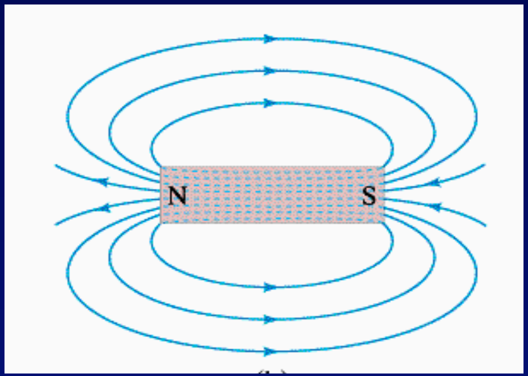
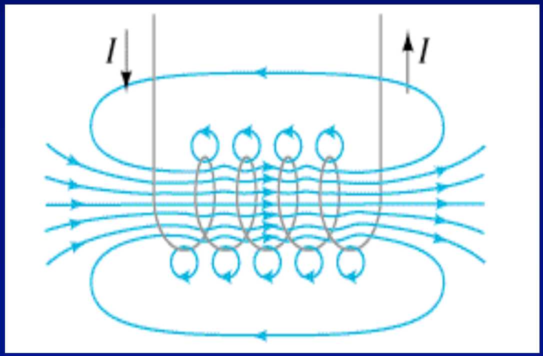
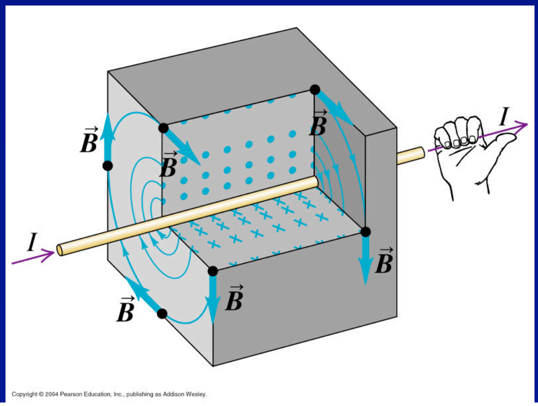
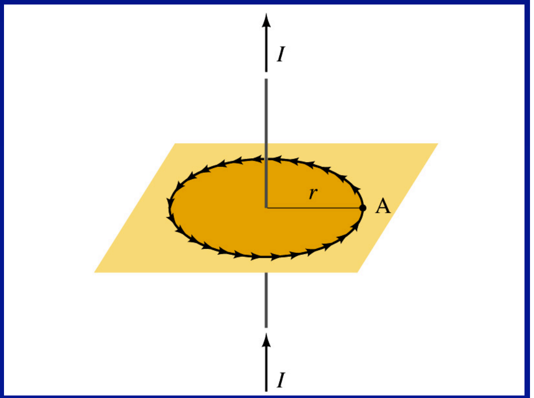
Chapter 28
Sources of Magnetic Field

- Electric Current generates Magnetic Field
- Biot-Savart Law
  - moving charge
  - A straight wire
  - Force between parallel wires
  - A current loop
- Ampere's Law
  - Solenoid and Toroid



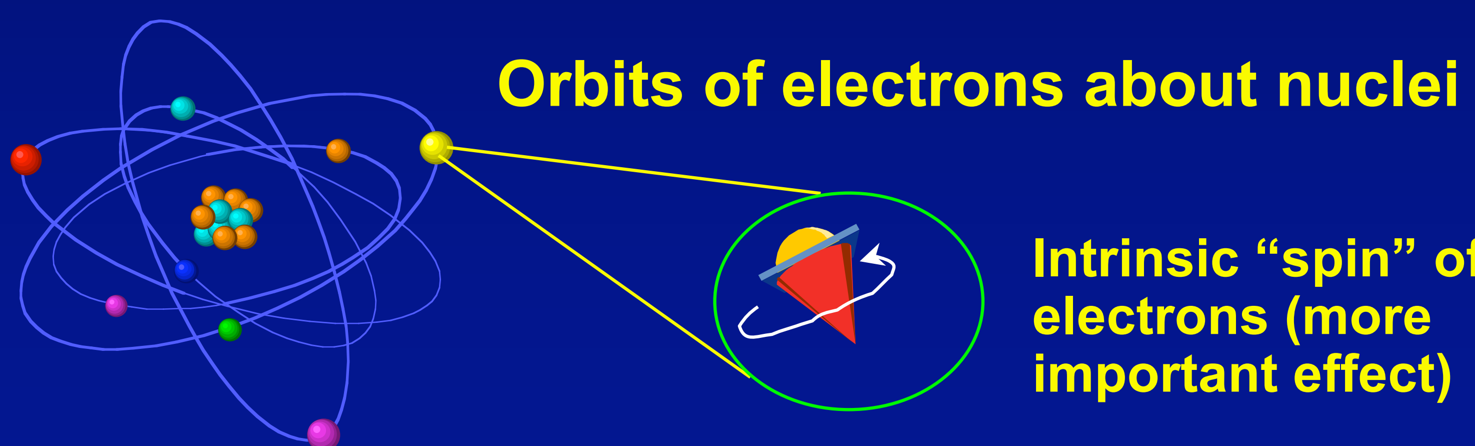






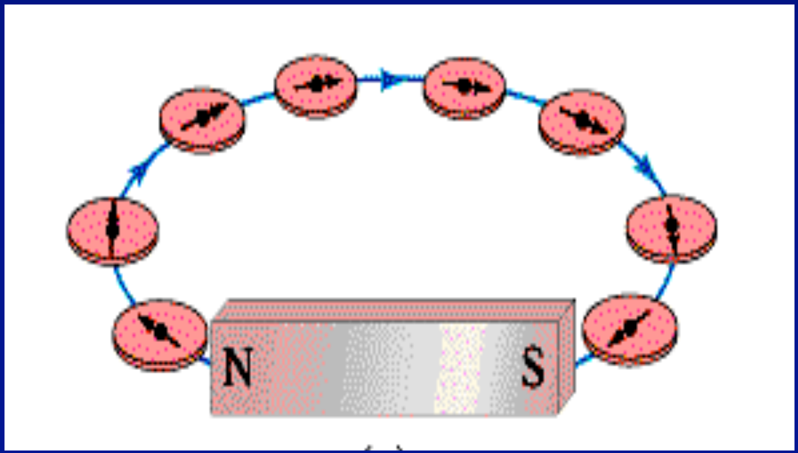
What causes magnetism?

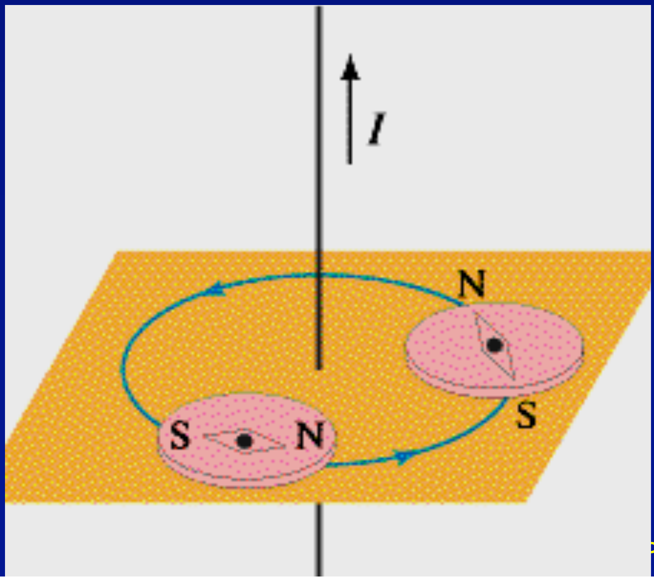
- What is the origin of magnetic fields?
- Answer: **electric charge in motion**!
  - For example, a current in a wire loop produces a field very similar to that of a bar magnet (as we shall see).
- To understand the field of a bar magnet one must understand electric currents at the atomic level!



Connection between *Electricity* and *Magnetism*

- We have seen that magnetic fields affects *moving* electric charges.
- In 1820, Oersted discovered another connection:
  - if a compass is placed near a wire and a current begins to flow through it, the compass needle deflects
  - *electric current produces a magnetic field!*





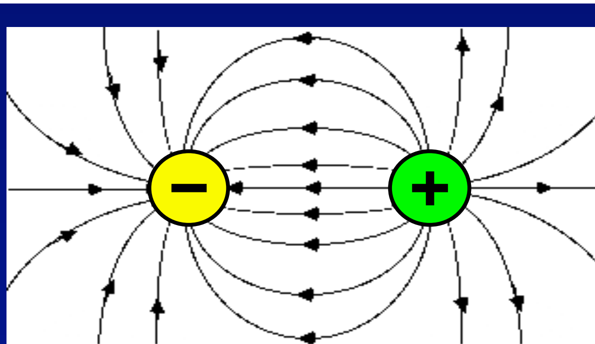
Electric Flux

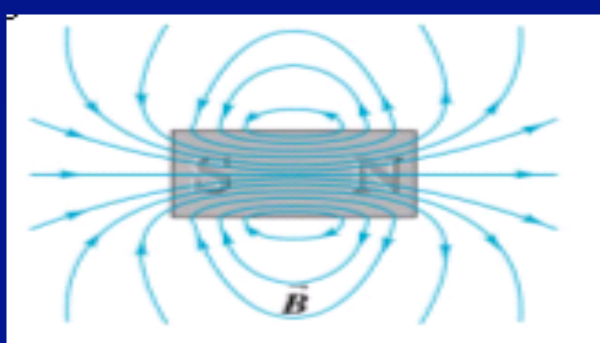
$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

Unit: N·m²/C

Gauss’s Law for electric field

$$\oint \vec{E} \cdot d\vec{A} = Q_{enclosed} / \varepsilon_0$$





Magnetic Flux

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Unit: 1 Wb = 1 T·m^2 = 1 N·m/A

Gauss’s Law for magnetic field

$$\oint \vec{B} \cdot d\vec{A} = 0$$

The total magnetic flux through a closed surface is always zero.

Reason: no isolated magnetic charges (magnetic monopoles).

Consequence: magnetic field lines are always closed.

Biot-Savart Law

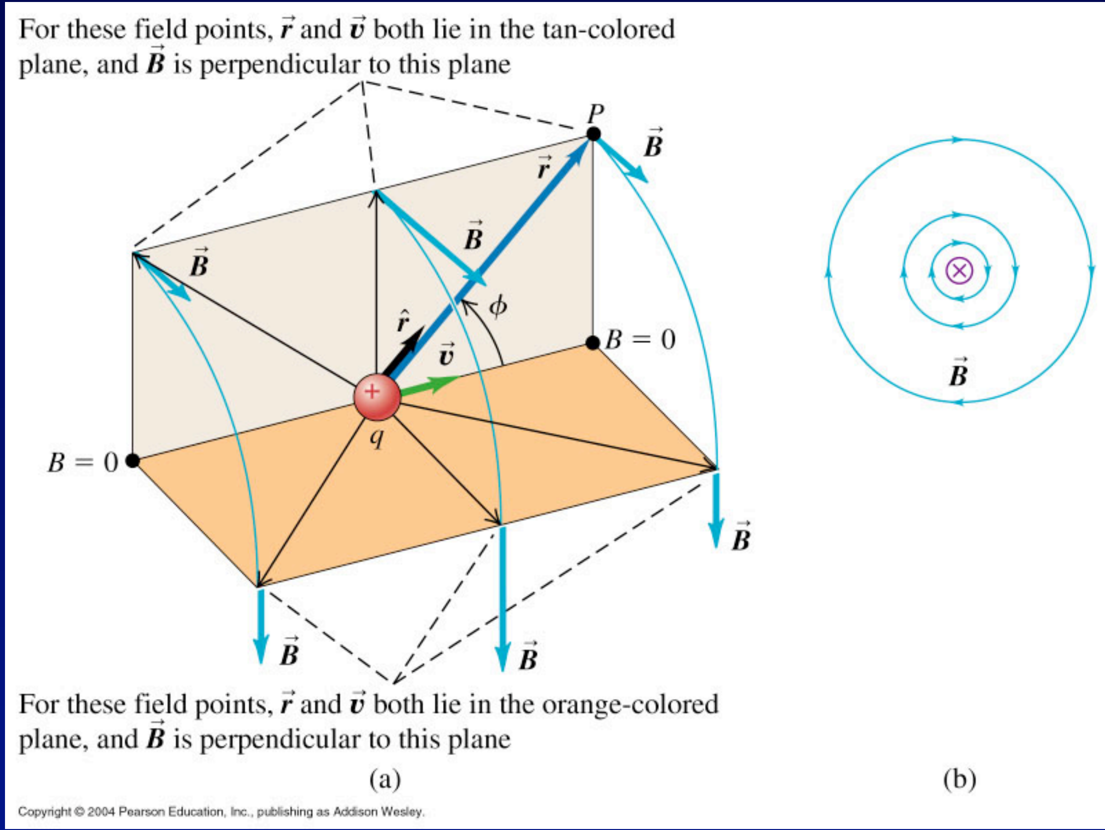
New Topic

Magnetic Field of a Moving Charge

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

$\mu_0 = 4\pi \times 10^{-7}$ T.m/A is called permeability of free space



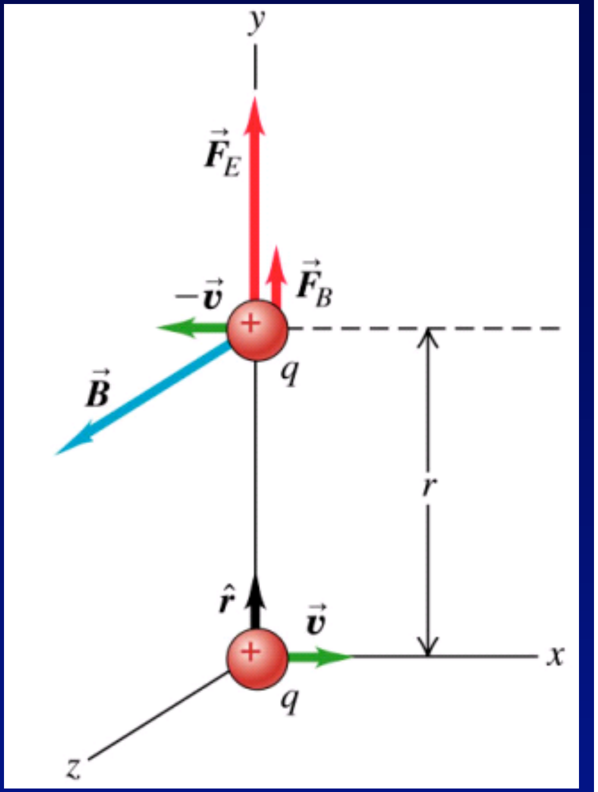
where $\hat{r}$ is the unit vector from the source point to the field point.

Compare with the Coulomb force: $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}\hat{r}$

Example: Force between two protons moving at same speed but opposite directions

The magnetic field by the lower proton is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(v\hat{i})\times\hat{j}}{r^2} = \frac{\mu_0}{4\pi}\frac{qv}{r^2}\hat{k}$$



The magnetic force felt by the upper proton is

$$\vec{F} = q(-\vec{v})\times\vec{B}$$

$$= q(-v\hat{i})\times\frac{\mu_0}{4\pi}\frac{qv}{r^2}\hat{k} = \frac{\mu_0}{4\pi}\frac{q^2v^2}{r^2}\hat{j}$$

Compare with the Coulomb force:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\hat{j}$$

$$\frac{F_B}{F_E} = \varepsilon_0\mu_0 v^2 = \frac{v^2}{c^2}$$

The magnetic force is much smaller than the electric force for small v.

Biot-Savart Law

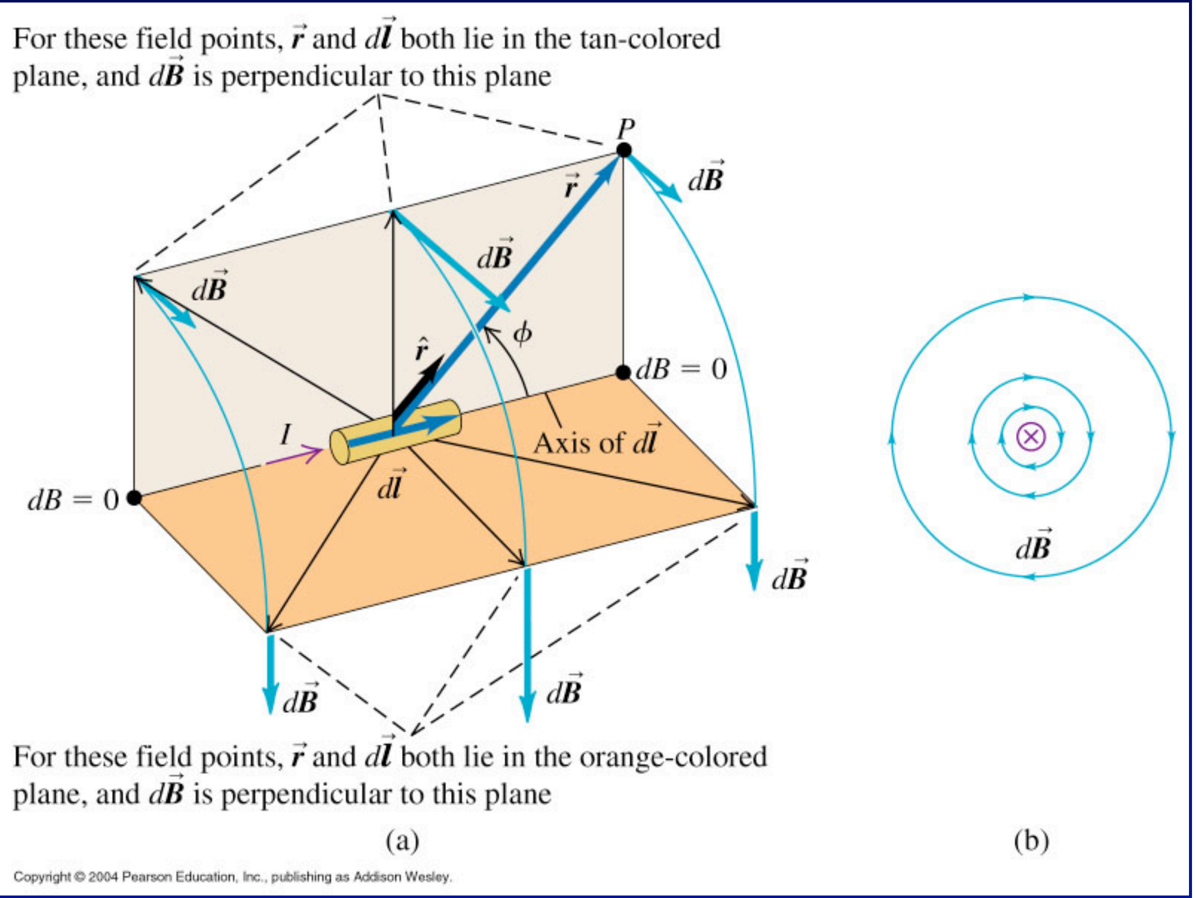
- Question: how to find B field (both direction and magnitude) due to a current segment ?

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2}$$

The total magnetic field is an integral over the entire wire:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{Id\vec{l} \times \hat{r}}{r^2}$$



Example: straight wire

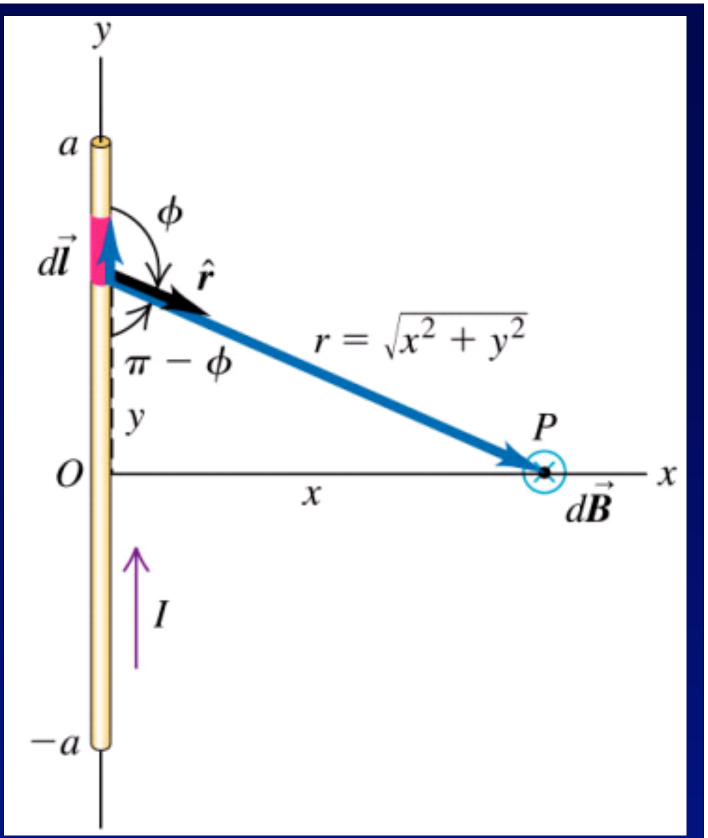
- Set up the coordinate system as shown. The field at point P due to a small segment:

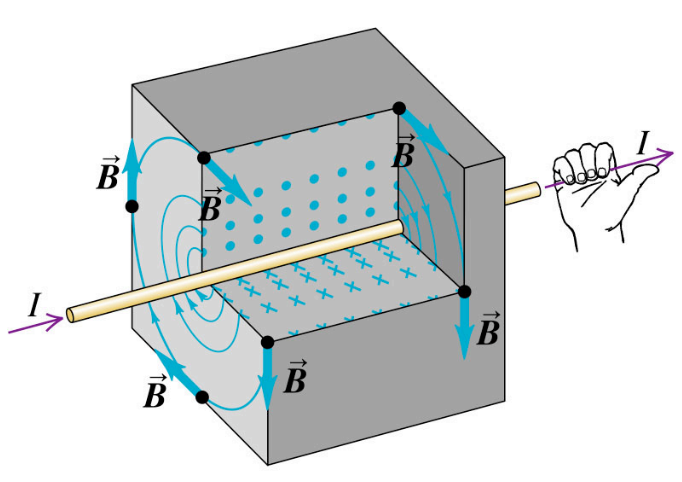
$$dB = \frac{\mu_0 I}{4\pi} \frac{dy \sin\phi}{r^2}$$

- The direction is the same from any segment, so the total is

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{+a} \frac{x\,dy}{\left(x^2 + y^2\right)^{3/2}} = \frac{\mu_0 I}{2\pi} \frac{2a}{x\sqrt{x^2 + a^2}}$$

For a long wire (a→ ∞): $B = \dfrac{\mu_0 I}{2\pi x}$



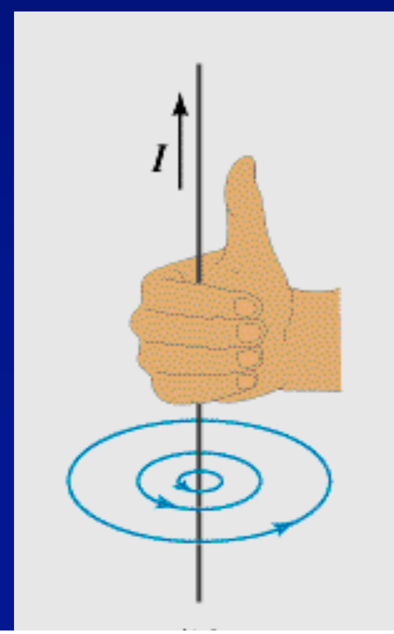


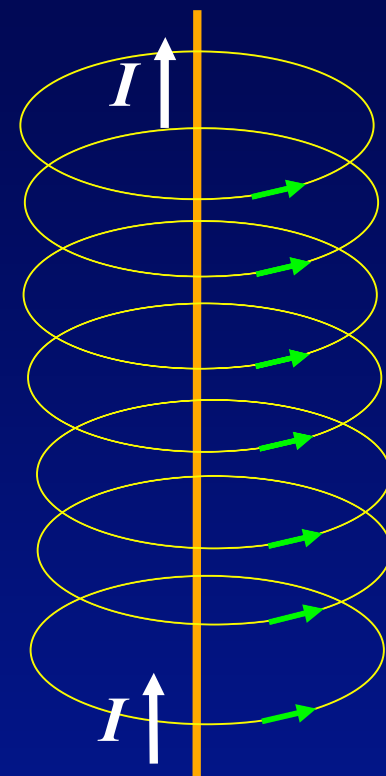
Magnetic Field due to a long straight wire

- For a long straight wire which is carrying current:
  - magnetic field forms circles around the wire
  - magnitude of B field:

$$B = \frac{\mu_0 I}{2\pi r}$$

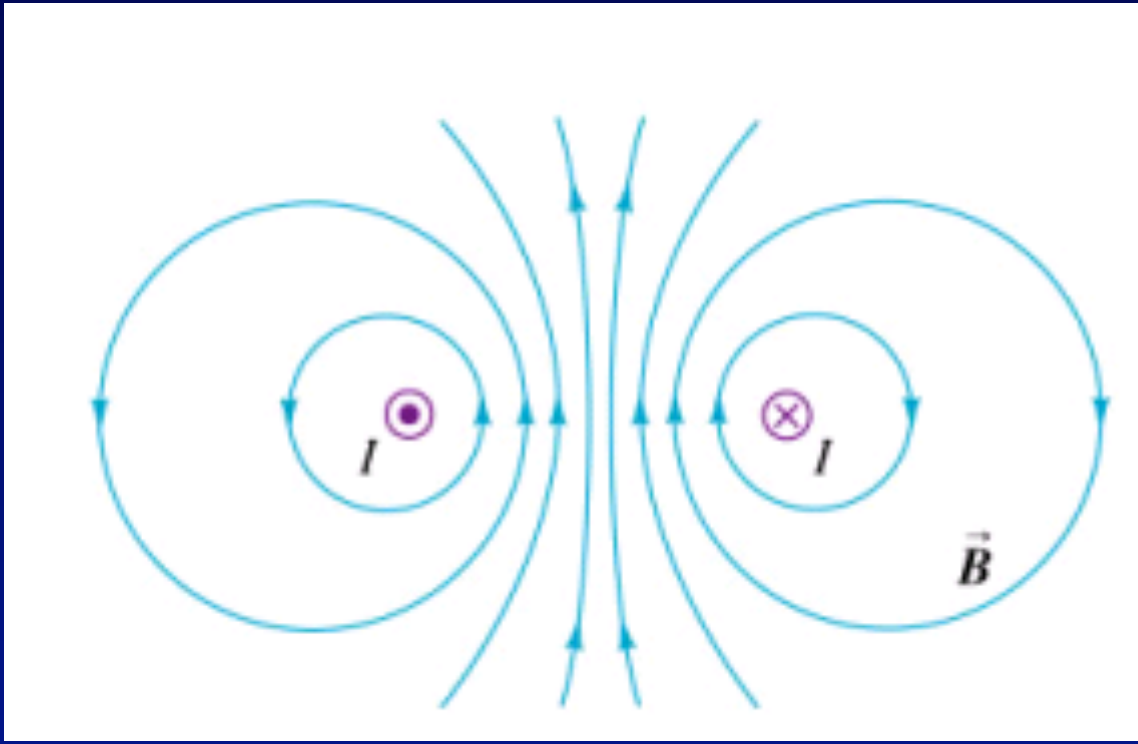
  - direction can be found by a right-hand rule





Magnetic field lines around two parallel wires

Opposite direction:

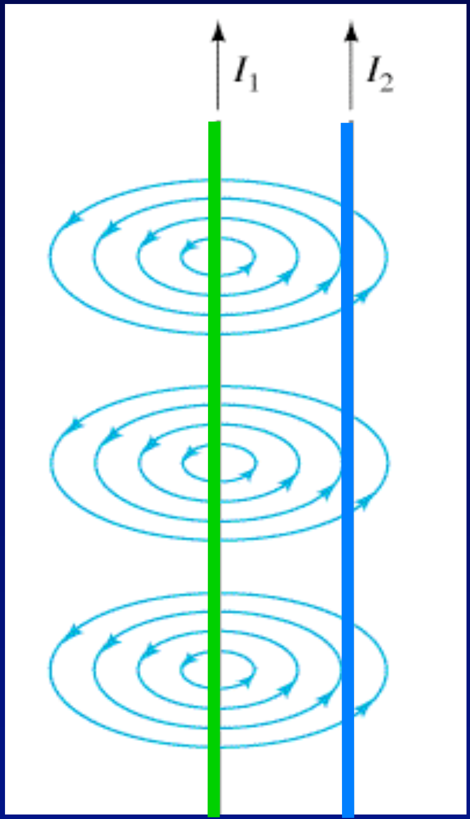


What about same direction?

Force Between Two Parallel Current-Carrying Wires

- **Field at wire 2 due to wire 1:** $$B_1 = \frac{\mu_0 I_1}{2\pi d}$$
- **Force on wire 2:** $$F_2 = I_2 L B_1 = \frac{\mu_0 I_1 I_2}{2\pi d} L$$
- **Force per unit length:**

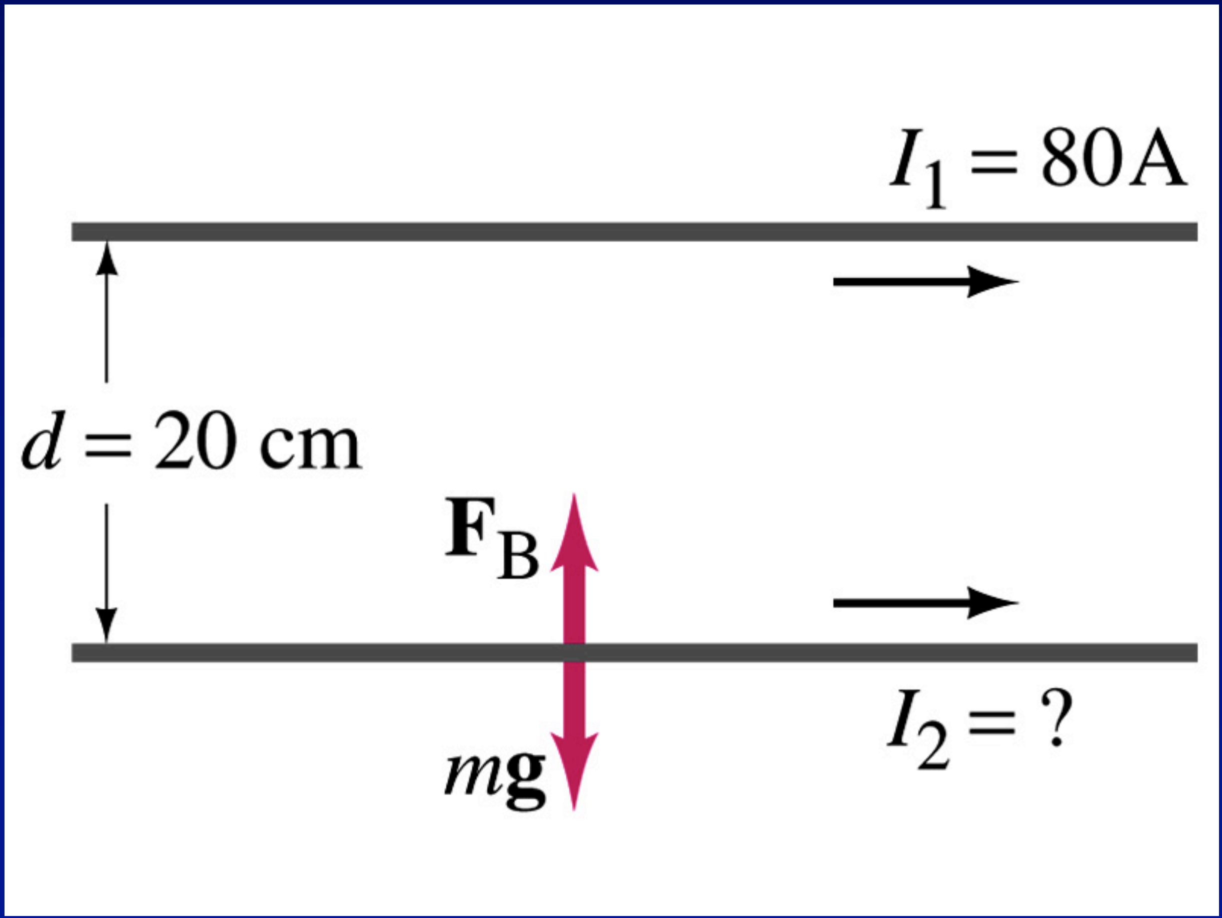
$$\frac{F_2}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$



SI definition of ampere:
One ampere is that unvarying current that, if present in each of two parallel conductors of infinite length and one meter apart in empty space, causes each conductor to experience a force of exactly 2 x 10^{-7} newtons per meter of length.

Example: Suspending a current with a current

How much current must the lower wire carry so that it does not fall due to gravity? The lower wire has a mass of 0.12 g per meter of length.

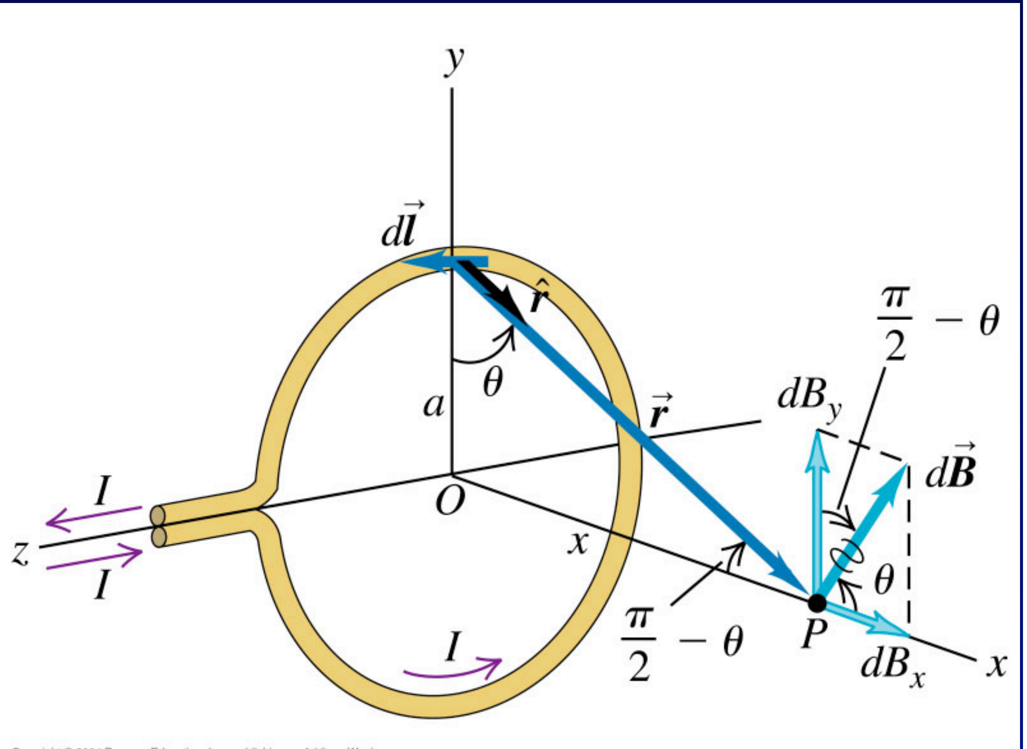


Example: a current loop

- Set up the coordinate system as shown. For a point on the axis

$$dB = \frac{\mu_0 I}{4\pi}\frac{dl}{r^2}$$

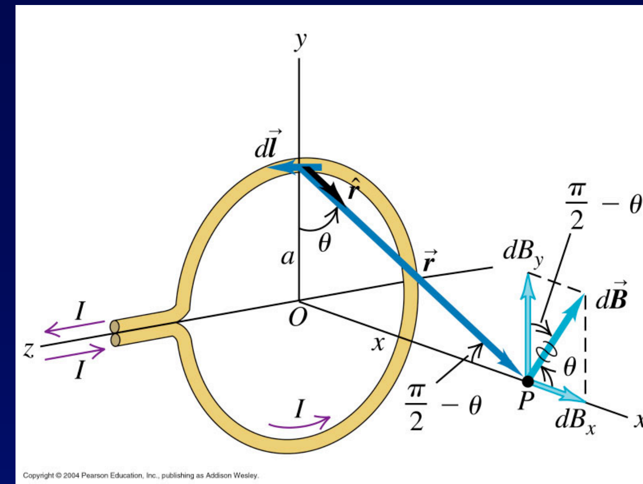
- By symmetry, total $B_\perp$ is zero, so total $B = B_{||}$



$$B = \int dB_x = \int dB\cos\theta = \int dB\frac{a}{r} = \frac{\mu_0 I}{4\pi}\frac{a}{r^3}\int dl = \frac{\mu_0 I}{2}\frac{a^2}{r^3}$$

current loop (continued)

$$B = \frac{\mu_0 I}{2} \frac{a^2}{(a^2 + x^2)^{3/2}}$$

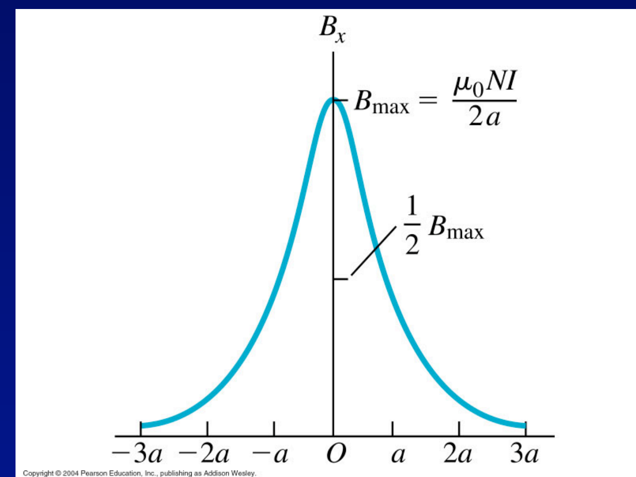


- At the center of loop, x=0

$$B = \frac{\mu_0 I}{2a}$$

- Far away, x >> a,

$$B \approx \frac{\mu_0 I}{2} \frac{a^2}{x^3}$$

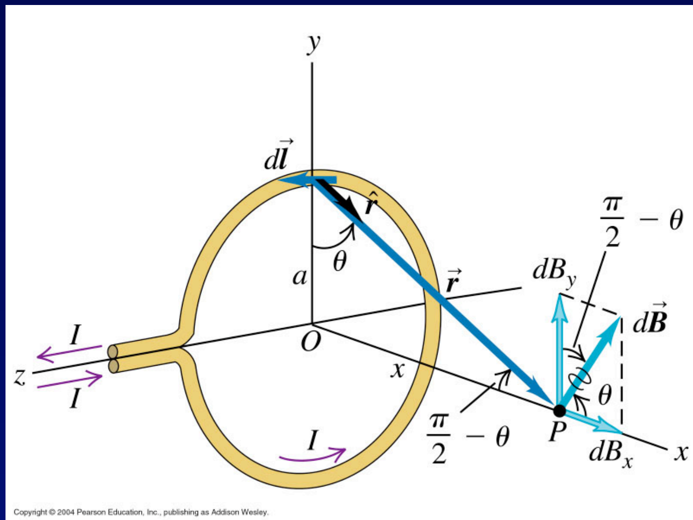


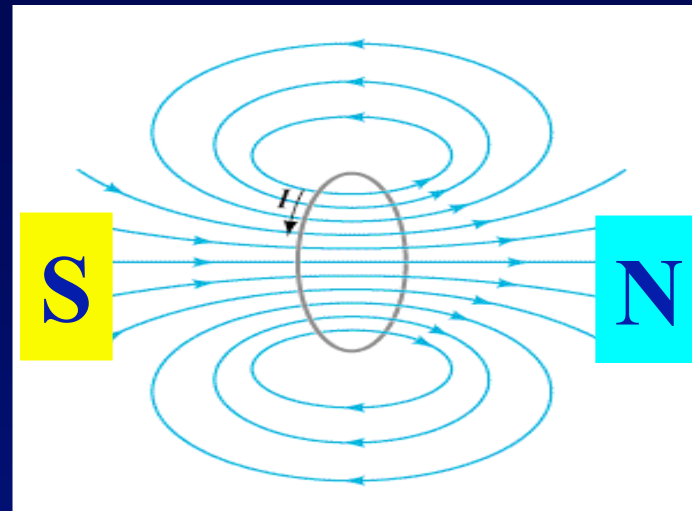
Or $B \approx \frac{\mu_0}{2\pi} \frac{\mu}{x^3}$, where $\mu = I(\pi a^2)$ is the magnetic dipole moment of the loop.

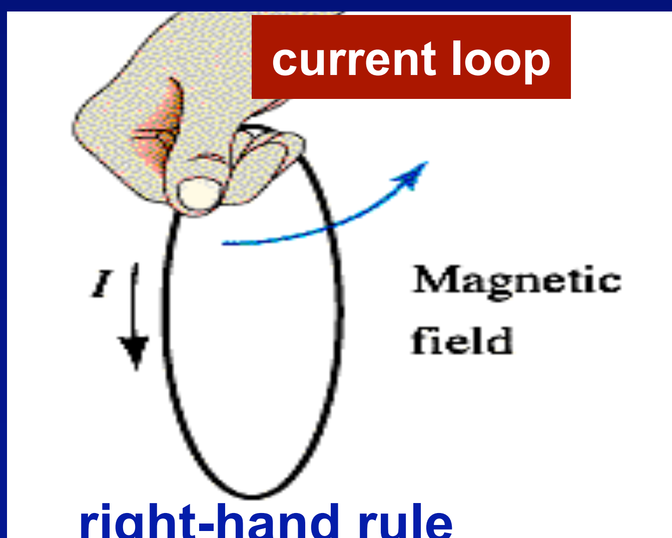
Recall electric dipole

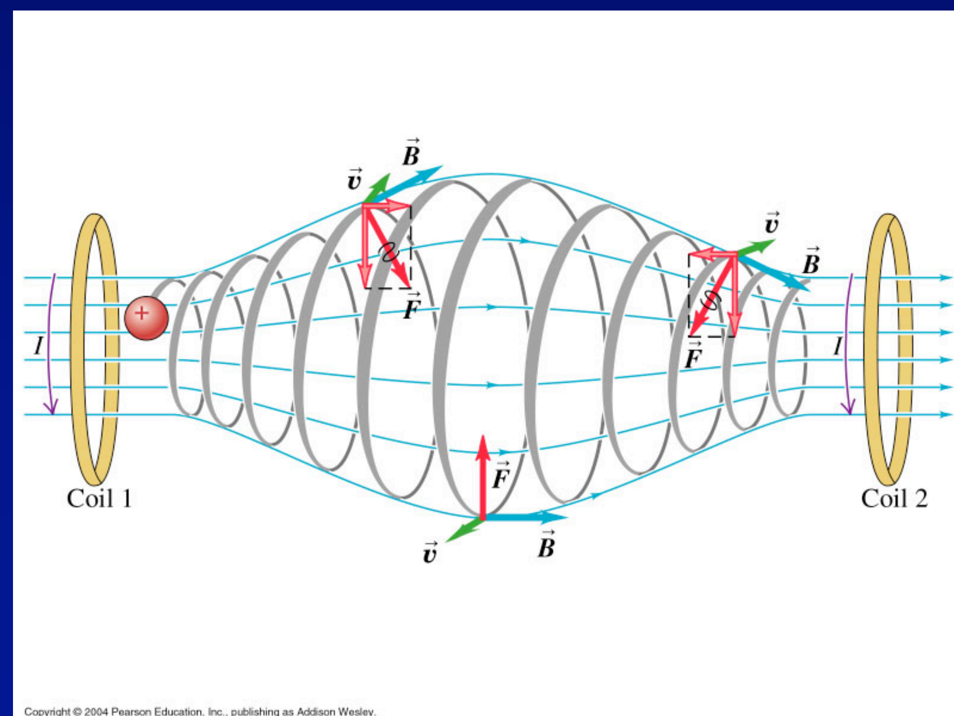
$$E \approx \frac{1}{4\pi\varepsilon_0} \frac{p}{x^3}$$

Current Loops



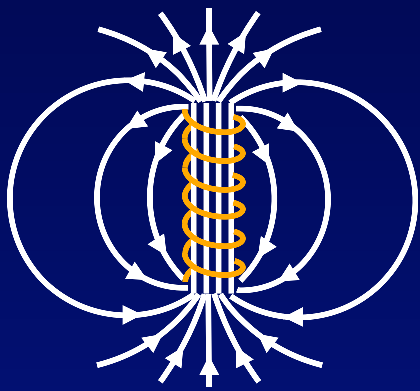






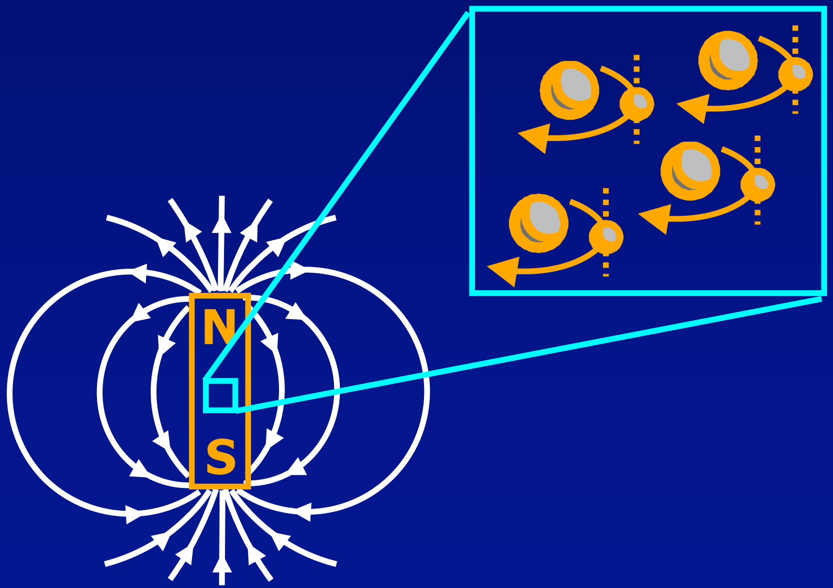
Bar Magnets and Solenoids

Magnetic field of a solenoid looks very similar to that of a ...

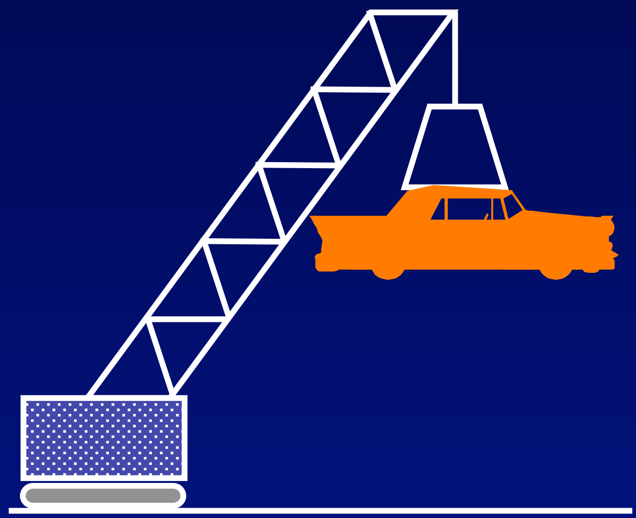


...bar magnet.

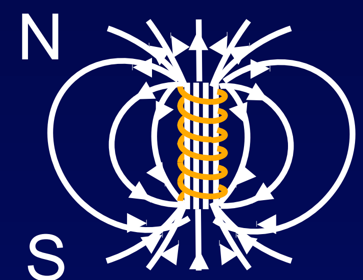
In a permanent magnet the spins and orbits of all electrons are aligned.

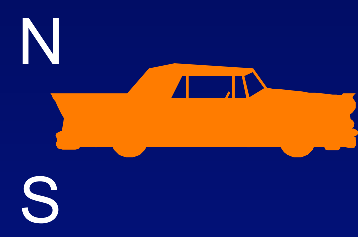


Electromagnet



Metal can be magnetized temporarily by strong magnetic field.



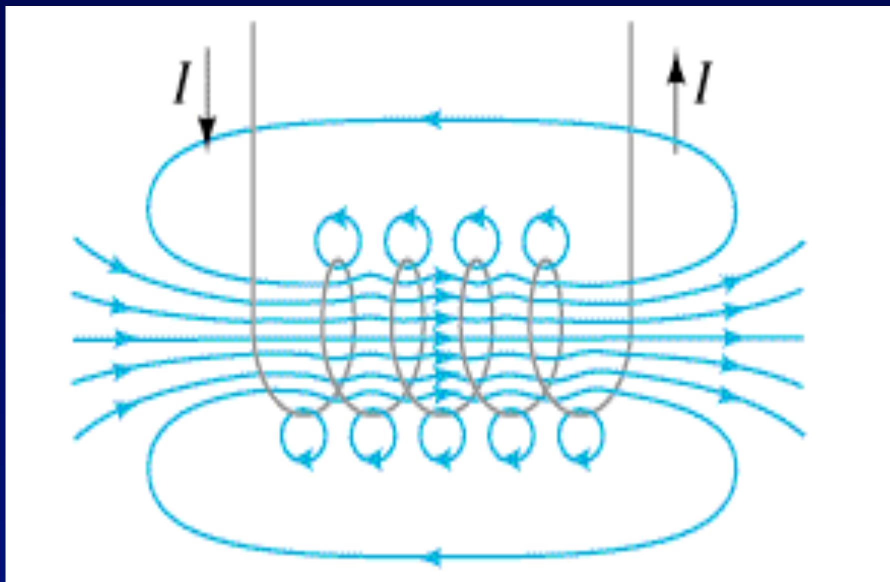


Benefits of electromagnets:

They are adjustable. You can turn them on and off.

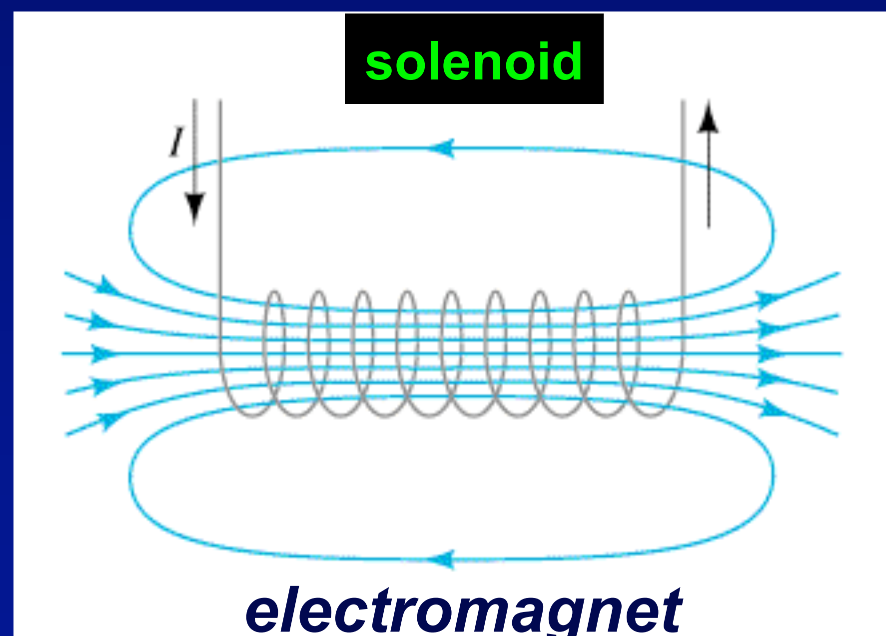
They can produce stronger fields than permanent magnets

Solenoids

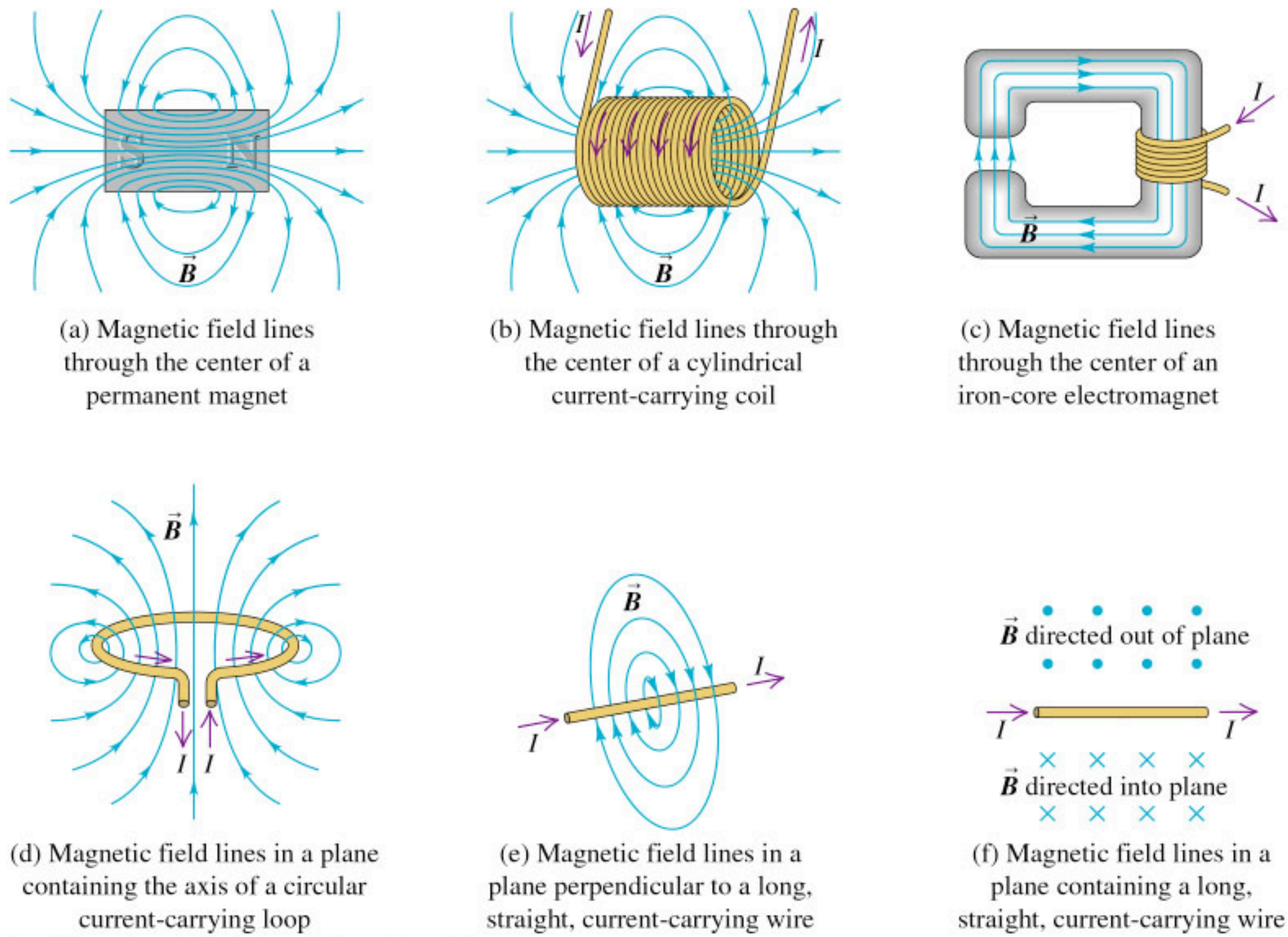


coil a long wire into
many current loops
so that the magnetic
fields all add together

- When the loops are very closely spaced:
  - strong field inside
    - weak outside
  - uniform field inside
    - non-uniform outside



Visualizing Magnetic Fields



(a) Magnetic field lines through the center of a permanent magnet

(b) Magnetic field lines through the center of a cylindrical current-carrying coil

(c) Magnetic field lines through the center of an iron-core electromagnet

(d) Magnetic field lines in a plane containing the axis of a circular current-carrying loop

(e) Magnetic field lines in a plane perpendicular to a long, straight, current-carrying wire

(f) Magnetic field lines in a plane containing a long, straight, current-carrying wire

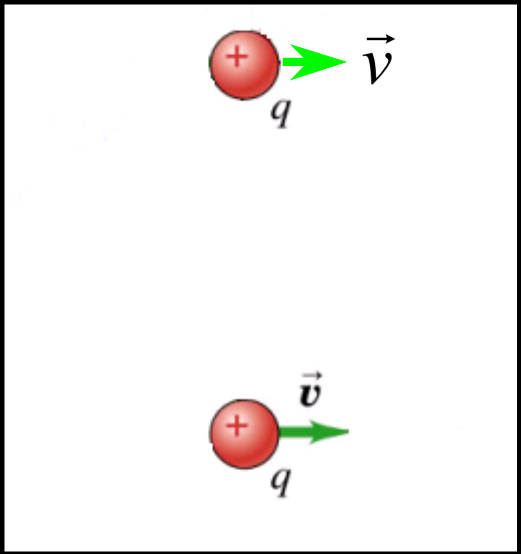
Use right-hand-rule

Note that magnetic field lines are always closed.

ConcepTest 28.1 Magnetic force

- If two protons are traveling parallel to each other in the same direction and at the same speed, what is the nature of the magnetic force between them?

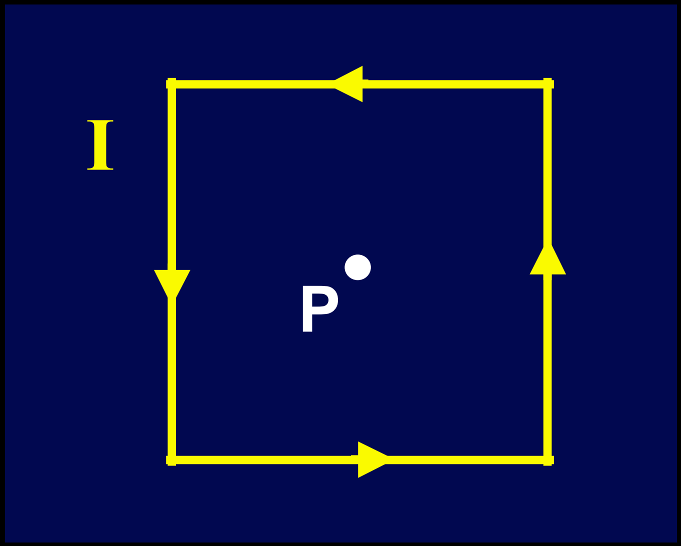
1) repulsive
2) attractive
3) zero



ConcepTest 28.2 Magnetic field

- What is the direction of the magnetic field at point P?

  (1) left
  (2) right
  (3) zero
  (4) into the page
  (5) out of the page



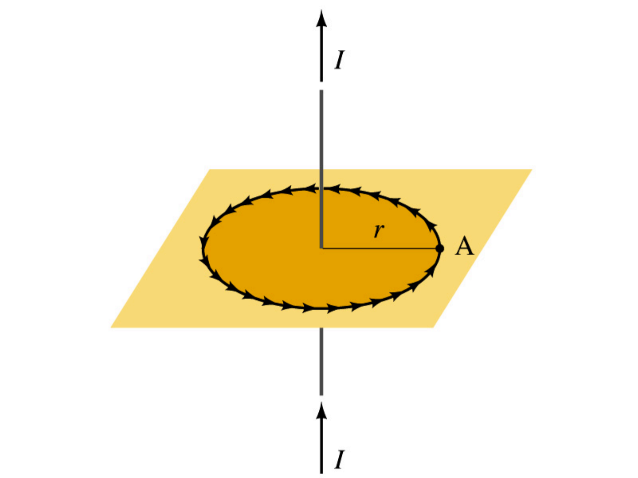
Ampere's Law

New Topic

Ampere's Law

- Question: Is there a general relation between a current in a wire of any shape and the magnetic field around it?

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$



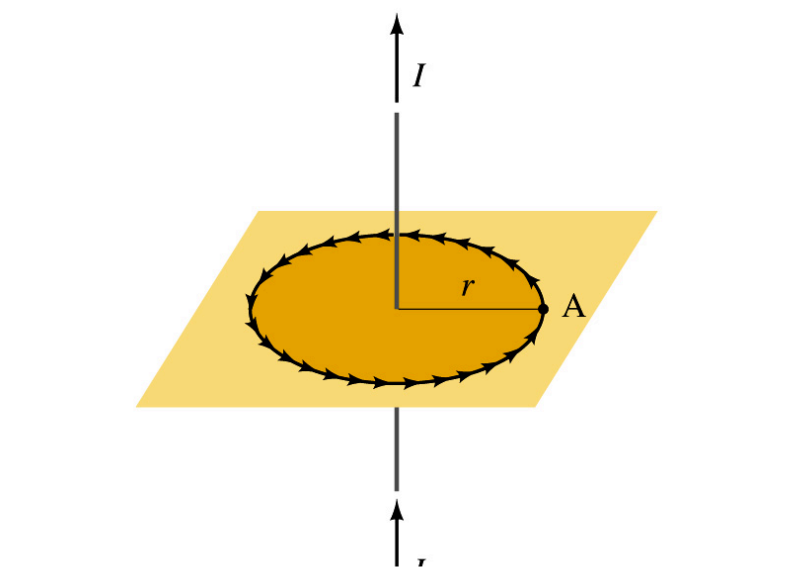
- Ampere's Law:
  The line integral of the magnetic field around any closed loop is equal to μ_0 times the total current enclosed by the loop

compare with Gauss's Law $\oint \vec{E} \cdot d\vec{A} = Q_{enclosed} / \varepsilon_0$

Example: a long straight wire

- Consider a circular path of radius r around the wire.
  - The plane of the path is perpendicular to the wire.

- By symmetry, the B field has the same magnitude at every point along the path, with a direction tangential to the circle by the right-hand-rule.

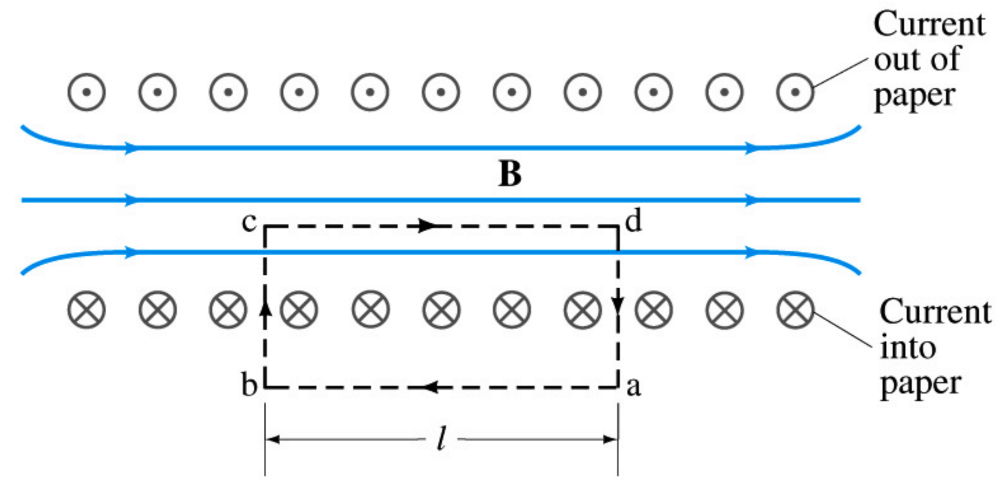


$$\mu_0 I = \oint \vec{B} \cdot d\vec{l} = B \oint dl = B(2\pi r)$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Example: Solenoid

- Consider a rectangular path as shown



- By symmetry, the only non-zero contribution comes from the segment cd:

$$\oint \vec{B} \cdot d\vec{l} = \int_c^d \vec{B} \cdot d\vec{l} = Bl$$

$$Bl = \mu_0 NI$$

$$B = \mu_0 nI$$

Where n = N / l is the number loops per unit length

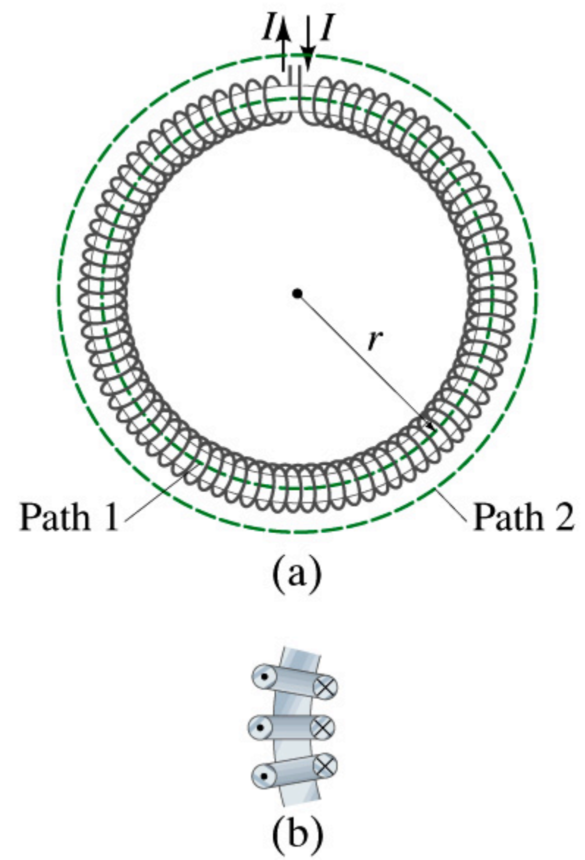
Example: Toroid

- Inside:

  consider path 1

$$B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$



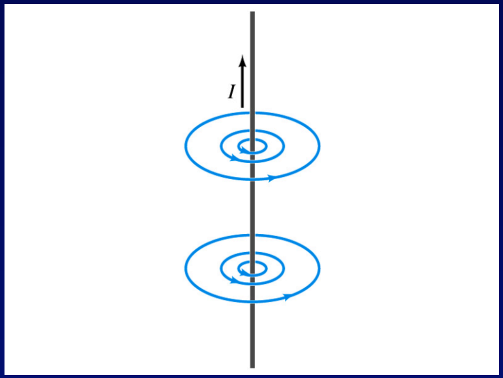
- Outside: consider path 2
  - The net current passing through is zero

$$B(2\pi r) = 0$$

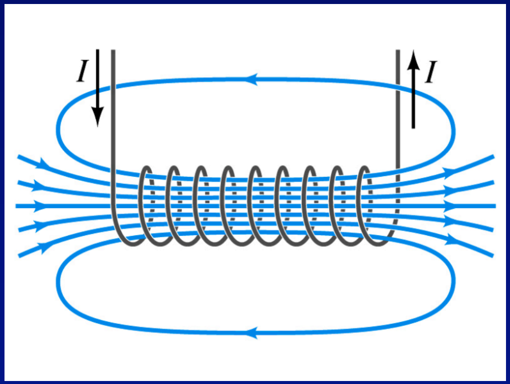
$$B = 0$$

Summary: electric current produces magnetic field

$$B = \frac{\mu_0 I}{2\pi r}$$



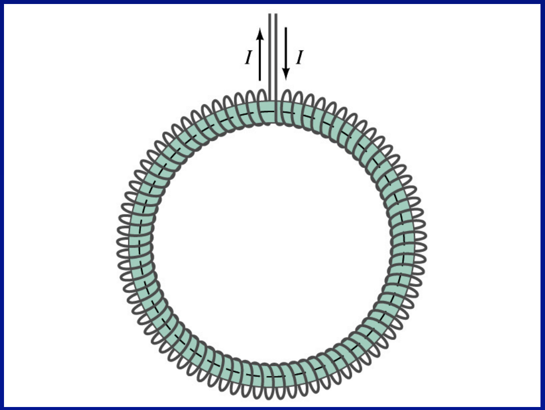
Straight wire



Solenoid

$$B = \mu_0 n I$$

$$B = \frac{\mu_0 N I}{2\pi r}$$



Toroid

ConcepTest 28.3 Coaxial Cable

- **The cable carries currents of equal magnitude, but opposite directions.**
  - **what is the magnetic field in between (in the yellow area)?**

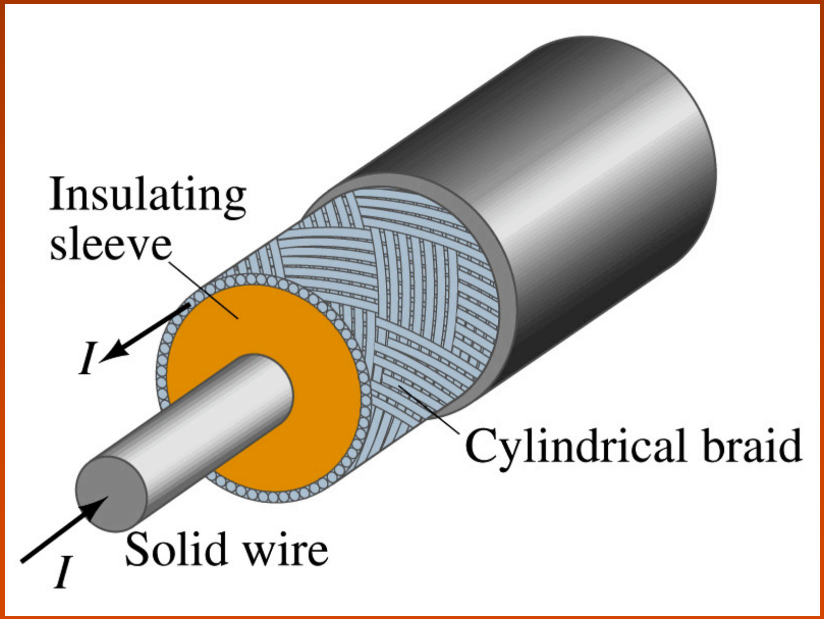
(1) zero

(2) $\mu_0 I/(2\pi r)$, clockwise

(3) $\mu_0 I/(\pi r)$, clockwise

(4) $\mu_0 I/(2\pi r)$, counter-clockwise

(5) $\mu_0 I/(\pi r)$, counter-clockwise



ConcepTest 28.4 Coaxial Cable

- The cable carries currents of equal magnitude, but opposite directions.
  - what is the magnetic field outside the cable?

(1) zero

(2) $\mu_0 I/(2\pi r)$, clockwise

(3) $\mu_0 I/(\pi r)$, clockwise

(4) μ0I/(2πr), counter-clockwise

(5) μ0I/(πr), counter-clockwise