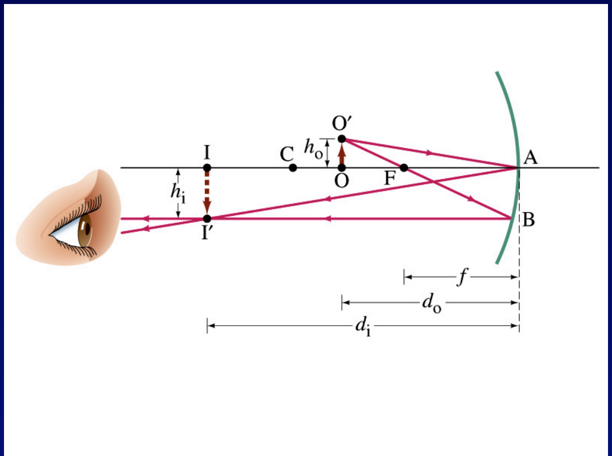
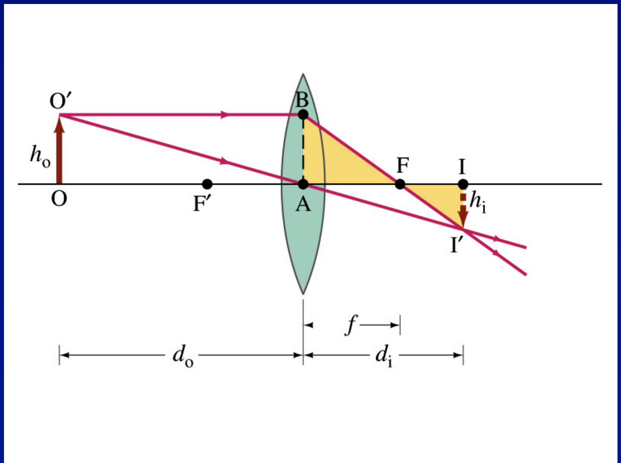
Chapter 34: Geometrical Optics

- **Mirrors**
  - Plane
  - Spherical (convex or concave)
- **Lenses**
  - The lens equation
  - Lensmaker's equation
  - Combination of lenses

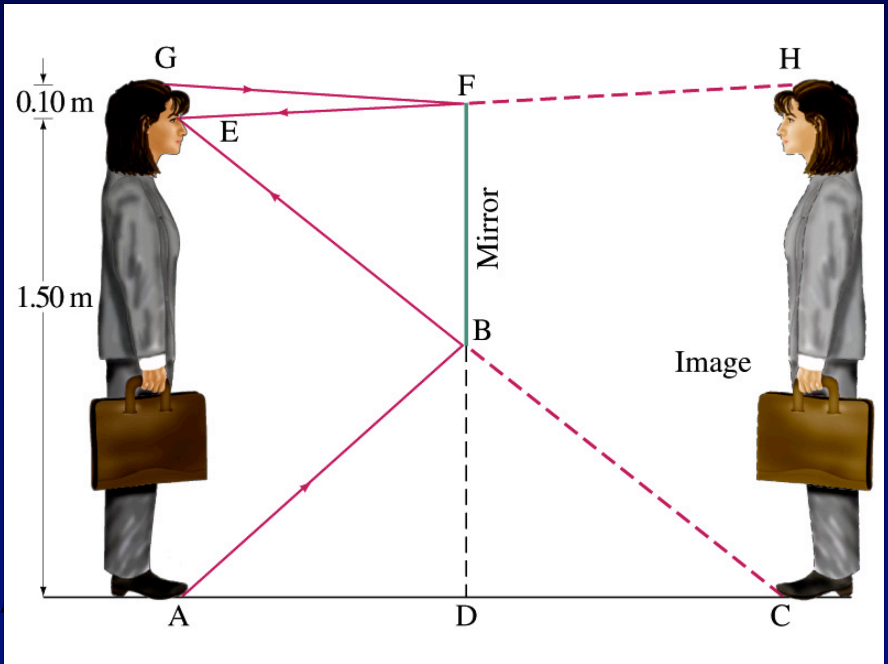




Mirrors

New Topic

Looking in the mirror



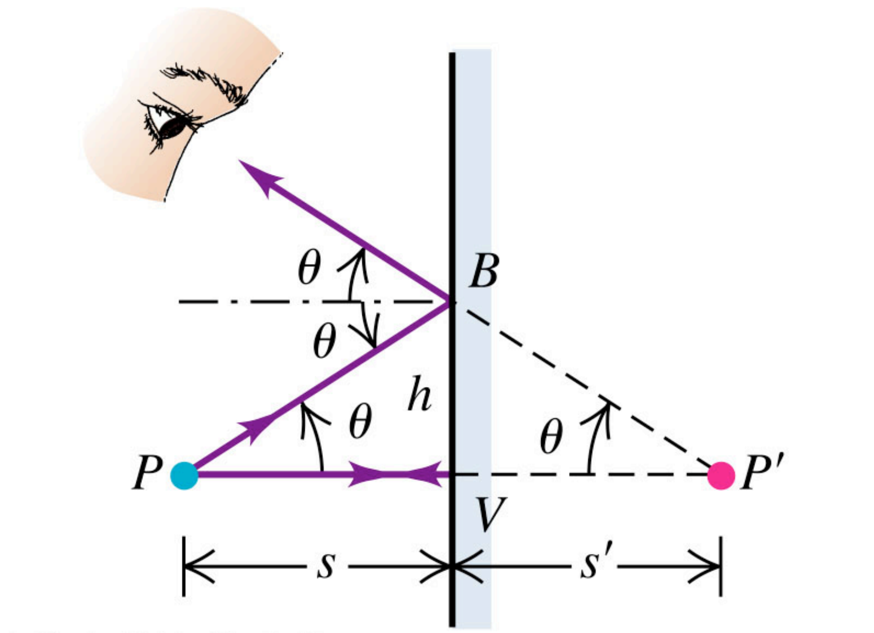
The image by a plane mirror is virtual, upright, equal size, equal distance, and with left and right reversed.

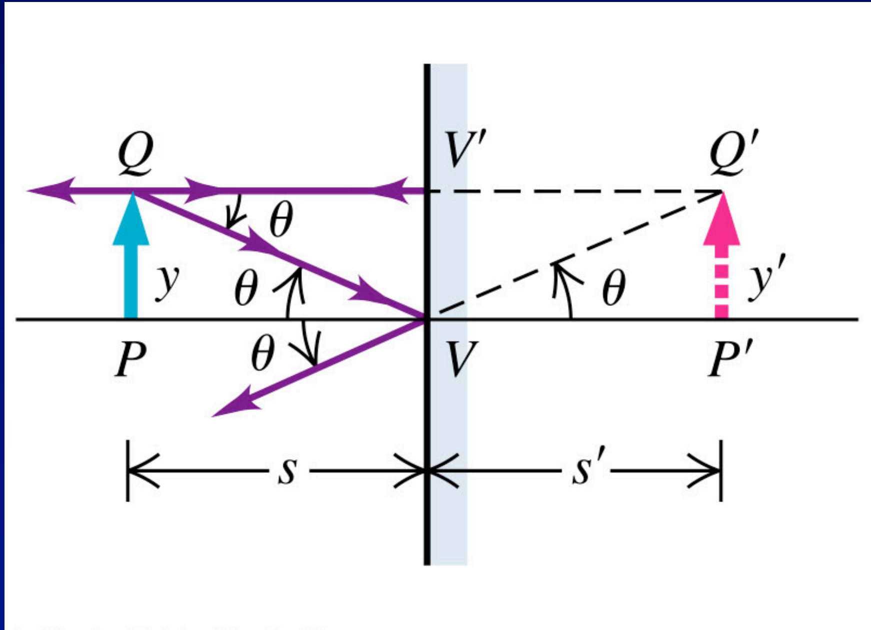
How to find the image?
Ray diagram (or ray tracing)

Click here for AP15.4 simulation

How to find an image?

- Trace rays from object to mirror to eye.
  - Brain assumes that light travels in a straight line !
  - Extend rays back behind mirror to form image.



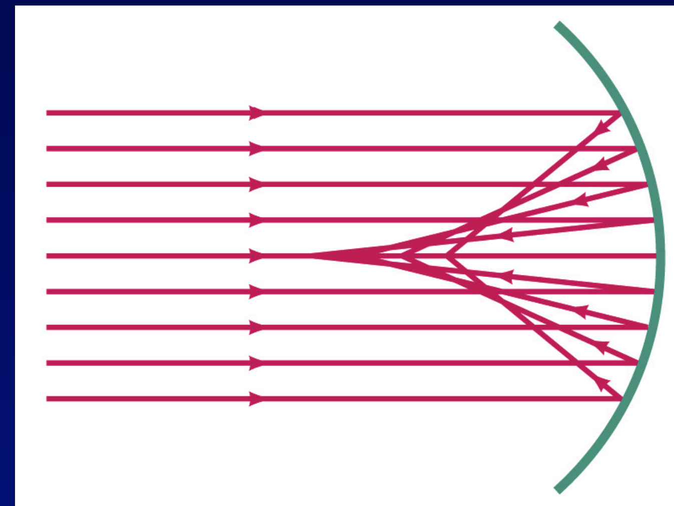


This is a "virtual" image since it does not actually exist behind the mirror.

The image by a plane mirror is virtual, upright, equal size, equal distance, and with left and right reversed.

Spherical Mirrors

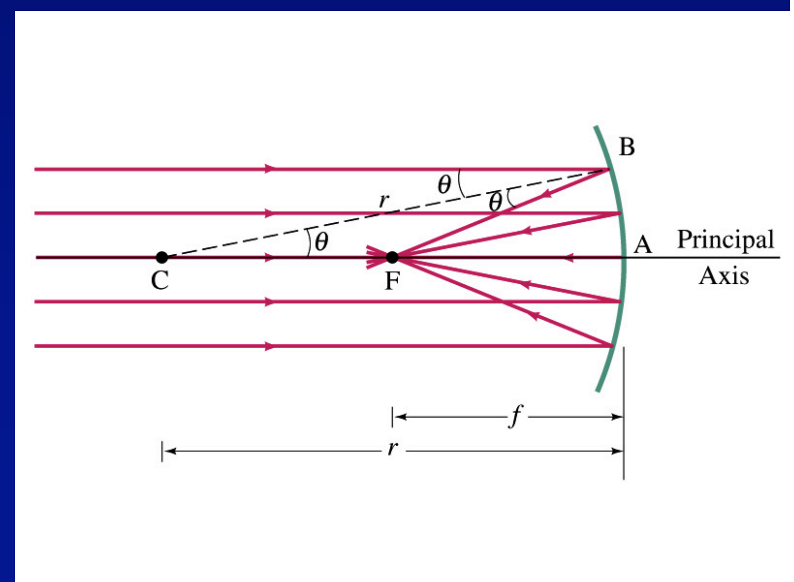
In general spherical mirrors do not focus at a single point. (aberration)



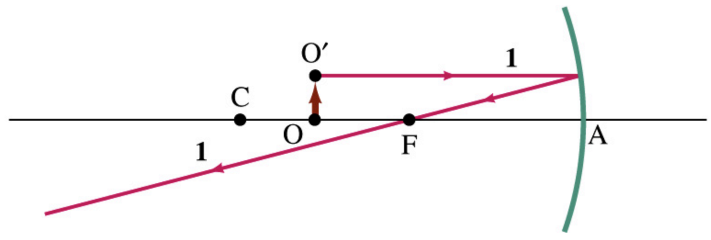
(Parabolic mirrors do have a perfect focus.)

Paraxial approximation (small angle approximation): mirror is small compared to its radius of curvature so a single focal point exists. The focal length is

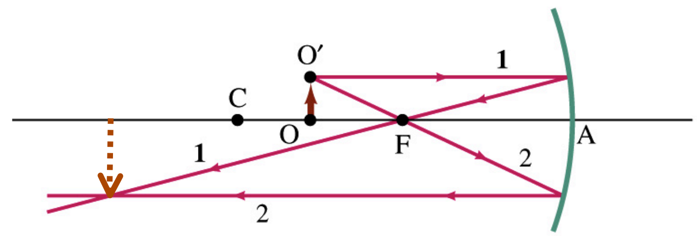
$$\boxed{f = \frac{r}{2}}$$



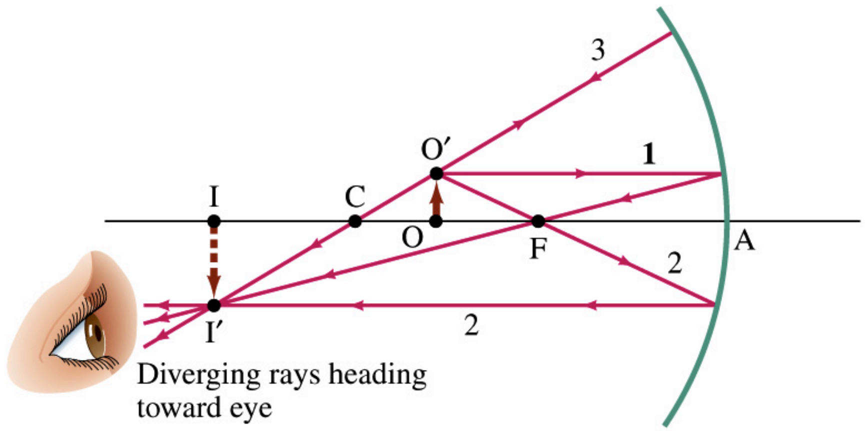
Concave Spherical Mirrors: image formation



(a) Ray 1 goes out from O′ parallel to the axis and reflects through F.



(b) Ray 2 goes through F and then reflects back parallel to the axis.

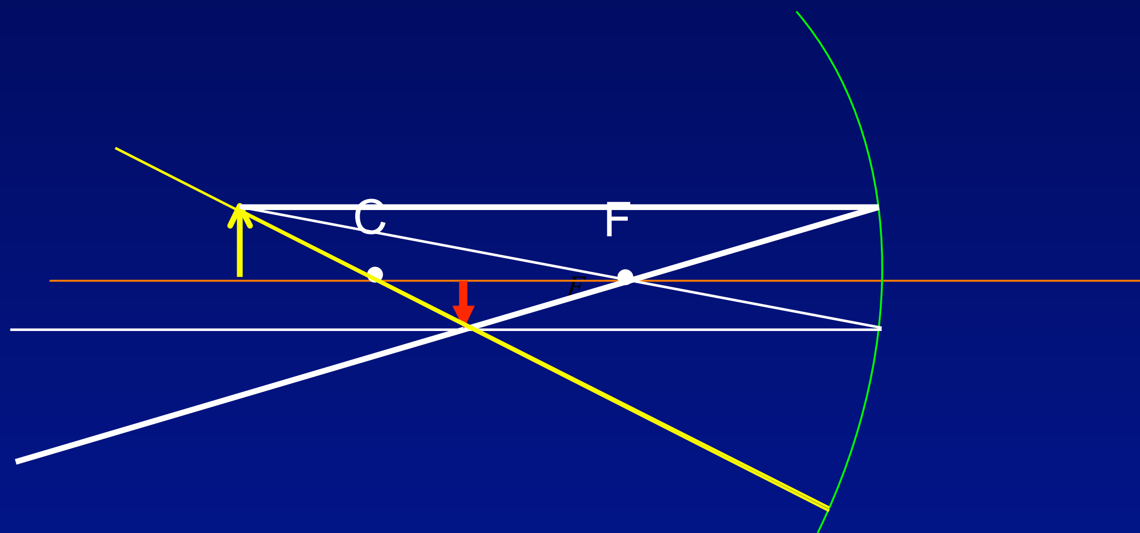


(c) Ray 3 heads out perpendicular to mirror and then reflects back on itself and goes through C (center of curvature)

If object is in between center and focal point, the image is real (light passes though), inverted, and enlarged.

Example: Ray Tracing

- The object is placed beyond the center point. What is the image?



The image is real, inverted, reduced.

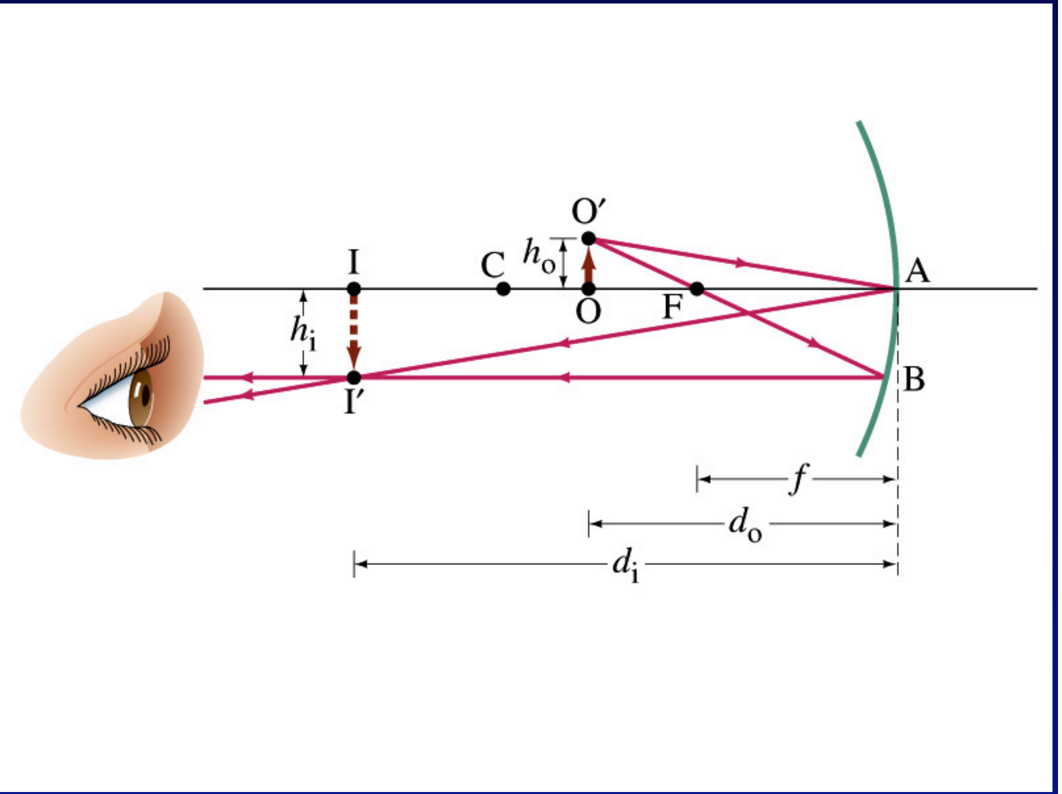
Spherical Mirrors: calculations

Mirror equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Lateral magnification:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$



Sign conventions:

h_i is positive if upright, negative if inverted.

d_o and d_i are positive if on the reflecting side, negative if on the other side.

Summary: Concave Mirrors
(assume f = 2 cm, then r = 4 cm)

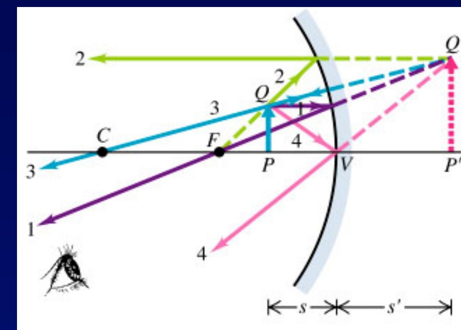
$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$$

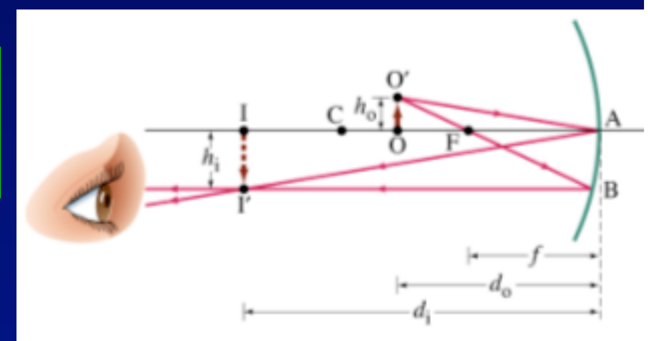
$$m=\frac{h_i}{h_o}=-\frac{d_i}{d_o}$$

Object: inside focal point (d_o=1.5 cm)
Image: virtual, upright, enlarged

$$\frac{1}{1.5}+\frac{1}{d_i}=\frac{1}{2},\quad d_i=-6\text{ cm}$$

$$m=-\frac{-6}{1.5}=4$$





Object: in between center and focal point (d_o=3 cm)
Image: real, inverted, enlarged

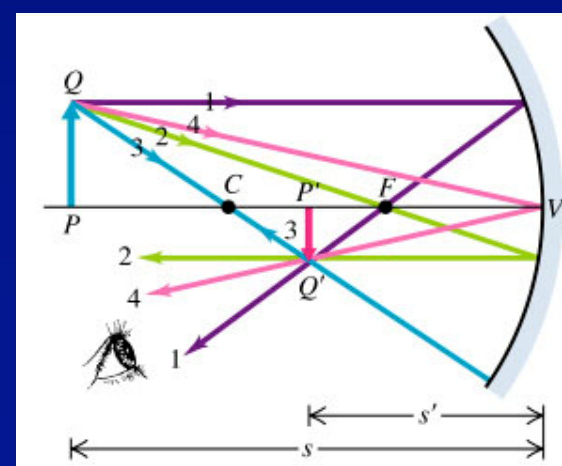
$$\frac{1}{3}+\frac{1}{d_i}=\frac{1}{2},\quad d_i=6\text{ cm}$$

$$m=-\frac{6}{3}=-2$$

Object: outside center point (d_o=6 cm)
Image: real, inverted, reduced

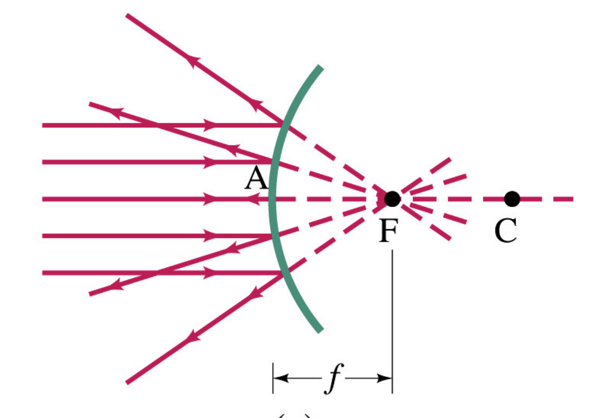
$$\frac{1}{6}+\frac{1}{d_i}=\frac{1}{2},\quad d_i=3\text{ cm}$$

$$m=-\frac{3}{6}=-0.5$$



Convex Spherical Mirrors

Convex mirrors produce virtual, upright and smaller images.



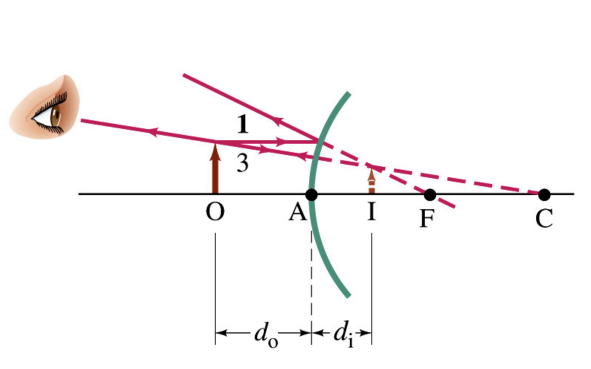
(a)

Mirror equation still holds, but f is negative.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$$

Magnification equation also holds

$$m=\frac{h_i}{h_o}=-\frac{d_i}{d_o}$$



(b)

For example, a convex rearview car mirror has curvature r =16 m. The image of an object 10 m away from the mirror is

$$\frac{1}{10}+\frac{1}{d_i}=\frac{1}{-8}, \text{ so } d_i=-4.4m$$

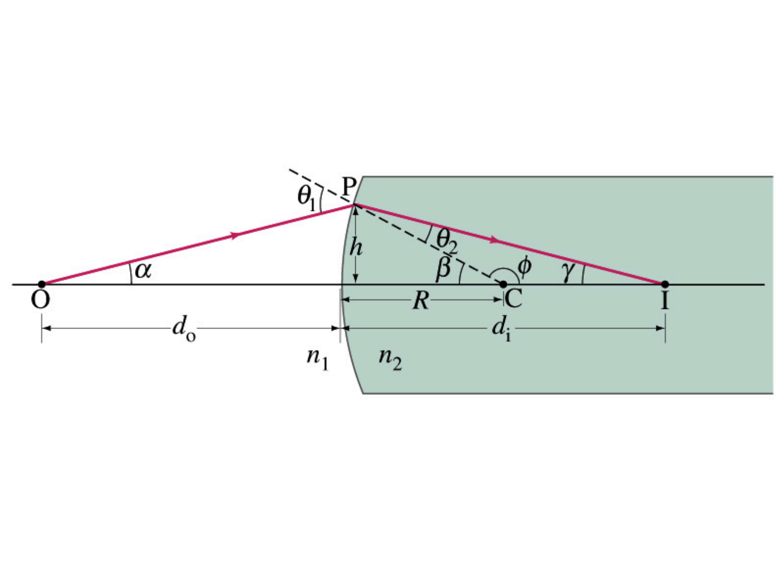
$$m=-\frac{d_i}{d_o}=-\frac{-4.4}{10}=0.44$$

Warning: objects in mirror may be closer than they appear.

Refraction at a spherical surface

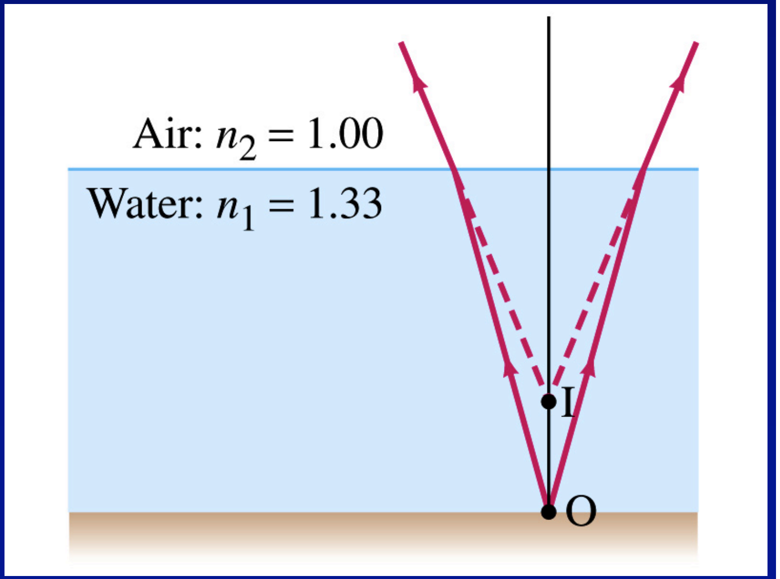
Convex surface

$$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}$$



Equation also holds for concave surface, but R is negative

For example, A person looks vertically down into a 1-m deep pool. How deep does the pool appear to be?



$$\frac{1.33}{1.0} + \frac{1}{d_i} = \frac{1 - 1.33}{\infty}$$

$$d_i = -0.75 \text{ m}$$

ConcepTest 34.1 Plane mirror

- **If you walk directly towards a plane mirror at a speed v, your image will move**

1) towards you with v

2) towards you with 2v

3) away from you with v

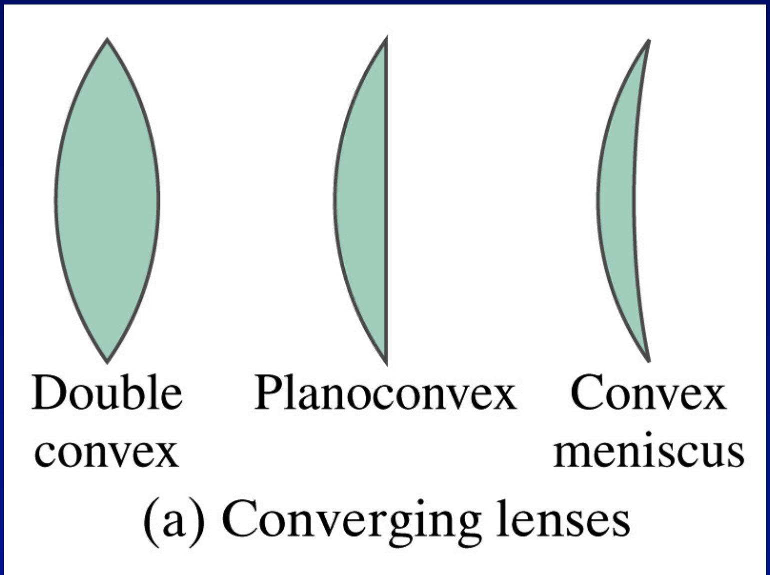
4) away from you with 2v

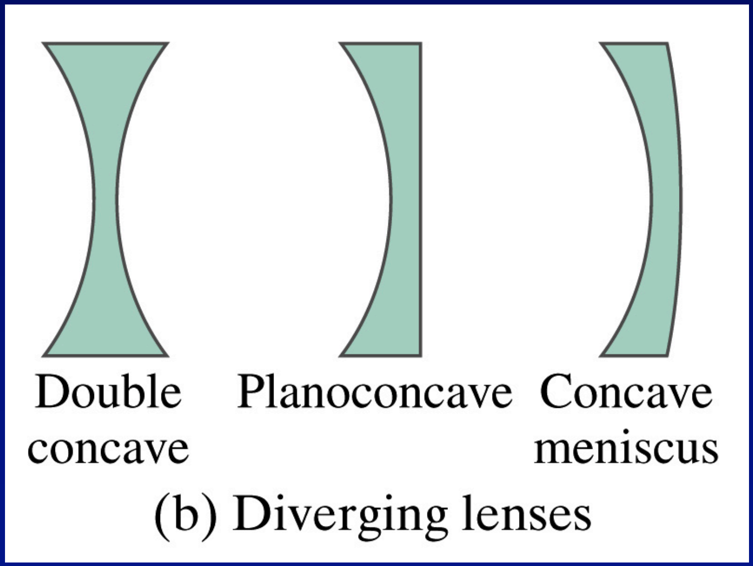
Lenses

New Topic

Thin Lenses

'Thin' in the sense that the thickness is much smaller than the diameter.

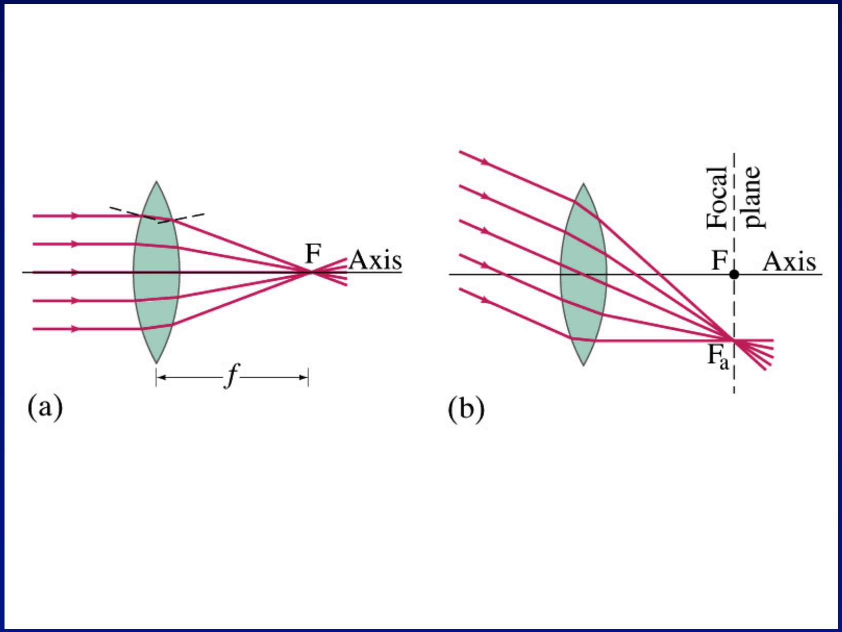


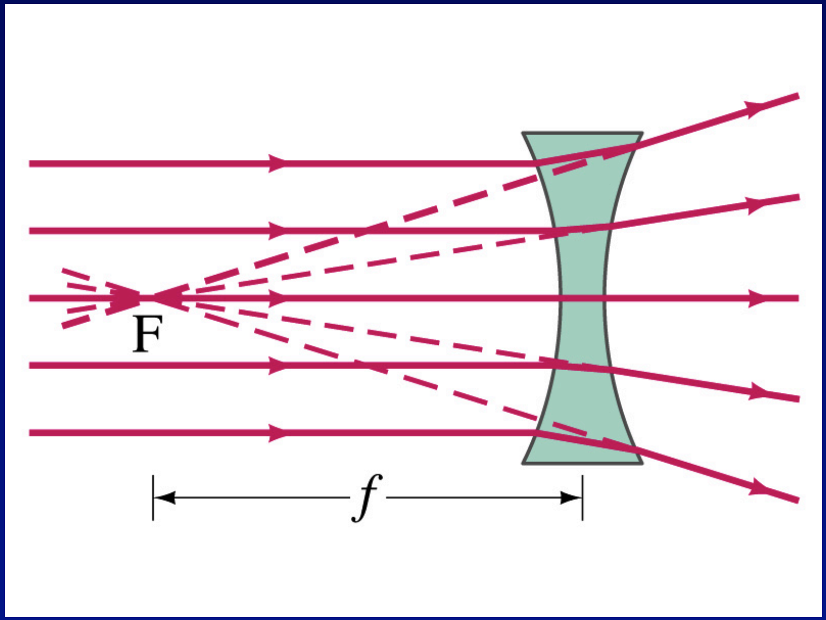


They form the basis of optical instruments: eyeglasses, cameras, telescopes, …

Thin Lenses

Focal point, focal length, focal plane.

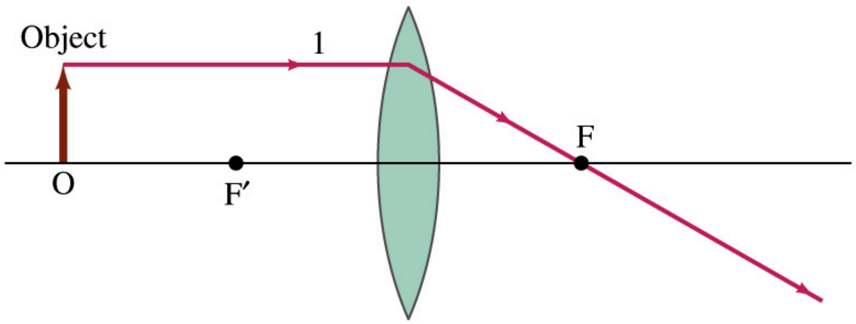




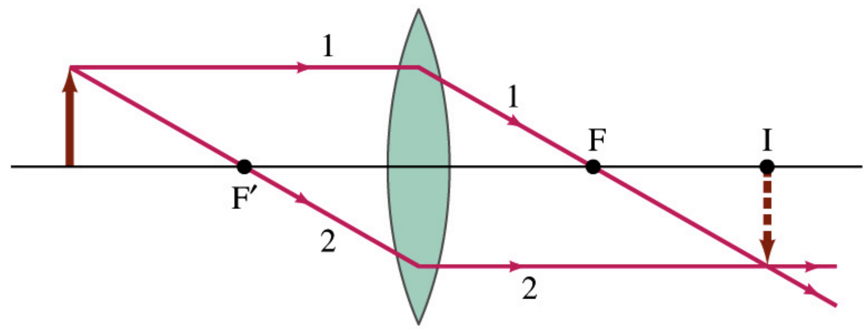
Optometrists like to use the reciprocal of the focus length, called power, $P = 1/f$. Unit is diopter (D). 1D = 1 m^{-1}.

For example, a 20-cm-focal-length converging lens has a power of P = 1/0.2 = 5.0 D. If it's diverging, P = -5.0 D

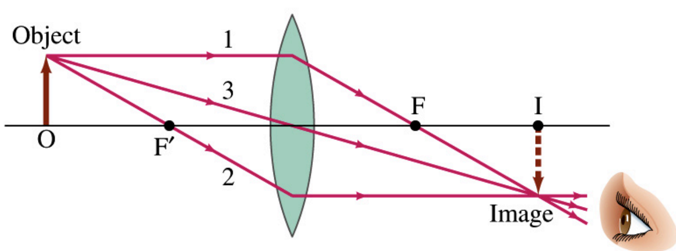
Thin Lenses: Ray Diagrams



(a) Ray 1 leaves top point on object going parallel to the axis, then refracts through focal point.



(b) Ray 2 passes through F′; therefore it is parallel to the axis beyond the lens.

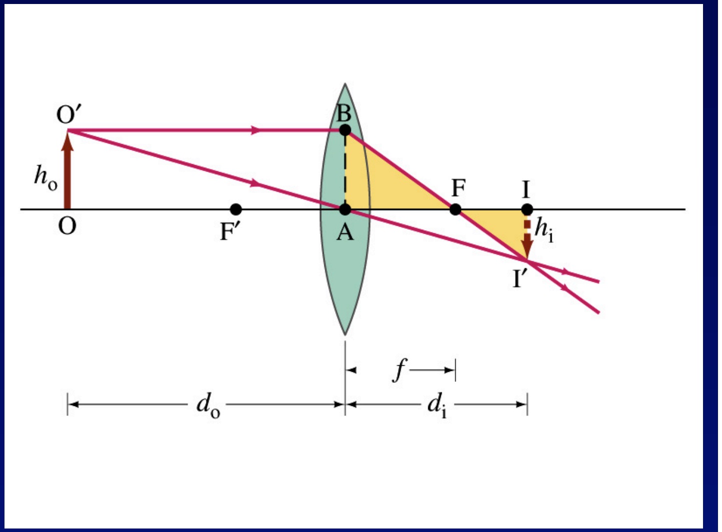


(c) Ray 3 passes straight through the center of the lens (assumed very thin).

The Lens Equation

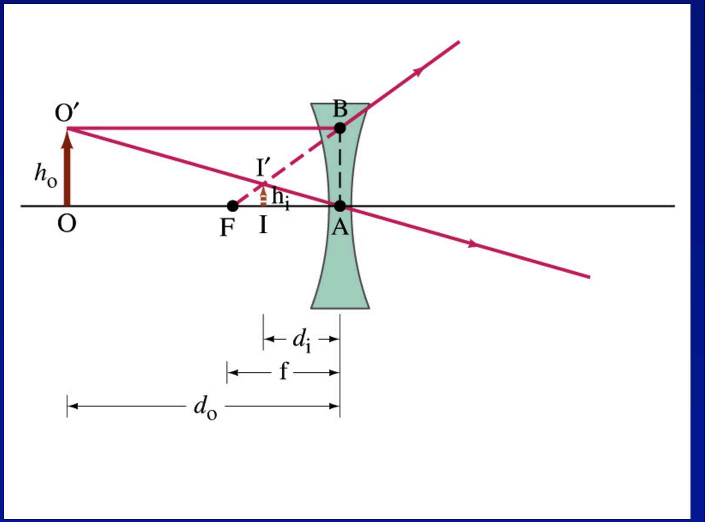
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$



Sign convention:

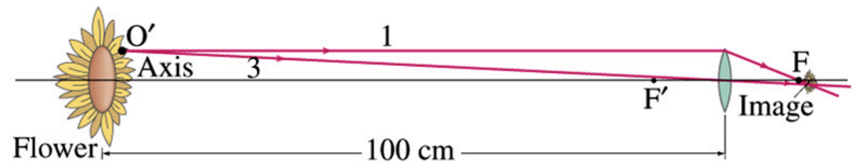
1) f is positive for converging lens, negative for diverging lens.
2) d_o is positive if it is on the same side of incident light, negative otherwise.
3) d_i is positive if it is on the opposite side of incident light, negative otherwise
4) h_i (or m) is positive if the image is upright and negative if inverted, relative to the object. (h_o is always taken as positive)



Click here for thin lens simulation AP15.9

Example: What is the position and size of the image of a 7.6-cm high flower placed 1 m from a +50-mm-focal-length camera lens?

$$\frac{1}{100}+\frac{1}{d_i}=\frac{1}{5}$$



$$d_i = 5.26\ cm$$

$$m=\frac{h_i}{h_o}=-\frac{d_i}{d_o}=-\frac{5.26}{100}=-0.0526$$

$$h_i = mh_o = -0.0526\times 7.6 = -0.4\ \text{cm}$$

Real, inverted, reduced

More Examples: Converging Lens (assume f = 20 cm)

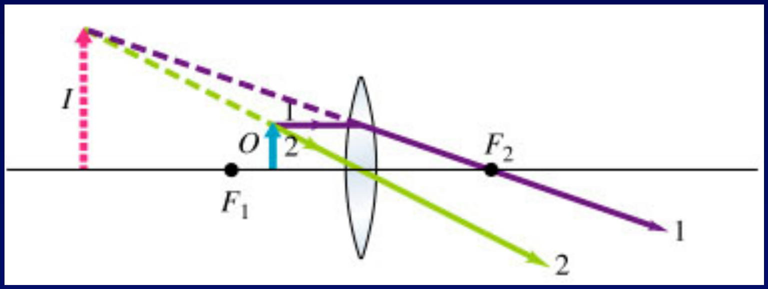
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Object: inside focal point (d_o=15 cm)
Image: virtual, upright, enlarged

$$\frac{1}{15} + \frac{1}{d_i} = \frac{1}{20}, \quad d_i = -60 \text{ cm}$$

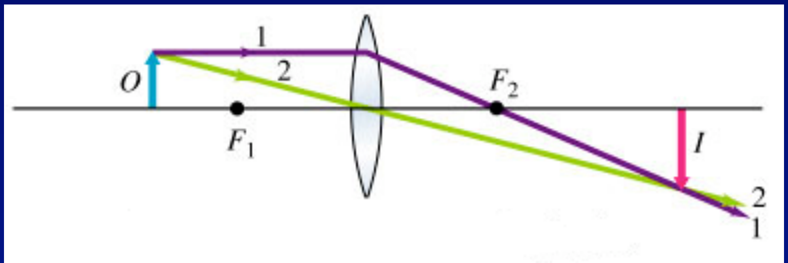
$$m = -\frac{-60}{15} = 4$$



Object: outside focal point (d_o=30 cm)
Image: real, inverted, enlarged

$$\frac{1}{30} + \frac{1}{d_i} = \frac{1}{20}, \quad d_i = 60 \text{ cm}$$

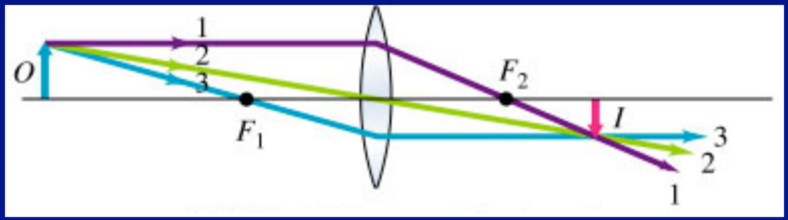
$$m = -\frac{60}{30} = -2$$



Object: outside focal point (d_o=60 cm)
Image: real, inverted, reduced

$$\frac{1}{60} + \frac{1}{d_i} = \frac{1}{20}, \quad d_i = 30 \text{ cm}$$

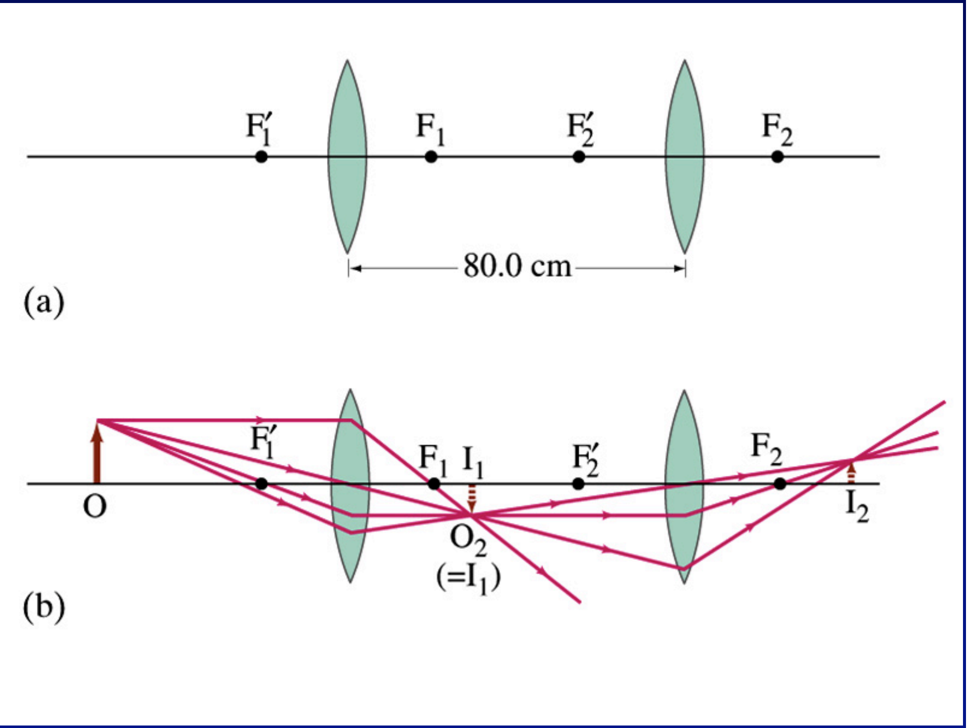
$$m = -\frac{30}{60} = -0.5$$



Combination of Lenses

Basic idea: the image formed by the 1st lens is the object for the 2nd lens, and so on. (Watch out for signs!)

Example: f_1=20 cm, f_2=25 cm, object is placed 60 cm in front.



$$\frac{1}{60}+\frac{1}{d_{i1}}=\frac{1}{20}, \quad d_{i1}=30\text{ cm}$$

$$\frac{1}{80-30}+\frac{1}{d_{i2}}=\frac{1}{25}, \quad d_{i2}=50\text{ cm}$$

$$m_1=-\frac{d_{i1}}{d_{o1}}=-\frac{30}{60}=-0.5$$

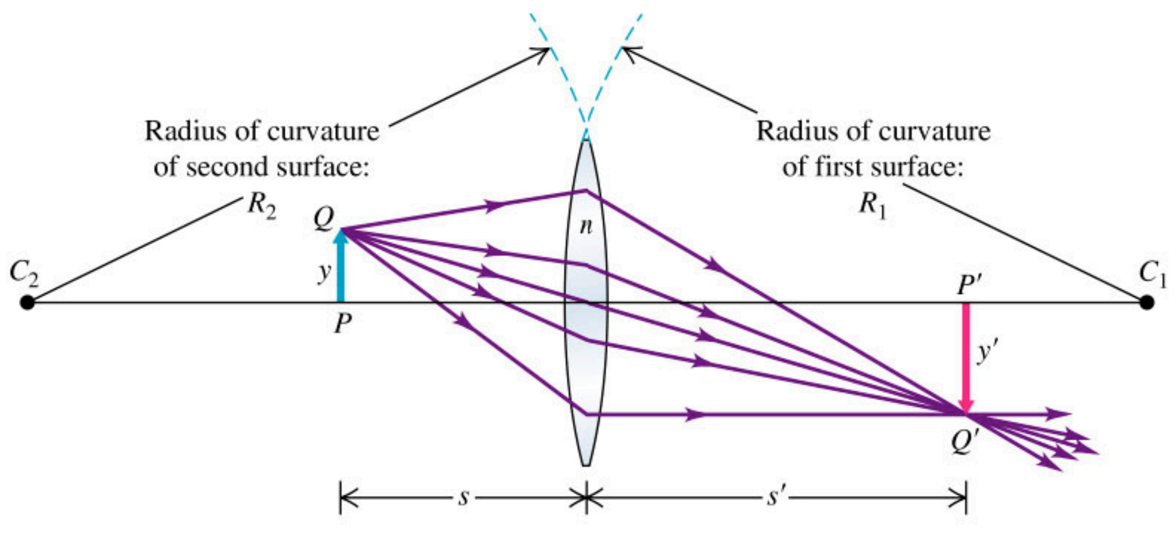
$$m_2=-\frac{d_{i2}}{d_{o2}}=-\frac{50}{50}=-1$$

$$m=m_1m_2=(-0.5)\times(-1)=0.5$$

Lensmaker's Equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Symmetric in R_1 and R_2 : so f is the same both ways.



Example: double-convex, n=1.52, absolute radius 10 cm for both surfaces, then the focal length is

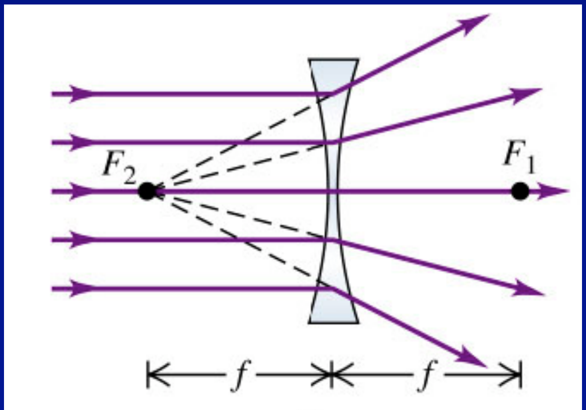
$$\frac{1}{f} = (1.52-1)\left(\frac{1}{10} - \frac{1}{-10}\right)$$

$$f = 9.6 \text{ cm}$$

Example: double-concave, n=1.52, absolute radius 10 cm for both surfaces,

$$\frac{1}{f} = (1.52-1)\left(\frac{1}{-10} - \frac{1}{10}\right)$$

$$f = -9.6 \text{ cm}$$



ConcepTest 34.2 Lens

- A light source is held near a lens and its light focuses at point B, as shown in the top figure. If the light source is moved closer to the lens, then at which of the indicated points will the light focus?