Final Exam is coming !

- Thurs., May 4, 4:30 to 6:30 pm, in this room.
- 25 multiple-choice questions
  - Personalized exams
  - I will enter the grade on your Mastering Physics account (“Final”).
- “Old Part” is comprehensive. Problems from Exams I and II only!!
  - 11 problems total
  - 9 conceptual/numerical problems, 1 point each
  - 2 questions are numerical - 2 pts each.
- “New Part” covers 7 chapters (32-38)
  - 14 problems total
  - 12 conceptual/numerical problems, 1 point each
  - 2 questions are numerical - 2 pts each.
  - I will pass out a new formula6a (not finished yet) sheet at the exam. Please familiarize yourself with it. Any constants needed will be given.
- No recovery points
- What can I bring to the exam?
  - pencil
  - eraser
  - calculator
  - That’s all (no cell phones for example)

"New Part" on Final

- Chapter 32: Electromagnetic waves
  - Maxwell equations; waves in free space
  - Spectrum; energy flow
- Chapter 33: Nature and Propagation of Light
  - Reflection and refraction
  - Snell's Law; total internal reflection
- Chapter 34: Mirrors and Lenses
  - Mirrors; lenses; combinations
  - Optical instruments; aberrations
- Chapter 35: Interference
  - Wave properties
  - Double slit; thin films
- Chapter 36: Diffraction
  - Single slit; gratings
  - Limits on resolution; X-ray
- Chapter 37: Relativity
  - Postulates; simultaneity; Lorentz transformation; Doppler effect
- Chapter 36: Photons, Electrons and Atoms (not covered here)
  - Blackbodies; photoelectric effect; Compton effect

10 lectures (including this one)
9 homework sets
9 quizzes

More than 1/3 of the work

Chapter 32: Electromagnetic waves

- Maxwell's equations
- Electromagnetic waves in free space
- The electromagnetic spectrum
- Energy in electromagnetic waves
  - The Poynting vector
  - Radiation pressure

Maxwell's Equations

Integral forms

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Q is charge and I is current

Differential forms

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

ρ is charge density and
J is current density

Review of Wave Properties

- The one-dimensional wave equation:

$$\frac{\partial^2 h}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 h}{\partial t^2}$$

has a general solution of the form:

$$h(x,t) = h_1(x - vt) + h_2(x + vt)$$

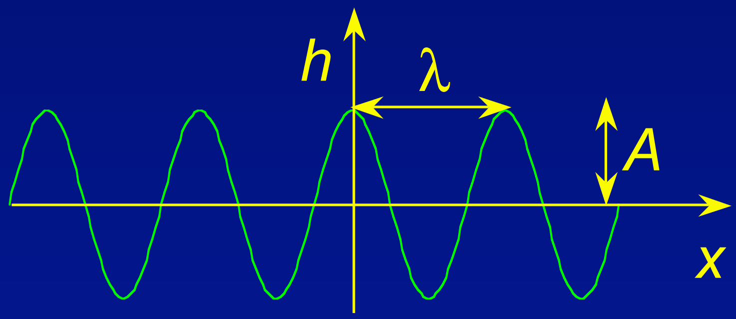
where h_1 represents a wave traveling in the +x direction and h_2 represents a wave traveling in the -x direction. The wave velocity is given by v.

- A specific solution for harmonic waves traveling in the +x direction is:

$$h(x,t) = A\sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

$$v = \lambda f = \frac{\omega}{k}$$



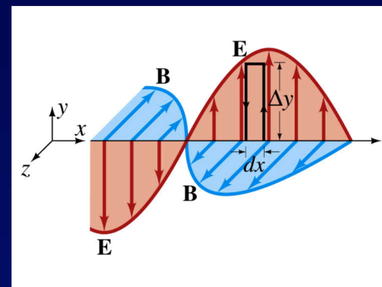
A = amplitude
λ = wavelength
f = frequency
v = speed
k = wave number

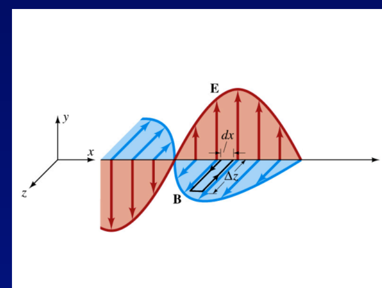
Electromagnetic Waves in Free Space (Q=0, I=0)

Differential forms

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$





$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Take partial derivatives

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial x \partial t}$$

$$\frac{\partial^2 B}{\partial t \partial x} = -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\text{so} \quad \frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

Read off wave speed from the wave equation:

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8 \text{ m/s } = c$$

Electromagnetic Waves in Free Space

Wave equations

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \qquad \frac{\partial^2 B}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}$$

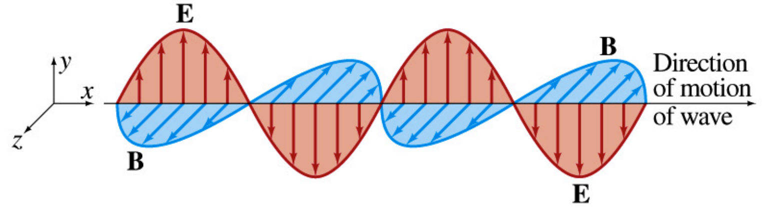
Plane-wave solution

$$E = E_y = E_0 \sin(kx - \omega t)$$
$$B = B_z = B_0 \sin(kx - \omega t)$$

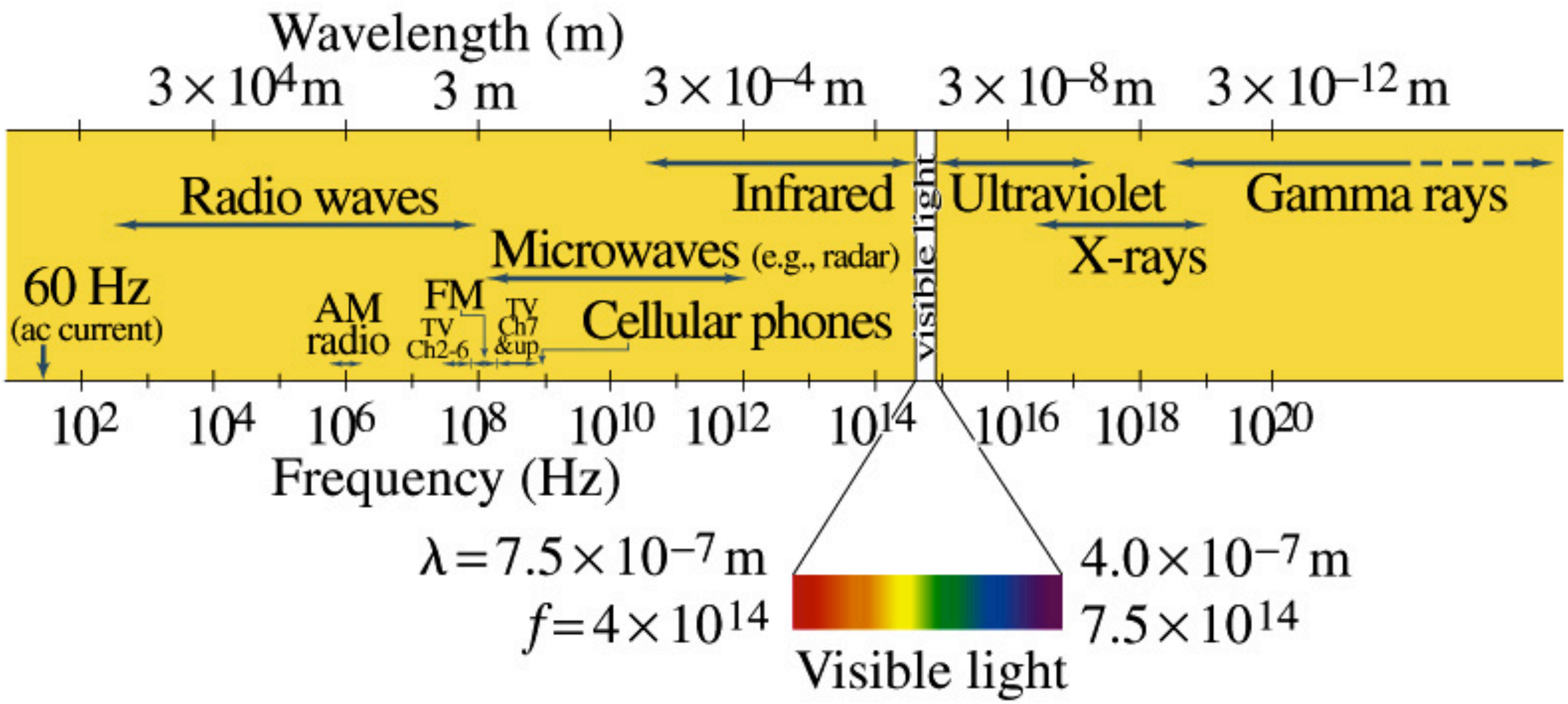
where

$$k = \frac{2\pi}{\lambda}, \omega = 2\pi f, c = \frac{\omega}{k} = \lambda f$$

- E and B are in phase.
- $\frac{E}{B} = c$
- The directions of E and B and wave travel form a right-hand-rule.



The electromagnetic spectrum



$$c = \lambda f$$

Energy in EM Waves

EM waves carry energy from one region of space to another. The energy density

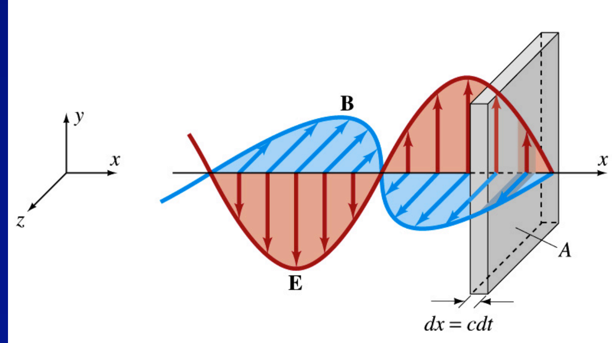
$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0} = \varepsilon_0 E^2 = \frac{B^2}{\mu_0} = \sqrt{\frac{\varepsilon_0}{\mu_0}} EB$$

is shared equally between E and B fields.

Poynting vector: the energy transported by the EM wave per unit time per unit area (W/m^2).

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

which also defines the direction of the wave.



$$\overline{S} = \frac{1}{2}\varepsilon_0 c E_0^{\,2} = \frac{cB_0^{\,2}}{2\mu_0} = \frac{E_0 B_0}{2\mu_0} = \frac{E_{rms}B_{rms}}{\mu_0}$$

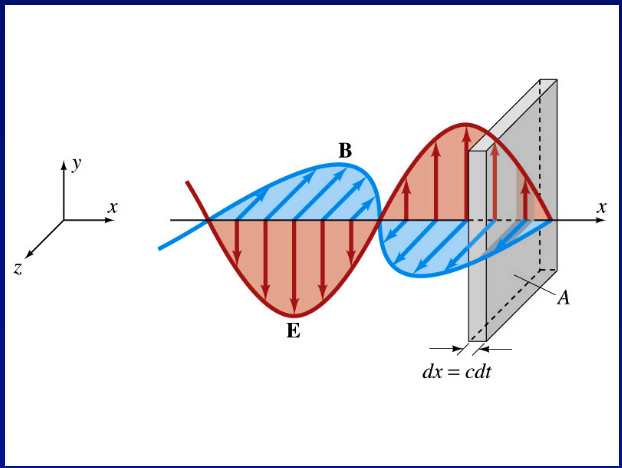
Recall intensity: $\overline{S} = \dfrac{\text{Power}}{4\pi r^2}$

Radiation Pressure

When an EM wave encounters the surface of a material, it exerts pressure on the surface.

Absorbed $$P = \frac{\overline{S}}{c}$$

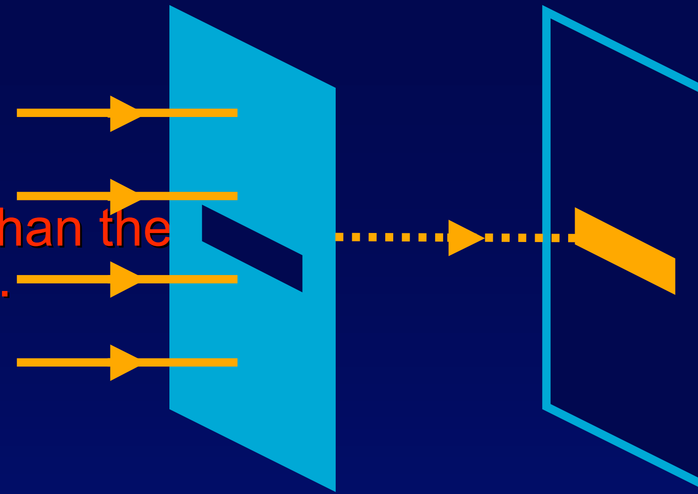
Reflected $$P = \frac{2\overline{S}}{c}$$



Chapter 33: The Nature and Propagation of Light

- Light rays
- Reflection
- Refraction
  - Index of refraction
  - Snell's Law
- Total Internal Reflection

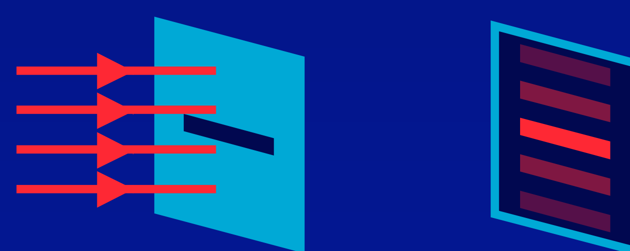
Geometrical Optics



- Assumption: the dimensions are much larger than the wavelength of the light waves (400 to 700 nm).
  - light follows straight-line paths (rays)
- Changes occur when a ray hits a boundary
  - ray may bounce off (reflection)
  - ray may bend into the other medium (refraction)
  - ray may be absorbed (light energy ⇒ thermal energy)

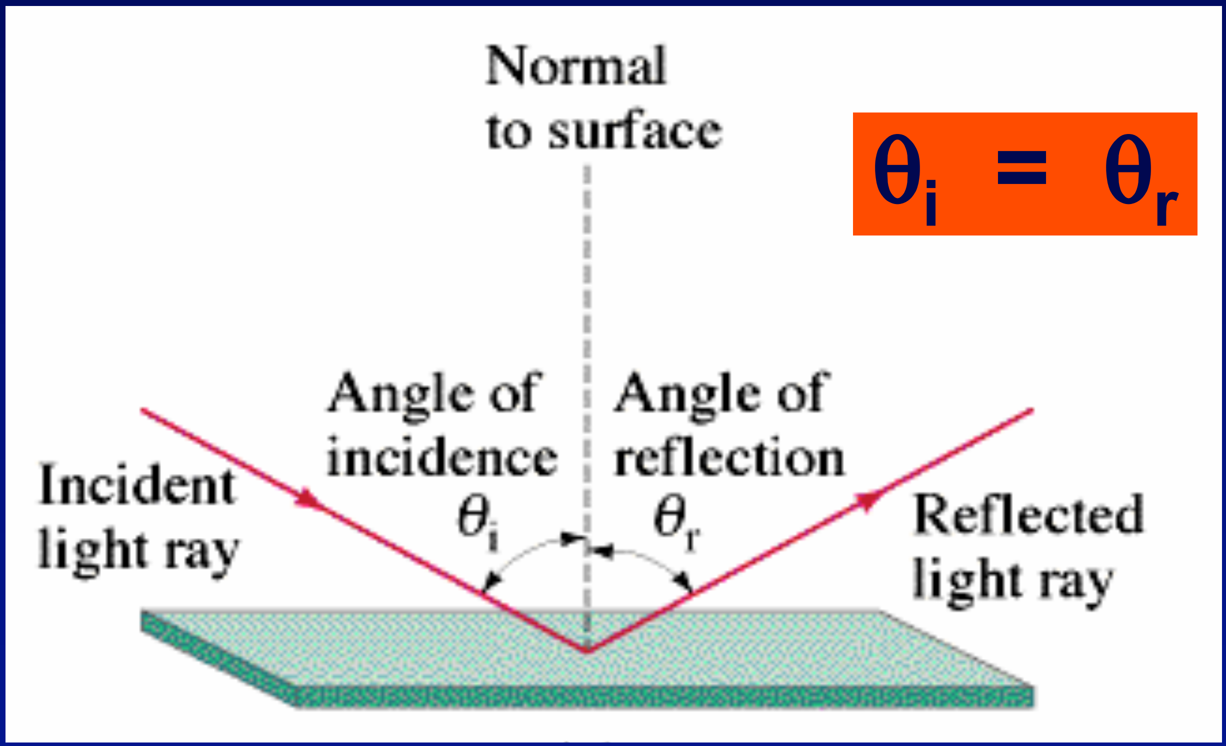
Physical Optics

- Assumption: the dimensions are comparable to the wavelength of the light waves.
  - light must be considered as waves
- Waves exhibit
  - interference
  - diffraction



Reflection

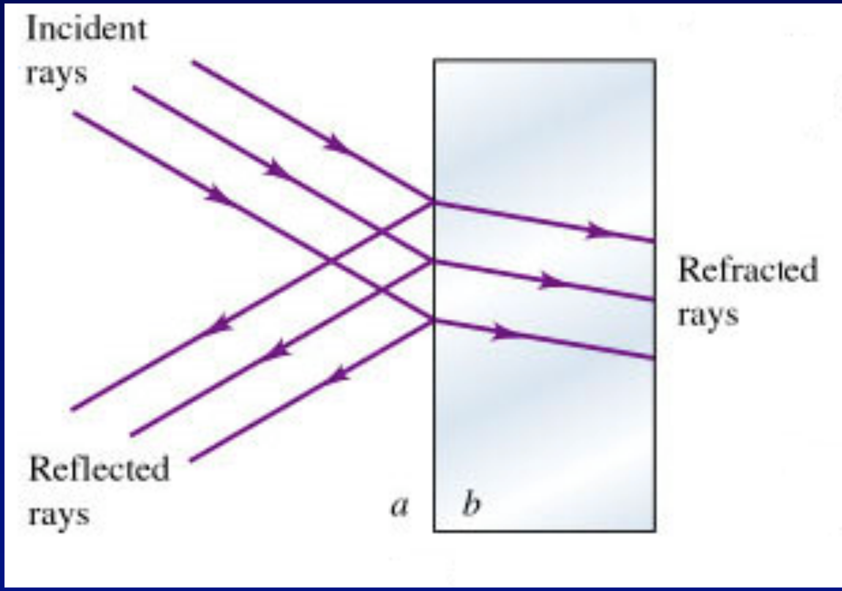
Angle of incidence = Angle of reflection



Notice the angles are measured with respect to the normal!

Refraction of Light

What happens when light goes through a boundary?



It bends!

medium	n
vacuum	1.0
air	~1.0
water	1.3
glass	1.5
Diamond	2.4

Why ? ⇒⇒ light travels *slower* in a medium than in air:

- **index of refraction:** $n = c / v$

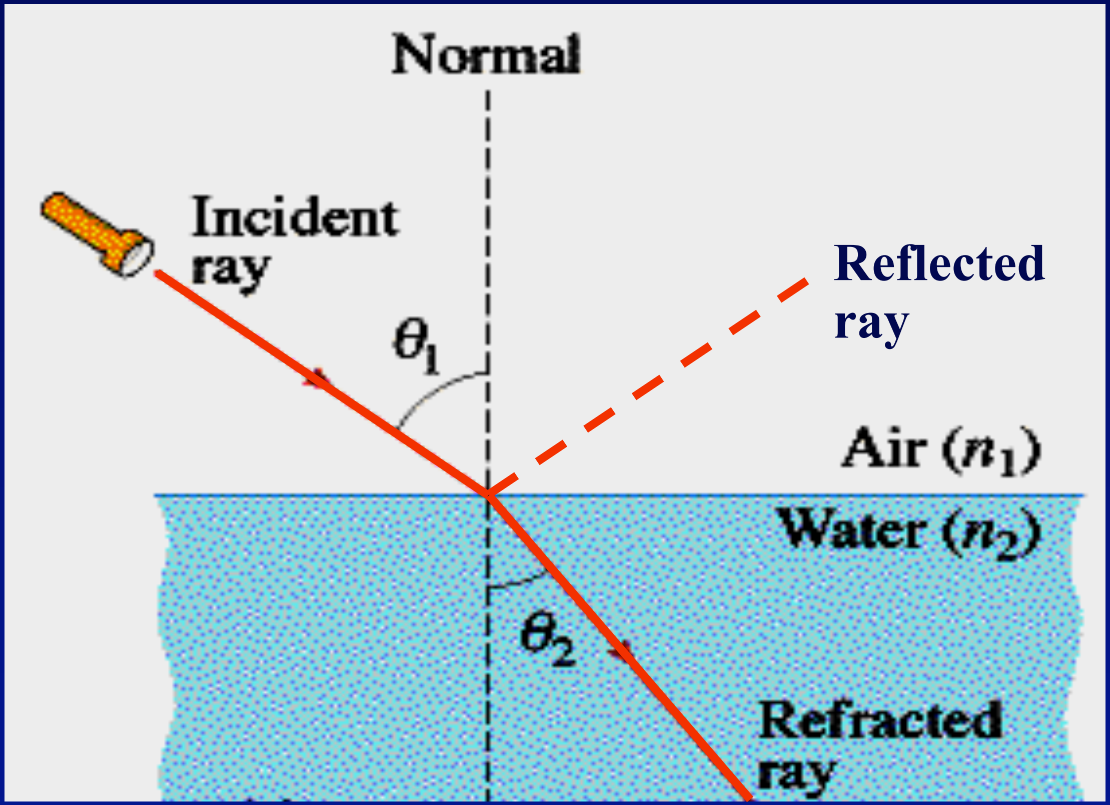
c = speed in vacuum

v = speed in medium

Refraction of Light

Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

W. Snell (1591-1626)

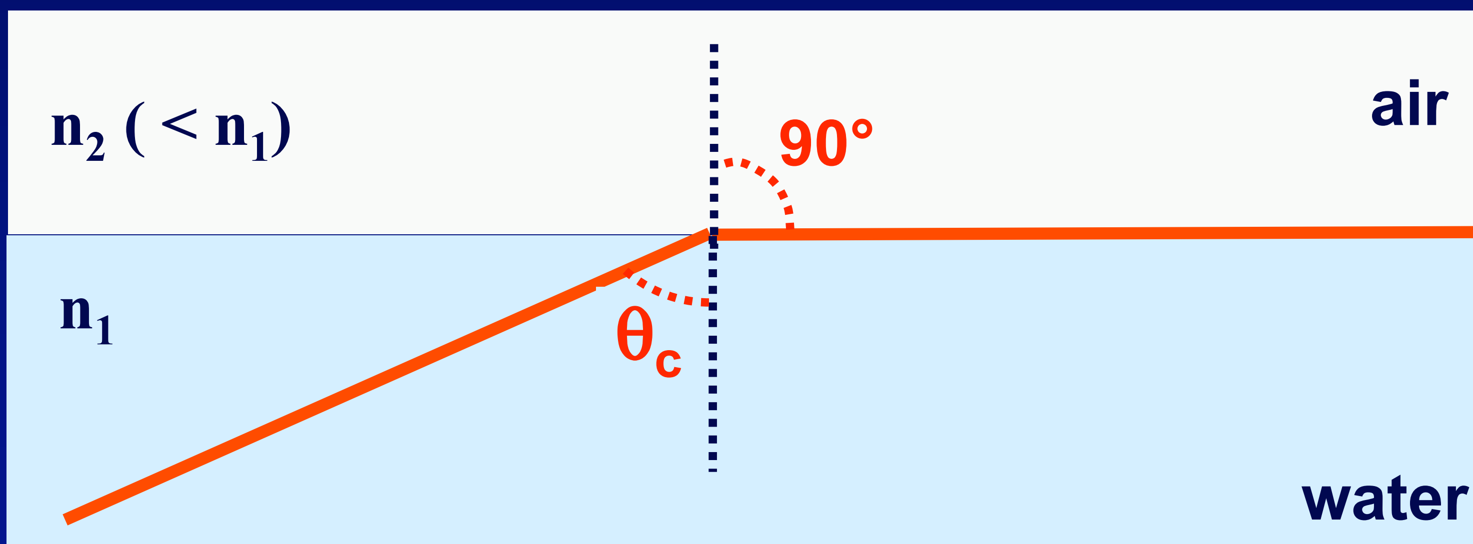


Note: the angles are defined relative to the normal.

Total Internal Reflection

- What is the condition for total internal reflection?
  - when $\theta_i = \theta_c \longrightarrow$ refracted angle is 90°

$$n_1 \sin\theta_c = n_2 \sin 90^\circ = n_2 \quad \Rightarrow \quad \sin\theta_c = \frac{n_2}{n_1}$$



- Remember: this only works when the ***incident medium*** has the ***higher index of refraction***.

Chapter 34: Geometrical Optics

- Mirror equation
- Lenses
  - The lens equation
  - Lensmaker's equation
  - Combination of lenses
- Instruments
  - Camera
  - The eye
  - Telescopes
  - Microscope

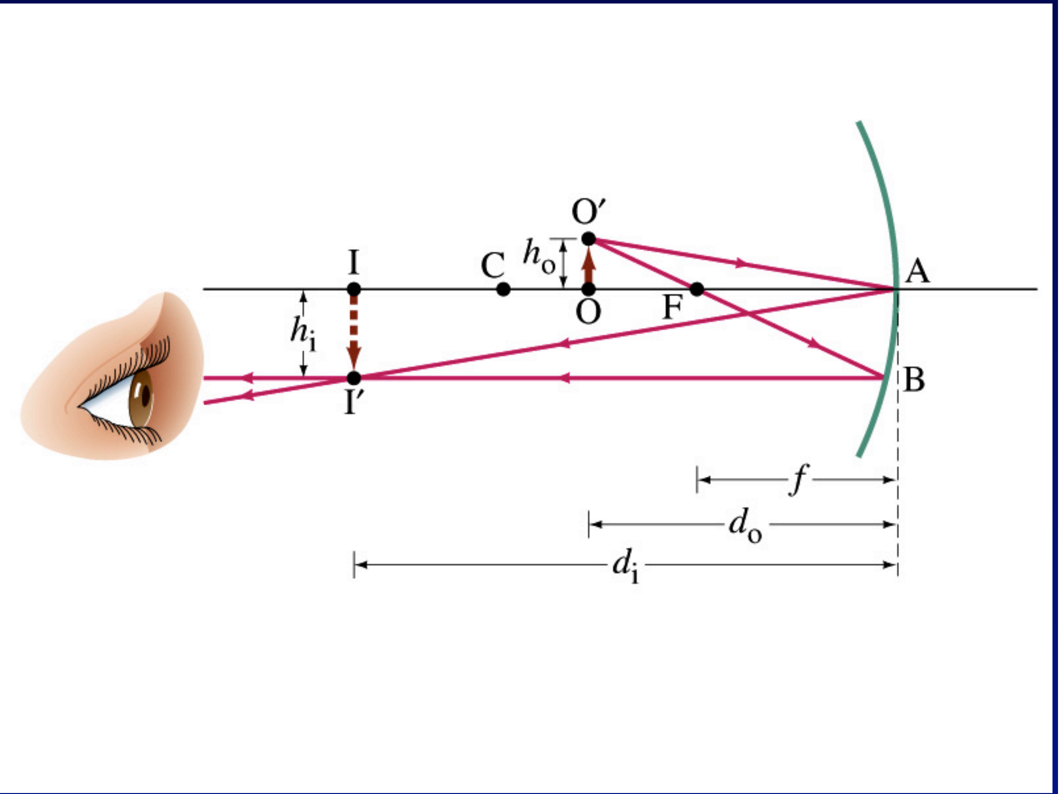
Spherical Mirrors: calculations

Mirror equation: $(f = \frac{r}{2})$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Lateral magnification:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$



Sign conventions:

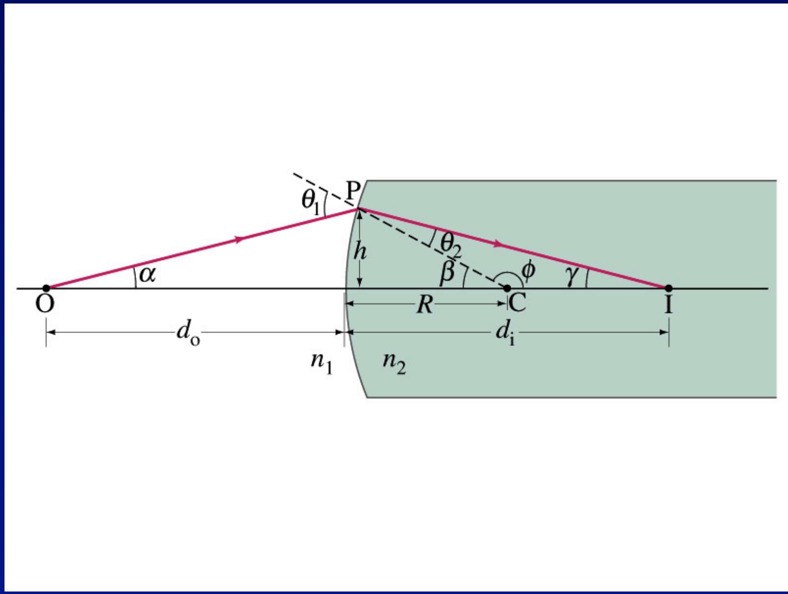
h_i is positive if upright, negative if inverted.
d_o and d_i are positive if on the reflecting side, negative if on the other side. f is positive for concave, negative for convex.

Refraction at a spherical surface

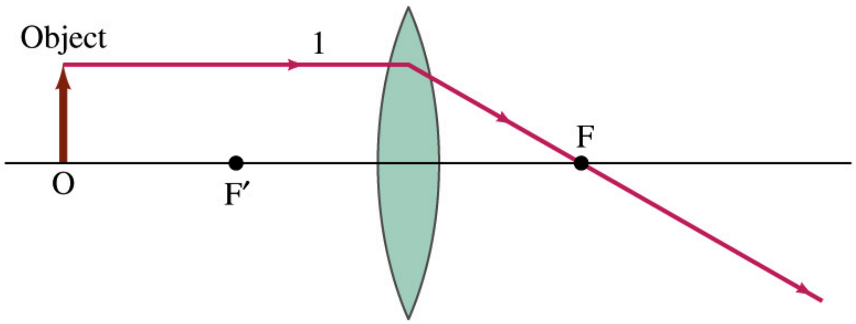
Convex surface

$$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}$$

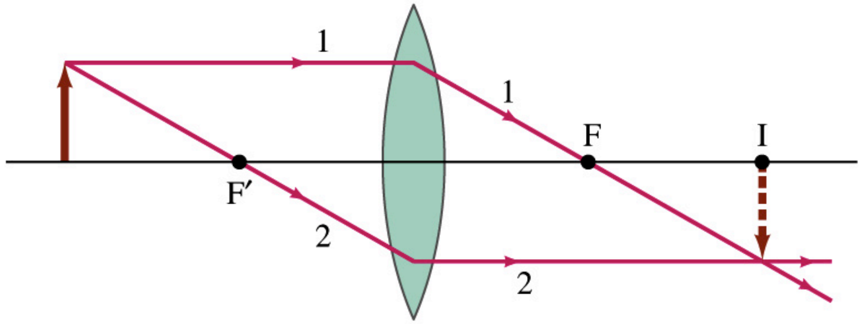
Equation also holds for concave surface, but R is negative



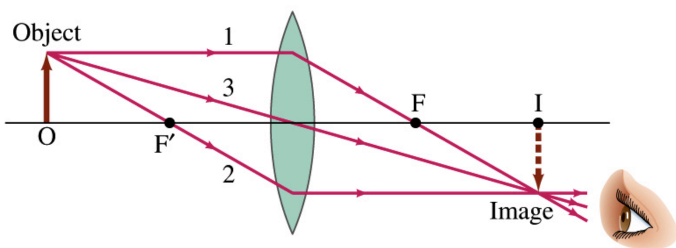
Thin Lenses: Ray Diagrams



(a) Ray 1 leaves top point on object going parallel to the axis, then refracts through focal point.



(b) Ray 2 passes through F′; therefore it is parallel to the axis beyond the lens.

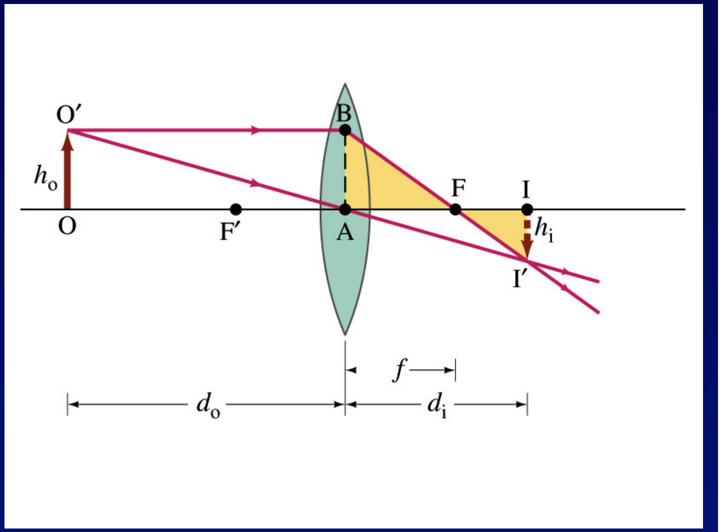


(c) Ray 3 passes straight through the center of the lens (assumed very thin).

The Lens Equation

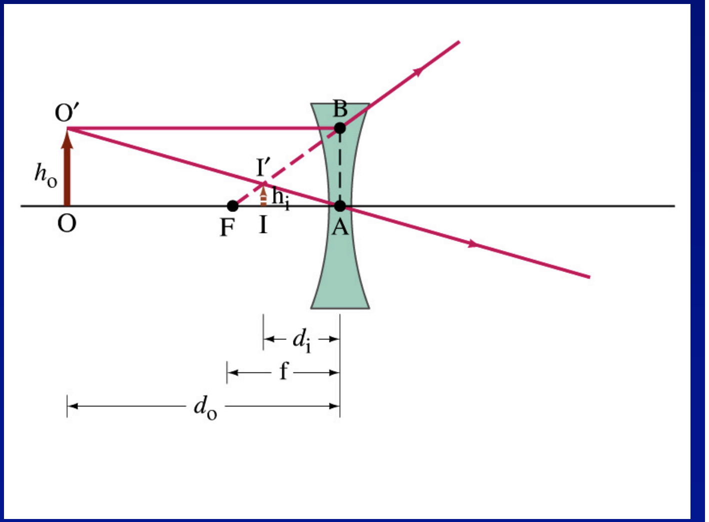
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$



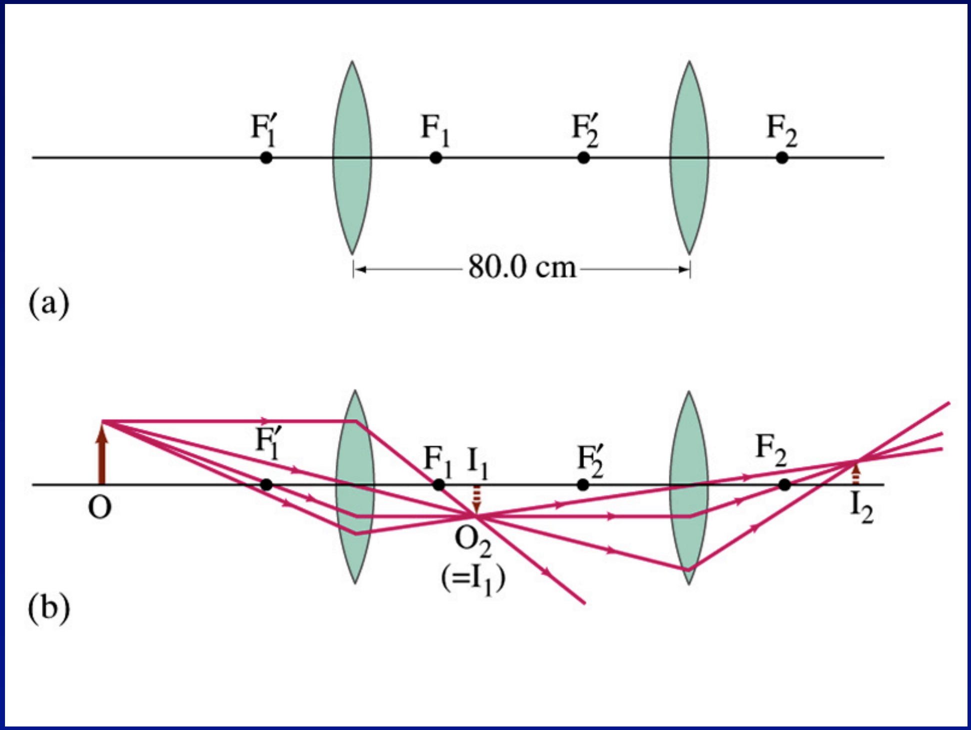
Sign convention:

1) f is positive for converging lens, negative for diverging lens.
2) d_o is positive if it is on the same side of incident light, negative otherwise.
3) d_i is positive if it is on the opposite side of incident light, negative otherwise
4) h_i (or m) is positive if the image is upright and negative if inverted, relative to the object. (h_o is always taken as positive)



Combination of Lenses

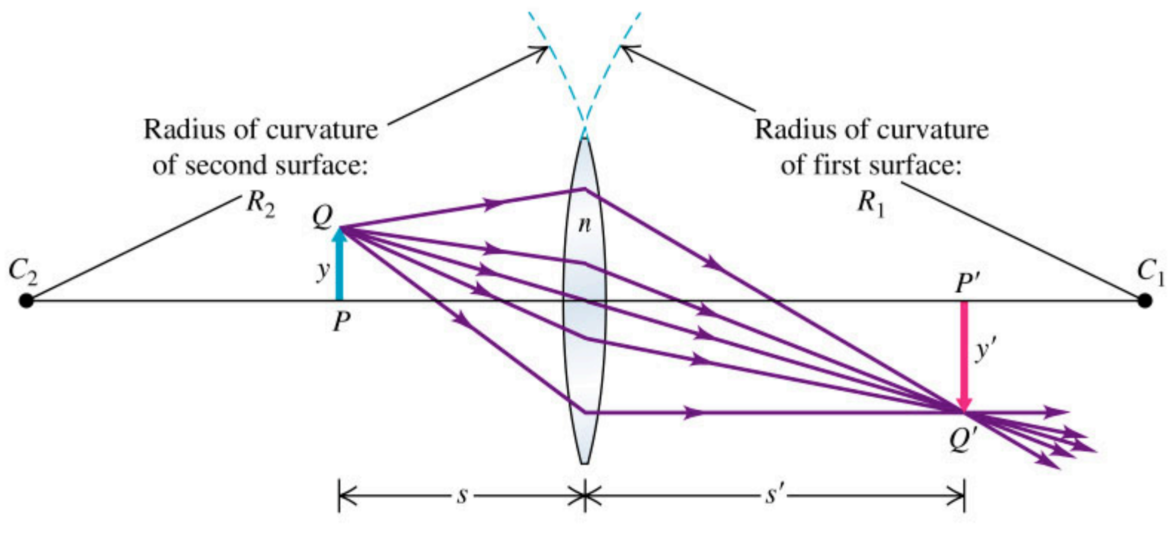
Basic idea: the image formed by the 1st lens is the object for the 2nd lens, and so on. (Watch out for signs!)



Final magnification is a combination of the individual magnifications of the two lenses; $m = m_1 \times m_2$.

Lensmaker's Equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



Symmetric in R_1 and R_2 : so f is the same both ways.

Example: double-convex, n=1.52, absolute radius 10 cm for both surfaces, then the focal length is

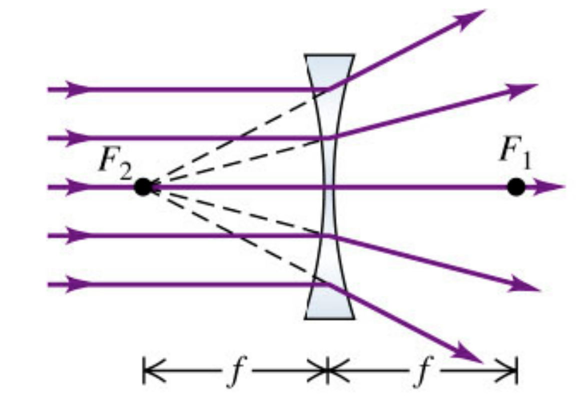
$$\frac{1}{f} = (1.52-1)\left(\frac{1}{10} - \frac{1}{-10}\right)$$

$$f = 9.6\text{ cm}$$

Example: double-concave, n=1.52, absolute radius 10 cm for both surfaces,

$$\frac{1}{f} = (1.52-1)\left(\frac{1}{-10} - \frac{1}{10}\right)$$

$$f = -9.6\text{ cm}$$



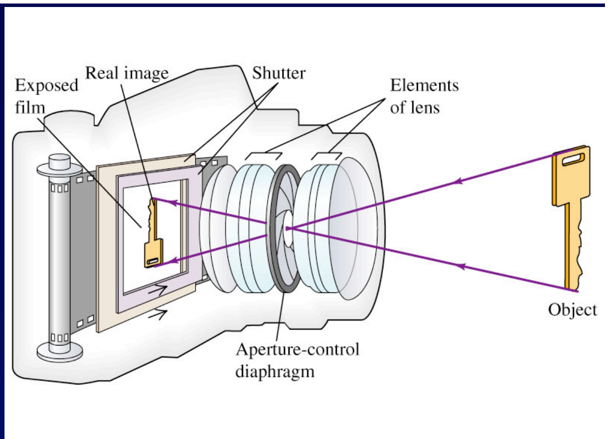
Cameras

Exposure: a measure of the amount of light reaching the film

Exposure depends on
- focal length of lens
- area of the lens opening
- duration for which shutter is open (shutter speed)

(focal length of lens and area of the lens opening: f-stop)

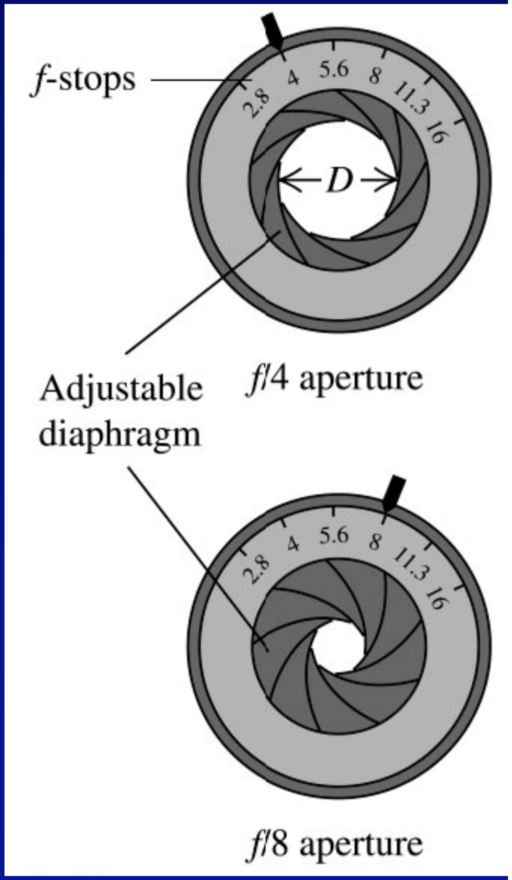
$$\text{f-stop} = \frac{\text{focal length}}{\text{lens diameter}}$$



Example:

For a 50 mm lens	f	= 50 mm
with diameter	D	= 12.5 mm
the f-stop is	f-stop	= 4

This is written as f/4



$$\text{Exposure} \propto \text{time} / \text{f-stop}^2$$

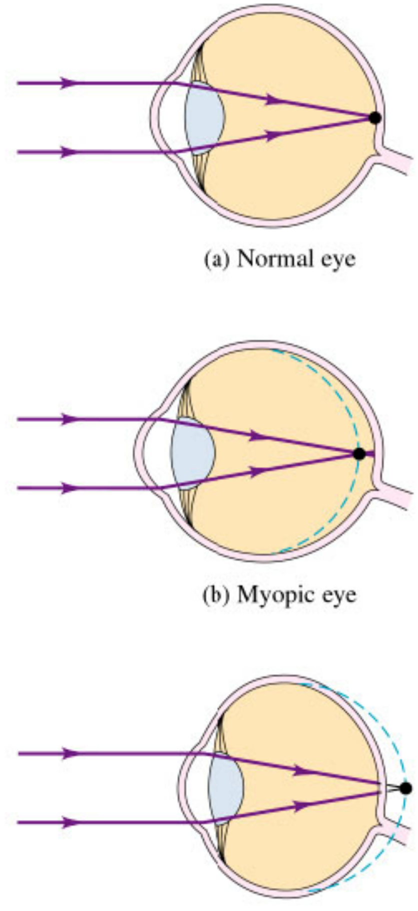
The Eye: Optics

Near point: **closest** distance at which the eye can focus clearly

Far point: **furthest** distance at which the eye can focus clearly

"Normal" eye: near point = 25 cm

far point = ∞



(a) Normal eye

(b) Myopic eye

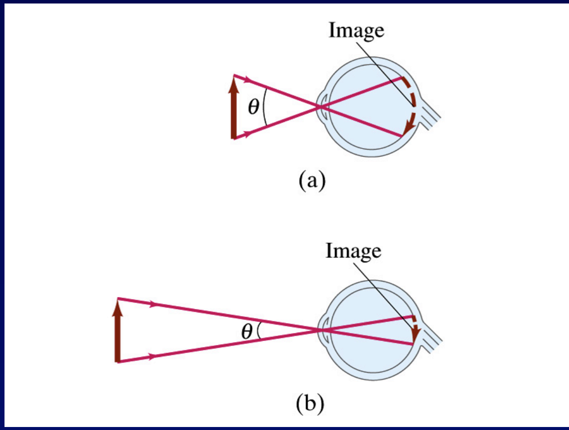
(c) Hyperopic eye

Common Defects

Nearsightedness	Eye cannot focus on distant objects. Far point < ∞
Farsightedness	Eye cannot focus on nearby objects. Near point > 25 cm
Astigmatism	Lens of eye is not spherical

Magnifying Glass

The same object subtends a larger angle at a closer distance:

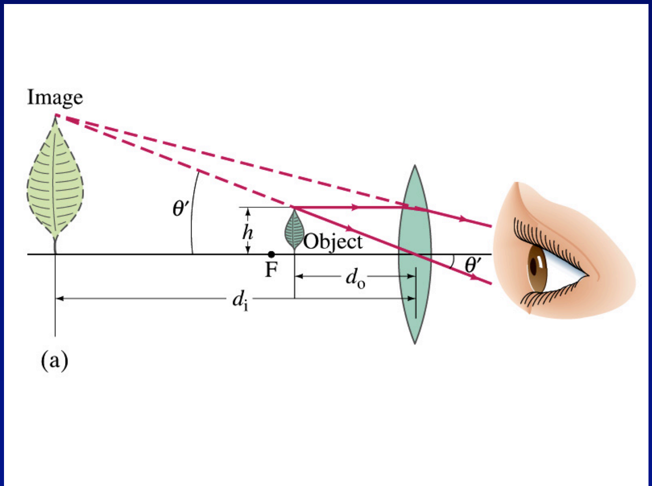


For eye focused at ∞ (relaxed eye), angular magnifying power

$$M = \frac{N}{f}$$

For eye focused at near point, angular magnifying power

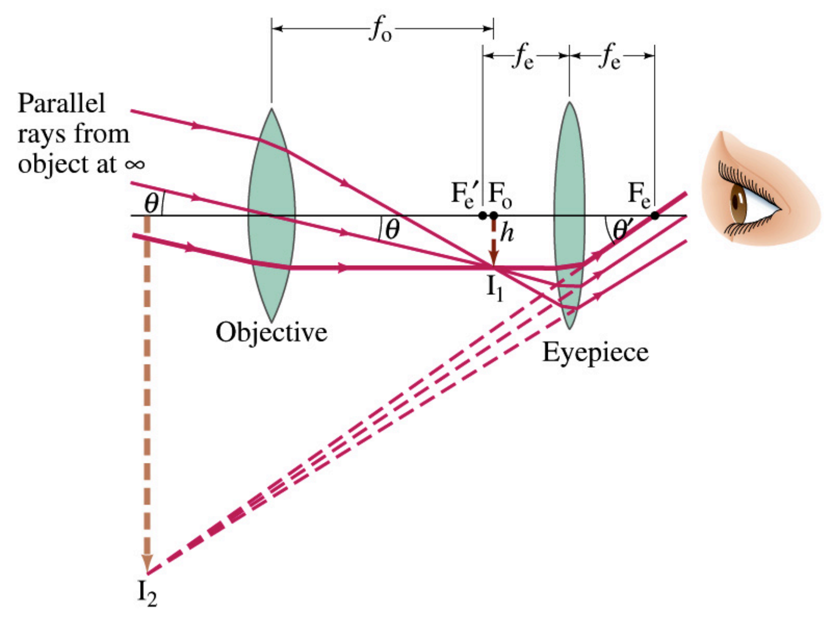
$$M = 1 + \frac{N}{f}$$



The near point for normal eye is N=25 cm.

Refracting Telescopes

Two converging lens at the two ends: objective lens and eyepiece lens.



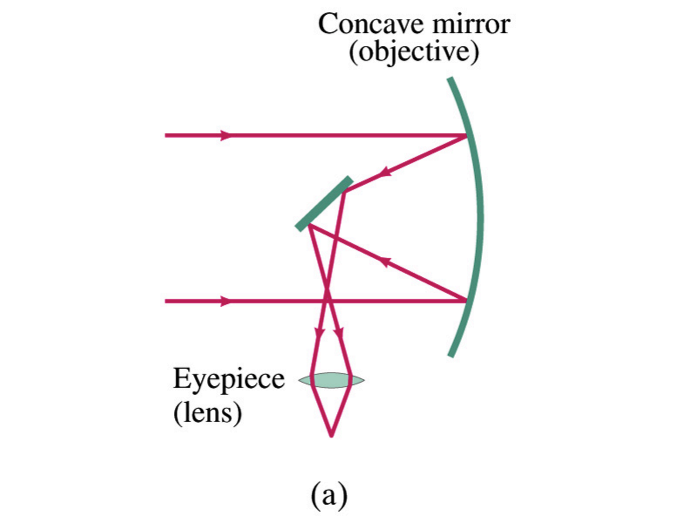
Magnifying power

$$M = -\frac{f_0}{f_e}$$

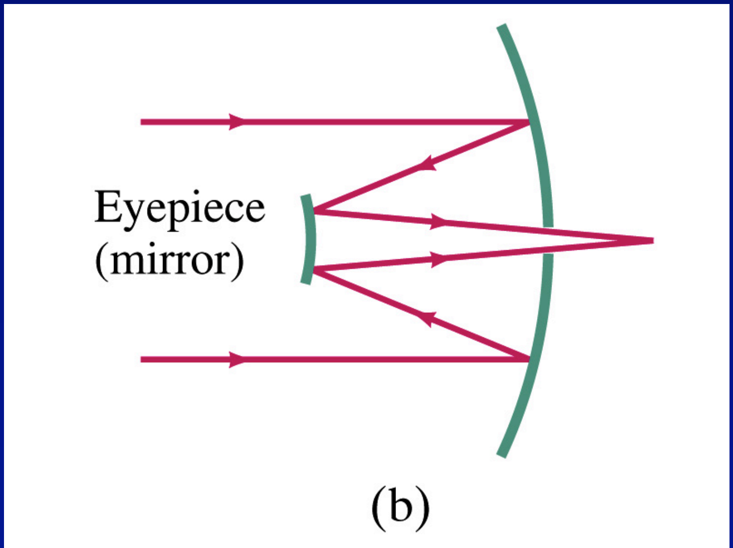
For example: the largest one is located in Wisconsin, a 40-in telescope. Its f_o=19 m, f_e=10 cm. So the magnifying power is
M = -19/0.1 = -190

Reflecting Telescopes

Advantages:
Only one surface has to be ground, unlike a lens.
Can be made large and supported over the entire surface.
Parabolic surface can reduce aberration



The largest optical telescopes are of this kind.

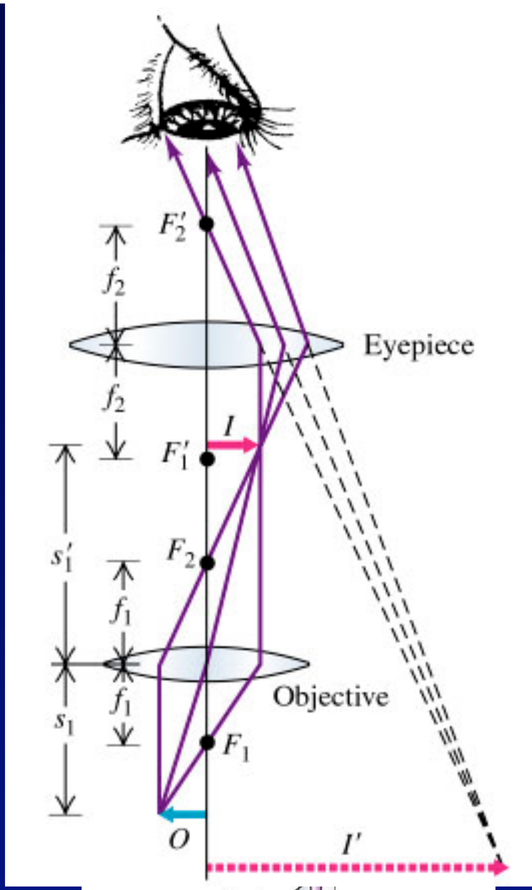


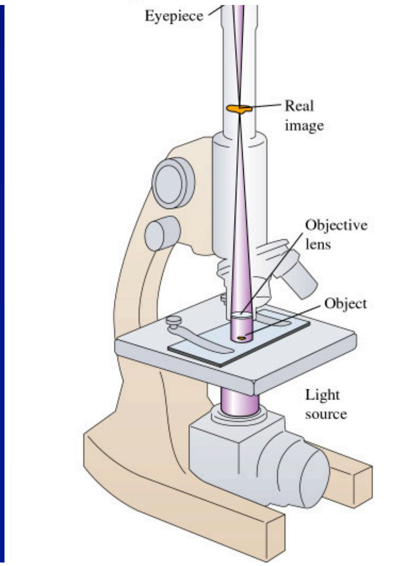
Microscope

Magnifying power

$$M \approx \left(\frac{N}{f_e} \right) \left(\frac{l}{f_o} \right)$$

Slightly different from Eq.(34.23) of the book.





Chapter 35: Interference

- Some properties of waves:
  - Superposition
  - Coherence
- Interference
  - Young's double-slit experiment
  - Thin-film interference

The Principle of Superposition

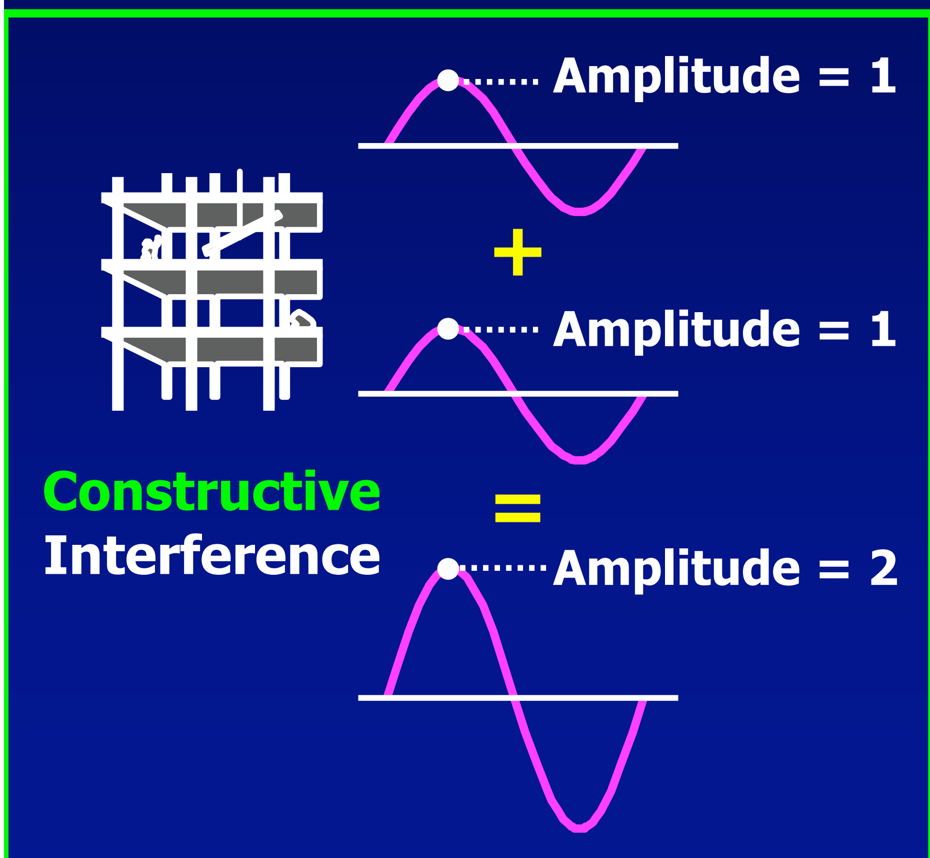
What happens when two **particles** are in the same place at the same time?

They collide!

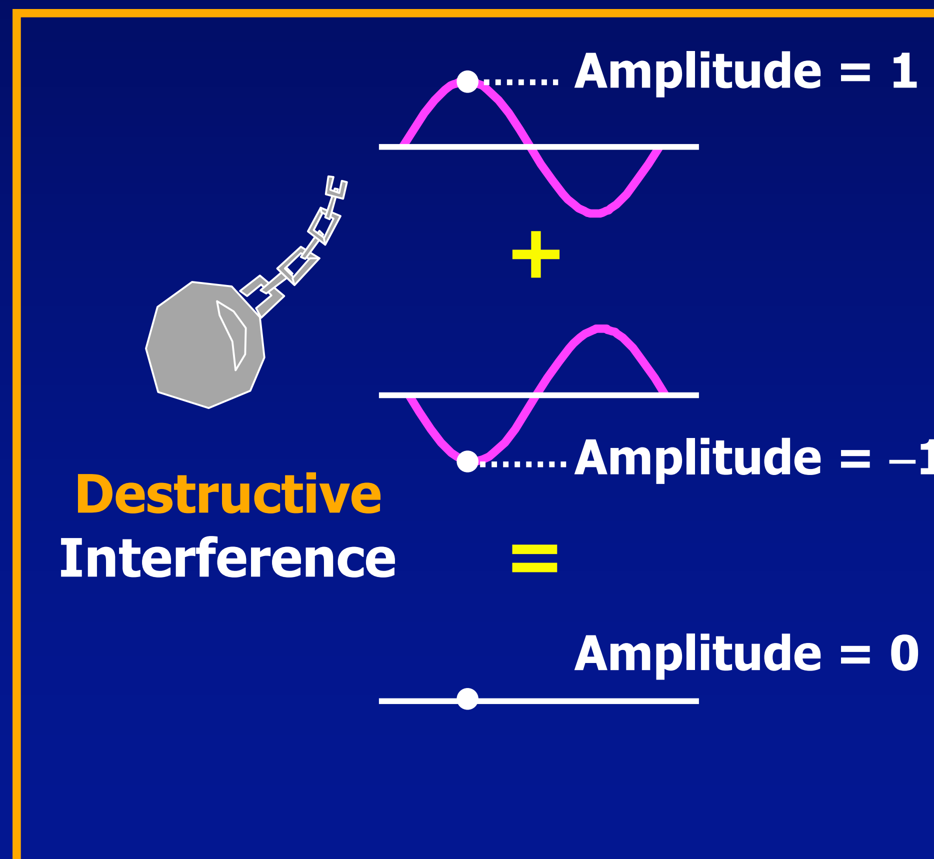
What happens when two **waves** are in the same place at the same time?

They "superpose"!

Their amplitudes add to give *one* new wave!



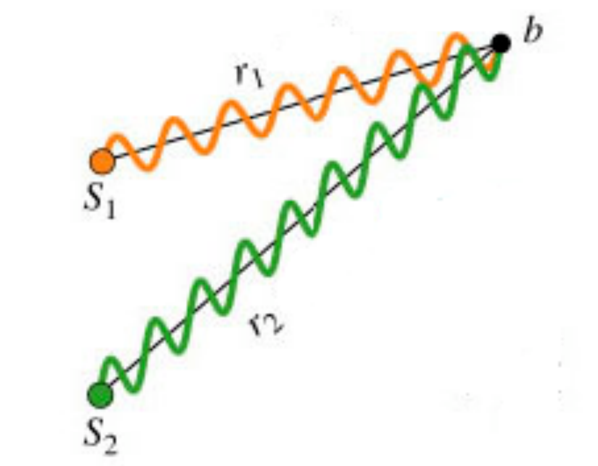
in phase



out of phase

Phase difference and path difference

$$\frac{\phi_2 - \phi_1}{2\pi} = \frac{r_2 - r_1}{\lambda}$$



$\Delta\phi = \phi_2 - \phi_1$	$\Delta r = r_2 - r_1$
0	0
$\pi/2$	$\lambda/4$
π	$\lambda/2$
$3\pi/2$	$3\lambda/4$
2π	λ
4π	2λ
…	…

I will refer to path and phase difference interchangeably.

Coherence

How is light produced? **Oscillating electrons!**

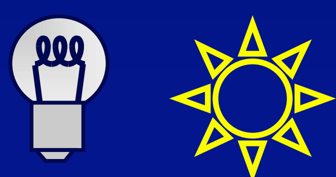
In a light bulb, billions of electrons are oscillating.

Question: is the phase difference between the light from each electron always the same?

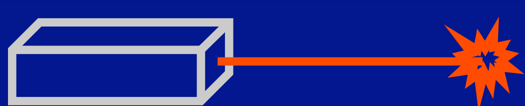
In general, NO!

Two sources of light are said to be

- ***coherent*** **if they have the same frequency and same phase difference**
- ***incoherent*** **if the frequency and phase difference between the waves emitted are random.**



Everyday light sources are not coherent



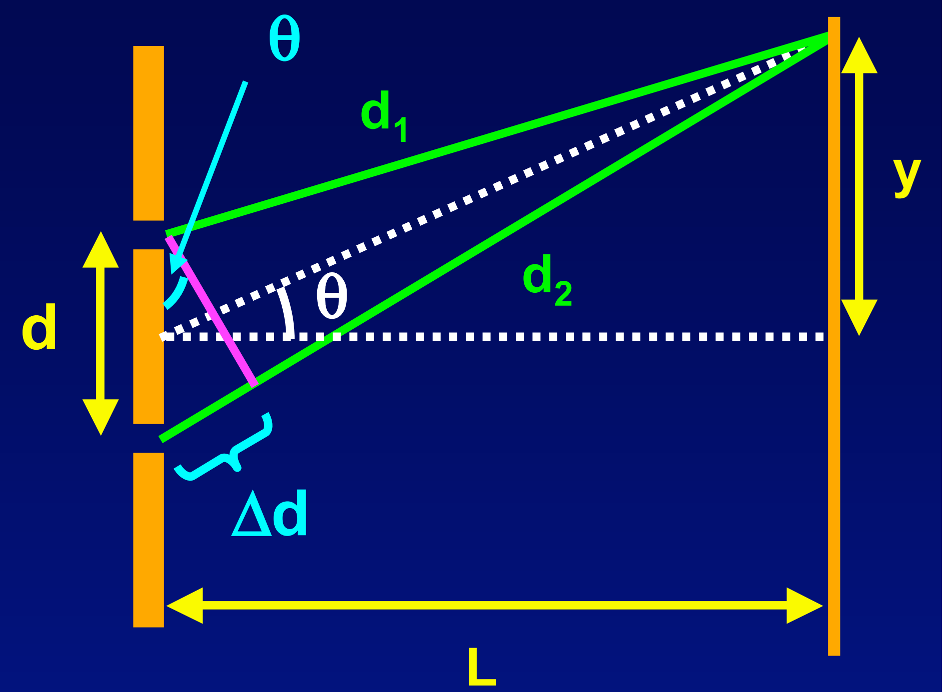
Lasers DO produce coherent light

No interference patterns appear for incoherent light.

Double-Slit Interference: The Math

Path difference Δ between the two waves is:

$$\boxed{\Delta = d_2 - d_1 = d \sin\theta}$$



For Destructive Interference

$\Delta = {}^1/_2\lambda,\ {}^3/_2\lambda,\ {}^5/_2\lambda,\ {}^7/_2\lambda,\ \ldots$

$= (m + {}^1/_2)\,\lambda$

$$d \sin\theta = (m + {}^1/_2)\,\lambda$$

For Constructive Interference

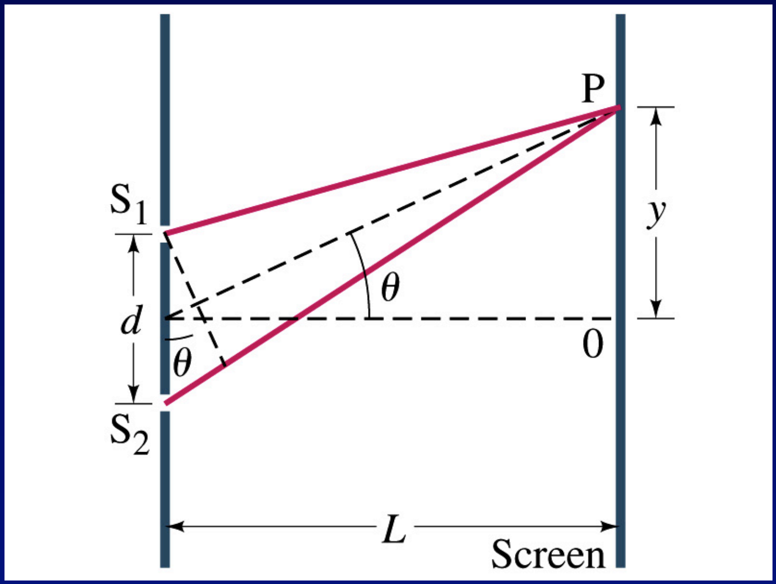
$\Delta = 0, 1\lambda,\ 2\lambda,\ 3\lambda,\ 4\lambda,\ \ldots$

$= m\,\lambda$

$$d \sin\theta = m\,\lambda$$

m is an integer: $m = 0, \pm 1, \pm 2, \ldots$

Double-Slit Interference



Calculate the distance of the bright fringes from the axis:

Note that tan θ = y / L.

Usually, L >> y, so

tan θ ≈ sin θ to a good approximation.

So the bright fringes will be at:

$$\sin\theta = \frac{m\lambda}{d} = \frac{y}{L}$$

therefore:

$$y_{bright} = \frac{m\lambda L}{d}$$

So the bright fringes are evenly spaced a distance $\Delta y = \lambda L/d$ apart.

What about the dark fringes ?

$$y_{dark} = \frac{(m+1/2)\lambda L}{d}$$

Two-source interference: Intensity

$$E_1 = E_0 \cos(\omega t + \phi)$$

$$E_2 = E_0 \cos \omega t$$

Intensity $\propto$ amplitude^2

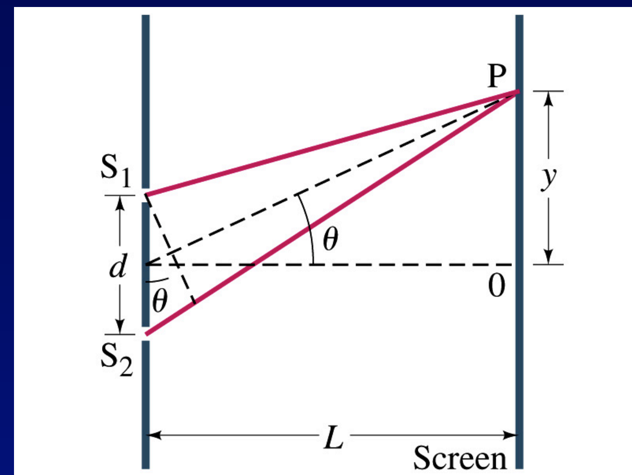
Incoherent light:

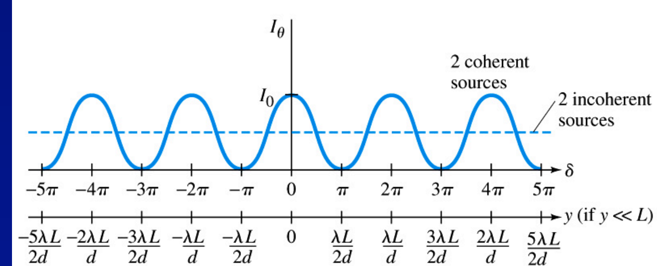
$$I \propto E_1^2 + E_2^2 = 2E_0^2$$

Coherent light:

$$I \propto (E_1 + E_2)^2 = 4E_0^2 \cos^2\left(\frac{\phi}{2}\right)$$

But
$$\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = \frac{2\pi}{\lambda} d \sin\theta$$



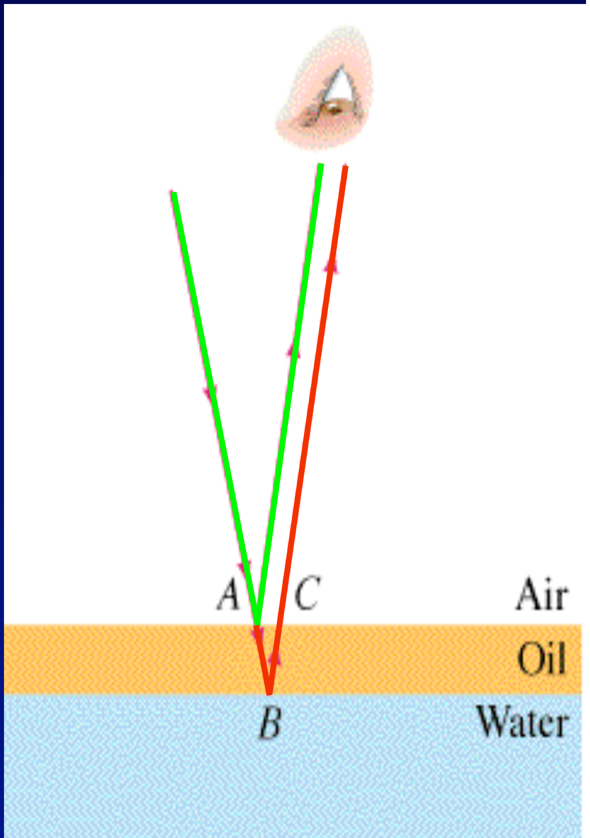


$$d \sin\theta = m\lambda, \text{ bright}$$

$$d \sin\theta = (m + 1/2)\lambda, \text{ dark}$$

Interference by Thin Films

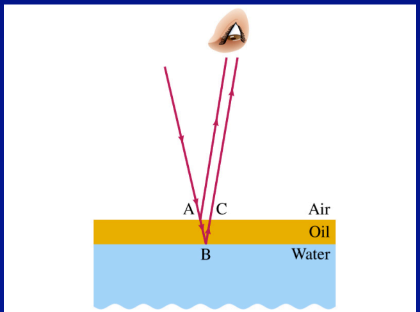
- **Example -- thin oil film on water:**
  - **Part of the incoming light is reflected off the top surface (point A), part at the lower surface (point B).**
  - **Light traveling through oil travels extra distance 2t (which is twice the thickness of the film).**
  - **If 2t is $\lambda, 2\lambda, 3\lambda, 4\lambda, \ldots$**
    - » ***constructive interference*!**
  - **If 2t is $\lambda/2, 3/2\lambda, 5/2\lambda, \ldots$**
    - » ***destructive interference*!**



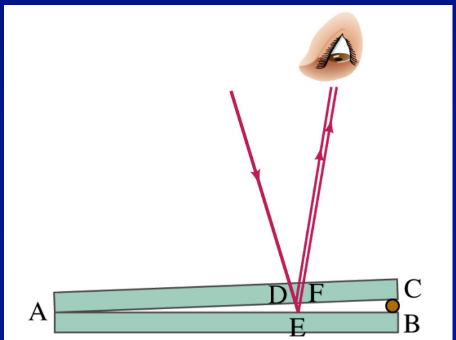
But watch out for possible phase changes at the boundaries.

Thin-film Interference: summary

- The deciding factor is the total path difference between the two waves reflected from the two surfaces of the thin film of thickness t:
  - $2t + \delta$
  - where δ is the wavelength shift due to phase change from the reflections
- So if δ is zero, then
  - $2t = m\lambda_n$ gives constructive interference (bright)
  - $2t = (m+1/2)\lambda_n$ gives destructive interference (dark)
  - here λ_n is wavelength in the thin film: $\lambda_n = \lambda/n$
- If δ is half-wavelength, then the above situation is reversed.
- The phase change at the reflection depends on the situation at the surface:
  - small n to big n: half-wavelength
  - big n to small n: 0

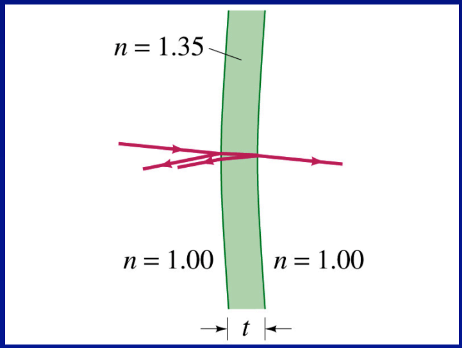


Thin film: oil layer: $\delta = 0$



Thin film: air gap

δ = half-wavelength - 0



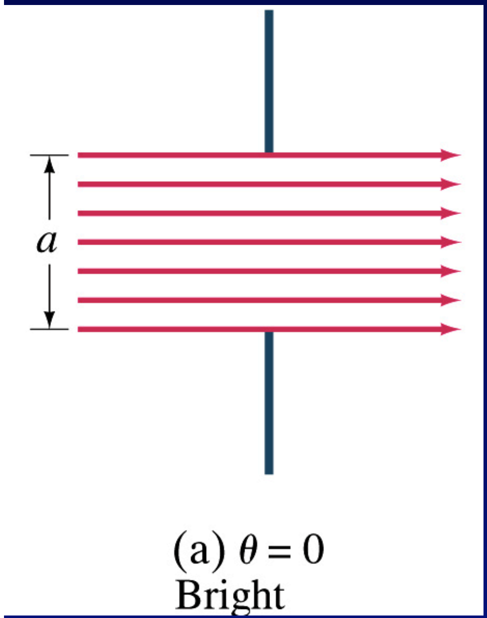
Thin film: soap bubble

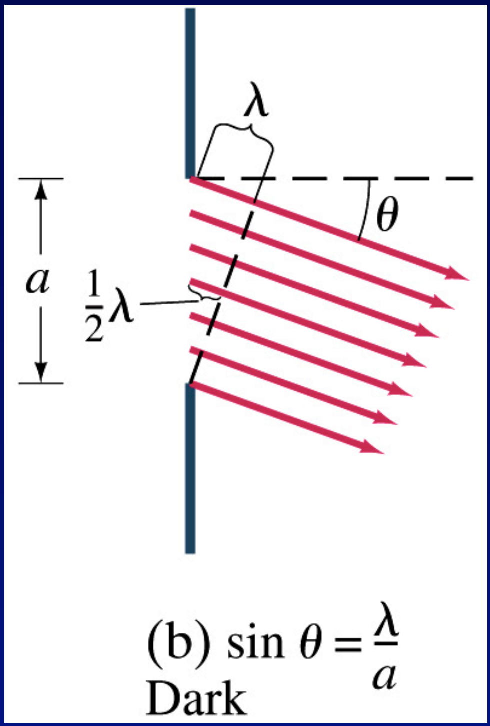
δ = 0 - half-wavelength

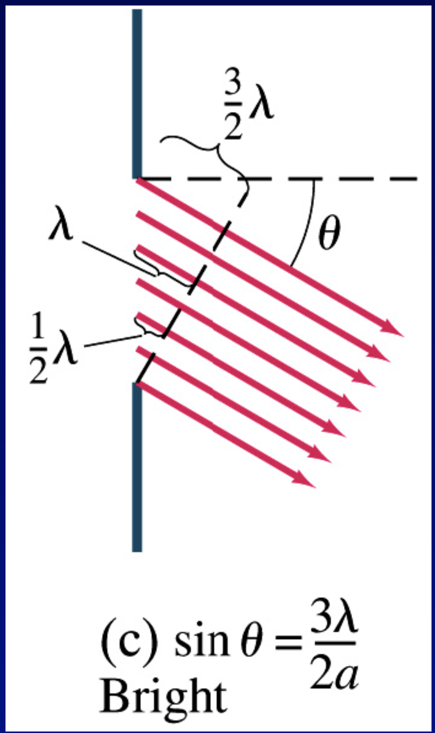
Chapter 36: Diffraction

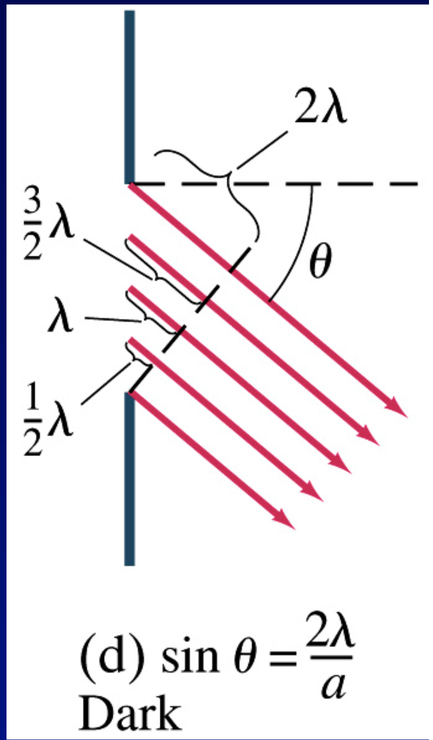
- Diffraction
  - Single-slit
  - Double-slit
  - Diffraction grating
  - Limit on resolution
  - X-ray diffraction

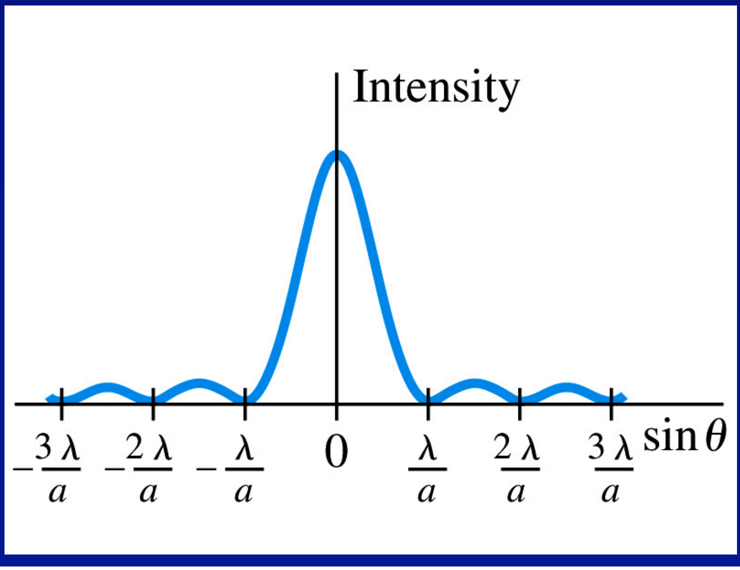
Single-slit Diffraction











dark: $$a\sin\theta = m\lambda$$

$m = \pm 1, \pm 2, \ldots$

bright: $$a\sin\theta \approx \pm\left(m + \tfrac{1}{2}\right)\lambda$$

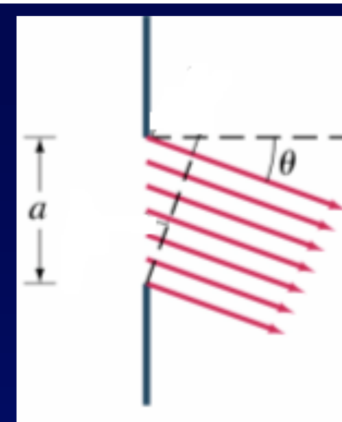
$m = 1, 2, 3\ldots$

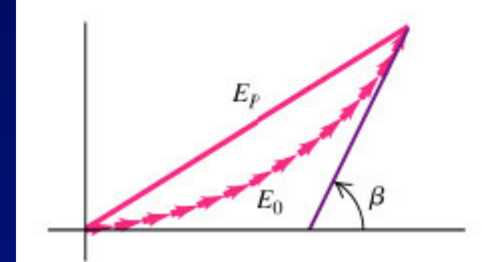
Intensity of Single-Slit Diffraction

Intensity $\propto$ amplitude^2

The total amplitude at a point P on the screen is the vector sum of all wavelets:

$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2}$$





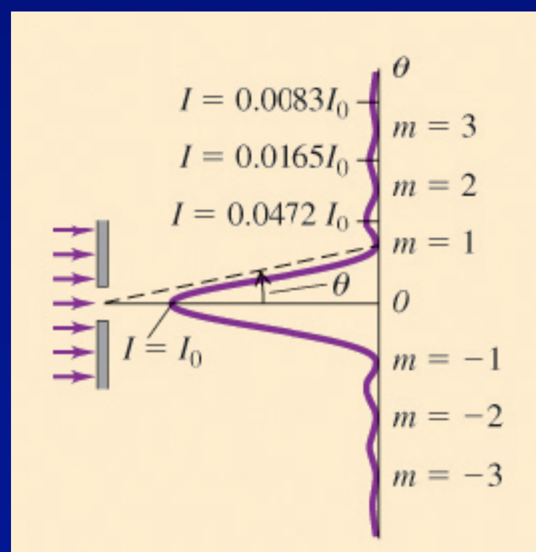
(b) Slightly off center of pattern: β = total phase difference between first and last phasors

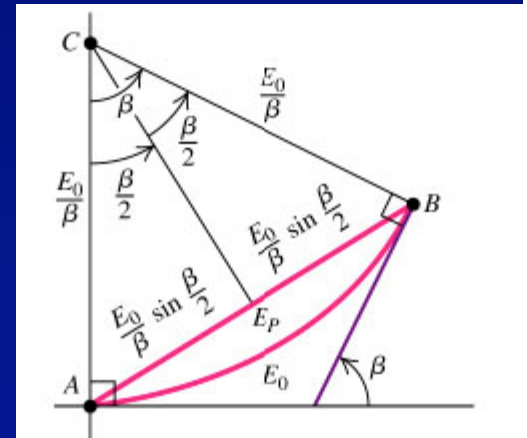
$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$ where $$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

The intensity is zero when β =2mπ, giving $$a\sin\theta = m\lambda$$

The intensity is approx. max. when β≅±(2m+1)π, giving

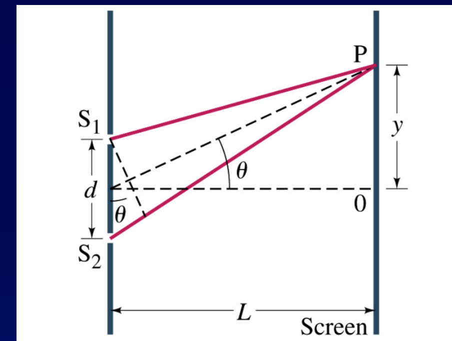
$$a\sin\theta \approx \pm(m+\tfrac{1}{2})\lambda$$





(c) As (b), but in the limit that the slit is subdivided into infinitely many strips

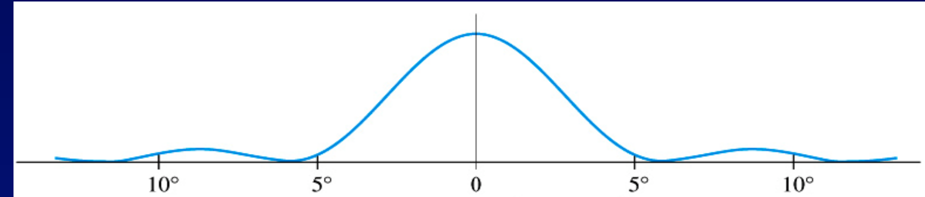
Intensity Pattern of Double-slit



Single-slit diffraction:

$$I = I_0 \left(\frac{\sin(\beta/2)}{\beta/2} \right)^2 \quad \text{where} \quad \beta = \frac{2\pi}{\lambda} a \sin\theta$$

which has minima at $a \sin\theta = m\lambda$

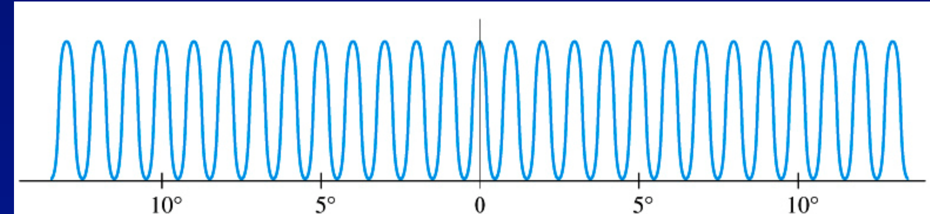


(a) Diffraction factor, $(\sin^2\beta/2)/(\beta/2)^2$ vs. θ

Double-slit interference:

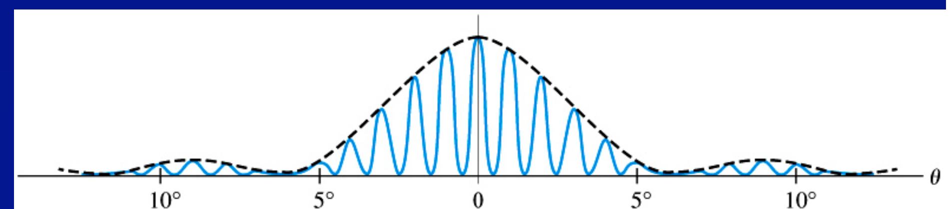
$$I = I_0 \cos^2\left(\frac{\delta}{2} \right) \quad \text{where} \quad \delta = \frac{2\pi}{\lambda} d \sin\theta$$

which has maxima at $d \sin\theta = m\lambda$



(b) Interference factor, $\cos^2 \frac{\delta}{2}$ vs. θ

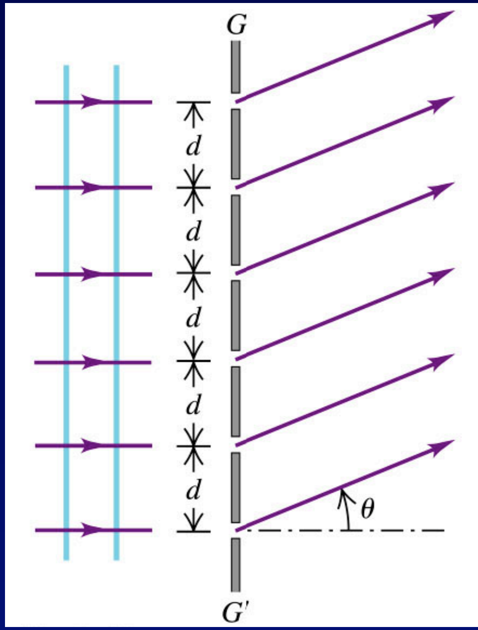
The pattern in Young's double-slit experiment is the product of the two intensities.

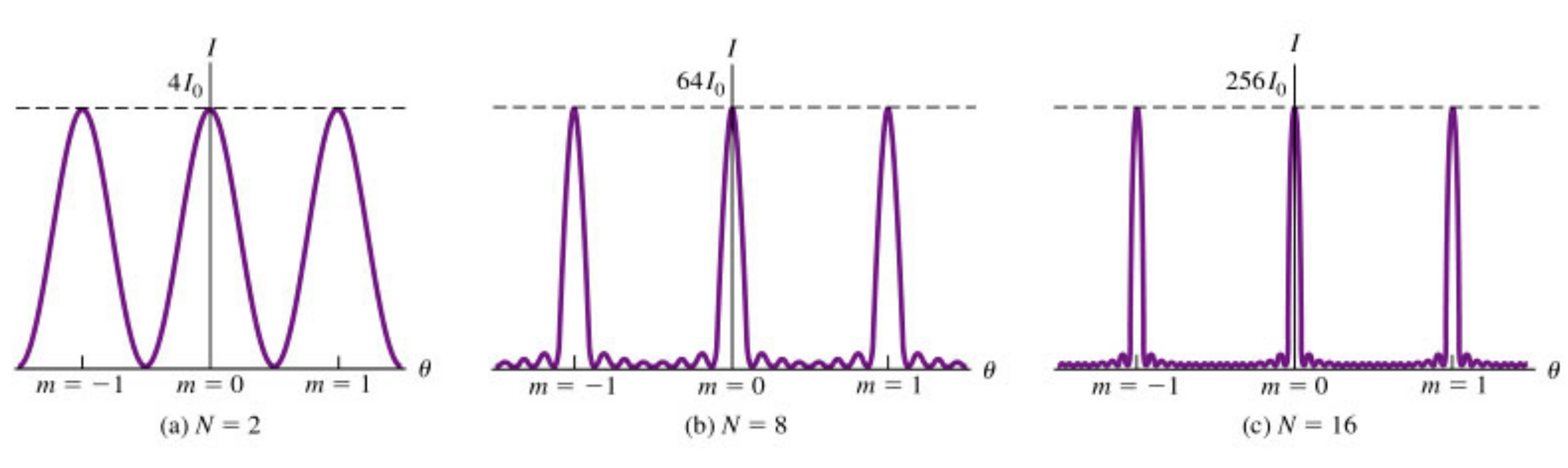


(c) Intensity, I_θ vs. θ

Diffraction Grating

- A large number of equally spaced slits (up to **N=10,000 per cm**) is called a *diffraction grating*
  - peaks are sharper, narrower
  - useful for high-precision measurements of wavelengths

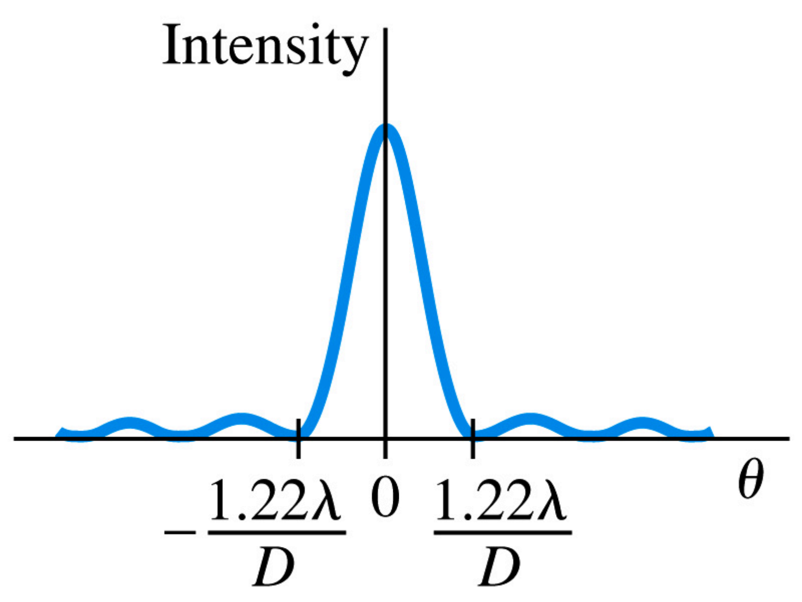




Principal maxima $$\boxed{d \sin\theta = m\lambda}$$ d=1/N

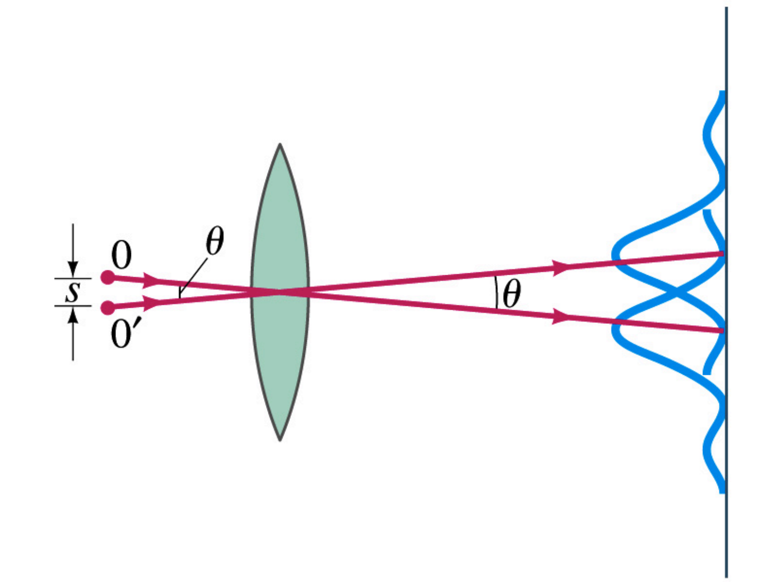
Limits of Resolution due to Diffraction

The intensity pattern of circular opening due to diffraction is



Rayleigh criterion: two objects are just resolvable if they are separated by an angle given by

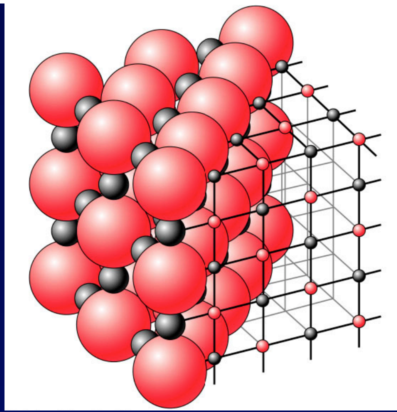
$$\boxed{\theta = \frac{1.22\lambda}{D}}$$



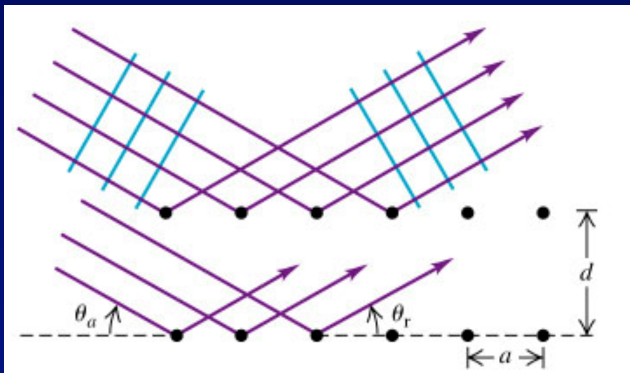
This is the limit of resolution on optical instruments set by the wave nature of light due to diffraction.

X-ray Diffraction

NaCl



- X-rays are EM waves of λ on the order of 0.1 nm, which is about the average distance between atoms.
- X-ray diffraction is a powerful tool to reveal the 3-D structure of materials
  - solids, organic molecules, DNA structure, molecular genetics.



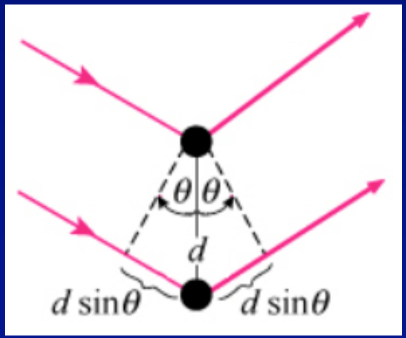
Conditions for constructive interference from the entire array are:

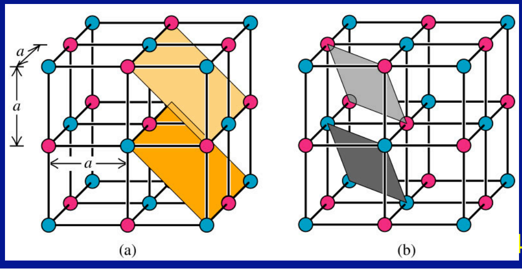
1) Angle of incidence must be equal to angle of scattering within the same row.

2) The path difference for adjacent rows must be

$$\boxed{2d\sin\theta = m\lambda}$$

(Bragg diffraction condition)





Chapter 37: Relativity

- Two postulates
- Relativity of simultaneity
- Time dilation; length contraction
- Lorentz transformations
- Doppler effect
- Relativistic kinematics

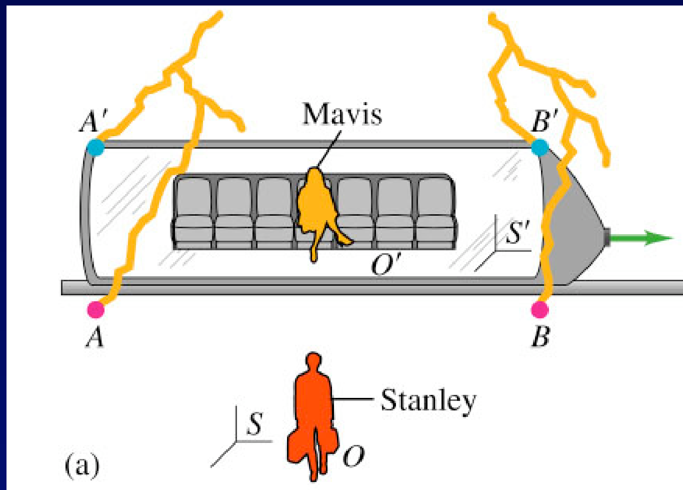
Two postulates

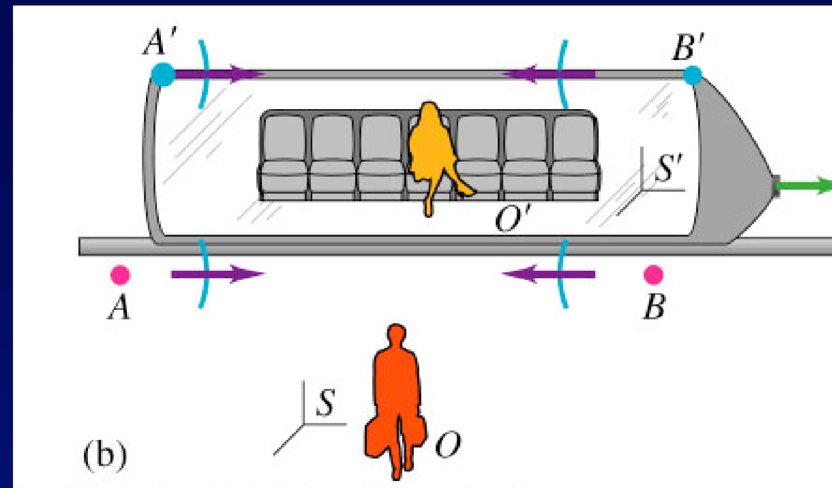
- The laws of physics are the same in every inertial frame of reference. (covariance)
  - Should be true for Maxwell's equations. It is!
- The speed of light is the same in all inertial reference frames and is independent of the motion of the source. (invariance)
  - What's wrong? Our implicit assumption that $t = t'$.

These postulates form the crux of the Special Theory of Relativity. There is also a General theory of Relativity, which deals only with gravitation, which we won't cover here.

We will assume that special relativity must reduce to Galilean relativity for $v \ll c$. This is called the "correspondence principle".

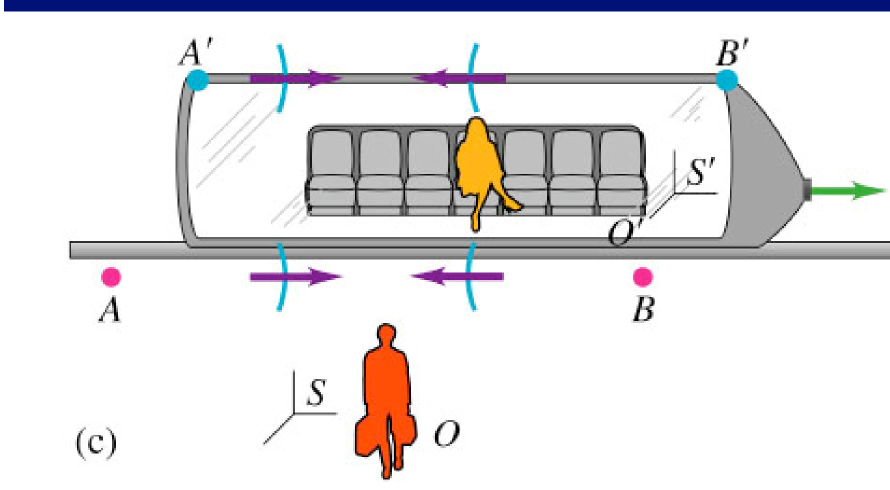
Simultaneity: Stanley and Mavis

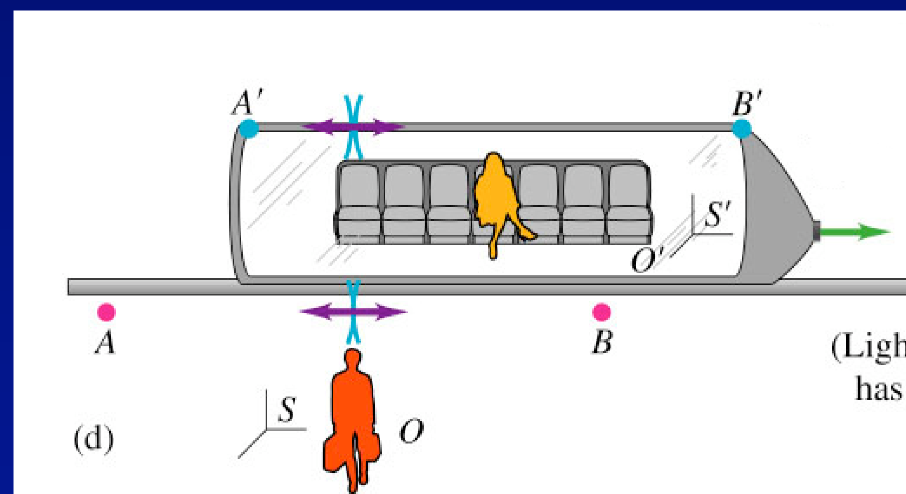




A‘ and B’ are simultaneous to Mavis

Light start to spread out (Stanley POV)





Light from B’ reaches Mavis first

Events are simultaneous for Stanley

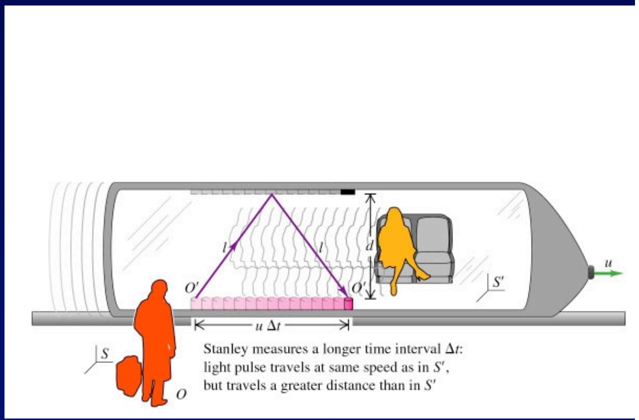
Moral: Simultaneity is relative!

Time dilation equations

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

$$\Delta t = \gamma \, \Delta t_0$$



Proper time (Δt_0): Time measured by an observer at rest with respect to two events.

Example: A particle known as a muon is generated high in the earth's atmosphere with a speed of 0.96c relative to the earth. The muon's average lifetime, measured at rest, is 2.2 x 10-6 sec. How far does such a muon travel through the Earth's atmosphere before decaying?

Length contraction equation

Twin paradox redux: If the traveling twin knows 1) the distance to the star is 9.5 light years away and 2) that he/she is traveling at a speed of 0.95c with respect to the Earth/star system, how can he/she possibly agree that it takes only 6.24 years to complete the journey??

Ans: Length contraction!

$$l = l_0\sqrt{1 - \frac{u^2}{c^2}} = \frac{l_0}{\gamma}$$

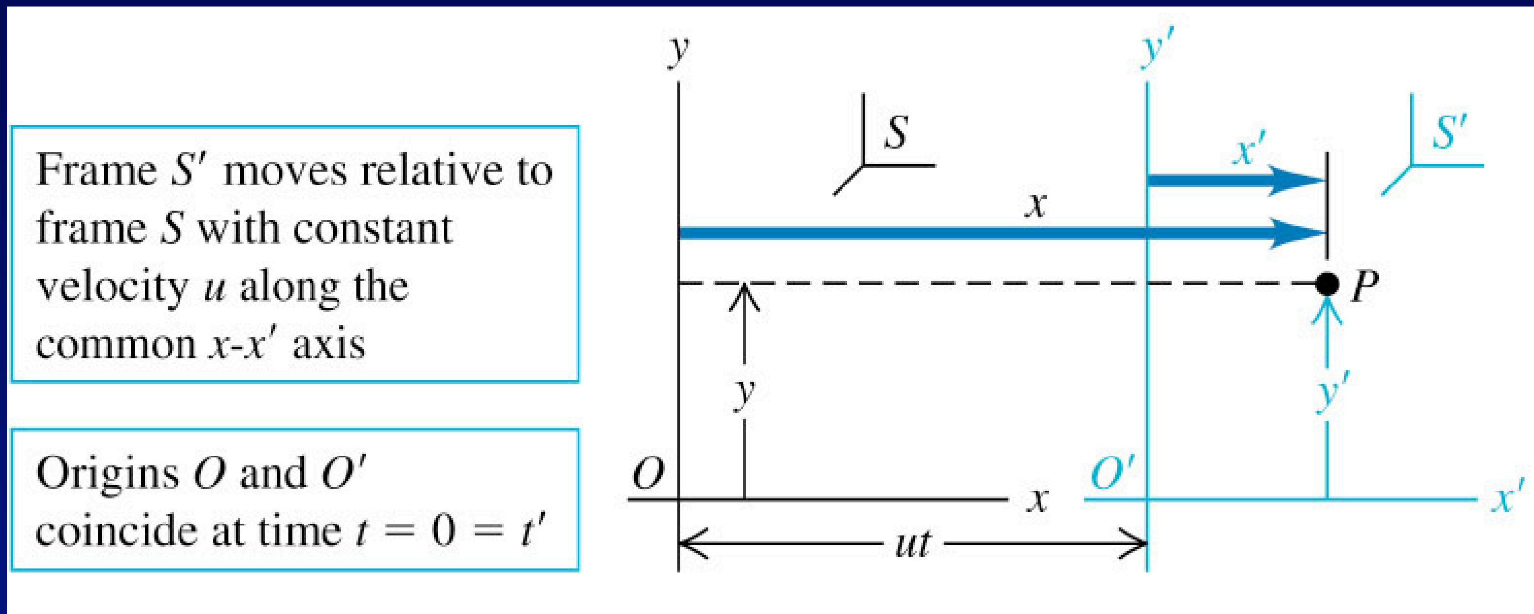
Contraction is only along the direction of motion, but distortions appear because of the finite speed of light.

$$\frac{1}{\gamma} = \sqrt{1-(.95)^2} = .3122\ !!$$

The earth/star distance is contracted !!

Proper length (l_0): Length measured by an observer at rest with respect to the object.

Transformation equations



$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}} = \gamma(x - ut)$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} = \gamma(t - ux/c^2)$$

"Galilean":

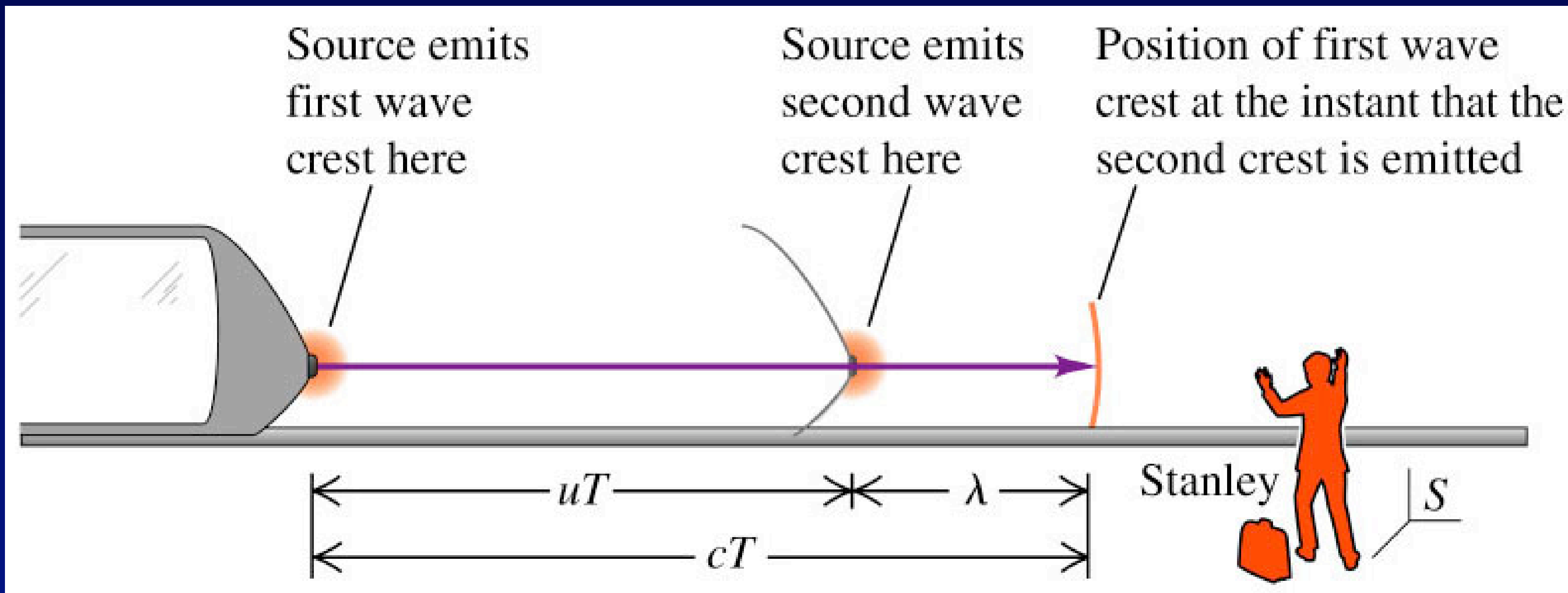
x' = x - ut,

y' = y,

z' = z,

t' = t

Doppler effect



$$f = \sqrt{\frac{c+u}{c-u}}\, f_0$$

Note: When the source moves away from the observer, change the sign of u.

Toward: u > 0 => $f > f_0$ "higher frequency"

Away: u < 0 => $f < f_0$ "lower frequency"

What if u << c ?? Then we obtain

$$f \approx \left(1+\frac{u}{c}\right) f_0$$

$$\Delta f \approx \frac{u}{c}$$

Relativistic momentum, work, etc.

Energy and momentum still conserved, but their expressions change!

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$$

$$\vec{p} = \gamma m \vec{v}$$

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = (\gamma - 1)mc^2$$

$$E = K + mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = \gamma mc^2$$

$$E^2 = (mc^2)^2 + (pc)^2$$

old connection :

$$E = \frac{\vec{p}^2}{2m}.$$

"Photon": (m=0) $E = pc$