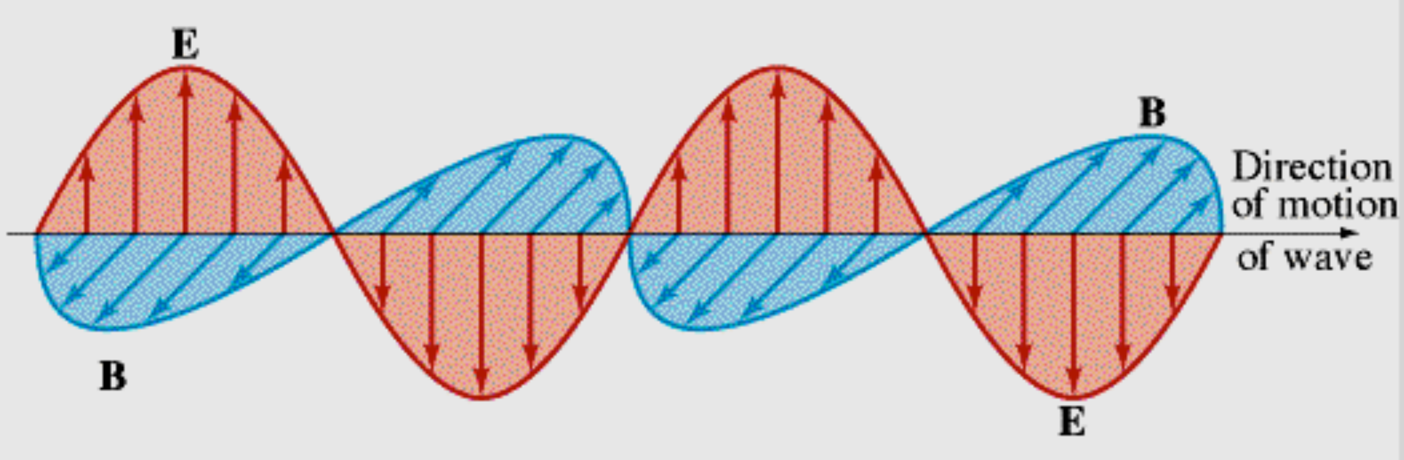
Chapter 32: Electromagnetic Waves

- Maxwell's equations
- Electromagnetic waves in free space
- The electromagnetic spectrum
- Energy in electromagnetic waves
  - The Poynting vector
  - Radiation pressure
  - Radio and Television



Maxwell’s Equations

New Topic

Maxwell's Insight

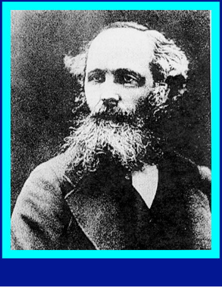
Faraday: A changing B field creates a E field

It doesn't just induce a current,
it produces an E field!

So let's summarize what we know so far…

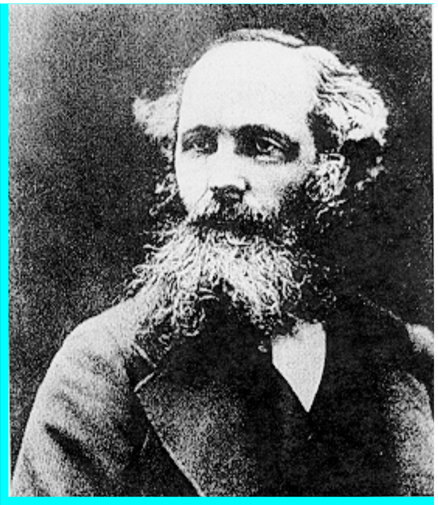
E Field produced by	B Field produced by
electric charge	*moving* electric charge
changing B field	*changing* E field

Maxwell looked at this table and, appealing to symmetry, postulated…



Maxwell: A changing E field creates a B field

Maxwell's Equations



J. C. Maxwell (1831 - 1879)

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

Gauss's law for electric field: electric charges produce electric fields.

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss's law for magnetic field: but there're no magnetic charges.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Faraday's law: changing B produces E.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampere's law as modified by Maxwell: electric current or changing E produces B.

All of electromagnetism is contained in this set of four equations.

Perfect Symmetry ?

What would Maxwell's Equations look like if there were isolated magnetic charges?

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_e}{\varepsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_e}{\varepsilon_0}$$

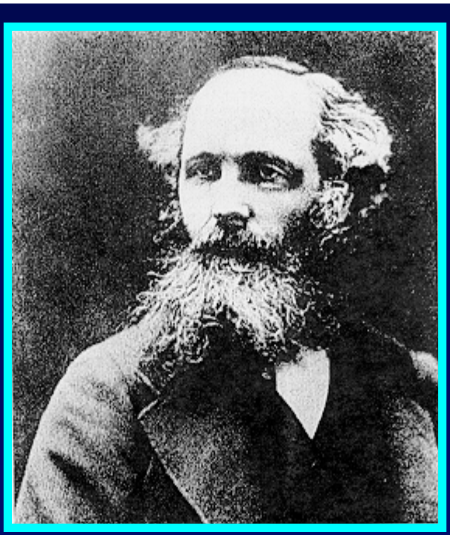
$$\oint \vec{B} \cdot d\vec{A} = \mu_0 Q_m$$

$$\oint \vec{E} \cdot d\vec{l} = \mu_0 \frac{dQ_m}{dt} - \frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \frac{dQ_e}{dt} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

E Field produced by	**B Field produced by**
electric charge	**magnetic charge**
moving magnetic charge	***moving* electric charge**
***changing* B field**	***changing* E field**

Maxwell's Predictions



J. C. Maxwell (1831 - 1879)

- There exist electromagnetic waves (EM waves) that can travel in empty space
- EM waves travel at the speed of light
- Light is an EM wave

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_e}{\varepsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

The rest was history.

telegraph, radio, television, cell-phone, other wireless communications,…

ConcepTest 32.1 Maxwell's Equations

- The Maxwell modification of Ampere's law describing the creation of a magnetic field is the analog of

1) Gauss's law on electric fields and charges
2) Gauss's law on magnetic fields and poles
3) the Lorentz equation
4) Faraday's Law

Electromagnetic Waves in Free Space

New Topic

The production and propagation of electromagnetic waves

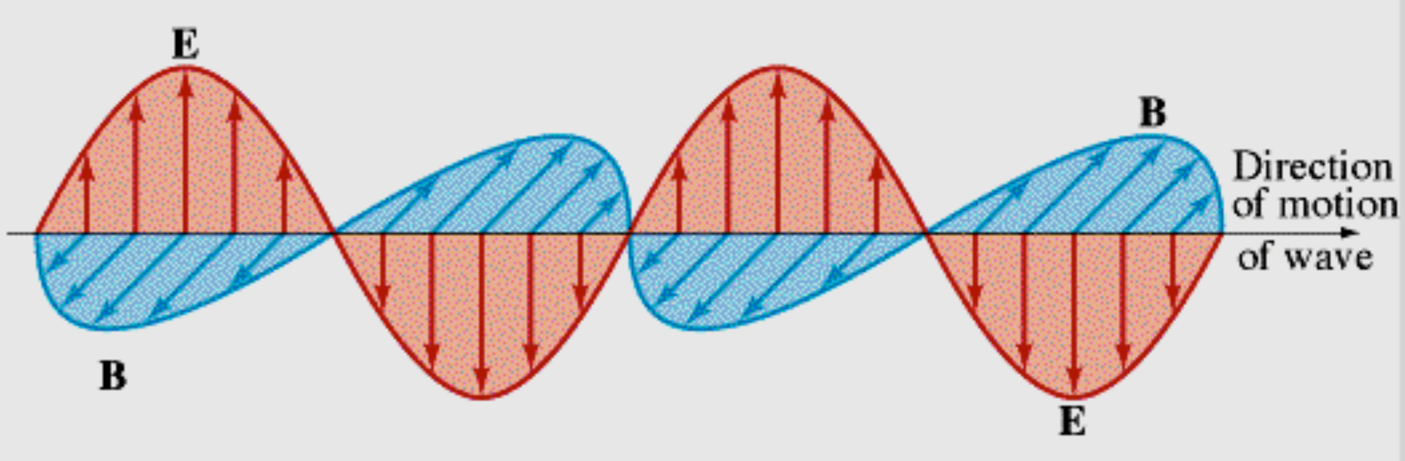
- **Let's put them both together:** ***we obtain changing electric and magnetic fields that continuously produce each other!***

oscillating charge

oscillating E field

oscillating B field

→ ***Electromagnetic waves***



Brief Review of Wave Properties

- The one-dimensional wave equation:

$$\frac{\partial^2 h}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 h}{\partial t^2}$$

has a general solution of the form:

$$h(x,t) = h_1(x - vt) + h_2(x + vt)$$

where h_1 represents a wave traveling in the +x direction and h_2 represents a wave traveling in the -x direction. The wave velocity is given by v.

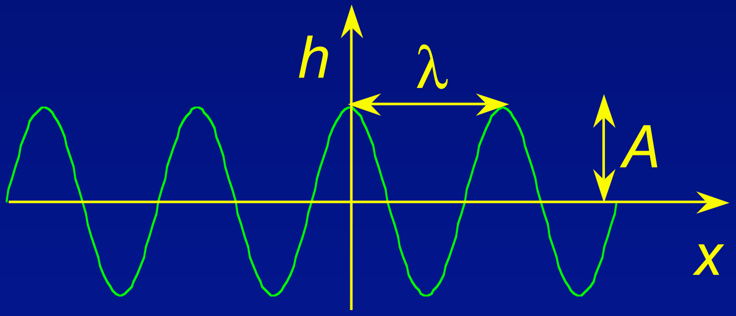
- A specific solution for harmonic waves traveling in the +x direction is:

$$h(x,t) = A\sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

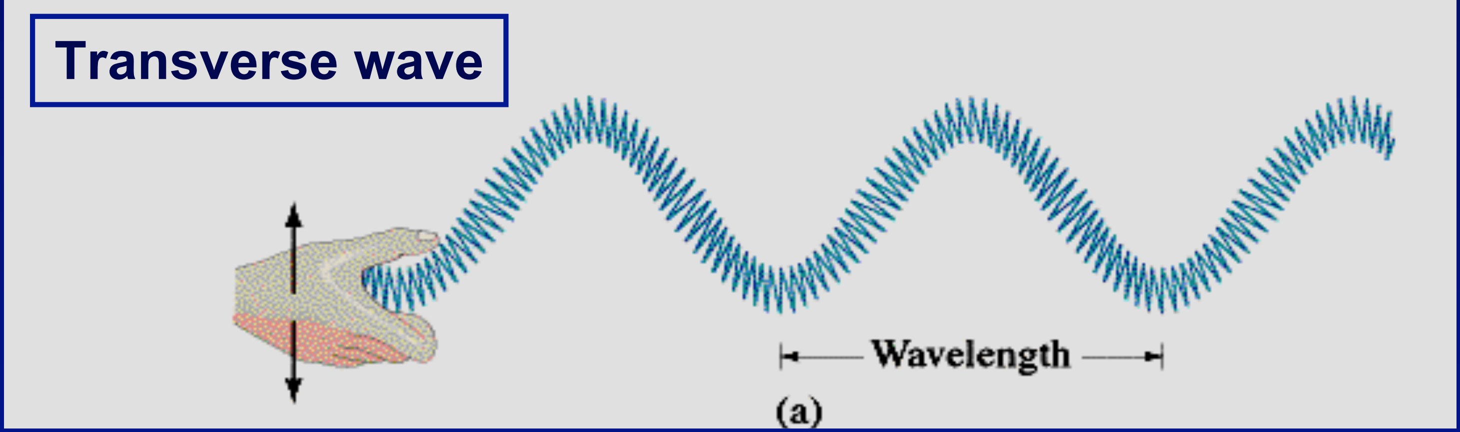
$$v = \lambda f = \frac{\omega}{k}$$

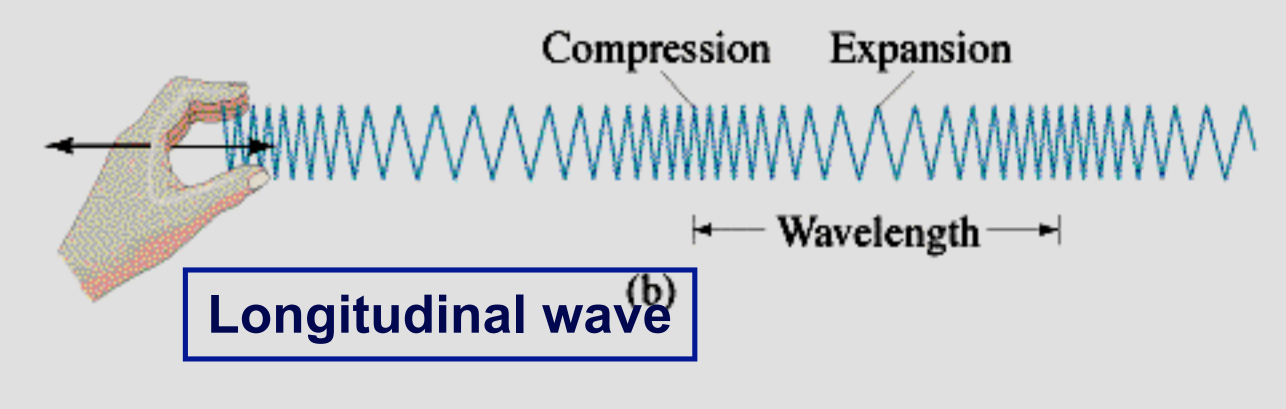


A = amplitude
λ = wavelength
f = frequency
v = speed
k = wave number

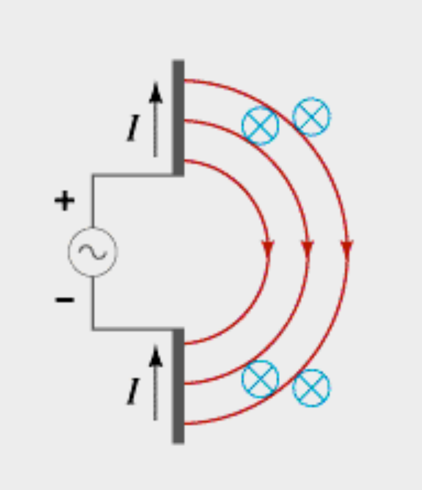
Waves

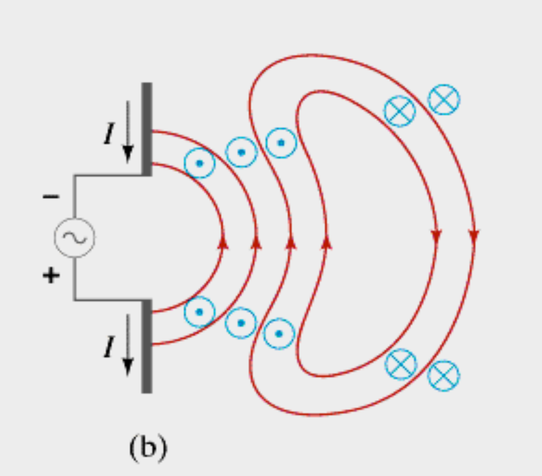
- are traveling disturbances that transport energy, *not matter*.
- are produced by oscillations.
- have a speed of the wave depends on the properties of the medium and not on the wavelength or frequency
- come in two basic flavors:

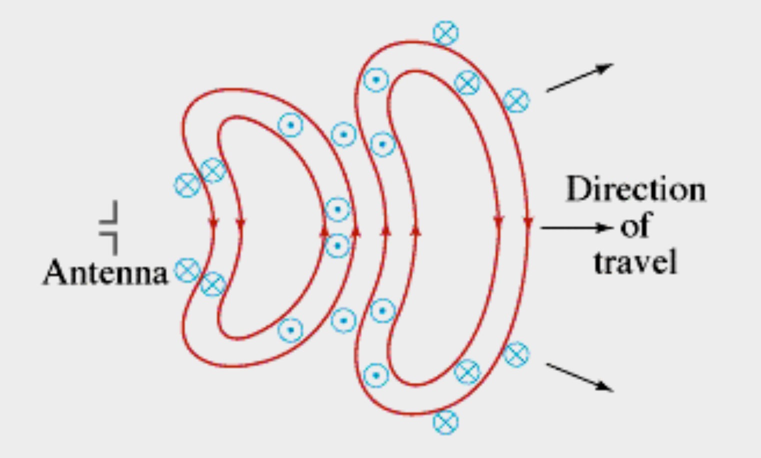


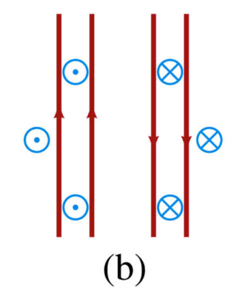


How does an electromagnetic wave propagate?



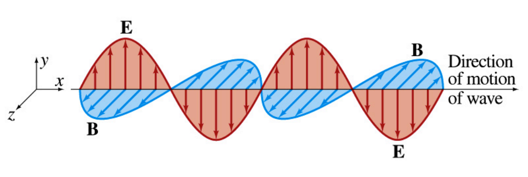






Properties of EM Waves:

- The E and B fields at any point are perpendicular to each other, and to the direction of wave propagation
- The E and B fields are in phase.
- Far away from the source, the EM waves are approximately plane waves



Maxwell's Equations in Differential Form

Integral forms

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Differential forms

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Q is charge and I is current

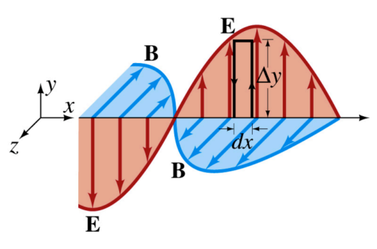
ρ is charge density and
J is current density

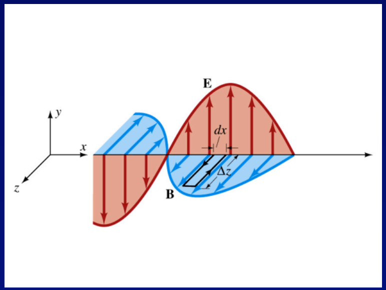
Electromagnetic Waves in Free Space (Q=0, I=0)

Differential forms

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$





$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Take partial derivatives

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial x \partial t}$$

$$\frac{\partial^2 B}{\partial t \partial x} = -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\text{so} \quad \frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

Read off wave speed from the wave equation:

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8 \text{ m/s } = c$$

Electromagnetic Waves in Free Space

Wave equations

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \qquad \frac{\partial^2 B}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}$$

Plane-wave solution

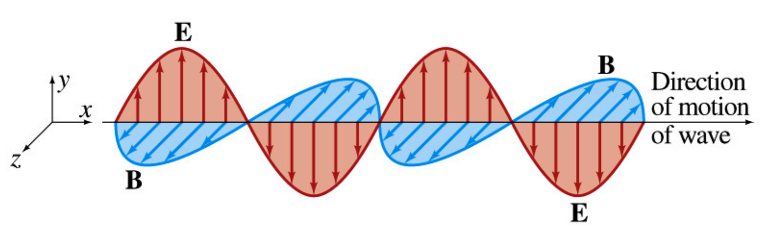
$$E = E_y = E_0 \sin(kx - \omega t)$$

$$B = B_z = B_0 \sin(kx - \omega t)$$

where

$$k = \frac{2\pi}{\lambda}, \omega = 2\pi f, c = \frac{\omega}{k} = \lambda f$$

- E and B are in phase.
- The directions of E and B and wave travel form a right-hand-rule.



Relation between magnitudes of E and B

Faraday’s law in differential form

$$\frac{\partial E}{\partial x}=-\frac{\partial B}{\partial t}$$

Apply to plane-wave solution

$$E=E_y=E_0\sin(kx-\omega t)$$

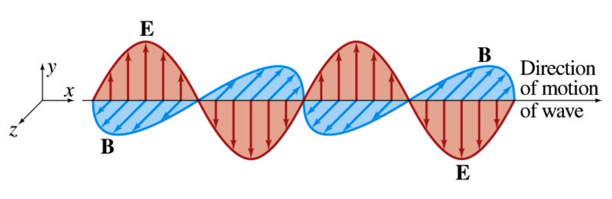
$$B=B_z=B_0\sin(kx-\omega t)$$

$$E_0k\cos(kx-\omega t)=B_0\omega\cos(kx-\omega t)$$

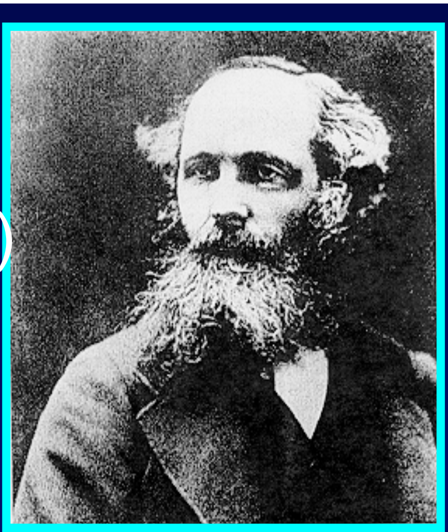
or $E_0k=B_0\omega$

But ω/k=c and E and B in phase, so

$$\frac{E}{B}=c$$

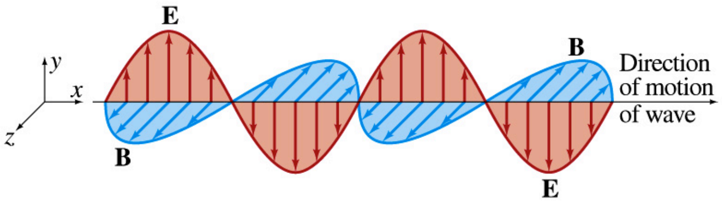


Maxwell's Predictions



J. C. Maxwell (1831 - 1879)

- There exist electromagnetic waves (EM waves) that can travel in vacuum
- EM waves travel at the speed of light
  - $$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8 \text{ m/s}$$
  - E and B are in phase (E / B = c)
  - $\vec{E} \times \vec{B} =$ direction of wave travel (right - hand - rule)
- Light is an EM wave

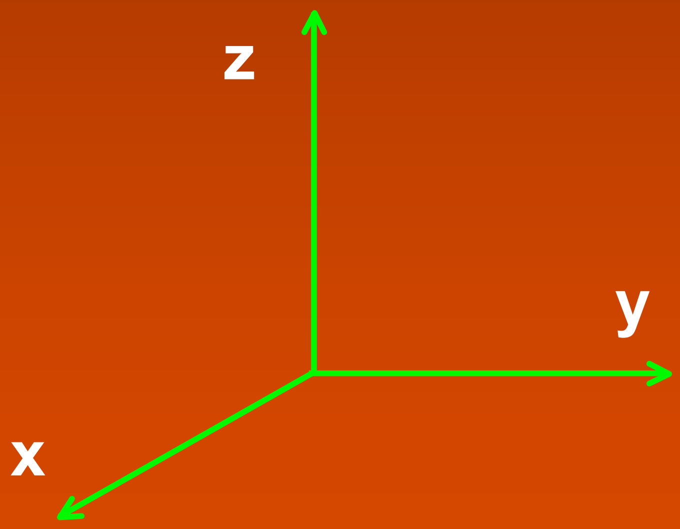


Physics that changed the world: telegraph, radio, television, cell-phone, satellite, electric power, ….

ConcepTest 32.2 EM Wave

- An electromagnetic wave with its electric field in the positive y direction propagates in the negative z direction. What is the direction of the magnetic field?

1) +x
2) -y
3) -x
4) +z
5) -z



Electromagnetic Spectrum

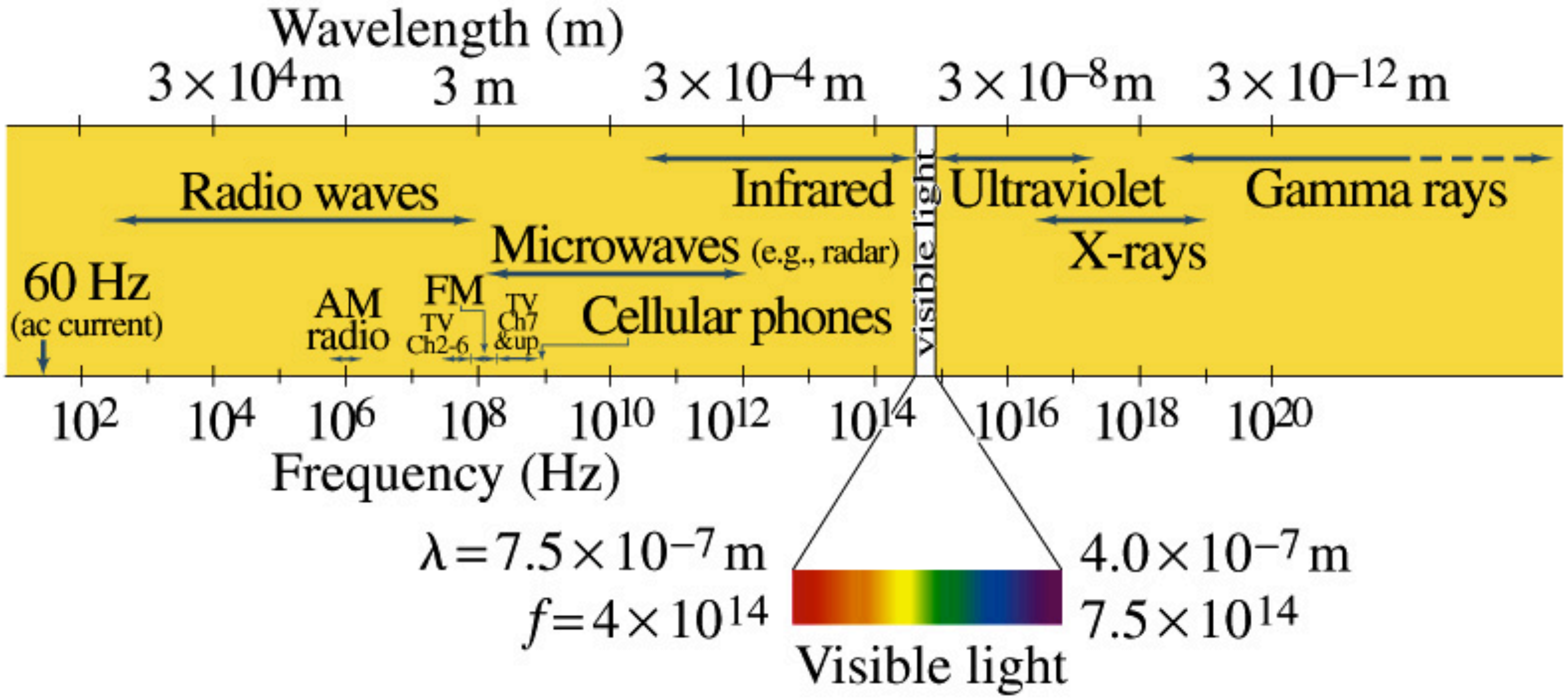
New Topic

A bit of history

- Long before J.C. Maxwell (1831-1879) predicted EM waves that can travel in vacuum, light was known to behave like a wave, but what kind of wave was it?
  - What's oscillating in a light wave?
  - It's E and B fields according to Maxwell.
- Eight years after Maxwell's death, H. Hertz (1857-1894) first generated and detected EM waves in his laboratory.
  - Spark-gap for generation, wire loop for detection
  - 10^9 Hz EM waves, but not visible to the eye.
  - Strong confirmation of Maxwell's theory.
- The wavelengths of visible light were measured long before anyone imagined that light was an EM wave.
  - 400 nm to 750 nm, or by $c = \lambda f$
  - $4.0\text{x}10^{14}$ Hz to $7.5\text{x}10^{14}$ Hz
  - But visible light is not the only EM waves

EM waves exist at all frequencies: the electromagnetic spectrum

The electromagnetic spectrum



$$c = \lambda f$$

ConcepTest 32.3 EM waves

- **Since Superman is from the planet Krypton his eyes are sensitive to the entire electromagnetic spectrum. Does that mean he can use x-ray vision to see that Lois Lane is being kidnapped in the other room?**

(1) Yes, no problem

(2) Nope, he can't

(3) Need more information

Energy in EM Waves

New Topic

Energy in EM Waves

EM waves carry energy from one region of space to another. The energy density

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0} = \varepsilon_0 E^2 = \frac{B^2}{\mu_0} = \sqrt{\frac{\varepsilon_0}{\mu_0}}EB$$

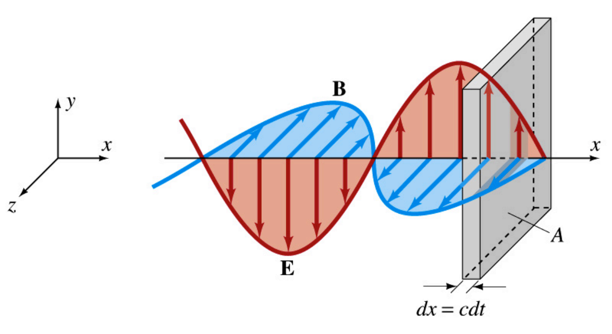
is shared equally between E and B fields.

Introduce the Poynting vector: the energy transported by the EM wave per unit time per unit area (W/m²).

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

which also defines the direction of the wave.

$$\overline{S} = \frac{1}{2}\varepsilon_0 c E_0^{\,2} = \frac{cB_0^{\,2}}{2\mu_0} = \frac{E_0 B_0}{2\mu_0} = \frac{E_{rms}B_{rms}}{\mu_0}$$



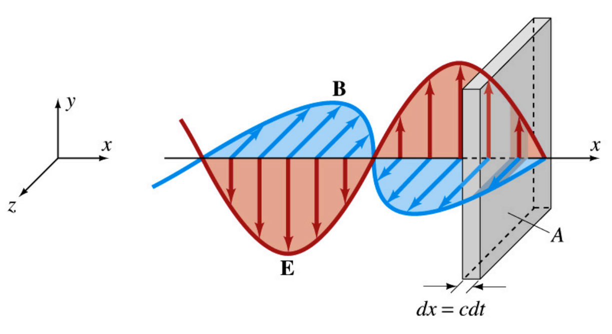
Recall intensity: $\overline{S} = \frac{\text{Power}}{4\pi r^2}$

Radiation Pressure

When an EM wave encounters the surface of a material, it exerts pressure on the surface.

Absorbed $P = \dfrac{\bar{S}}{c}$

Reflected $P = \dfrac{2\bar{S}}{c}$



For example: Radiation from the sun that reaches the Earth's surface transports energy at a rate of about 1000 W/m^2.. So the pressure

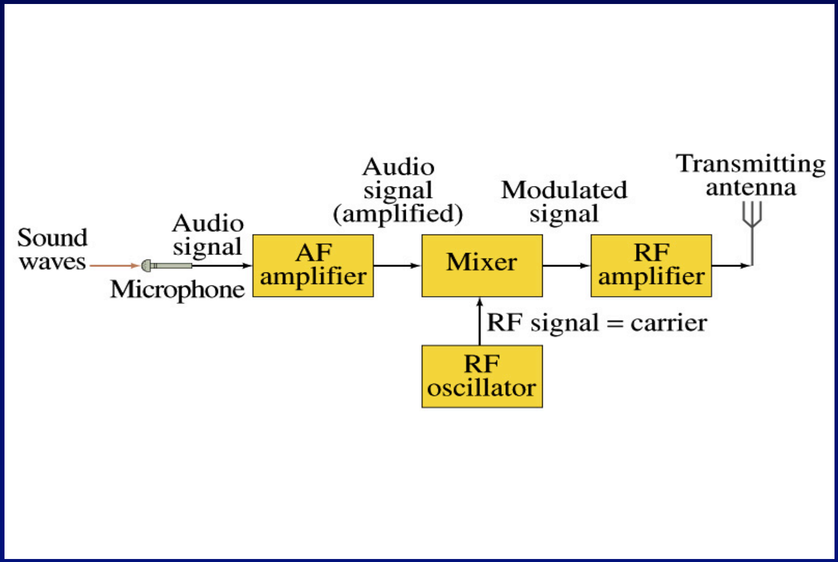
$$P \approx \frac{\bar{S}}{c} = \frac{1000}{3\times10^{8}} = 3.3\times10^{-6}\ \mathrm{N/m^2}$$

which is a force on your hand of

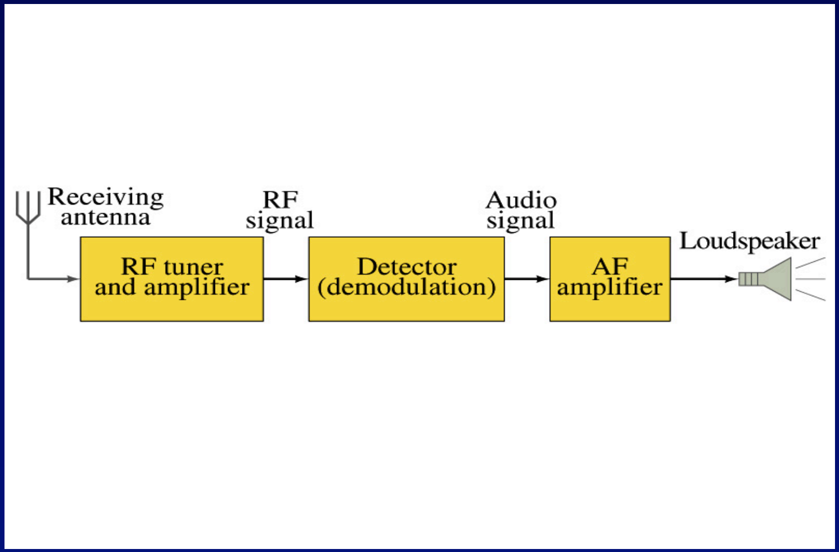
$$F = PA = 3.3\times10^{-6}\times0.02 = 6.6\times10^{-8}\,\mathrm{N}$$

How do Radio and Television Work?

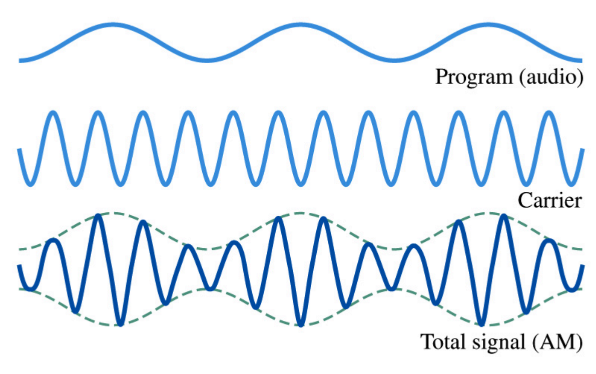
transmitter



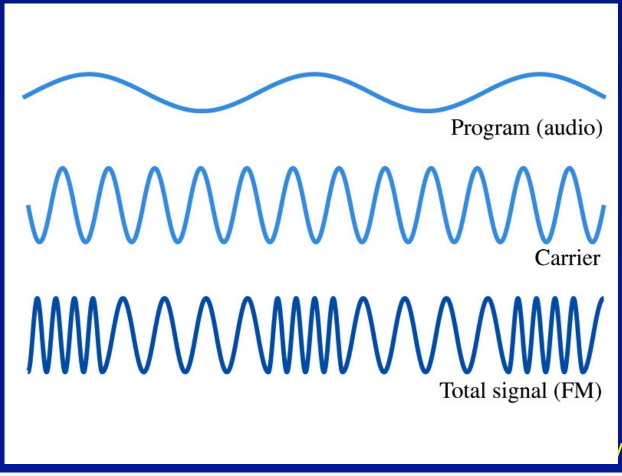
receiver



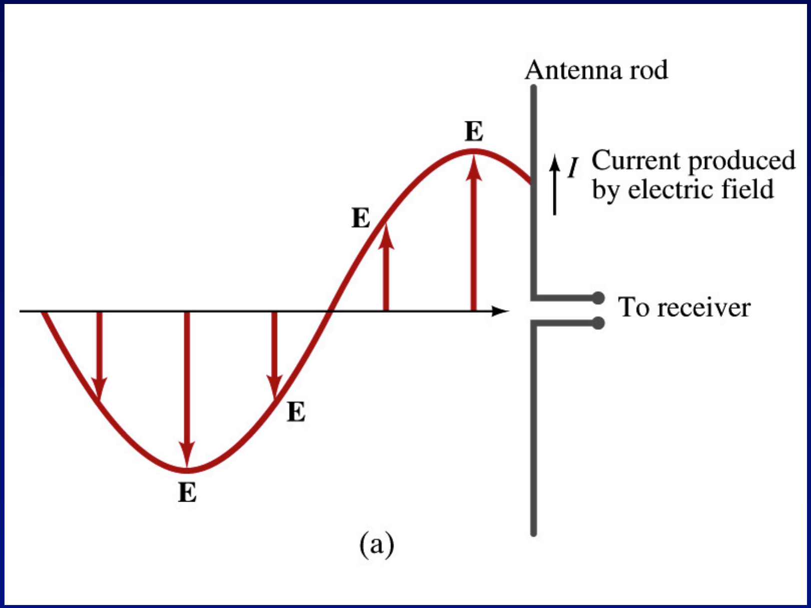
AM (amplitude modulation)
530 kHz to 1600 kHz



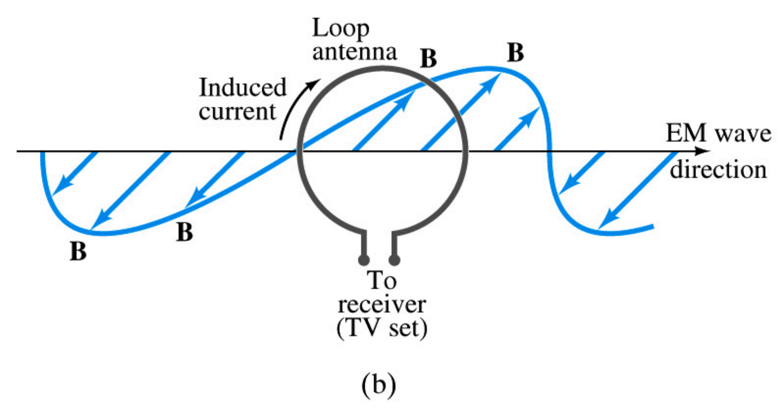
FM (frequency modulation)
88 MHz to 108 Mhz



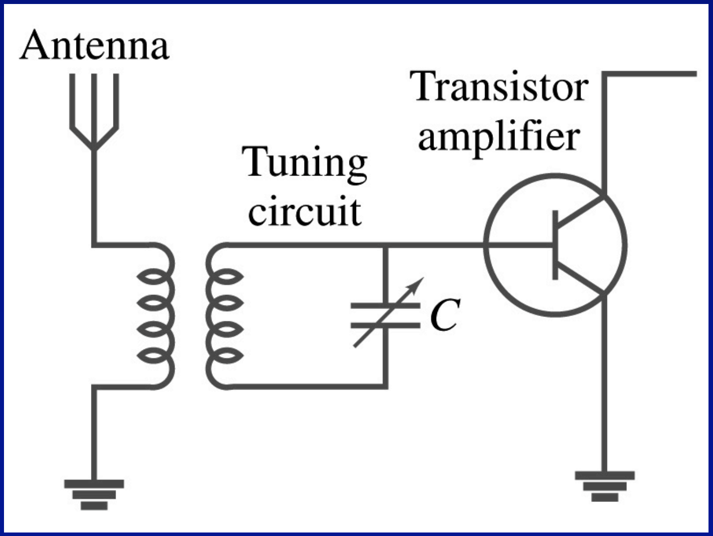
Antennas


Antenna rod
E
E
E
E
E
I
Current produced by electric field
To receiver

(a)


Loop antenna
B
B
Induced current
EM wave direction
B
B
To receiver (TV set)

(b)


Antenna
Transistor amplifier
Tuning circuit
C

ConcepTest 32.4 EM Wave

- **Which gives the largest average intensity at the distance specified and thus, at least qualitatively, the best illumination?**

1) a 50-W source at a distance R
2) a 100-W source at a distance 2R
3) a 200-W source at a distance 4R