Exam 2 is coming !

- Tues., March 21, 12:30 to 1:55 pm, in this room.
- Covering 5 chapters (27-31)
- 18 multiple-choice questions (just as before)
  - 15 conceptual/numerical problems, 1 point each
  - 3 questions are numerical (like homework problems) - 2 pts.
  - I will pass out formula5 sheets at the exam. Please familiarize yourself with it. Any constants needed will be given. (Only the equations up to "AC Circuits" will come into play.)
  - Personalized exams
  - I will enter the grade on your Mastering Physics account ("Exam 2").
- Recovery points
  - Set 2 on sPH2435-04 opens March 22 (Weds., noon) and closes March 24 (Fri., 11:59 pm). Use the "old interface".
  - You need a 4-digit CAPA ID to access it. Will get this from on-class exam.
  - You must try to recover everything, not just the ones you missed. You must get a higher grade on the recovery exam to get any points added to your class score.
- What can I bring to the exam?
  - Pencil
  - eraser
  - calculator
  - <u>That's all (no cell phones for example)</u>

Exam 2 coverage

- Chapter 27: Magnetic Field and Forces
  - Magnets
  - Magnetic Force
- Chapter 28: Sources of Magnetic Field
  - Biot-Savart Law
  - Ampere's law
- Chapter 29: Electromagnetic Induction
  - Induced EMF and applications
  - General form of Faraday's Law
- Chapter 30: Inductance
  - Mutual and Self-inductance
  - Energy storage and DC Circuits
- Chapter 31: Alternating Current
  - AC Circuits
  - Resonance and Transformers

7 lectures (including this one)
6 homework sets
6 quizzes
1 exam

About 1/3 of the work

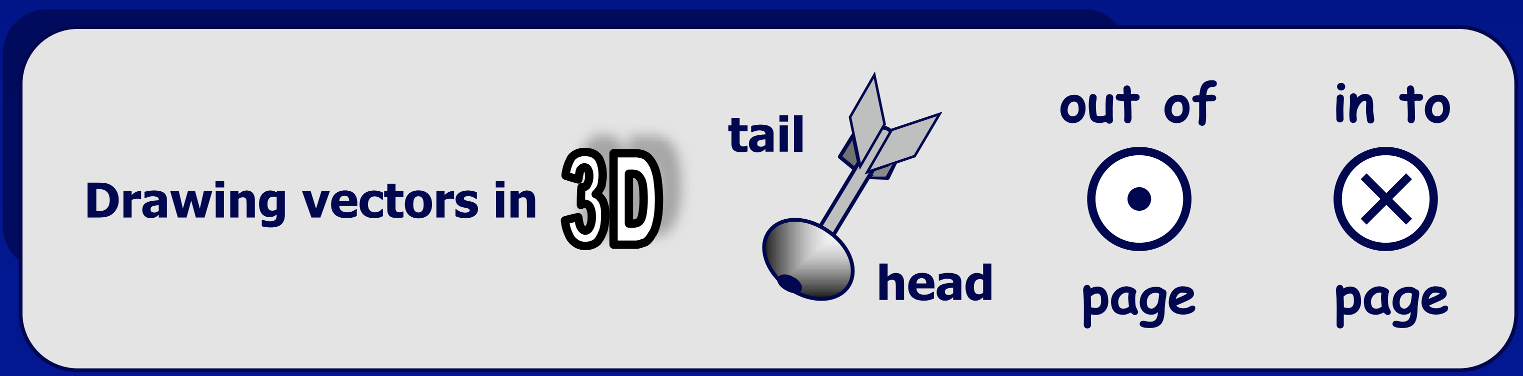
Chapter 27: Magnetic Field and Forces

- Magnets and Magnetic Fields
- Magnetic Force (Lorentz Force)
  - force on a moving charge
  - force on a current in a wire
  - torque on a current loop
  - mass spectrometer

Magnetism

- Natural magnets were observed by Greeks more than 2500 years ago in *"Magnesia"* (northern Greece)
  - certain type of stone (lodestone) exert forces on similar stones
  - Small lodestone suspended with a string aligns itself in a north-south direction due to ***Earth's magnetic field!***

Direction of Magnetic Field



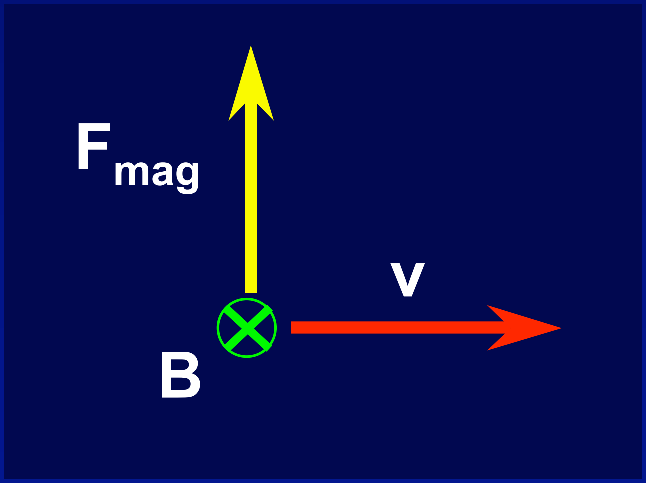
The Magnetic Force

- What happens if you put a charged particle in a magnetic field?
  - it experiences a *magnetic force*! (Lorentz force)
- Magnitude depends on
  - Charge q
  - Velocity v
  - Field B
  - Angle between v and B

$$F_{magnetic} = q\,v\,B\sin\theta$$

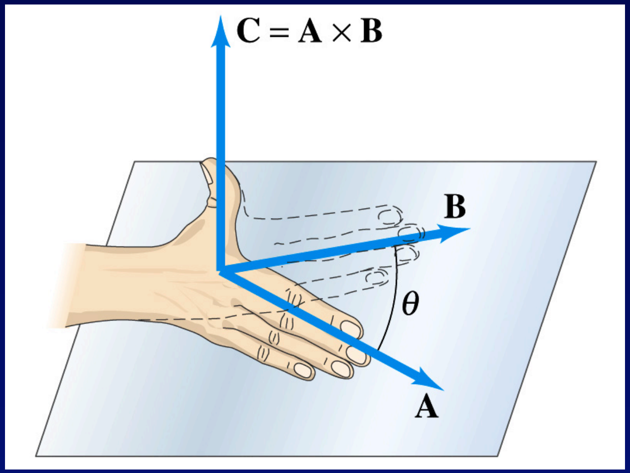
- Direction is *"sideways"*
  - force is perpendicular to both v and B!
- Vector cross product

$$\vec{F} = q\vec{v} \times \vec{B}$$

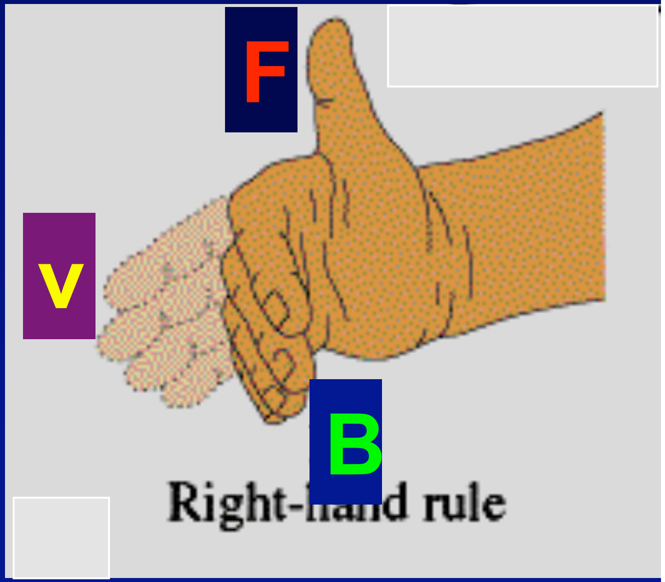


Direction of the Magnetic Force

$$\vec{F} = q\vec{v} \times \vec{B}$$



- Use the **right-hand rule**:
  - point your fingers along the direction of velocity
  - curl your fingers towards the magnetic field vector
  - your thumb will then point in the direction of the force



- **Reverse direction if it's a negative charge!**

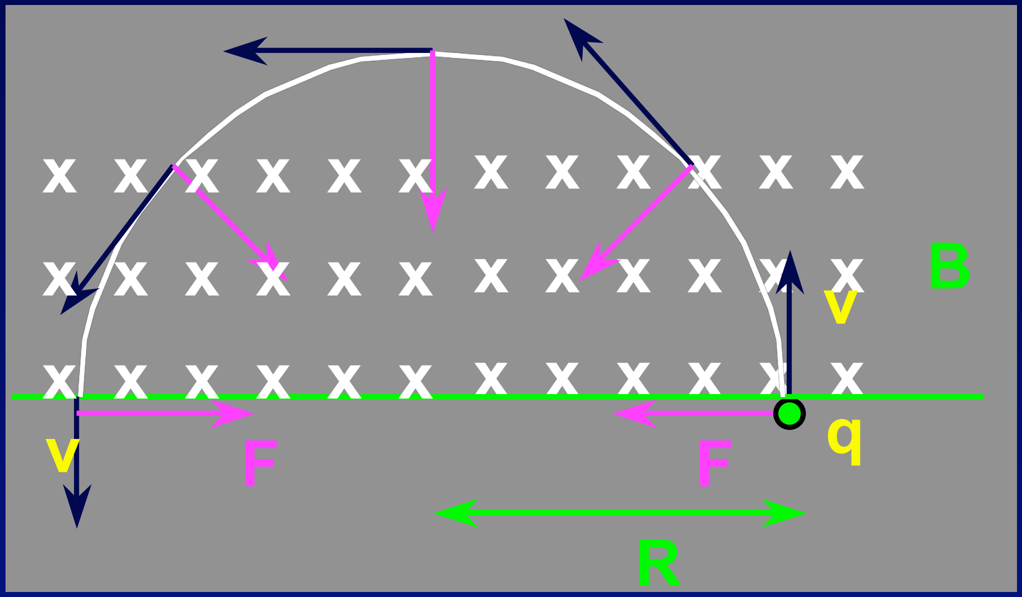
Radius of Circular Orbit

magnetic force:

$$F = qvB$$

centripetal force:

$$F = m\frac{v^2}{R}$$



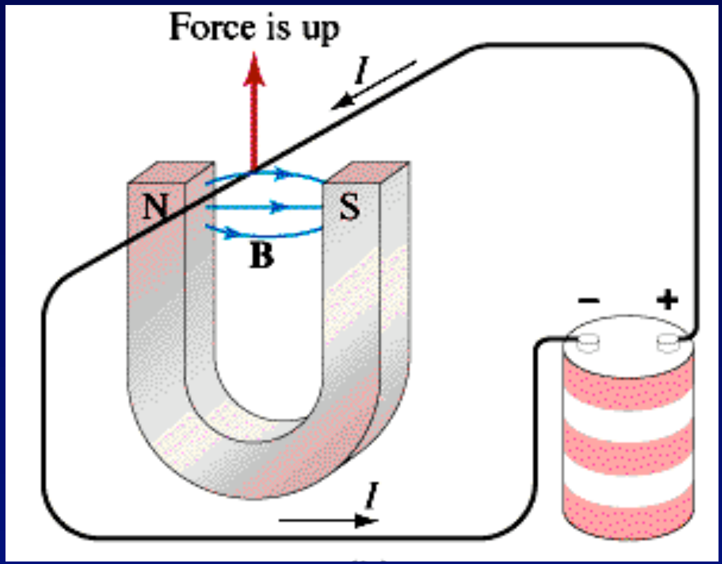
$$\Rightarrow \quad qvB = m\frac{v^2}{R}$$

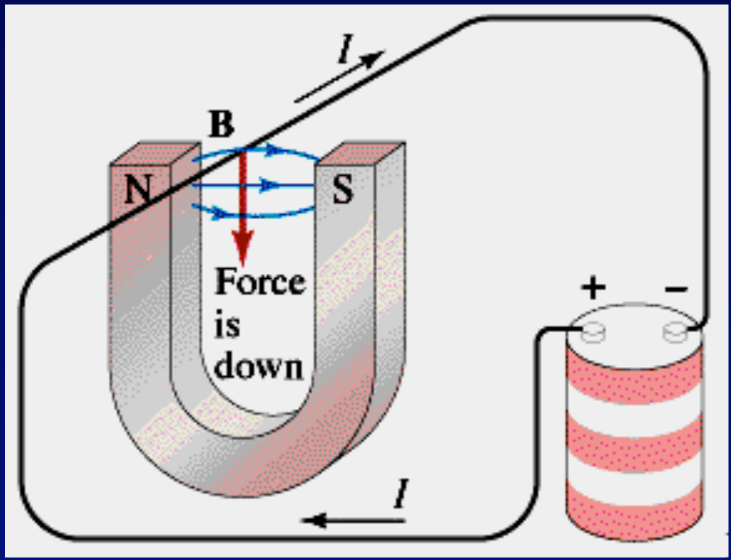
$$\Rightarrow \quad \boxed{R = \frac{mv}{qB}}$$

This is an important result:

It relates atomic quantities (m and q) to quantities we can measure (R, v, B) in a lab.

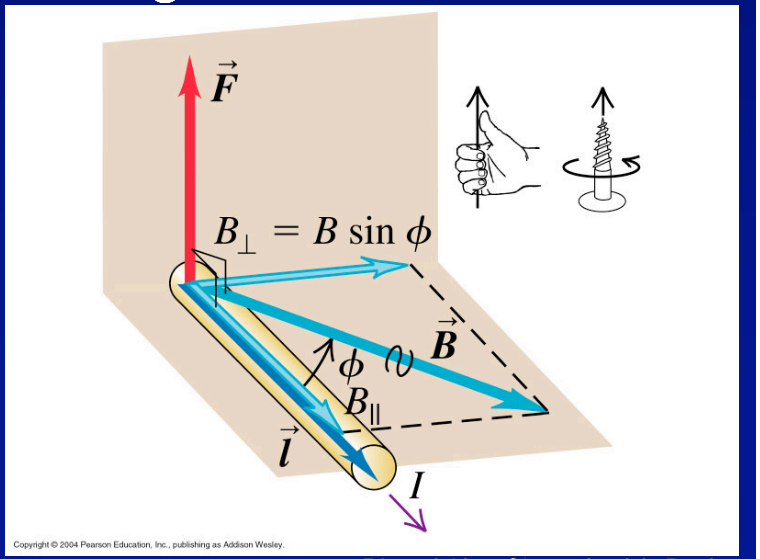
Magnetic Force on a Current-Carrying Wire





$$\vec{F} = I\vec{l} \times \vec{B}$$

The right-hand-rule

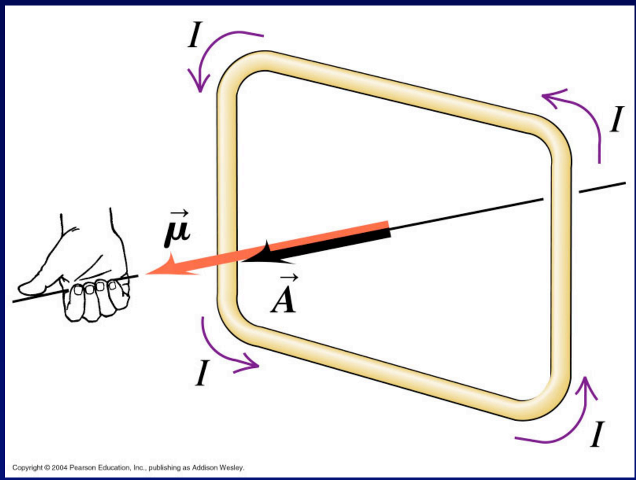


Torque on a Current Loop

$$\tau = NIAB\sin\theta$$

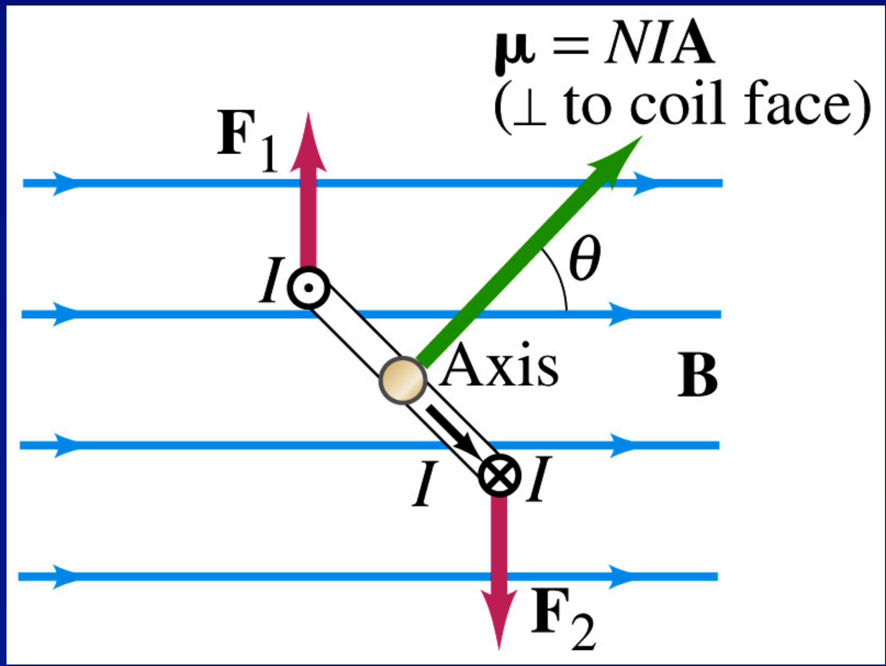
- Define magnetic dipole moment

$$\boxed{\vec{\mu} = NI\vec{A}}$$



- Then the torque can be written as a vector cross product

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

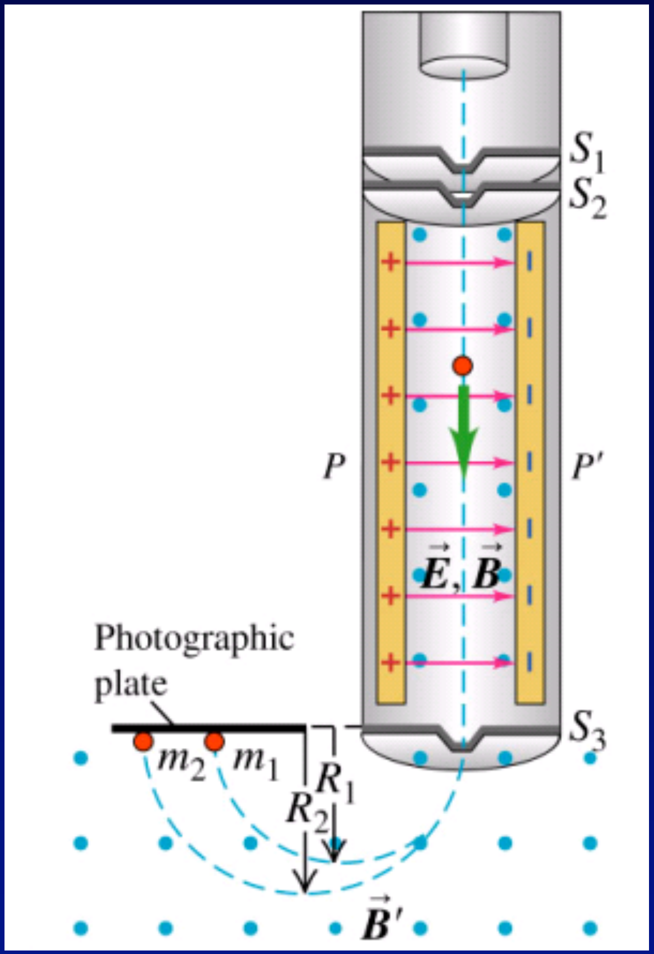


- The potential energy is

$$\boxed{U = -\vec{\mu} \cdot \vec{B}}$$

Mass Spectrometer

- *Velocity selector*
  - q E = q v B
  - v = E / B
- *Only those particles with v can pass through.*



- **Bending in a magnetic field**

$$qvB' = mv^2 / r$$

$$\boxed{m = qBB'r / E}$$

Chapter 28: Sources of Magnetic Field

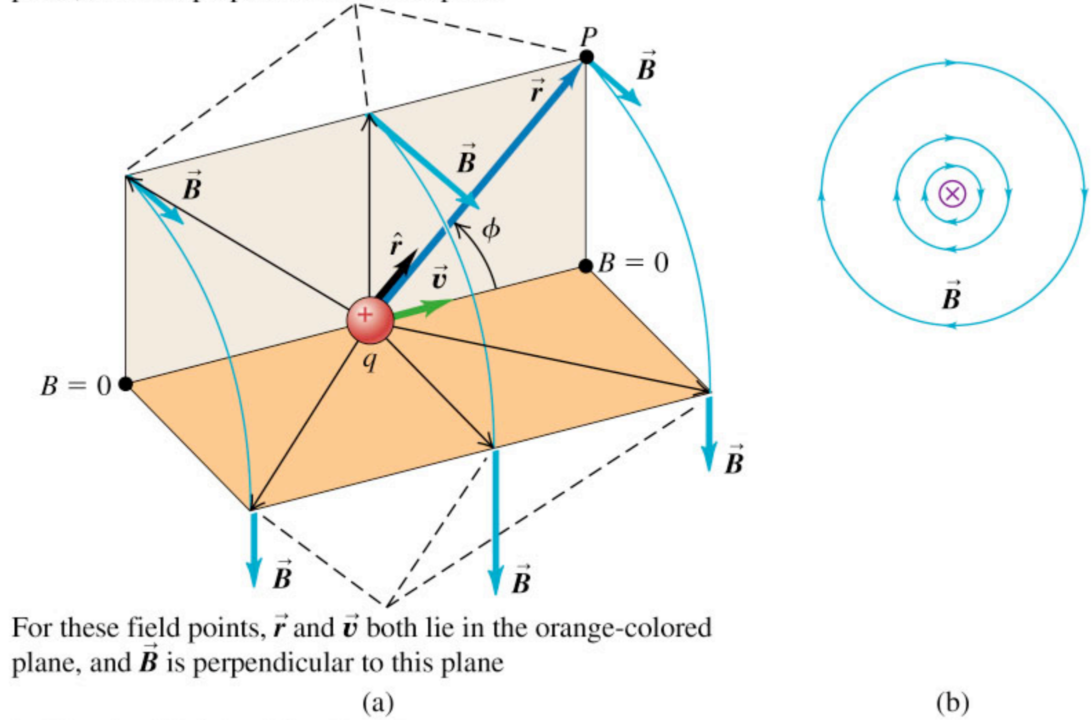
- Biot-Savart Law
  - moving charge
  - a straight wire
  - force between parallel wires
  - current loops
- Ampere’s Law
  - straight wire
  - solenoid
  - toroidal solenoid

Biot-Savart Law - Moving Charge

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

$\mu_0 = 4\pi x 10^{-7}$ T.m/A is called permeability of free space



where $\hat{r}$ is the unit vector from the source point to the field point.

Biot-Savart Law - Curent Segment

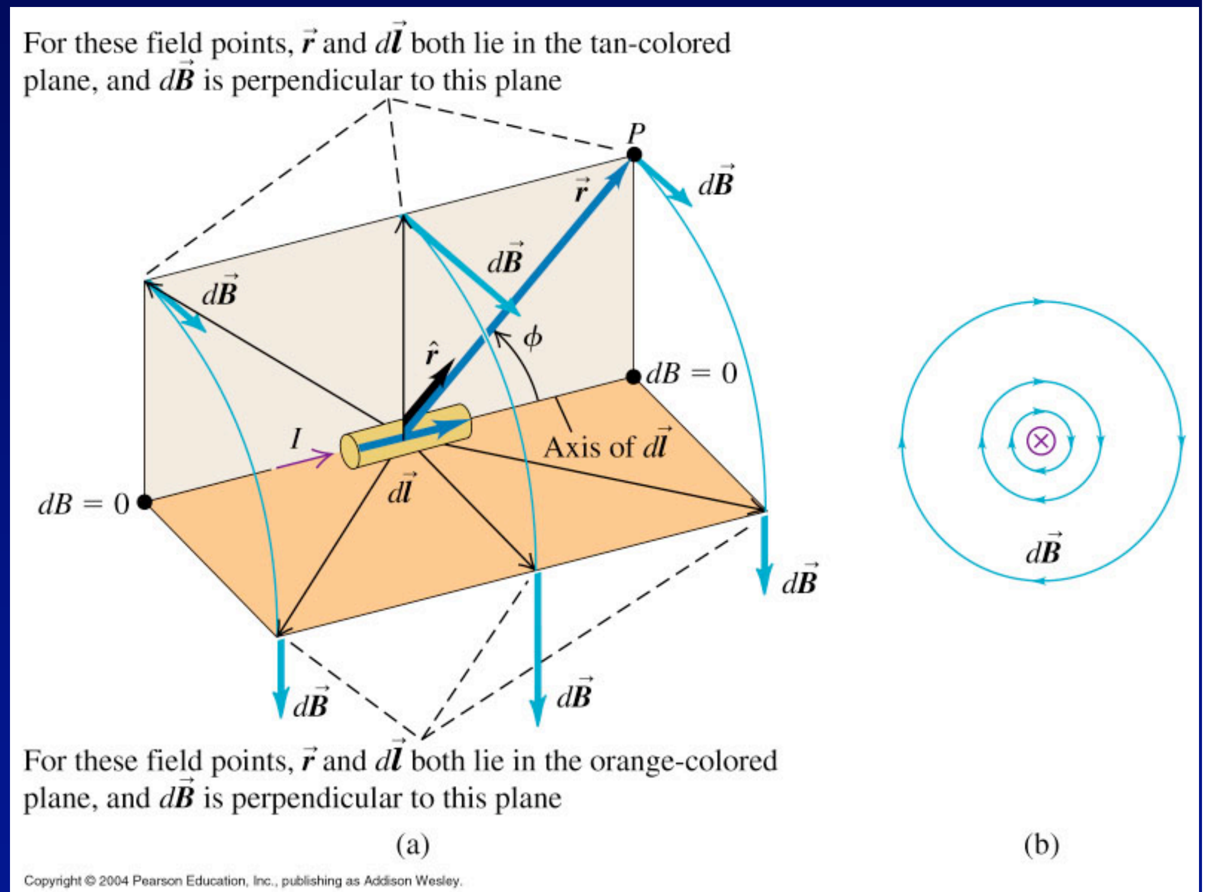
- Question: how to find B field (both direction and magnitude) due to a current segment ?

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2}$$

The total magnetic field is an integral over the entire wire:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{Id\vec{l} \times \hat{r}}{r^2}$$



Example: straight wire

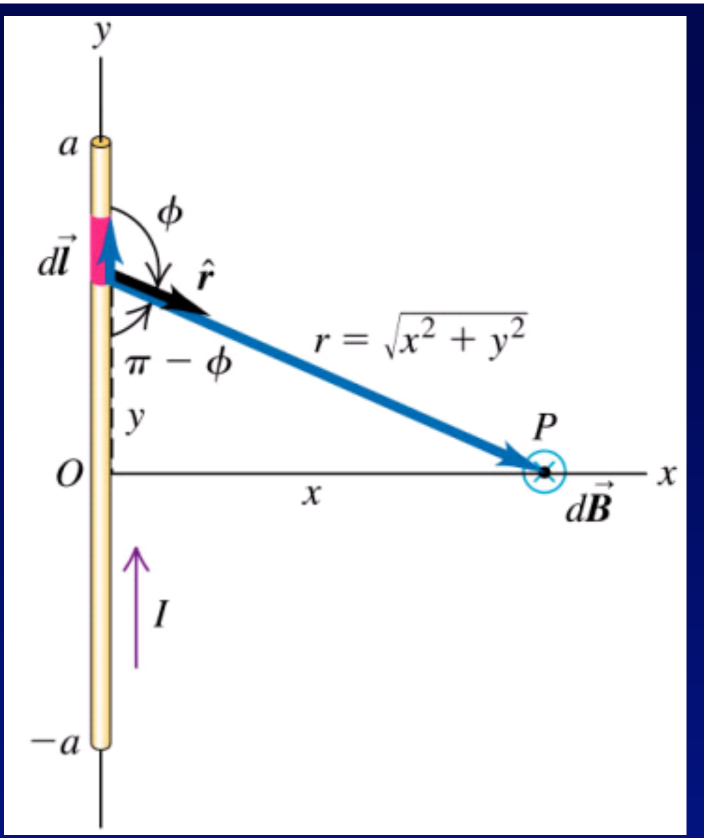
- Set up the coordinate system as shown. The field at point P due to a small segment:

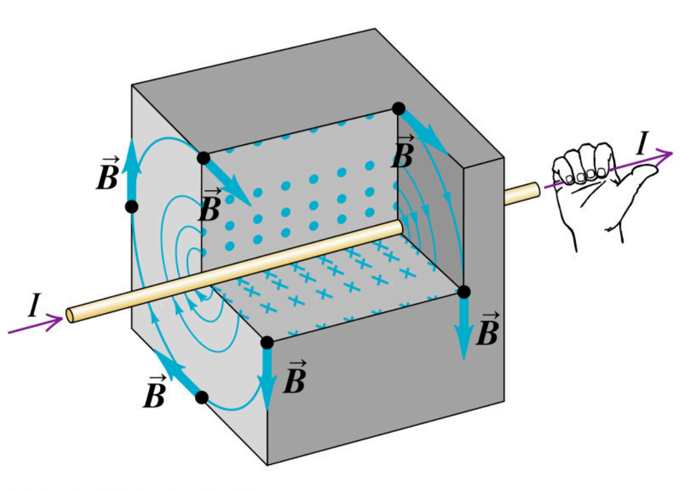
$$dB = \frac{\mu_0 I}{4\pi}\frac{dy \sin\phi}{r^2}$$

- The direction is the same from any segment, so the total is

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{+a}\frac{x\,dy}{\left(x^2+y^2\right)^{3/2}} = \frac{\mu_0 I}{2\pi}\frac{2a}{x\sqrt{x^2+a^2}}$$

For a long wire (a→ ∞): $B = \dfrac{\mu_0 I}{2\pi x}$

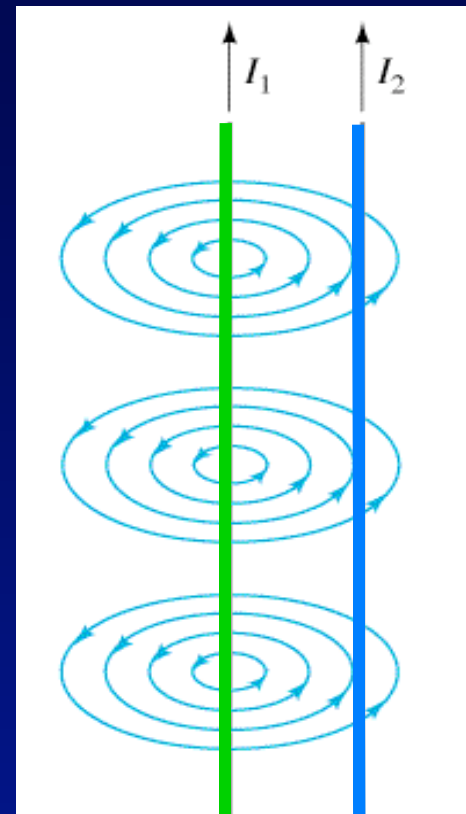




Force Between Two Parallel Current-Carrying Wires

- Field at wire 2 due to wire 1: $$B_1 = \frac{\mu_0 I_1}{2\pi d}$$
- Force on wire 2: $$F_2 = I_2 L B_1 = \frac{\mu_0 I_1 I_2}{2\pi d} L$$
- Force per unit length:

$$\boxed{\frac{F_2}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}}$$

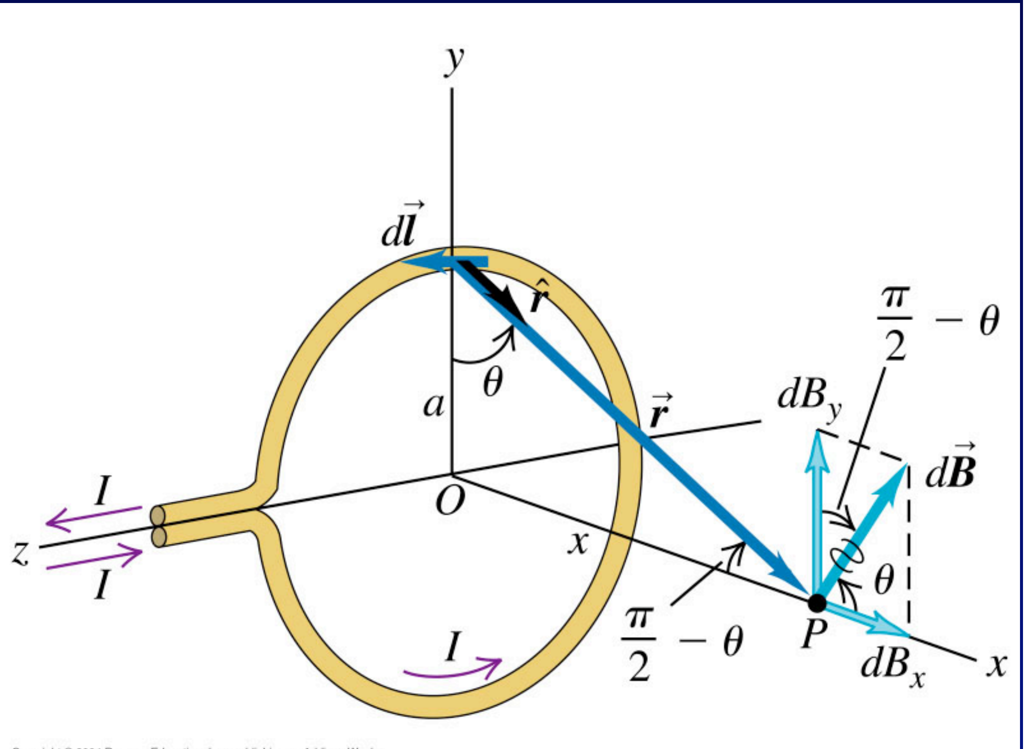


Example: a current loop

- Set up the coordinate system as shown. For a point on the axis

$$dB = \frac{\mu_0 I}{4\pi}\frac{dl}{r^2}$$

- By symmetry, total $B_\perp$ is zero, so total $B=B_{||}$

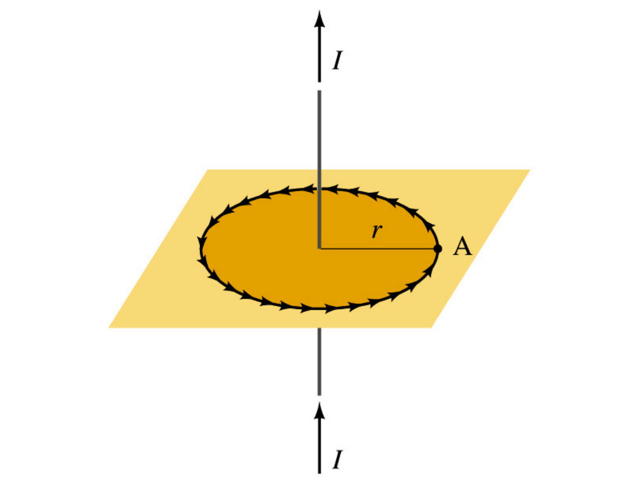


$$B = \int dB_x = \int dB\cos\theta = \int dB\frac{a}{r} = \frac{\mu_0 I}{4\pi}\frac{a}{r^3}\int dl = \frac{\mu_0 I}{2}\frac{a^2}{r^3}$$

Ampere's Law

- Question: Is there a general relation between a current in a wire of any shape and the magnetic field around it?

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$



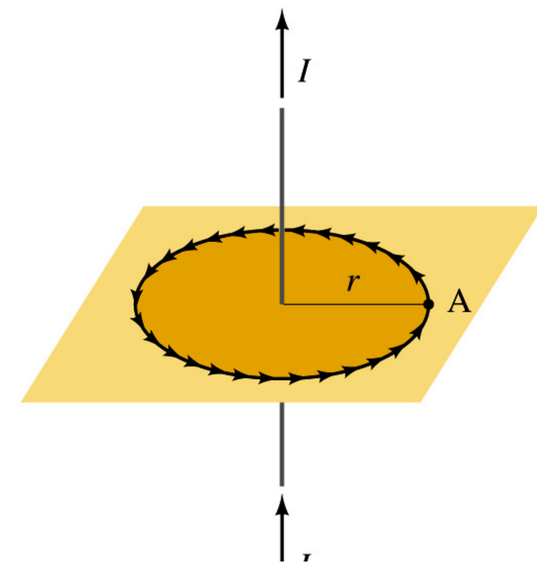
- Ampere's Law:

  The line integral of the magnetic field around any closed loop is equal to μ_0 times the total current enclosed by the loop

Example: a long straight wire

- Consider a circular path of radius r around the wire.
  - The plane of the path is perpendicular to the wire.

- By symmetry, the B field has the same magnitude at every point along the path, with a direction tangential to the circle by the right-hand-rule.

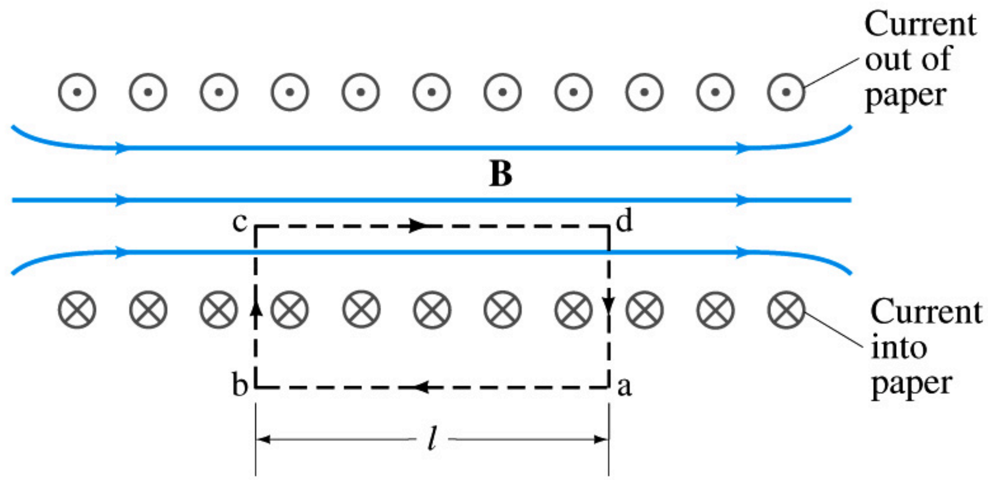


$$\mu_0 I = \oint \vec{B} \cdot d\vec{l} = B \oint dl = B(2\pi r)$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Example: Solenoid

- Consider a rectangular path as shown



- By symmetry, the only non-zero contribution comes from the segment cd:

$$\oint \vec{B} \cdot d\vec{l} = \int_c^d \vec{B} \cdot d\vec{l} = Bl$$

$$Bl = \mu_0 NI$$

$$B = \mu_0 nI$$

Where n = N / l is the number loops per unit length

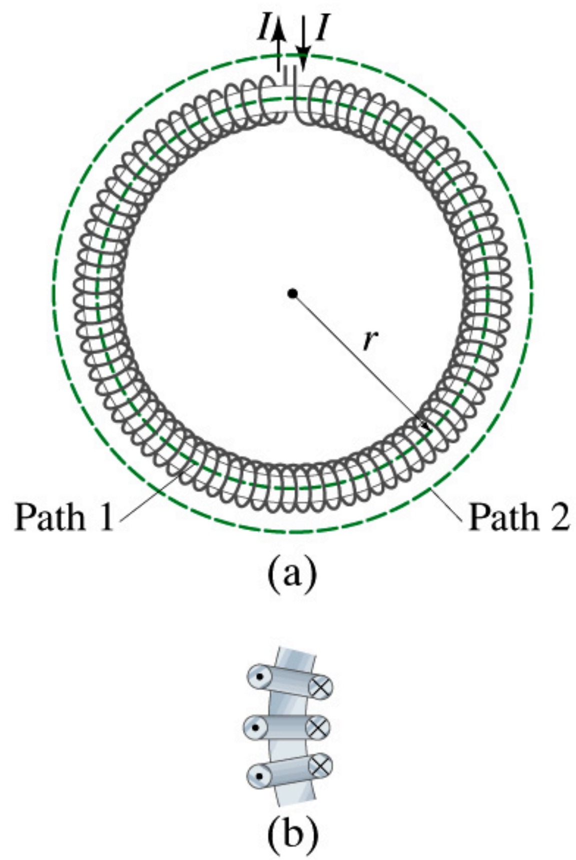
Example: Toroidal Solenoid

- Inside:

  consider path 1

$$B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$



- Outside: consider path 2
  - The net current passing through is zero

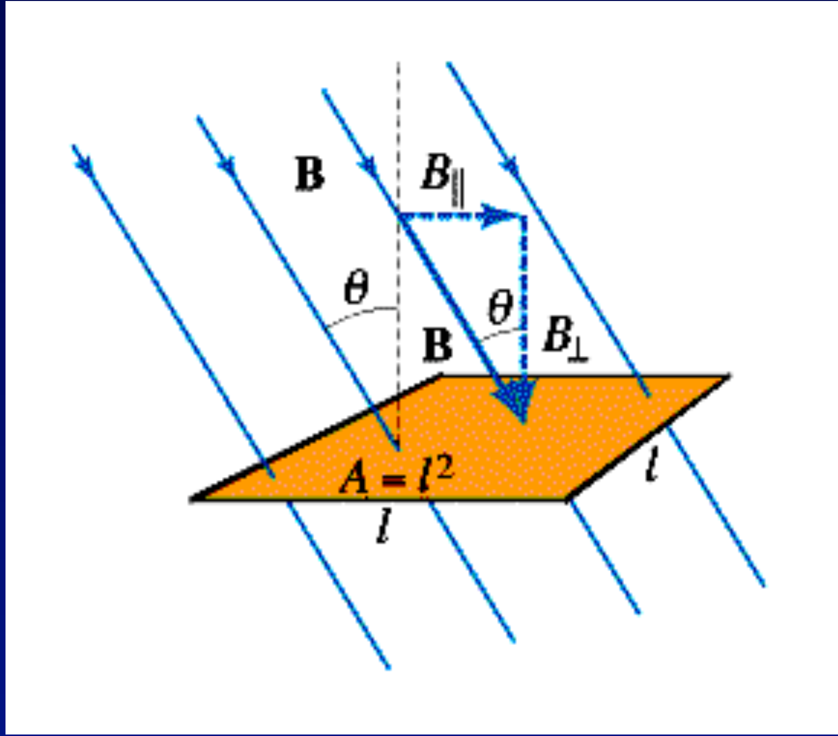
$$B(2\pi r) = 0 \qquad B = 0$$

Chapter 29: Electromagnetic Induction

- Magnetic Flux
- Induced EMF
  - Faraday’s law
  - Lenz’s law
- Motional EMF
- Applications of Induction
  - generators
  - motors
- Counter EMF
- Faraday Law (general form)
- Displacement Current and Maxwell Equations

Magnetic Flux

- Consider the B field lines that pass through a surface
  - define a quantity called the *magnetic flux Φ*



$$\boxed{\Phi \equiv B\,A\cos\theta}$$

where *θ* is angle between *magnetic field B* and the *normal to the plane.*

- units of magnetic flux are $T.m^2$ = Weber (Wb)

- Scalar product $$\Phi_B = \vec{B}\cdot\vec{A}$$

Faraday's Law of Induction

Lenz's Law

$$\boxed{\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t}}$$

induced emf

rate of change of flux with time

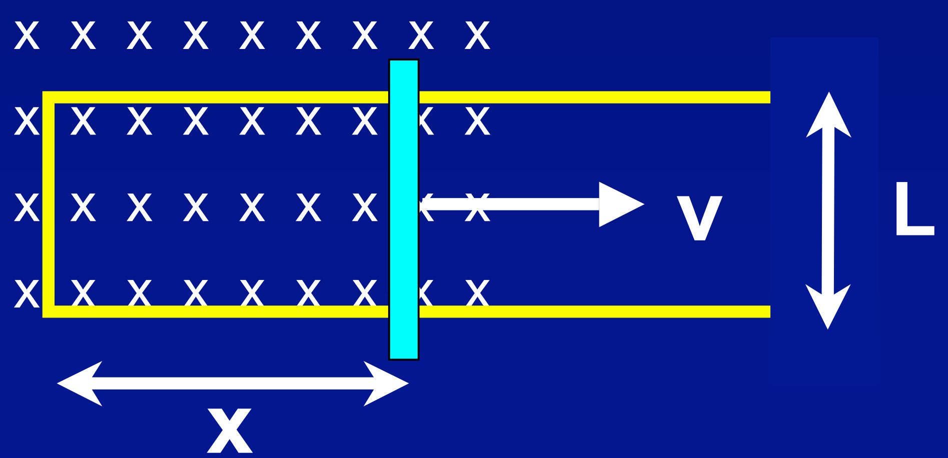
- minus sign comes from <u>Lenz's Law</u>:
  - The induced emf gives rise to a current whose magnetic field <u>*opposes*</u> the original <u>*change in flux.*</u>

Motional EMF

- Consider a conducting rod moving on metal rails in an uniform magnetic field:

$$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{\Delta(BA)}{\Delta t} = \frac{\Delta(BLx)}{\Delta t} = BL\frac{\Delta x}{\Delta t}$$

- Current will flow counter-clockwise in this “circuit”

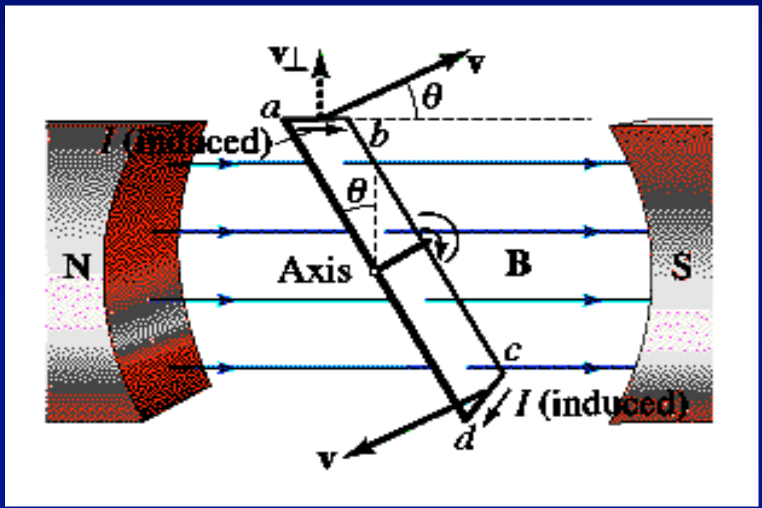


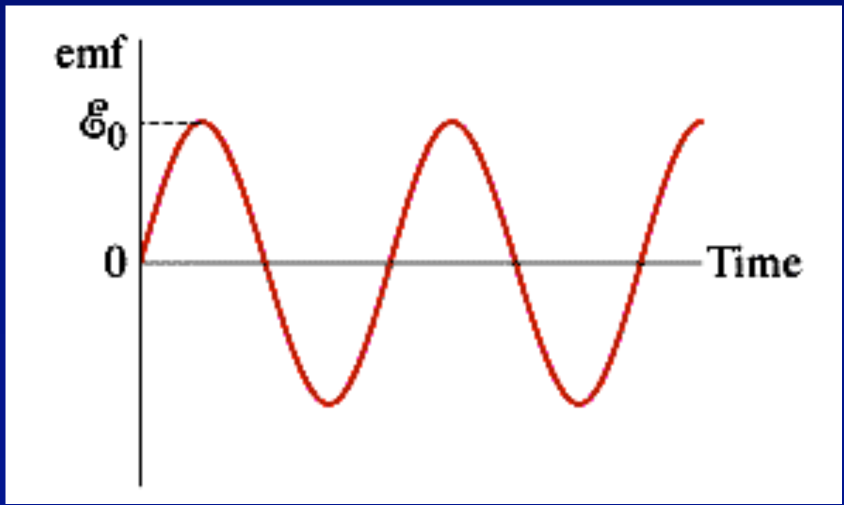
$$|\mathcal{E}| = BLv$$

Electric Generators

- Flux is changing in a sinusoidal manner:
  - $\Phi = B\,A \cos\theta = B\,A \cos(\omega t)$
  - this leads to an ***alternating emf*** (AC generator)

$$\varepsilon = N\frac{d\Phi_B}{dt} = NBA\frac{d\cos(\omega t)}{dt} = NBA\,\omega\sin(\omega t)$$





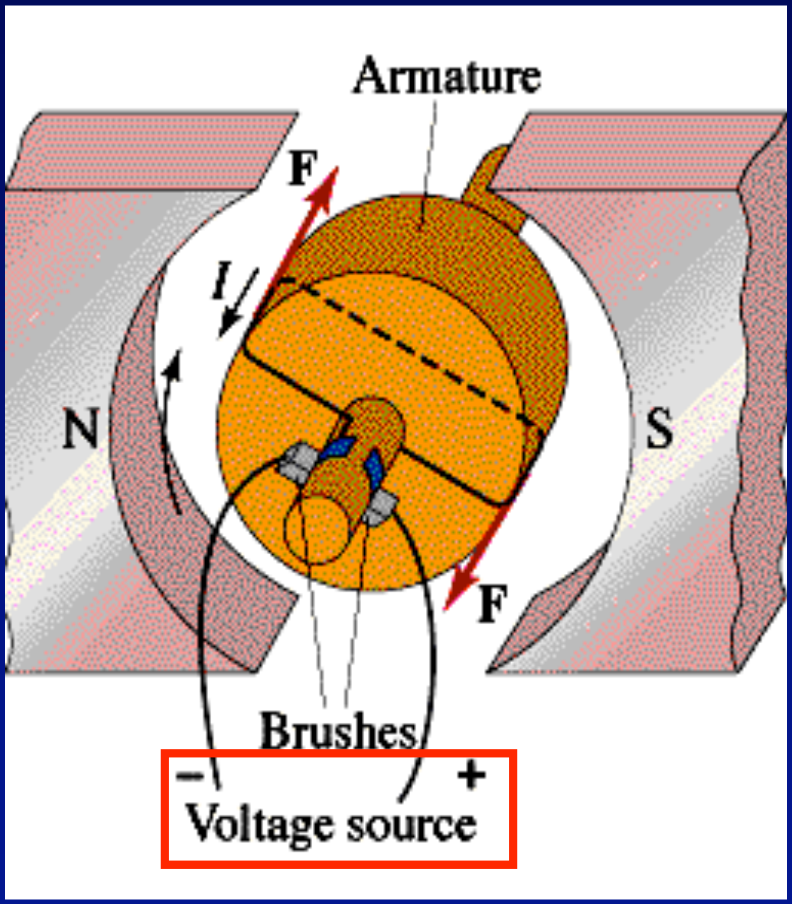
- This is how most of our electricity is generated !!
  - water or steam turns blades of a turbine which rotates a loop

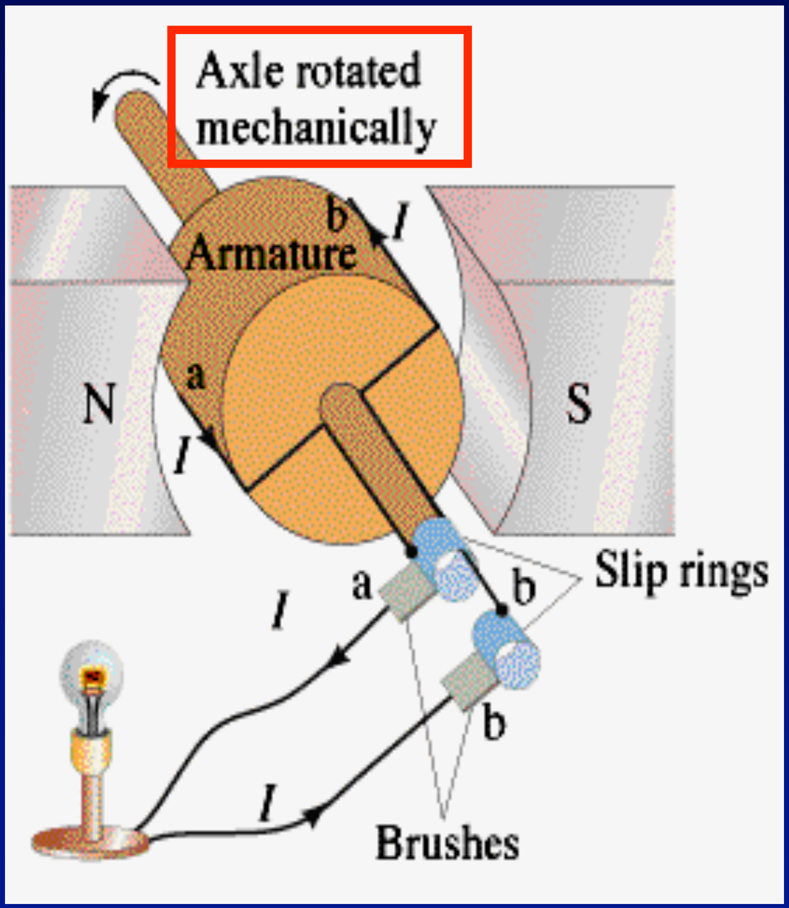
Motors

Generators

AC current + B field → rotation

rotation + B field → AC current



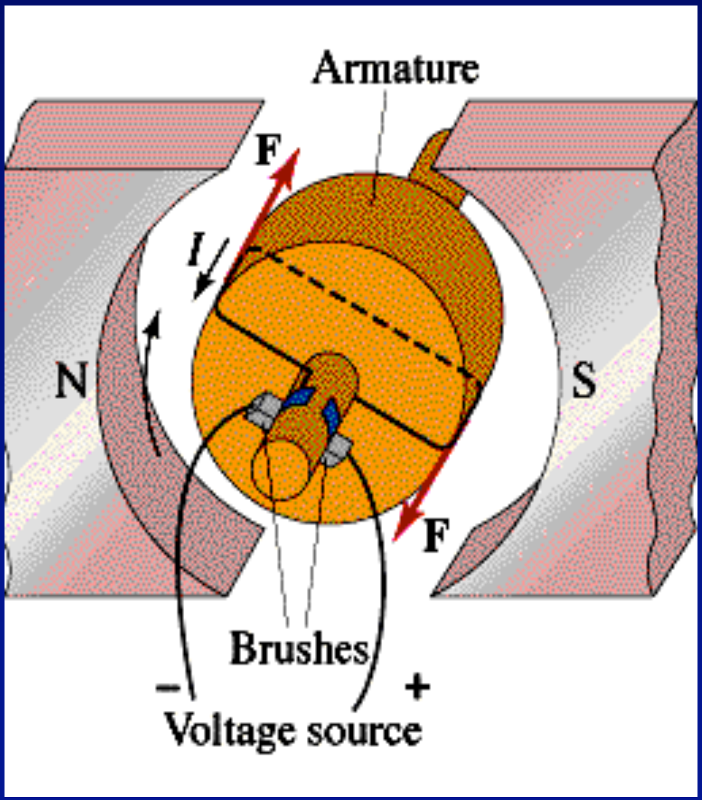


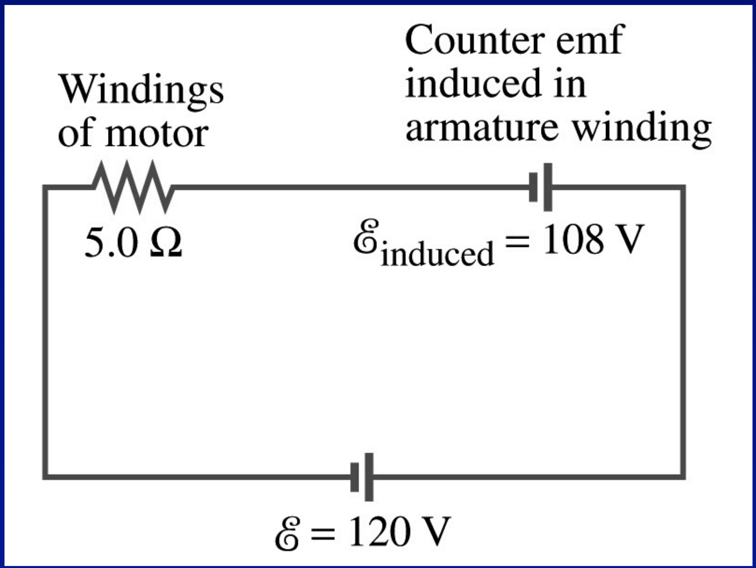
electrical ⇒ mechanical energy

mechanical ⇒ electrical energy

Counter EMF in a motor

- The armture windings of dc motor have a resistance of 5.0 Ω. The motor is connected to a 120-V line, and when the motor reaches full speed against its normal load, the counter emf is 108 V. Calculate
  - the current into the motor when it is just starting up
  - the current when it reaches full speed.





Faraday's Law (general form)

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

$$\varepsilon = \oint E \cdot dl$$

The integral is taken around the loop through which the magnetic flux is changing.

$$\oint E \cdot dl = -\frac{d\Phi_B}{dt}$$

- A changing magnetic flux produces an electric field.

It's a non-conservative field, because $\varepsilon = \oint E \cdot dl \neq 0$

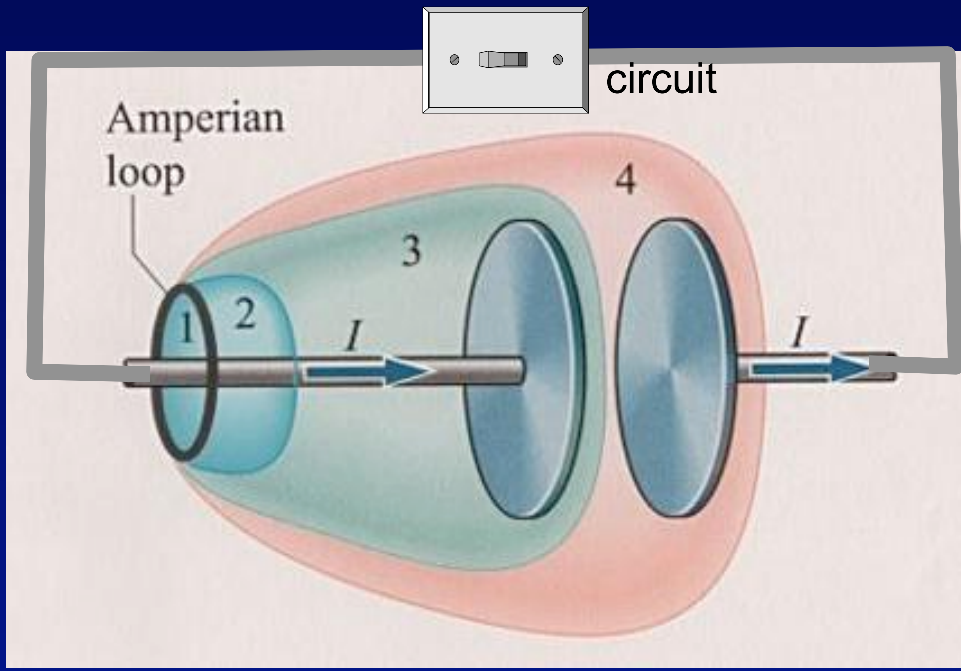
Maxwell's Displacement Current

- Can we understand why we must have a "displacement current"?

• Consider applying Ampere's Law to the current shown in the diagram.

  • If the surface is chosen as 1, 2 or 4, the enclosed current = I

  • If the surface is chosen as 3, the enclosed current = 0! (ie there is no current between the plates of the capacitor)



Big Idea: The added term is non-zero in this case, since the current I causes the charge Q on the capacitor to change in time which causes the Electric field in the region between the plates to change in time. The "displacement current" $I_D = \varepsilon_0 (d\phi_E/dt)$ in the region between the plates = the real current I in the wire.

Maxwell's Displacement Current

- In order to have $\oint \vec{B} \bullet d\vec{\ell}$ for surface 2 to be equal to $\oint \vec{B} \bullet d\vec{\ell}$ for surface 3, we want the displacement current in the region between the plates to be equal to the current in the wire.

- The Electric Field E between the plates of the capacitor is determined by the charge Q on the plate of area A: $E = Q/(A\varepsilon_0)$
- What we want is a term that relates E to I without involving A. The answer: the time derivative of the electric flux!!

Recall flux:

$$\phi_E = \oint E \cdot dS = \frac{1}{\varepsilon_0} Q$$

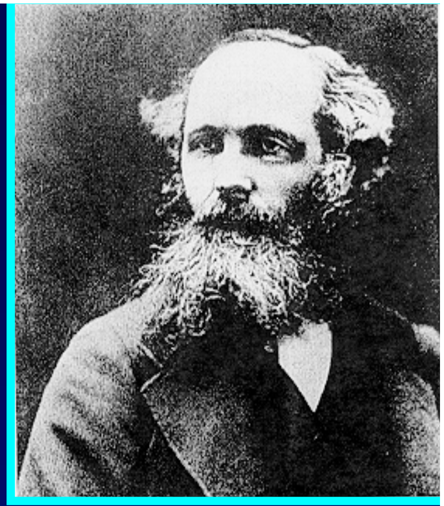
$$\frac{d\phi_E}{dt} = \frac{1}{\varepsilon_0}\frac{dQ}{dt} = \frac{1}{\varepsilon_0} I$$

Therefore, if we want $I_D = I$, we need to identify:

$$\boxed{I_D = \varepsilon_0 \frac{d\phi_E}{dt}}$$

$$\boxed{\oint \vec{B} \bullet d\vec{\ell} = \mu_0 (I + I_D)}$$

Maxwell's Equations



J. C. Maxwell (1831 - 1879)

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

Gauss's law for electric field: electric charges produce electric fields.

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss's law for magnetic field: but there're no magnetic charges.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Faraday's law: changing B produces E.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

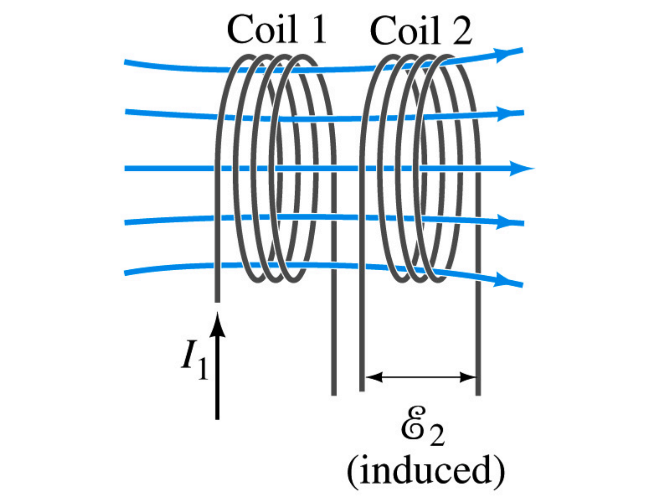
Ampere's law as modified by Maxwell: electric current or changing E produces B.

All of electromagnetism is contained in this set of four equations.

Chapter 30: Inductance

- Mutual Inductance
- Self-inductance
- Energy Stored in a Magnetic Field
- LR Circuits (DC)
- LC Circuits (DC)
- LRC Circuits (DC)

Mutual Inductance



The magnetic flux in coil 2 created by coil 1 is proportional to I_1. Define

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1}$$

M_{21} is called the mutual inductance. It depends only on the geometric factors, NOT on the currents.

Faraday's law:

$$\varepsilon_2 = -N_2 \frac{d\Phi_{21}}{dt}$$

$$\varepsilon_2 = -M_{21} \frac{dI_1}{dt}$$

The reverse situation is

$$\varepsilon_1 = -M_{12} \frac{dI_2}{dt}$$

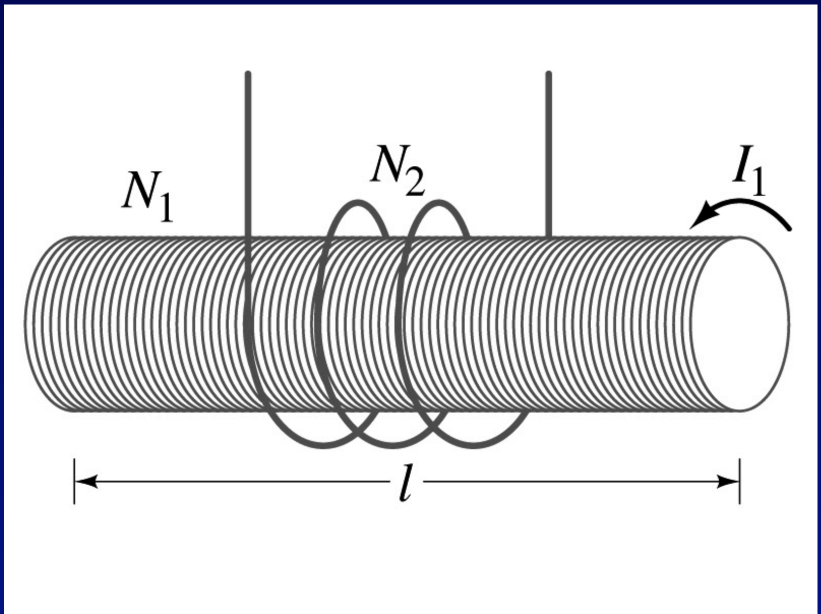
$$M_{12} = M_{21} = M$$

The SI unit for M is henry (H). 1 H = 1 V.s/A = 1 Ω.s

Example: Solenoid and coil

The magnetic field inside the solenoid is

$$B = \mu_0 \frac{N_1}{l} I_1$$



The magnetic flux through the coil is

$$\Phi_{21} = BA = \mu_0 \frac{N_1}{l} I_1 A$$

Hence the mutual inductance is

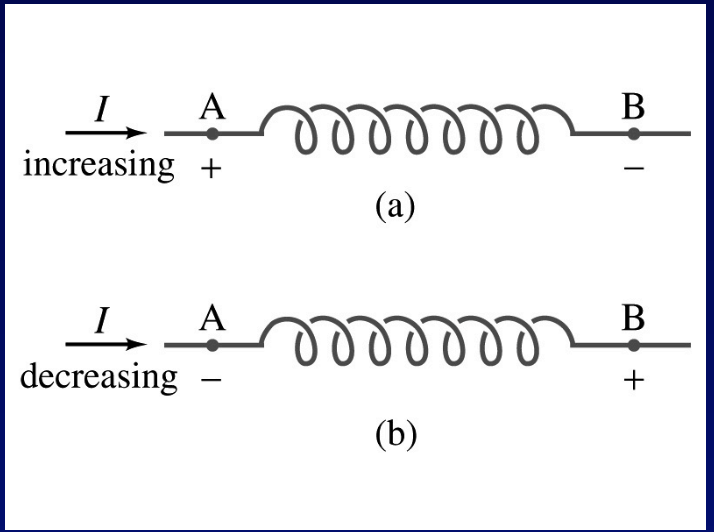
$$M = \frac{N_2 \Phi_{21}}{I_1}$$

$$M = \mu_0 \frac{N_1 N_2 A}{l}$$

Self-Inductance

The total magnetic flux in the coil is proportional to the current I. Define

$$L = \frac{N\Phi_B}{I}$$



L is called self-inductance. It depends only on the geometric factors, NOT on the current. Such coil is called an inductor.

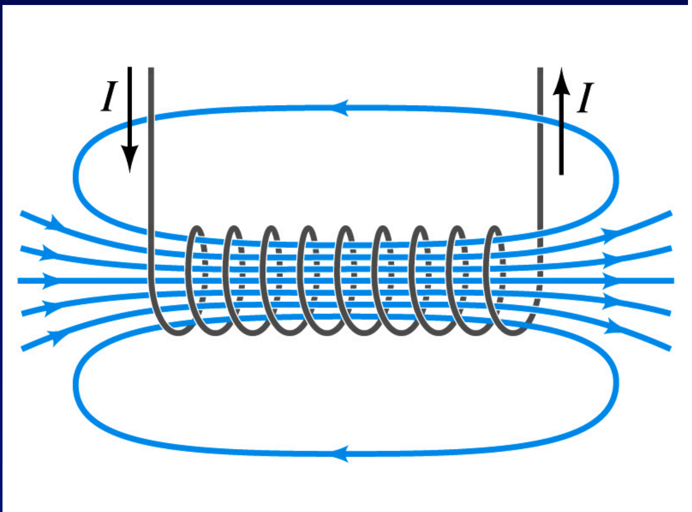
Use Faraday's law: $\varepsilon = -N\frac{d\Phi_B}{dt}$ $\quad \varepsilon = -L\frac{dI}{dt}$

The SI unit for L is also henry (H). 1 H =1 Ω.s.

Example: Solenoid inductance

The magnetic field inside the solenoid is

$$B = \mu_0 \frac{N}{l} I$$



The total magnetic flux through the coil is

$$\Phi_B = BA = \mu_0 \frac{N}{l} IA$$

Hence the self-inductance is $$L = \frac{N\Phi_B}{I} \text{ or } \quad \boxed{L = \mu_0 \frac{N^2 A}{l}}$$

For N=100, l=5 cm, A=0.3 cm^2, L=4π x10^{-7}x100^{2}x0.3x10^{-4}/0.05=7.5 μH.

If filled with an iron core (μ=4000 μ_0), L= 30 mH.

Voltage across an inductor

- Inductor does not oppose current that flows through it. It opposes the change in the current.
  - It's a current stabilizer in the circuit.

$$V_{ab} = iR$$

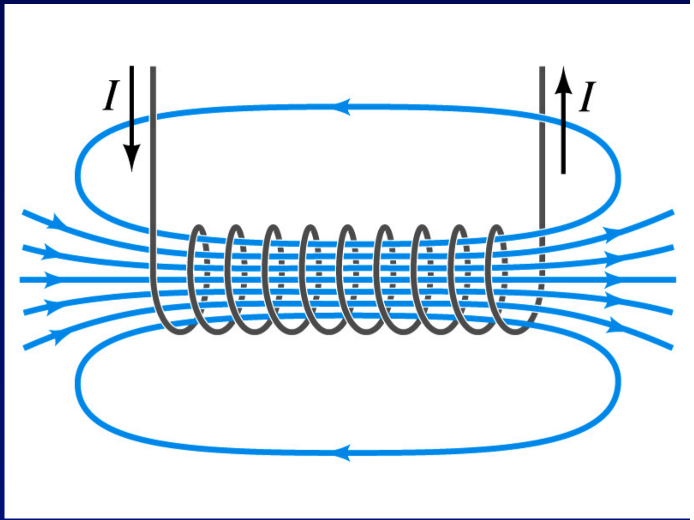
(a) Resistor with current i flowing from a to b: potential drops from a to b

$$\boxed{V_{ab} = L\frac{di}{dt}}$$

(b) Inductor with current i flowing from a to b:
- If $di/dt > 0$: potential drops from a to b
- If $di/dt < 0$: potential increases from a to b
- If i is constant ($di/dt = 0$): no potential difference

Energy Stored in a Magnetic Field

When an inductor is carrying a current which is changing at a rate dI/dt, the energy is being supplied to the inductor at a rate



$$P = I\varepsilon = LI\frac{dI}{dt}$$

The work needed to increase the current from 0 to I is

$$W = \int P dt = \int_0^I LI dI = \frac{1}{2}LI^2$$

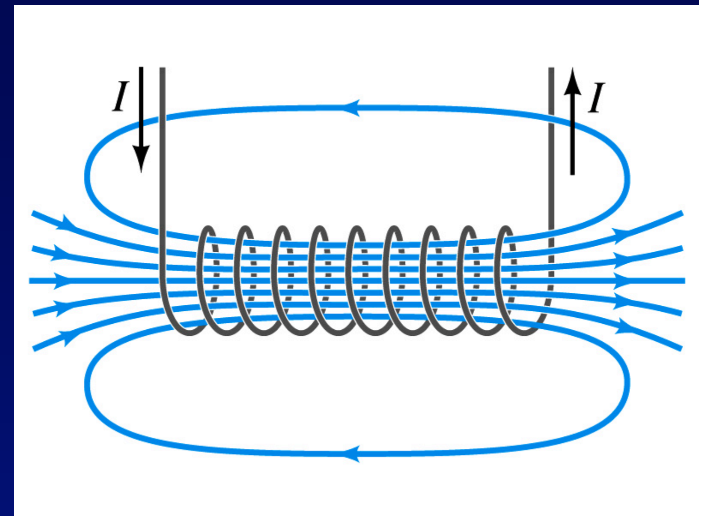
By energy conservation, the energy stored in the inductor is

$$U = \frac{1}{2}LI^2$$

Energy Stored in a Magnetic Field

Question: Where exactly does the energy reside?

$$U = \frac{1}{2} L I^2$$



Answer: It resides in the magnetic field.

Using $L = \mu_0 \dfrac{N^2 A}{l}$ and $B = \mu_0 \dfrac{N}{l} I$

$$U = \frac{1}{2}\left(\mu_0 \frac{N^2 A}{l}\right)\left(\frac{Bl}{\mu_0 N}\right)^2 = \frac{1}{2}\frac{B^2}{\mu_0} A l$$

Or energy density

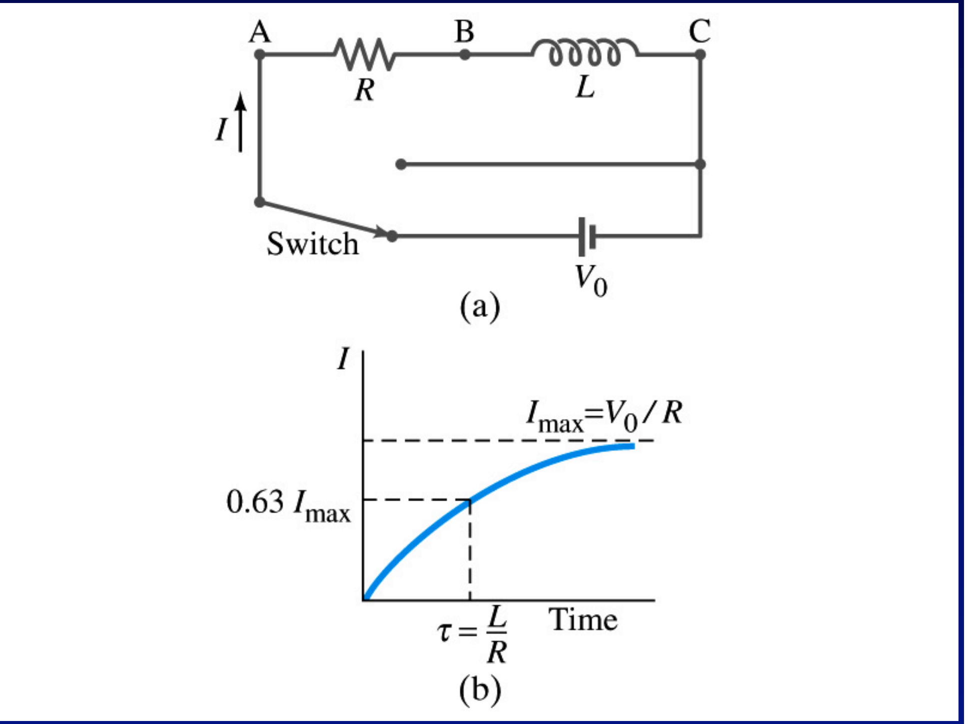
$$u = \frac{1}{2}\frac{B^2}{\mu_0}$$

The conclusion is valid for any region of space where a magnetic field exists.

Compare with the electric case: $U = \dfrac{1}{2} C V^2$ $u = \dfrac{1}{2}\varepsilon_0 E^2$

LR Circuits

$$\text{loop rule}: V_0 - IR - L\frac{dI}{dt} = 0$$



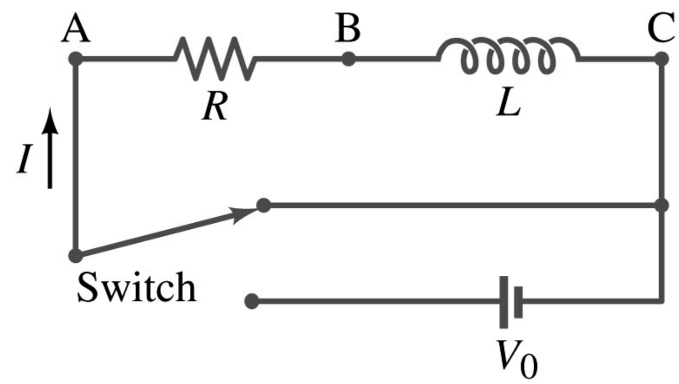
Solve differential equation:

$$I(t) = \frac{V_0}{R}\left(1 - e^{-t/\tau}\right)$$

Where $\tau = L/R$ is called the time constant.

LR Circuits

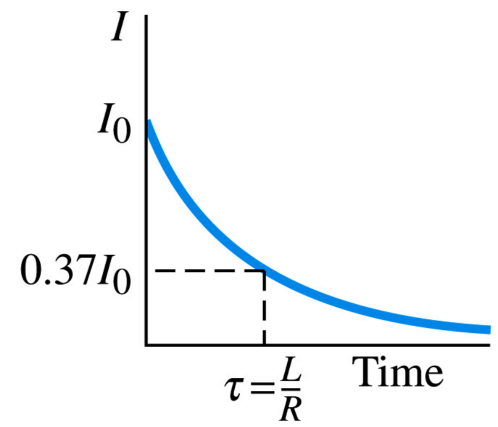
$$\text{loop rule}: -IR - L\frac{dI}{dt} = 0$$



Solve differential equation

$$I(t) = \frac{V_0}{R}e^{-t/\tau}$$

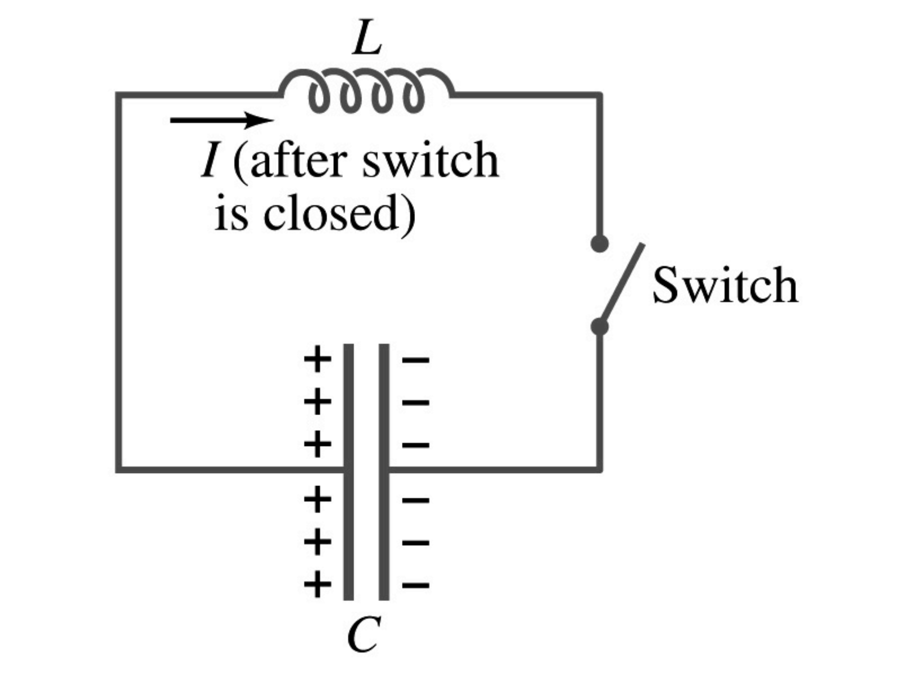
time constant τ=L/R



Summary: there is always some reaction time when a LR circuit is turned on or off. The situation is similar to RC circuits, except here the time constant is proportional to 1/R, not R.

LC Circuits

The capacitor is charged to Q_0.
At t=0, the circuit is closed.
What will happen?



$$\text{loop rule}: \frac{Q}{C} - L\frac{dI}{dt} = 0$$

Using I=-dQ/dt, one gets

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0$$

Solve differential equation

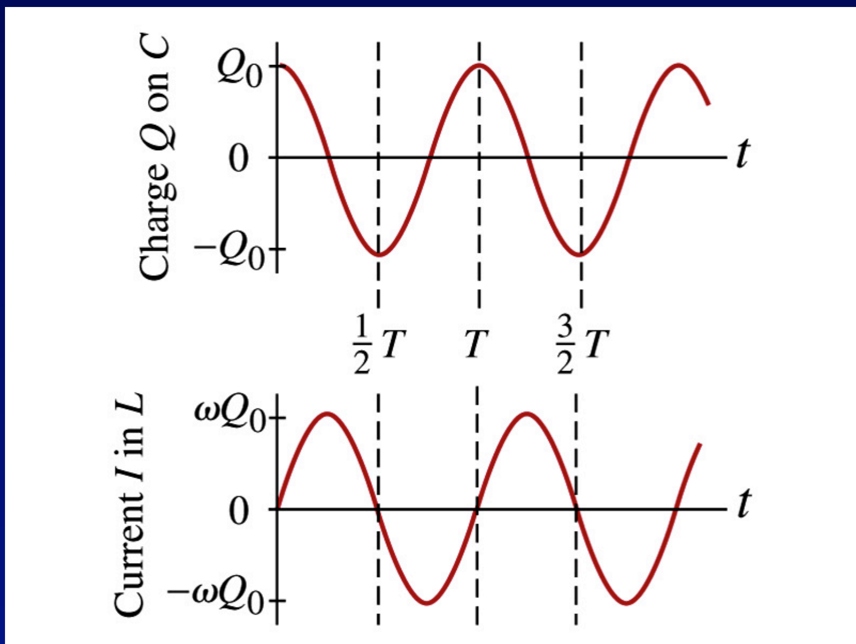
$$Q(t) = Q_0 \cos(\omega t + \phi)$$

where

$$\omega = 1/\sqrt{LC}$$

LC Circuits

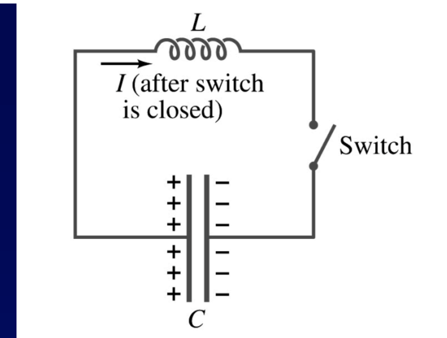
$$Q(t) = Q_0 \cos(\omega t + \phi)$$



$$I(t) = \frac{dQ}{dt} = -Q_0 \omega \sin(\omega t + \phi)$$

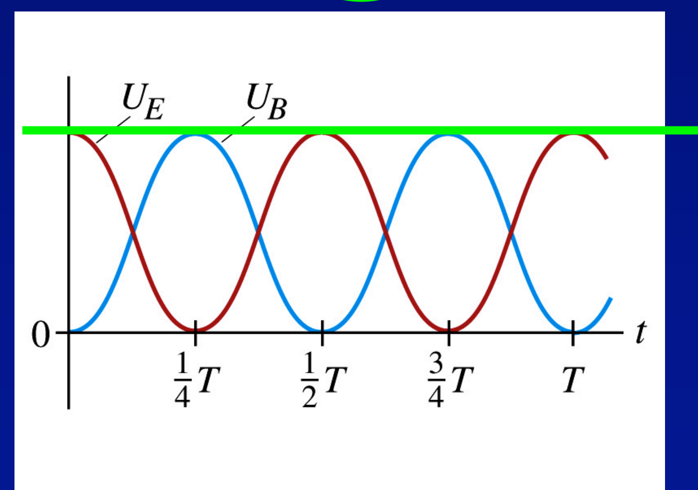
Charge oscillates! So does the current and voltage.

What about energy?



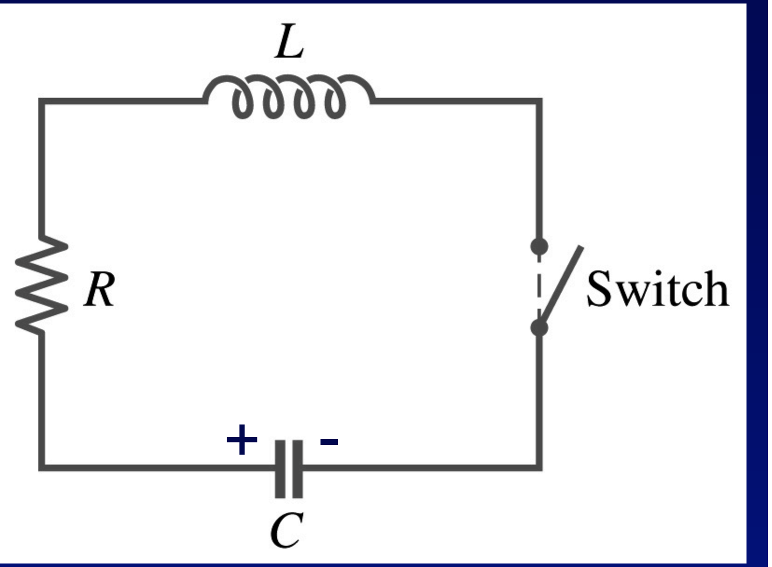
$$U_E = \frac{1}{2}\frac{Q^2}{C} = \frac{Q_0^2}{2C}\cos^2(\omega t + \phi)$$

$$U_B = \frac{1}{2}LI^2 = \frac{Q_0^2}{2C}\sin^2(\omega t + \phi)$$



The total energy is conserved.

LRC Circuits



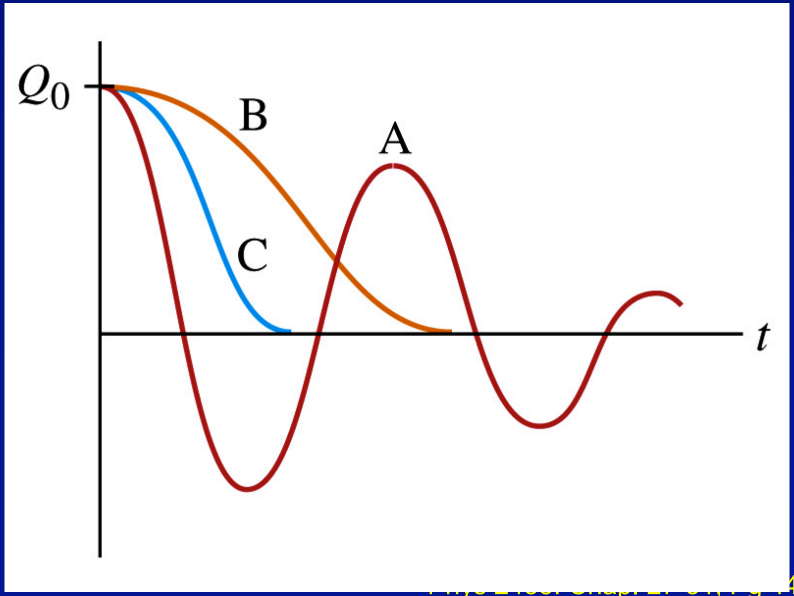
Damped oscillations! 3 scenarios.

- A) Under-damped if $R^2 < 4L/C$.
- B) Critical damping if $R^2 = 4L/C$.
- C) Over-damped if $R^2 > 4L/C$.

For under-damping:

$$Q(t) = Q_0 e^{-\frac{R}{2L}t} \cos(\omega' t + \phi)$$

$$T = \frac{2\pi}{\omega'} = \frac{2\pi\sqrt{LC}}{\sqrt{1 - \frac{R^2 C}{4L}}}$$



Summary of Various Direct Current Circuits

RC circuit, time constant $\tau = RC$ (transient)

LR circuit, time constant $\tau = L/R$ (transient)

LC circuit, oscillation period $T = 2\pi\sqrt{LC}$ (oscillator)

LRC circuit, damped oscillation period $T = 2\pi\sqrt{LC} \,/\, \sqrt{1 - \frac{R^2C}{4L}}$

(damped oscillator)

Chapter 31: Alternating Current

- AC Circuits (using phasors)
  - AC Circuit, R only
  - AC Circuit, L only
  - AC Circuit, C only
  - AC Circuit with LRC
- Resonance
- Transformers

RMS current and voltage

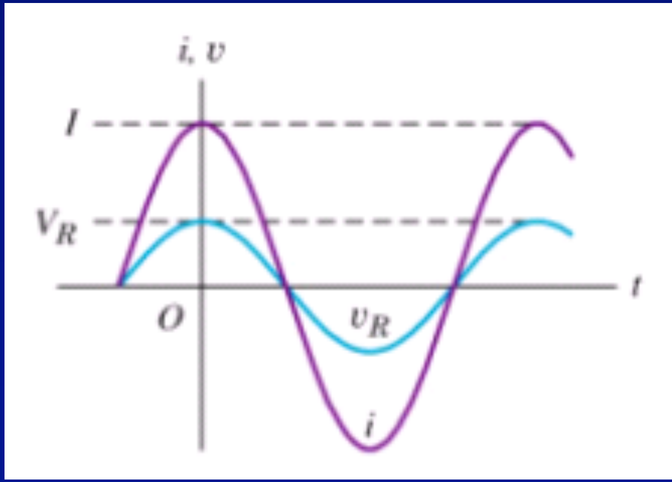
$$i = I\cos\omega t$$

root-mean-square: $$I_{rms} = \sqrt{\overline{i^2}}$$

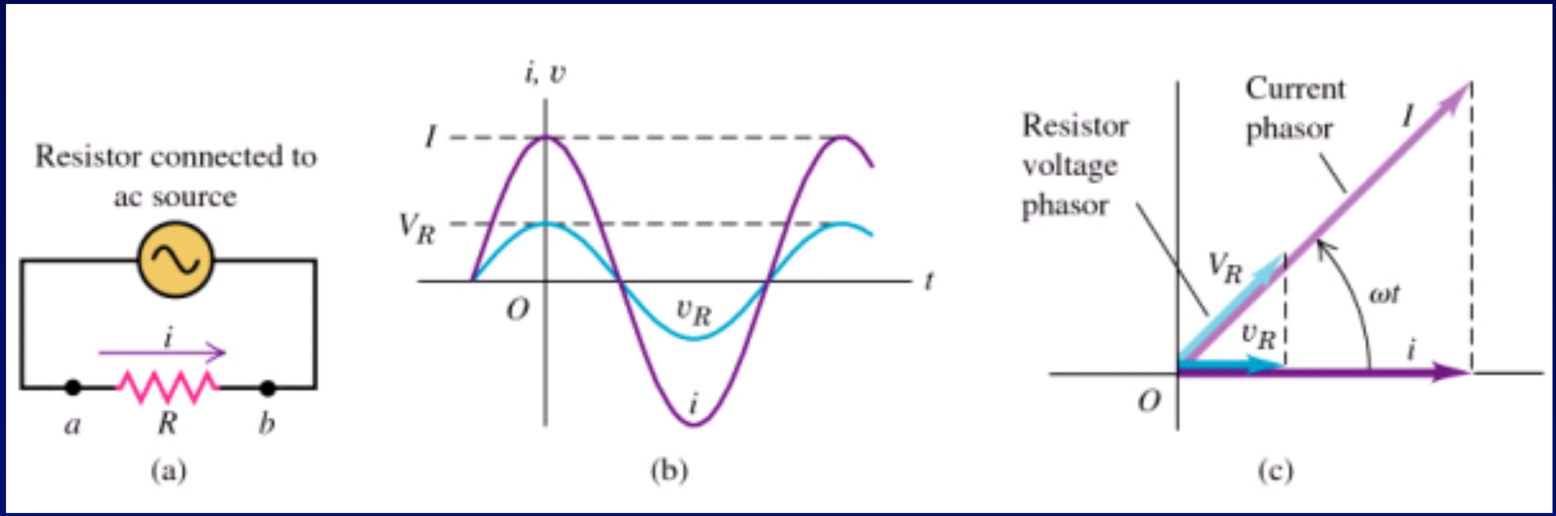
$$\overline{i^2} = \frac{1}{T}\int_0^T i^2 dt = \frac{I^2}{2T}\int_0^T (1+\cos 2\omega t)\,dt = \frac{I^2}{2}$$

$$I_{rms} = \frac{I}{\sqrt{2}}$$

$$V_{rms} = \frac{V}{\sqrt{2}}$$



AC Circuit Containing only Resistance



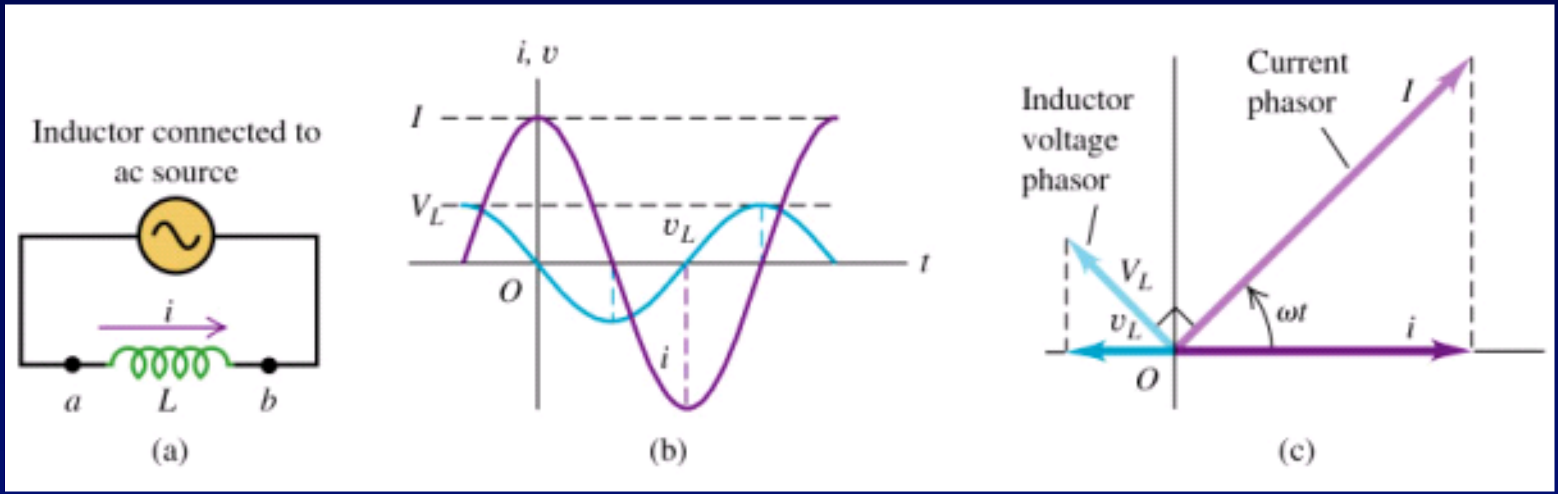
The AC source is given as $i = I\cos\omega t$

So the voltage across the resistor is $v_R = iR = IR\cos\omega t = V_R\cos\omega t$

The resistor voltage is in phase with the current.

The power dissipated in the resistor is p=iv, or at an average rate $\overline{P} = I_{rms}^2 R = \dfrac{V_{rms}^2}{R}$

AC Circuit Containing only Inductor



The AC source is given $i = I\cos\omega t$

So the voltage across the inductor is

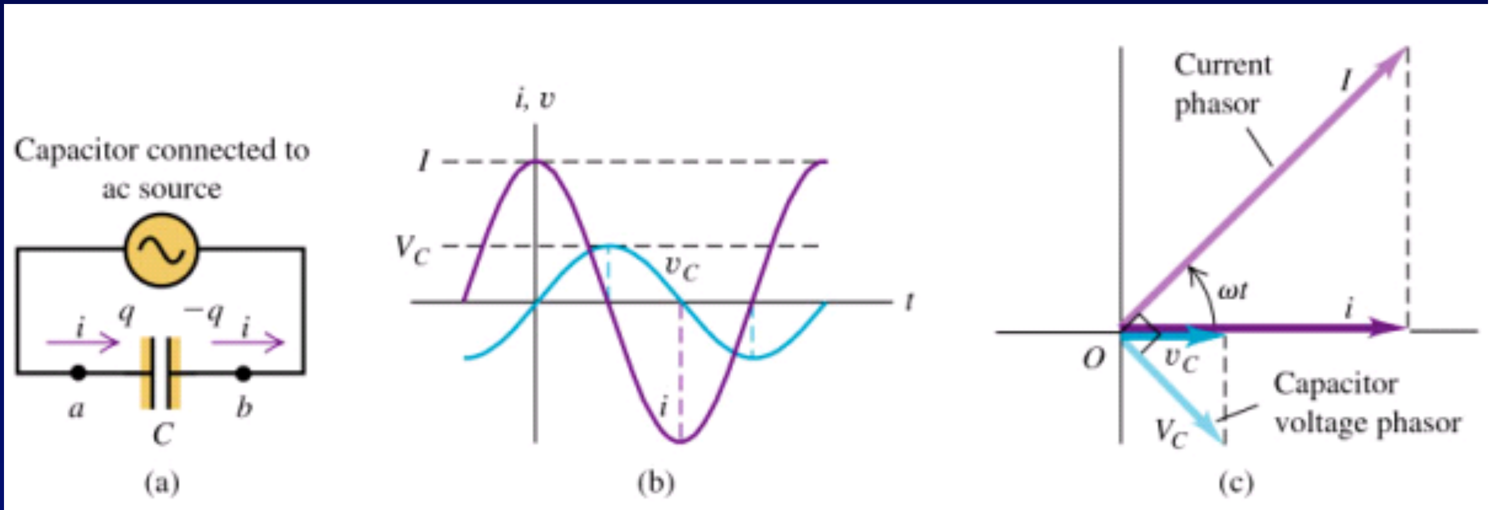
$$v_L = L\frac{di}{dt} = -IL\omega\sin\omega t = V_L\cos(\omega t + 90^0)$$

The inductor voltage leads the current by 90 degrees.

Define reactance of inductor $X_L=\omega L$, then $V_L = I X_L$.
Its unit is Ohm.

AC Circuit Containing only Capacitor

$$i = I\cos\omega t$$



The voltage across the capacitor is $v = q/C$

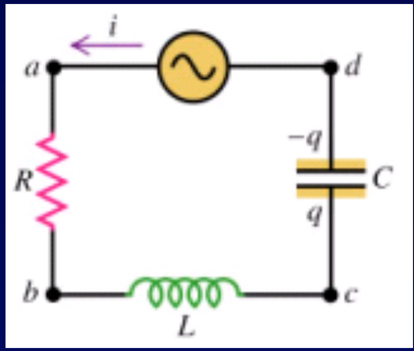
$$q = \int_0^t i\,dt = \frac{I}{\omega}\sin\omega t = \frac{I}{\omega}\cos(\omega t - 90^0)$$

$$v_C = \frac{I}{\omega C}\cos(\omega t - 90^0)$$

The capacitor voltage lags the current by 90 degrees.

Define reactance of capacitor $X_C = 1/\omega C$, then $V_C = I\,X_C$. Its unit is Ohm.

AC-driven LRC Circuit in Series
(Graphical Method)

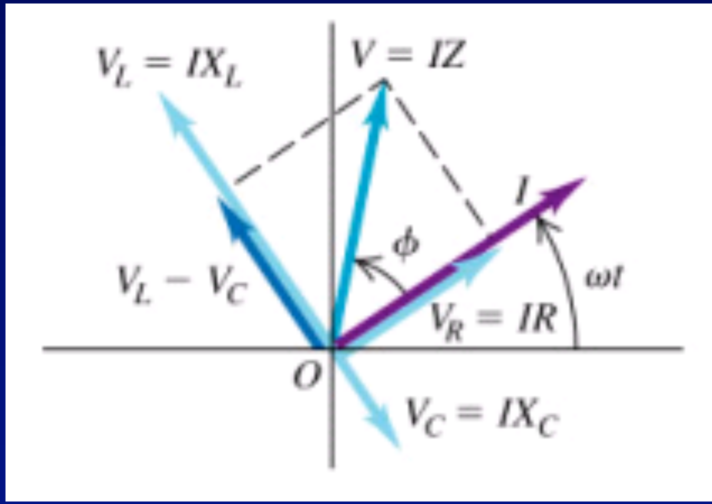


The goal is to cast the total voltage at any time into the form

$$i = I\cos\omega t$$

$$v = v_R + v_L + v_C = V\cos(\omega t + \phi)$$

where ϕ is the phase angle with which v leads i.



From the triangle:

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$
$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

The impedance

$$V = IZ \text{ with } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The phase difference

$$\tan\varphi = \frac{X_L - X_C}{R} \quad \text{or} \quad \cos\varphi = \frac{R}{Z}$$

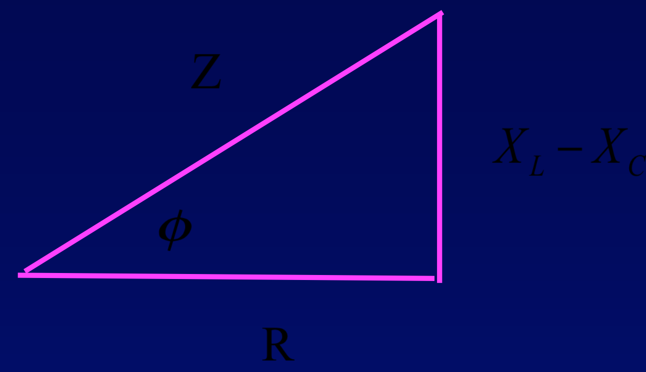
The Impedance Triangle

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$\tan\phi = \frac{X_L - X_C}{R} \quad \text{or } \cos\phi = \frac{R}{Z}$$



General results, valid for any combination of L, R, C in series.
For example, if LR circuit, set X_C=0.

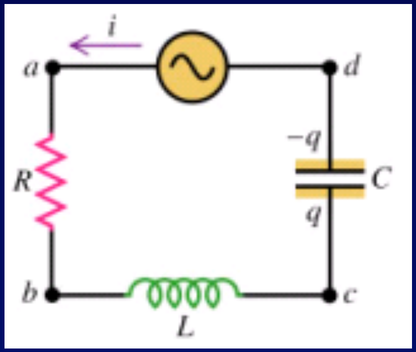
Check special cases:

$$R \text{ only} : \mathrm{X_L} = 0, \mathrm{X_C} = 0, \text{so } \mathrm{Z} = \mathrm{R}, \phi = 0$$

$$L \text{ only} : \mathrm{R} = 0, \mathrm{X_C} = 0, \text{so } \mathrm{Z} = \mathrm{X}_L, \phi = +90^0$$

$$C \text{ only} : \mathrm{R} = 0, \mathrm{X_L} = 0, \text{so } \mathrm{Z} = \mathrm{X}_C, \phi = -90^0$$

Frequency Dependence



$$i = I\cos\omega t$$

$$v = V\cos(\omega t + \phi)$$

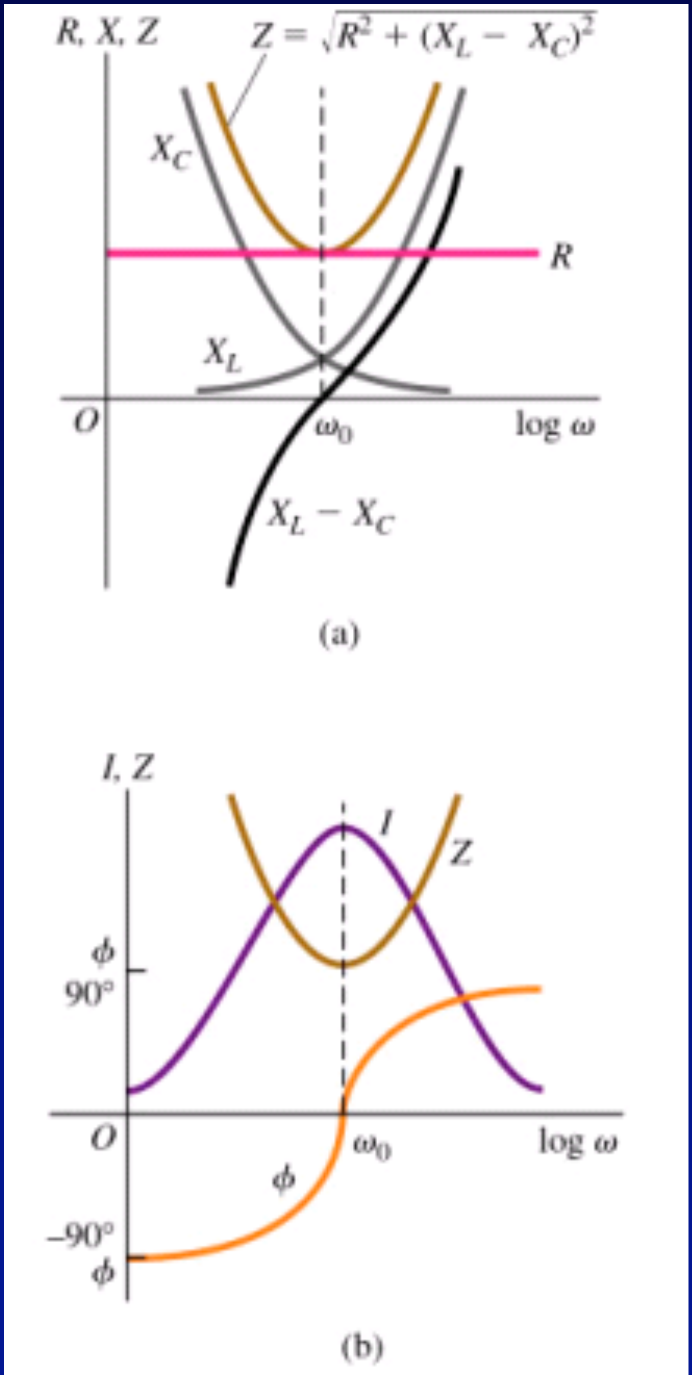
$$V = IZ$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

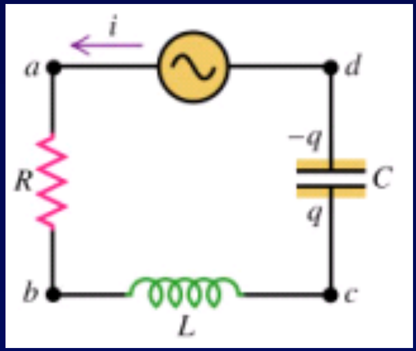
$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$\tan\varphi = \frac{X_L - X_C}{R} \quad \text{or } \cos\varphi = \frac{R}{Z}$$



The Power Factor



The power is only dissipated in the resistor, so the average power is

$$\overline{P} = I_{rms}^2 R$$

But

$$R = Z\cos\phi$$

$$\overline{P} = I_{rms}^2 Z\cos\phi = I_{rms} V_{rms}\cos\phi = \frac{1}{2} IV\cos\phi$$

The factor $\cos\phi$ is called the power factor.

For example:
For a pure resistor ($\phi=0$) , $\cos\phi=1$.
For a pure inductor ($\phi=90^0$) or capacitor ($\phi=-90^0$), $\cos\phi=0$.

Resonance in AC Circuit

Recall:

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

- For fixed R,C,L the current I will be a maximum at the resonant frequency ω_0 which makes the impedance Z purely resistive.

Z is minimum when:

$$\boxed{X_L = X_C}$$

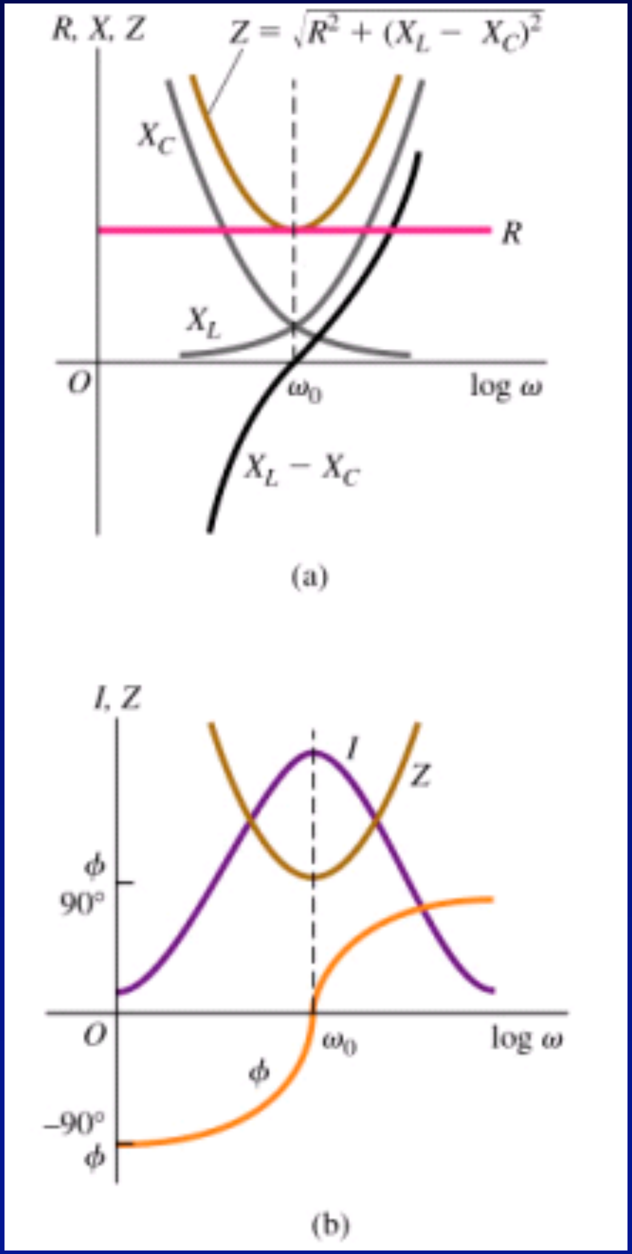
So the frequency at which this condition is satisfied is given from:

$$\omega_o L = \frac{1}{\omega_o C} \quad \Rightarrow \quad \boxed{\omega_o = \frac{1}{\sqrt{LC}}}$$

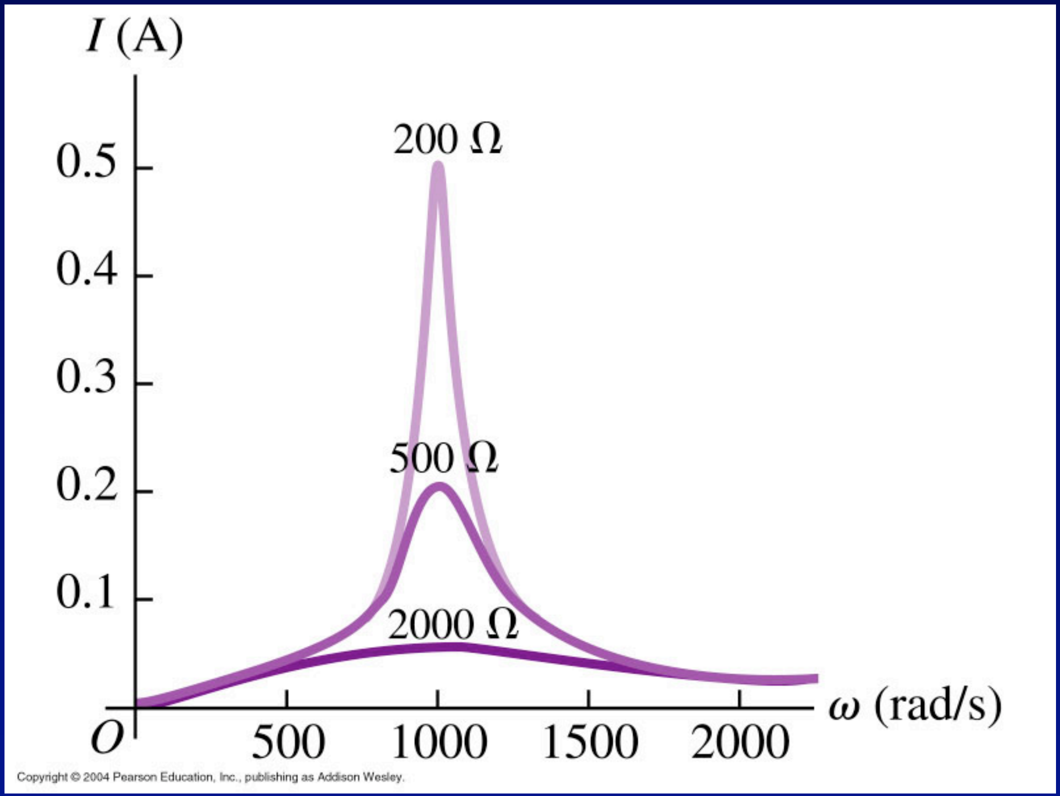
- **Note that this resonant frequency is identical to the natural frequency of the LC circuit by itself!**
- **At this frequency, the current and the driving voltage are in phase!**

$$\cos\phi = \frac{R}{Z} = 1, \ \text{so}\ \phi = 0$$

Resonance in AC Circuit



At resonance, I and V are in phase. V, I and P are at their maximum.



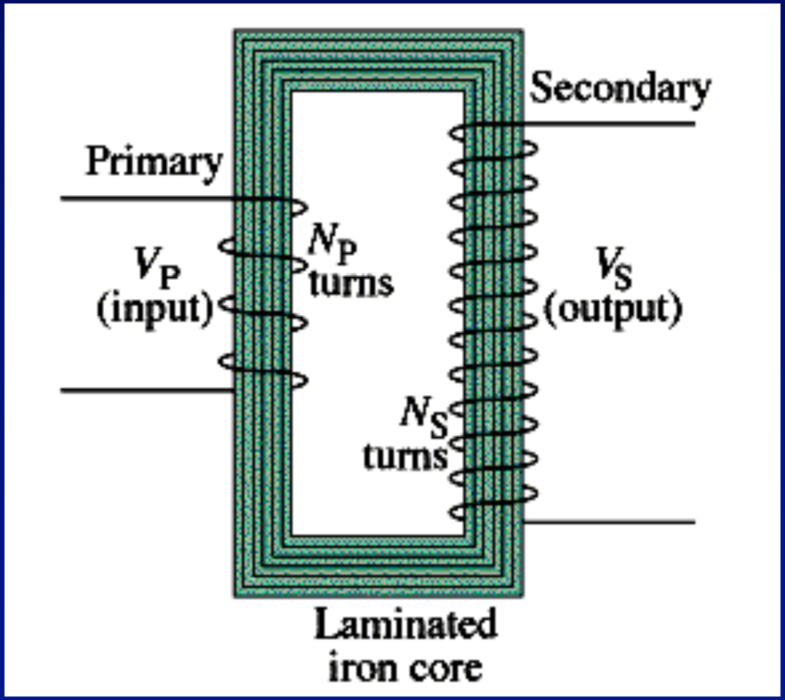
(V=100 V, L=2.0 H, C=0.5 μF)

Transformers

Transformers change alternating (AC) voltage to a bigger or smaller value

Input AC voltage V_p in the primary produces a flux

$$V_p = N_p \frac{\Delta\Phi_B}{\Delta t}$$



Changing flux in secondary induces emf V_s

$$V_s = N_s \frac{\Delta\Phi_B}{\Delta t}$$

Same $\Delta\Phi / \Delta t$!!

Transformer equation:

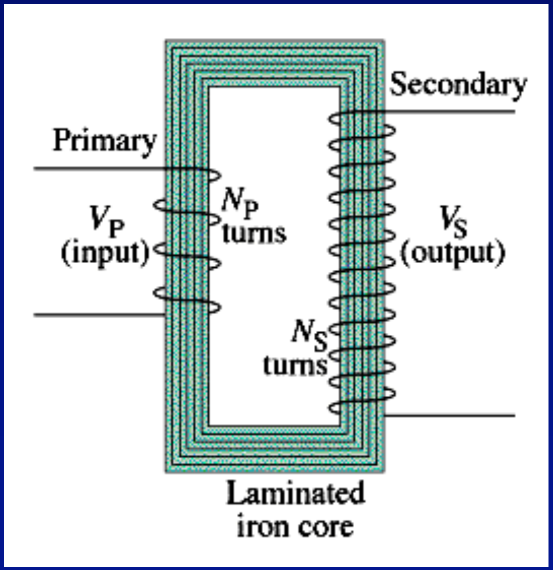
$$\boxed{V_s = V_p \frac{N_s}{N_p}}$$

Transformers

- Nothing comes for free, however!
  - voltage increase comes at the cost of current
  - output power cannot exceed input power
  - power in = power out (assume no heat loss)

$$I_p V_p = I_s V_s$$

$$\frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s}$$



If voltage increases, then current decreases
If voltage decreases, then current increases