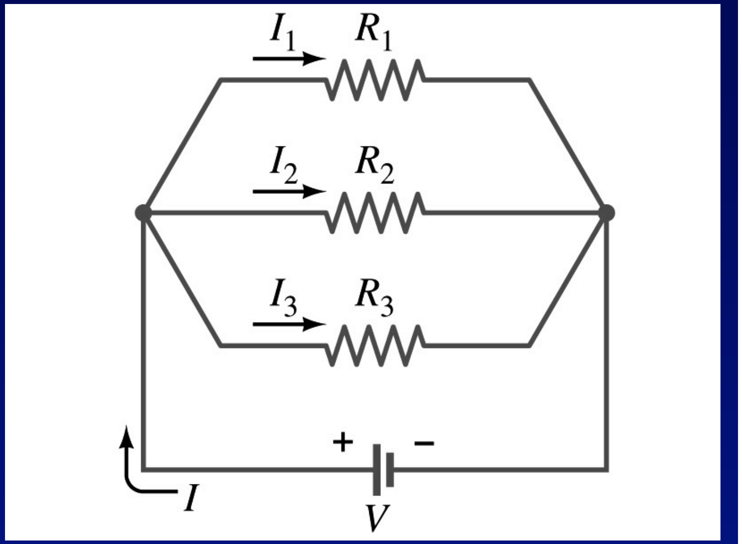
- Ohm's Law, V = I R, gives:

$$IR_{eq} = IR_1 + IR_2 + IR_3$$

$$R_{eq} = R_1 + R_2 + R_3$$

Resistors in parallel

- Current splits up into several branches. However, total current must be conserved!
  - currents add: $I = I_1 + I_2 + I_3$



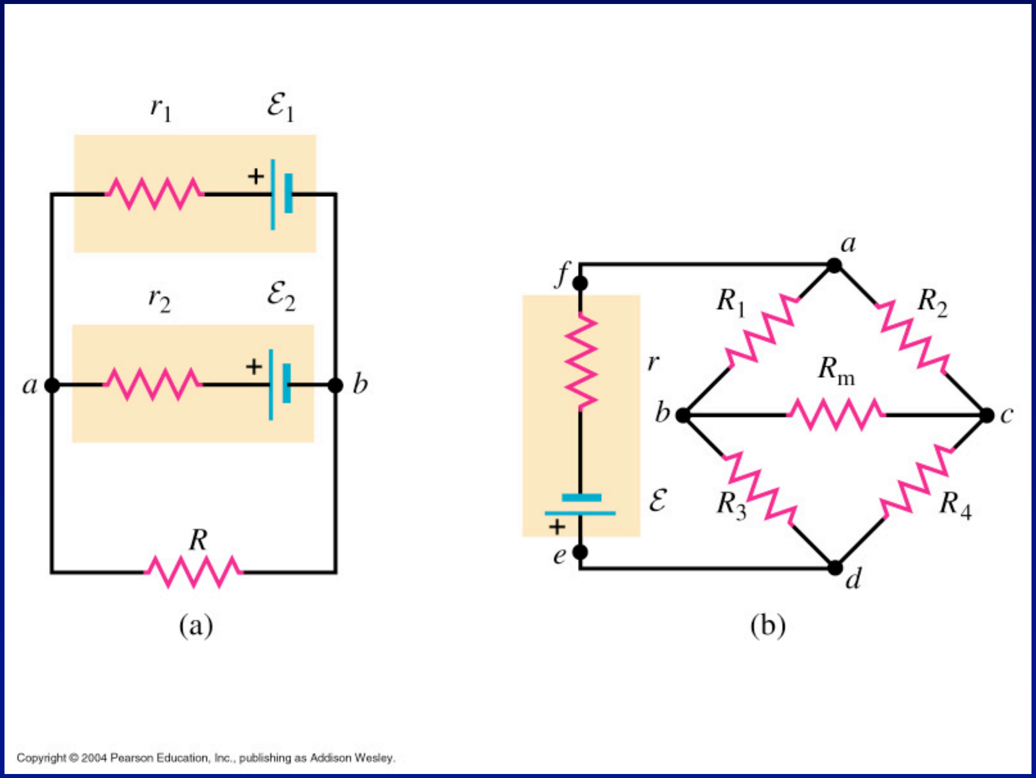
- But the voltage is the same across each resistor
  - $V = V_1 = V_2 = V_3$

- From Ohm's Law, V = I R, we find:

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

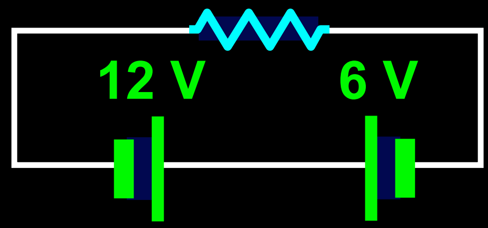
- What do we do with more complicated circuits?
  - Parallel and series rules are not enough!



- *use* ***Kirchhoff's rules!***

ConcepTest 26.1 batteries

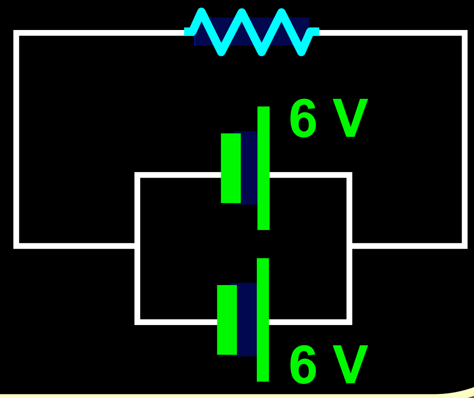
- The resistor is the same in both circuits. How does the current through the resistor in the upper circuit (I_1) compare with that in the lower one (I_2) ?



(1) $I_1 > I_2$

(2) $I_1 < I_2$

(3) $I_1 = I_2$



Kirchhoff's Rules

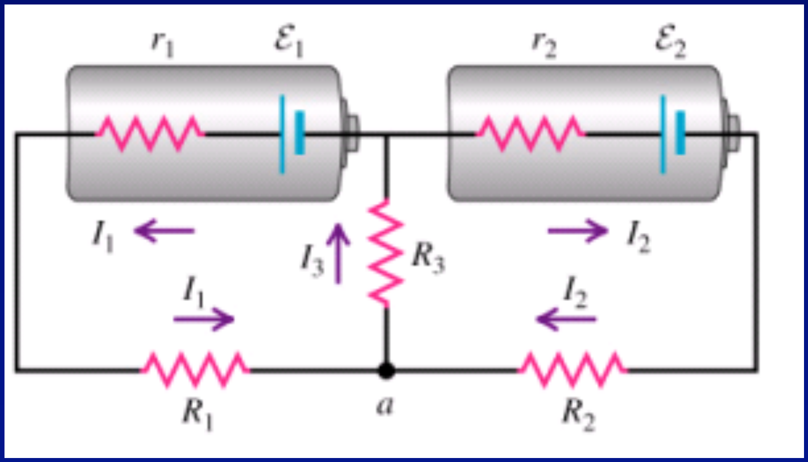
New Topic

Kirchhoff's Junction Rule

- **At any junction point, the sum of all currents entering the junction must equal the sum of all currents leaving.**

(or: what goes in has to come out!)

$$I_3 = I_1 + I_2$$



This rule follows from *conservation of charge* !

Kirchhoff's Loop Rule

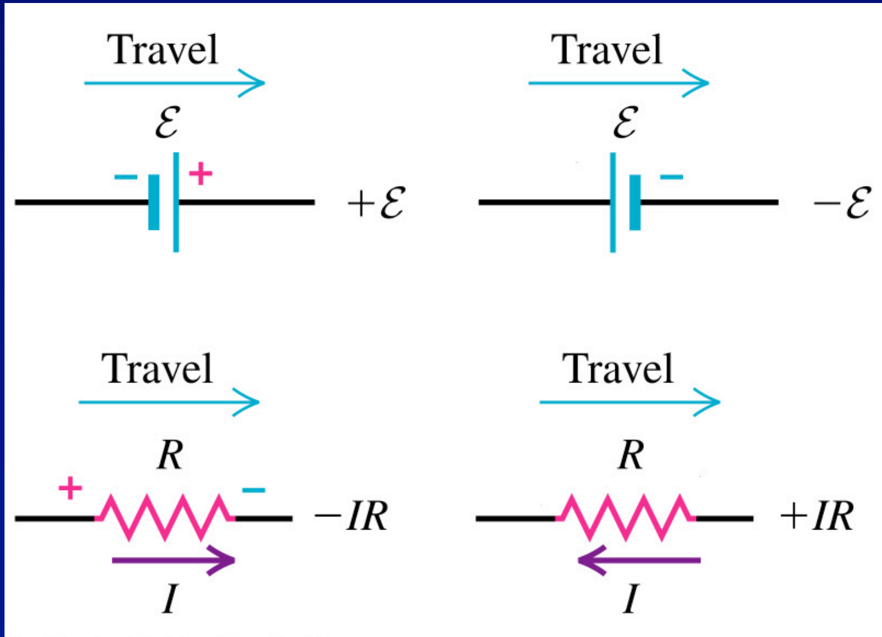
- "The sum of voltage drops and gains around any closed circuit loop must be zero"

$$\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 + \ldots = 0$$

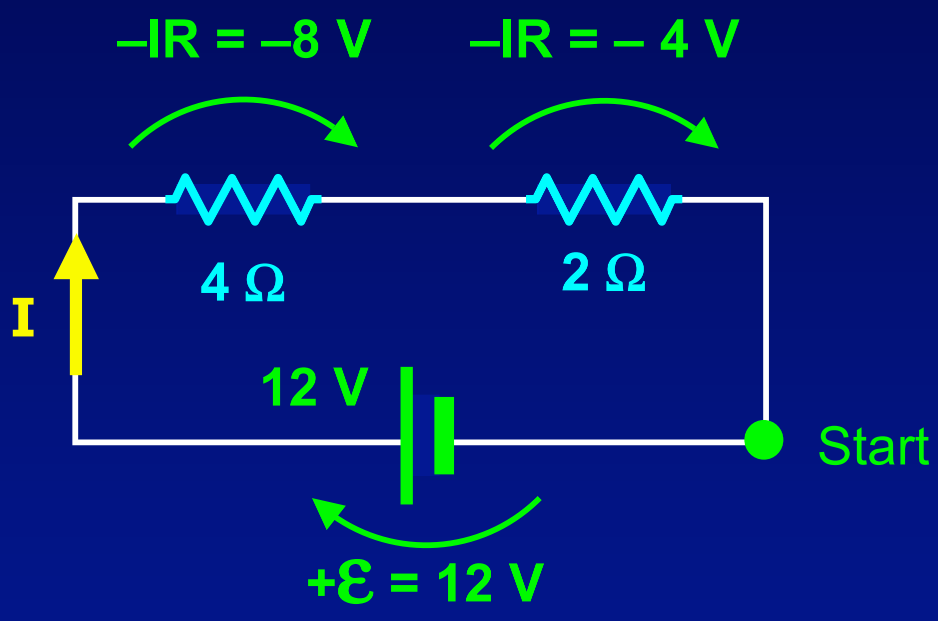
- This rule follows from *conservation of energy*

Sign convention:
– sign for voltage drop and
+ sign voltage gain.

It depends on several factors:
1. emf or resistor
2. direction of current
3. direction of loop travel



Loop Rule Example
(with numbers)



Current in each resistor?

Total resistance = 6 Ω

Ohm's Law: $I = V / R$
$= 12\ V / 6\ \Omega$
$= 2\ A$

Voltage drop across resistor?

$V = I R$

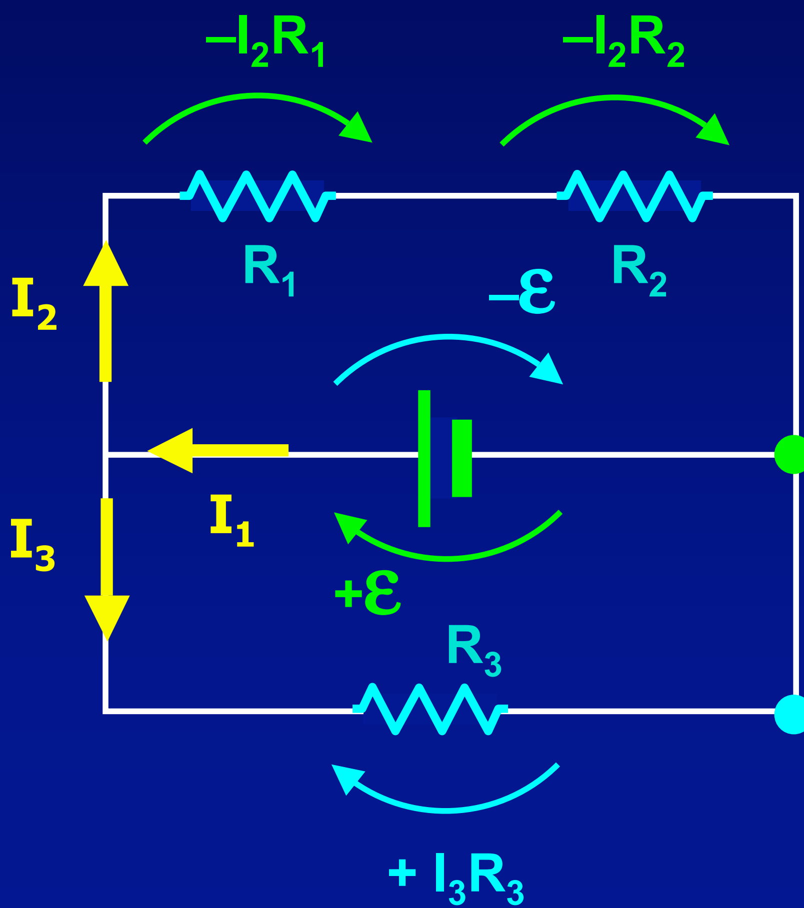
$$\Delta V = +12\ V - 8\ V - 4\ V = 0 \quad \checkmark$$

Loop Rule Example
(without numbers)

The first thing we have to do is?
Define currents!

Next: Define travel direction:
top loop (CW)
bottom loop (CW)

Now apply Loop rule to each loop



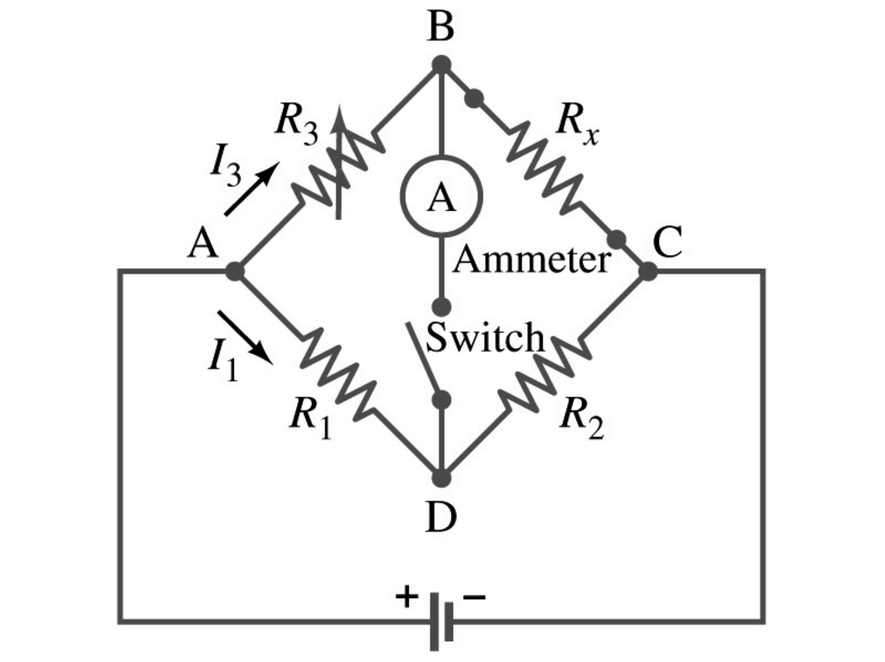
Start $\Delta V = +\,\varepsilon\; - I_2R_1 - I_2R_2 = 0$

Start $\Delta V = +\,I_3\,R_3\; - \varepsilon\; = 0$

Example: Wheatstone bridge

- A device to measure resistance. When the bridge is balanced by adjusting R_3, no current flows through the ammeter. Determine R_x.

Label each current with a symbol and a direction.



V_{AB} is same as V_{AD} :
$I_3 R_3 = I_1 R_1$ (1)

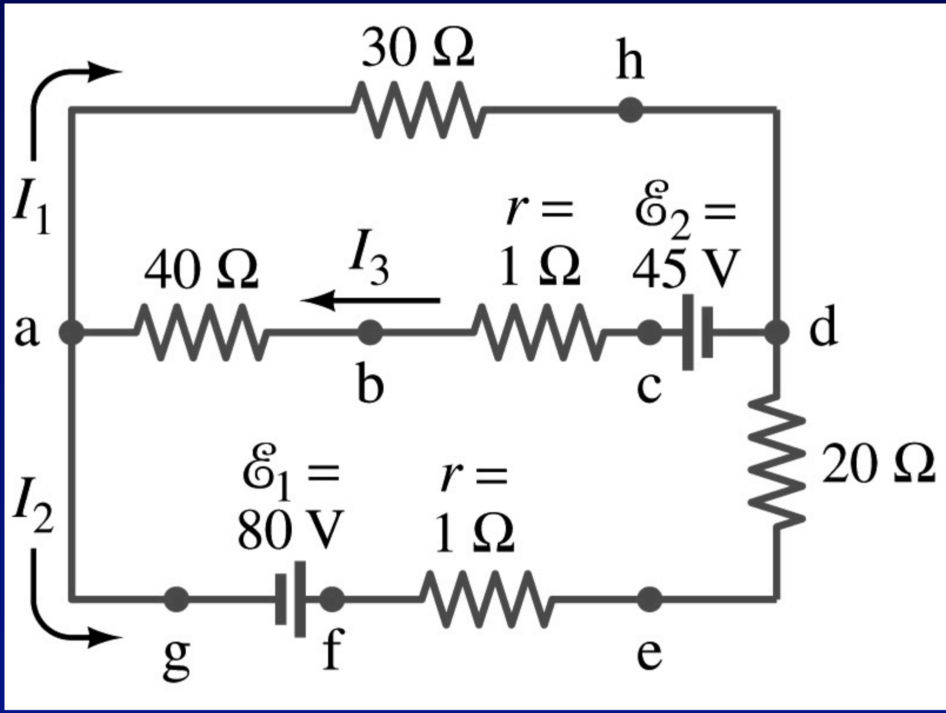
V_{BC} is same as V_{DC} :
$I_3 R_x = I_1 R_2$ (2)

Solve for R_x :
$R_x = R_2 R_3 / R_1$

If $R_1 = 630\ \Omega$
$R_2 = 972\ \Omega$
$R_3 = 65.7\ \Omega$,
then $R_x = 65.7\ \Omega$

Example: using Kirchhoff's Rules

- Determine the currents flowing through each resistor in the circuit shown.



Label each current with a symbol and a direction.

Junction rule at point a:
$I_3 = I_1 + I_2$ (1)

Upper loop (ahdcba):
$-30I_1 + 45 - I_3 - 40I_3 = 0$ (2)

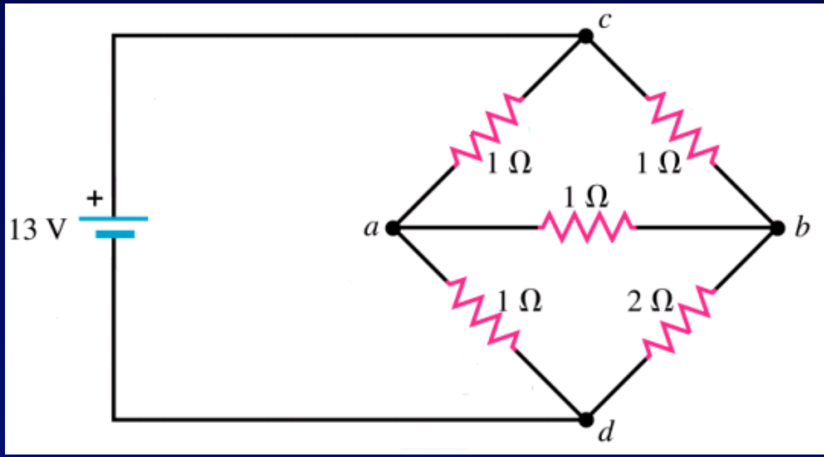
Outer loop (ahdefga):
$-30I_1 + 20I_2 + I_2 - 80 = 0$ (3)

Lower loop (agfedcba):
$+80 - I_2 - 20I_2 + 45 - I_3 - 40I_3 = 0$ (4)

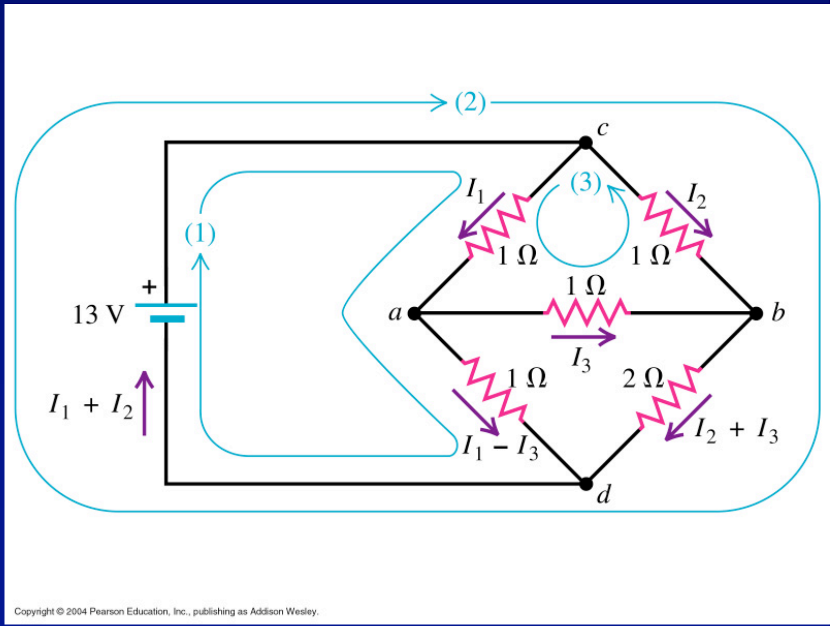
Solve from (1), (2), (3):
I_1 = - 0.87 A (opposite to assumed)
I_2 = 2.6 A
I_3 = 1.7 A

Example: a complex network

- The picture shows a bridge-type circuit with 5 equal resistors connected to a battery.
  - Find the current in each resistor
  - What's the equivalent resistance of the five resistors?



There are 5 different currents. Junction rule can reduce the number to 3. Choose currents and 3 loops as shown:



$$\text{loop 1 (ccw)}: \quad +13 - I_1 \times 1 - (I_1 - I_3) \times 1 = 0$$

$$\text{loop 2 (cw)}: \quad -I_2 \times 1 - (I_1 + I_3) \times 2 + 13 = 0$$

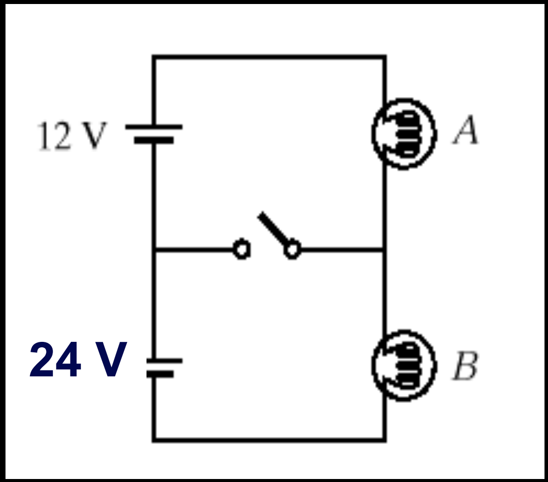
$$\text{loop 3 (ccw)}: \quad -I_1 \times 1 - I_3 \times 1 + I_2 \times 1 = 0$$

$$I_1 = 6\text{ A}, \; I_2 = 5\text{ A}, \; I_3 = -1\text{ A}$$

$$R_{eq} = 13/(5+6) = 1.2\,\Omega$$

ConcepTest 26.2 Kirchhoff's Rules

- The light bulbs in the circuit are identical. When the switch is closed, what happens?
  - (1) both bulbs go out
  - (2) intensity of both bulbs increases
  - (3) intensity of both bulbs decreases
  - (4) A gets brighter and B gets dimmer
  - (5) A gets dimmer and B gets brighter
  - (6) nothing changes

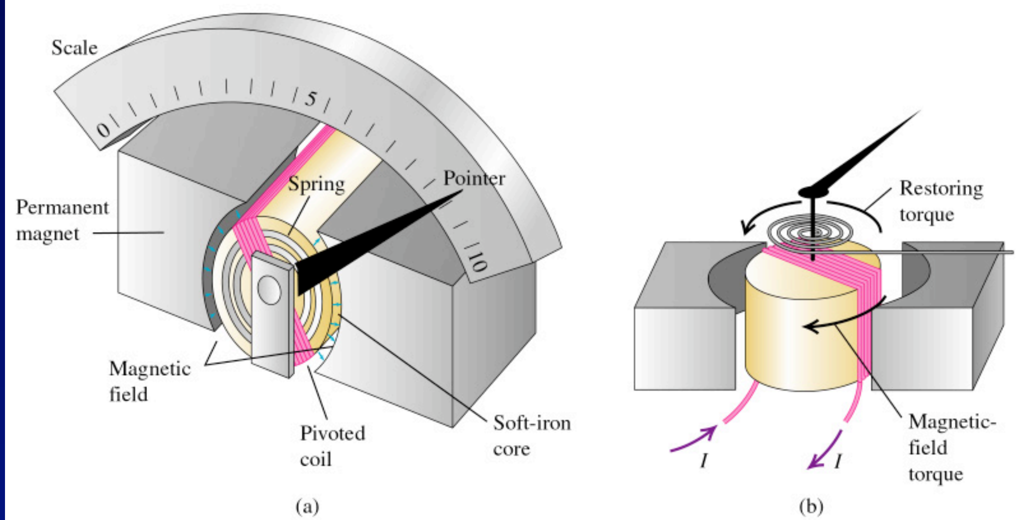


Electrical Measuring Instruments

New Topic

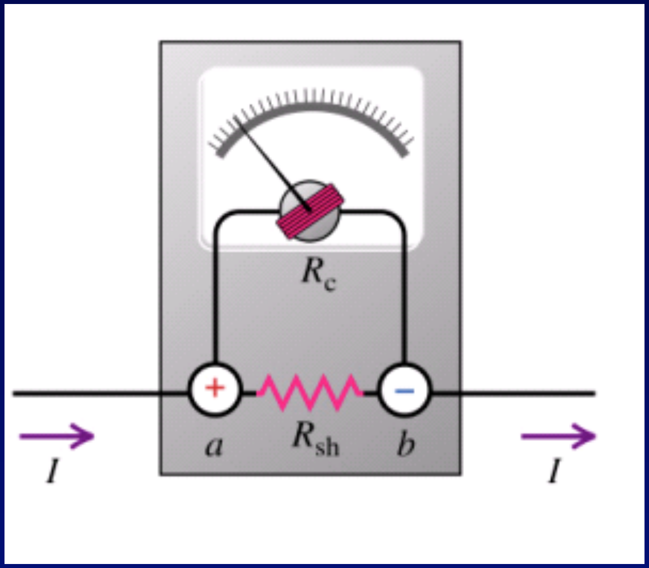
d’Arsonval Galvanometer

- No current, no movement.
- If there’s a current passing through the coil, then the permanent magnet will cause it to deflect.
- A spring brings it back.
- The amount of deflection is directly proportional to the current
- Full-scale deflection I_{fs} : the maximum current it can measure
- Resistance in coil R_c (as small as possible)
- So the voltage across the coil at full deflection is $V = I_{fs} R_c$



Ammeter

- Ammeter can be adapted to measure larger currents by connecting a shunt resistor in parallel as shown



Example: What shunt resistance is required to make the 1.00-mA, 20-Ω meter into an ammeter with a range of 0 A to 50.0 mA ? (Example 26.8)

$$I_{fs} R_c = (I_a - I_{fs}) R_{sh}$$

$$R_{sh} = \frac{I_{fs} R_c}{I_a - I_{fs}} = \frac{1 \times 20}{50 - 1} = 0.408\,\Omega$$

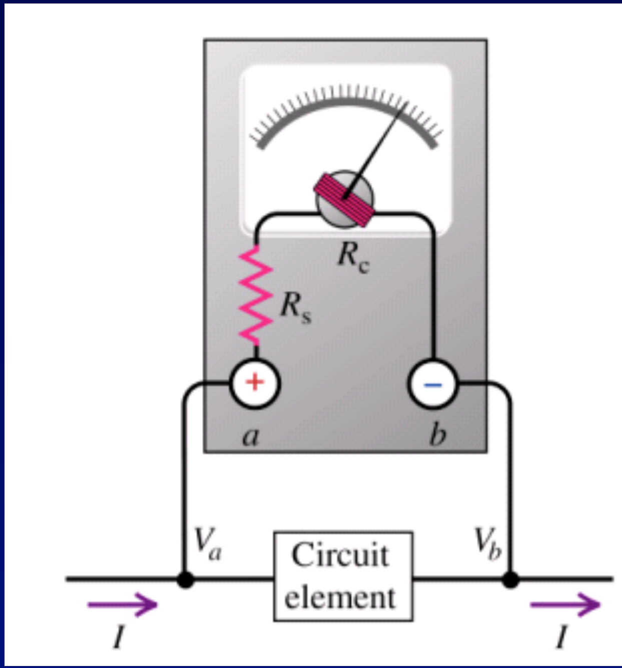
$$R_{eq} = 0.400\,\Omega$$

Question: how much current is in each resistor?

Answer: 1 mA in the coil and 49 mA in the shunt resistor.

Voltmeter

Question: How can we make a 1.00-mA, 20-Ω galvanometer into a voltmeter with a maximum range of 10.0 V ? (Example 26.9)



- Answer: by connecting a large resistor in series as shown.

$$V_{ab} = I_{fs}(R_c + R_s)$$

$$R_s = \frac{V_{ab}}{I_{fs}} - R_c = \frac{10}{0.001} - 20 = 9980\,\Omega$$

$$R_{eq} = 10000\,\Omega$$

The voltage across the coil is $I_{fs} R_c$ **= 0.02 V.**
The voltage across the added resistor is 9.98 V.

ConcepTest 26.3 Ammeter and Voltmeter

- The circuit in which the ammeter and the voltmeter are correctly arranged to measure the unknown resistance R is

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5