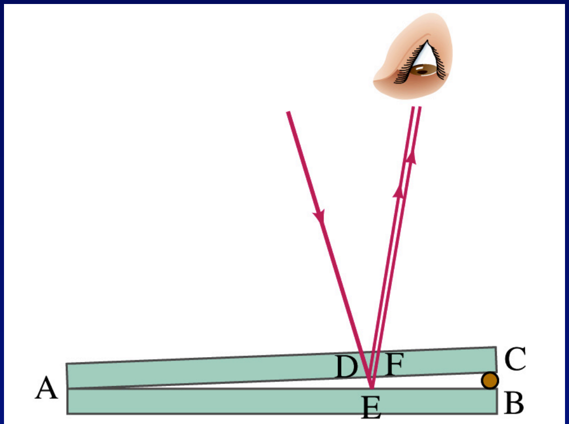
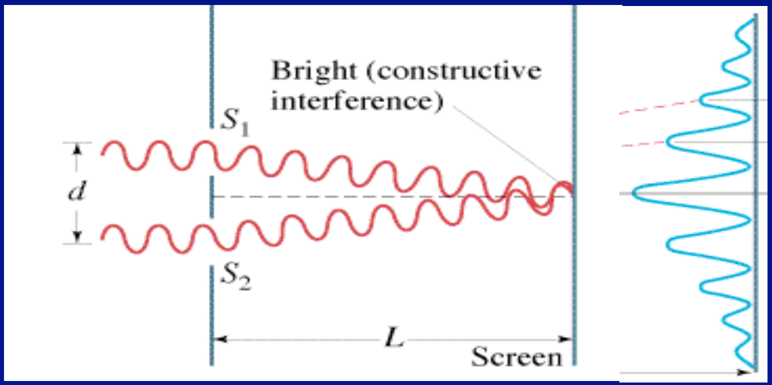
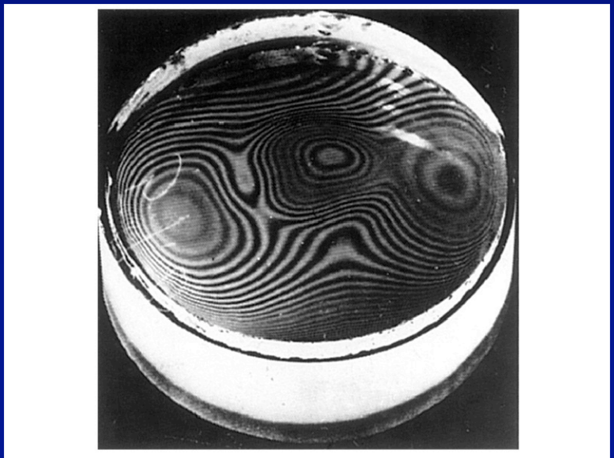
Chapter 35: Interference

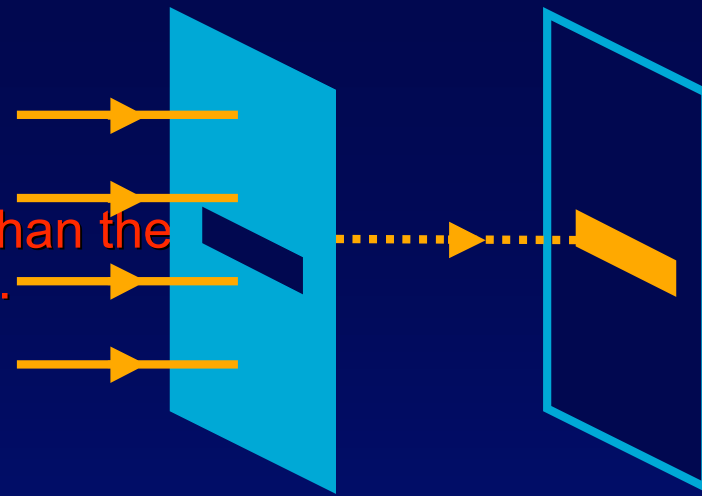
- Some properties of waves:
  - Huygens' principle
  - Superposition
  - Coherence
- Interference
  - Young's **double-slit** experiment
  - Thin-film interference







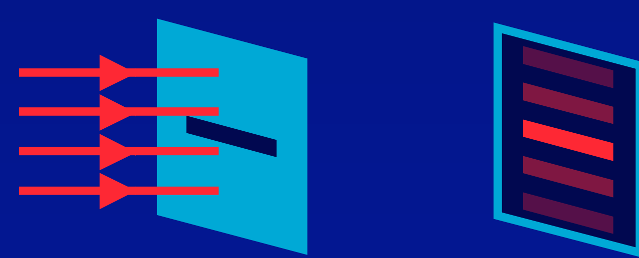
Geometrical Optics



- Assumption: the dimensions are much larger than the wavelength of the light waves (400 to 700 nm).
  - light follows straight-line paths (rays)
- Changes occur when a ray hits a boundary
  - ray may bounce off (reflection)
  - ray may bend into the other medium (refraction)
  - ray may be absorbed (light energy ⇒ thermal energy)

Physical Optics

- Assumption: the dimensions are comparable to the wavelength of the light waves.
  - light must be considered as waves
- Waves exhibit
  - interference
  - diffraction

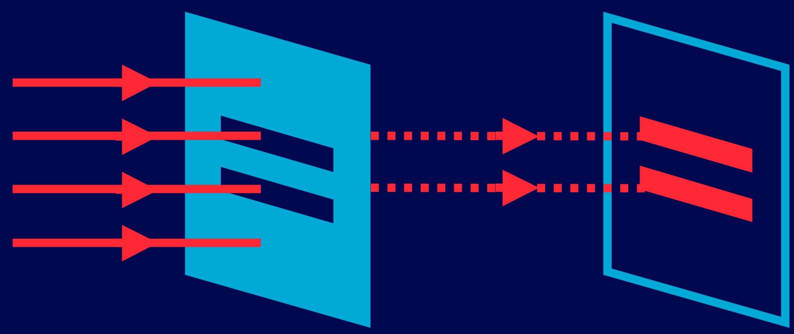


Interference

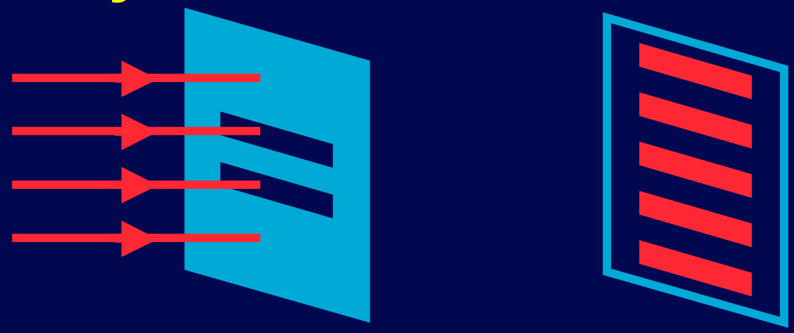
New Topic

Question: Suppose light falls onto a screen with two slits. What would you see on the wall behind the screen?

You would see two bright lines on the wall for large slits:



You would see many lines if the slits are small:

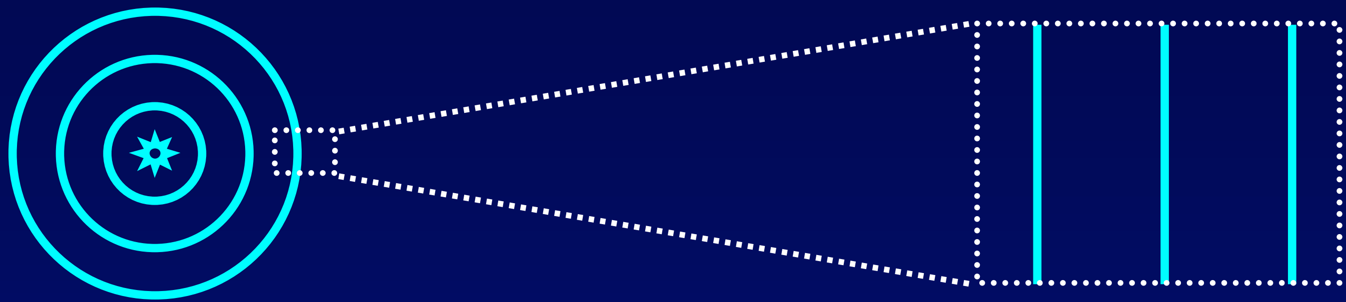


To understand this, we must understand these principles about waves

- **Huygens' Principle**
- **Superposition**
- **Coherence**

Recall that a point source of light emits a spherical wave

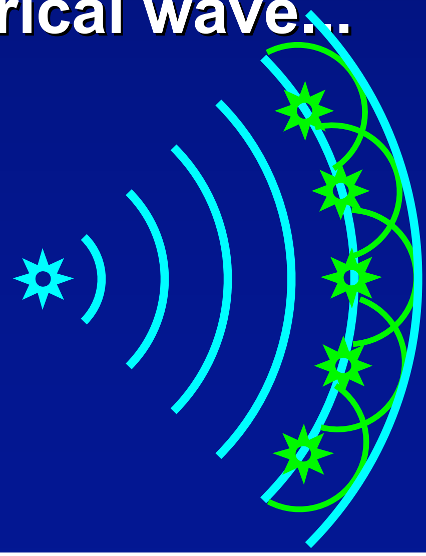
…and that far from the source, the wave is a plane wave



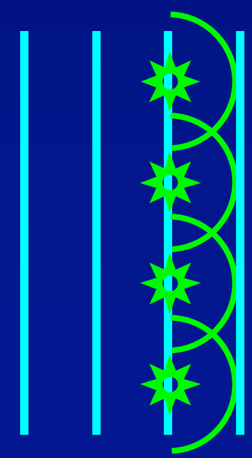
Huygens' Principle

All points on a wave front serve as point sources of spherical waves

Apply Huygens' Principle to a spherical wave...

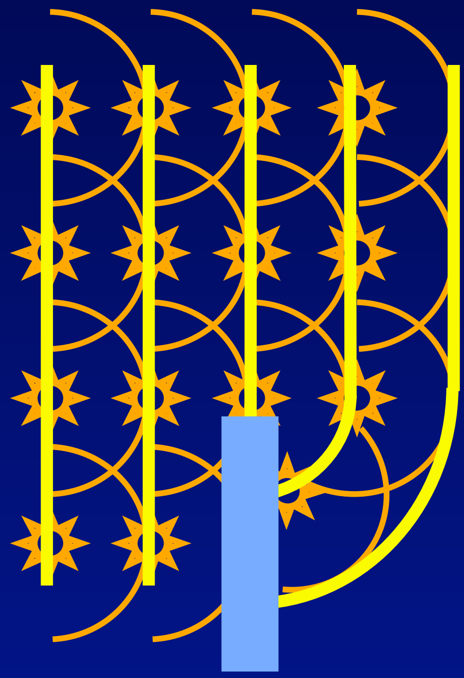


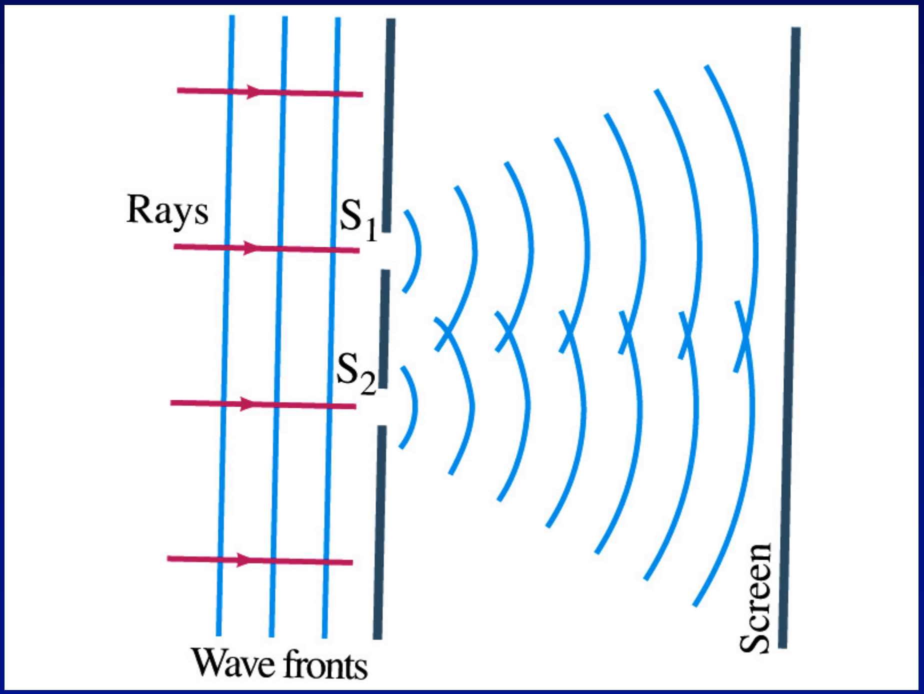
This also works for plane waves...



So What?

Waves can *bend* around corners!





This is a characteristic of *all* waves:

EM waves
sound waves
water waves

The Principle of Superposition

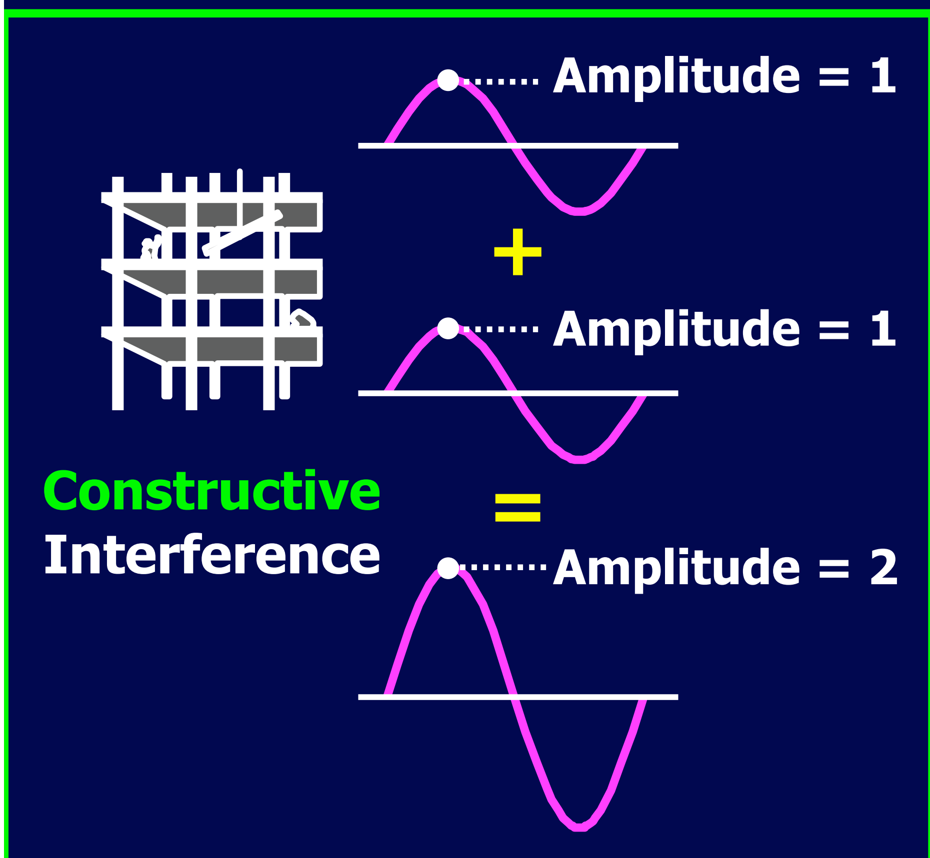
What happens when two particles are in the same place at the same time?

They collide!

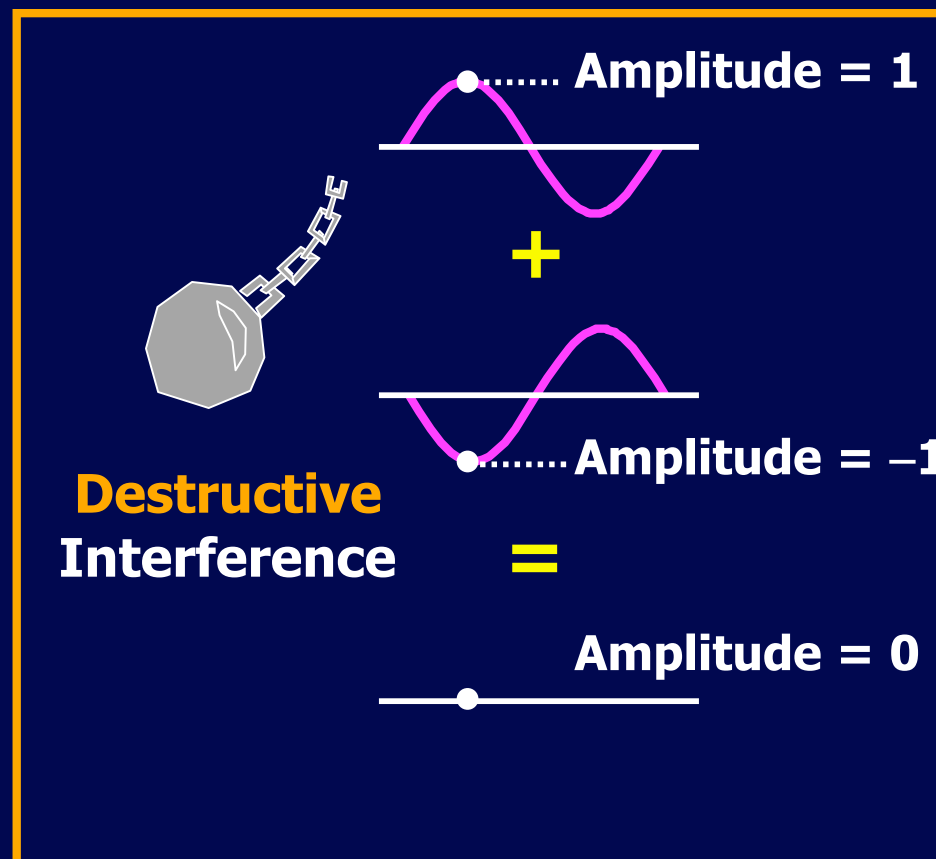
What happens when two waves are in the same place at the same time?

They "superpose"!

Their amplitudes add to give *one* new wave!



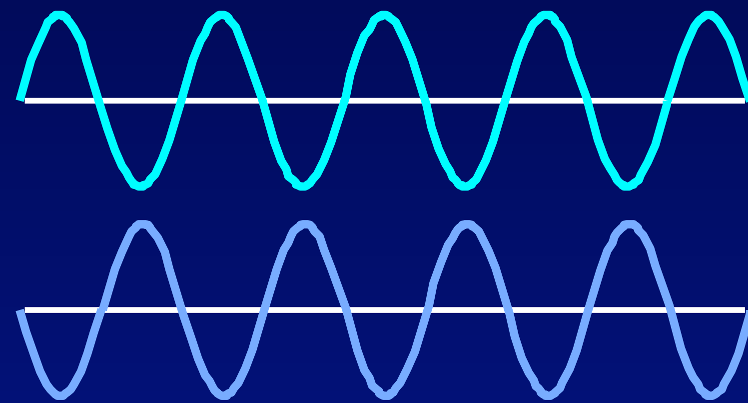
in phase



out of phase

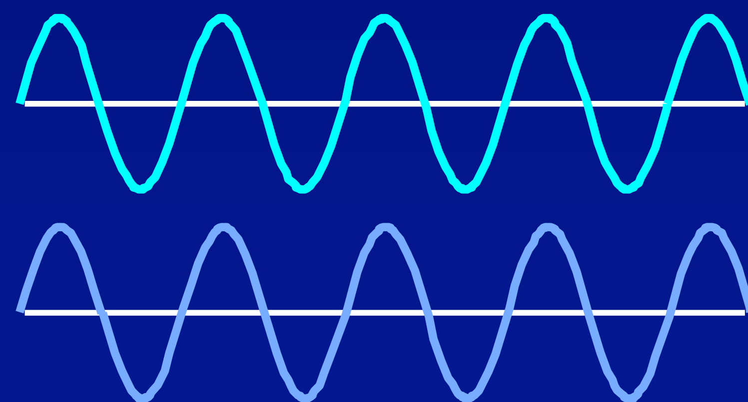
Phase difference refers to the relative position of the wave crests of two waves

Phase difference
= 180°
"Out of phase"



Phase difference
= 0°

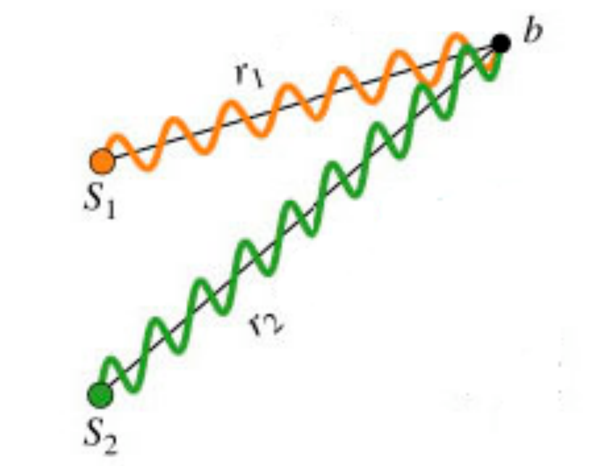
"In phase"



Phase difference and path difference

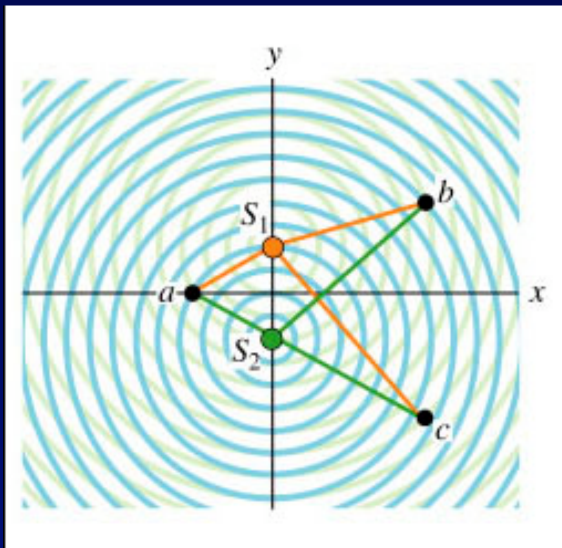
$$\frac{\phi_2 - \phi_1}{2\pi} = \frac{r_2 - r_1}{\lambda}$$

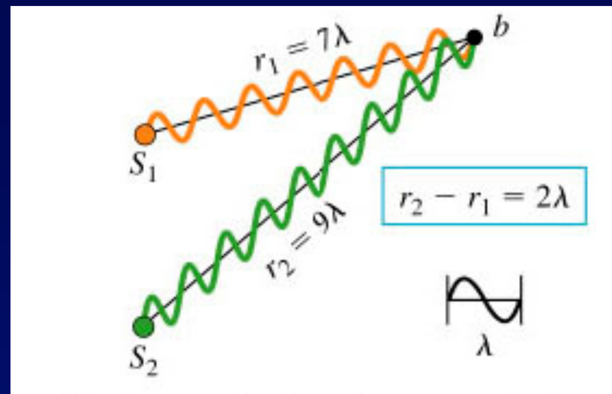
$\Delta\phi = \phi_2 - \phi_1$	$\Delta r = r_2 - r_1$
0	0
$\pi/2$	$\lambda/4$
π	$\lambda/2$
$3\pi/2$	$3\lambda/4$
2π	λ
4π	2λ
…	…



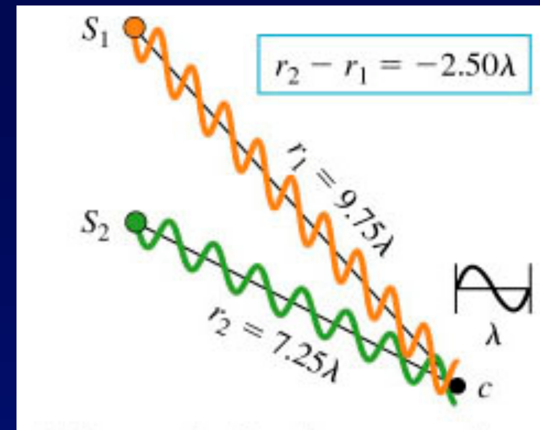
I will refer to path and phase difference interchangeably.

Interference





(b) Constructive interference at point *b*:
path difference =
a whole number of wavelengths



(c) Destructive interference at point *c*:
path difference =
a half-integral number of wavelengths

In general, when two coherent waves meet,

Constructive interference occurs if the path difference is integer multiple of the wavelength:

$$r_2 - r_1 = m\lambda \ \ (\mathrm{m} = 0,\ \pm 1,\ \pm 2,\ \cdots)$$

Destructive interference occurs if the path difference is half-integer multiple of the wavelength:

$$r_2 - r_1 = (m + 1/2)\lambda \ \ (\mathrm{m} = 0,\ \pm 1,\ \pm 2,\ \cdots)$$

Coherence

How is light produced? **Oscillating electrons!**

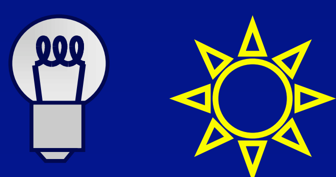
In a light bulb, billions of electrons are oscillating.

Question: is the phase difference between the light from each electron always the same?

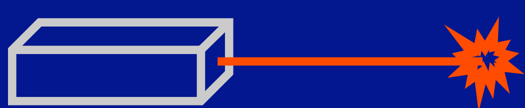
In general, NO!

Two sources of light are said to be

- ***coherent*** **if they have the same frequency and same phase difference**
- ***incoherent*** **if the frequency and phase difference between the waves emitted are random.**



Everyday light sources are not coherent

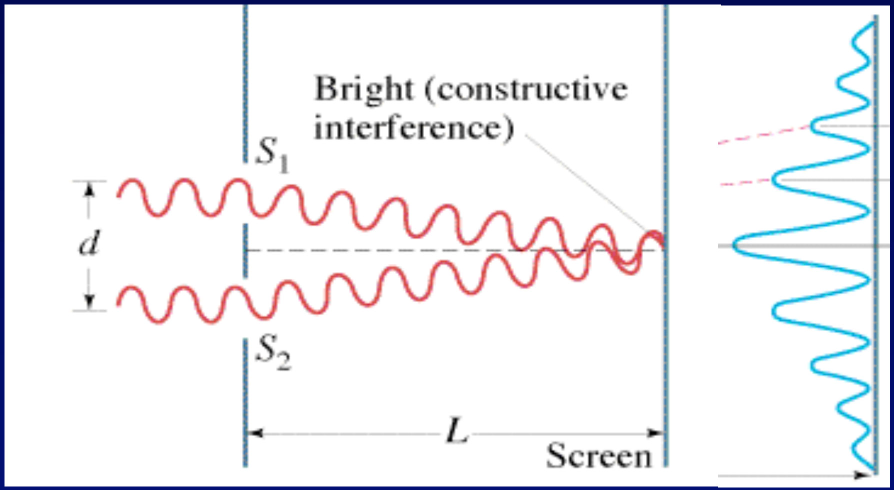


Lasers DO produce coherent light

No interference patterns appear for incoherent light.

Young's Double-slit Experiment

T. Young (1773-1829)



Questions:

Where do the dark and bright spots occur?

Why the intensity pattern?

How are they related to the light's wavelength?

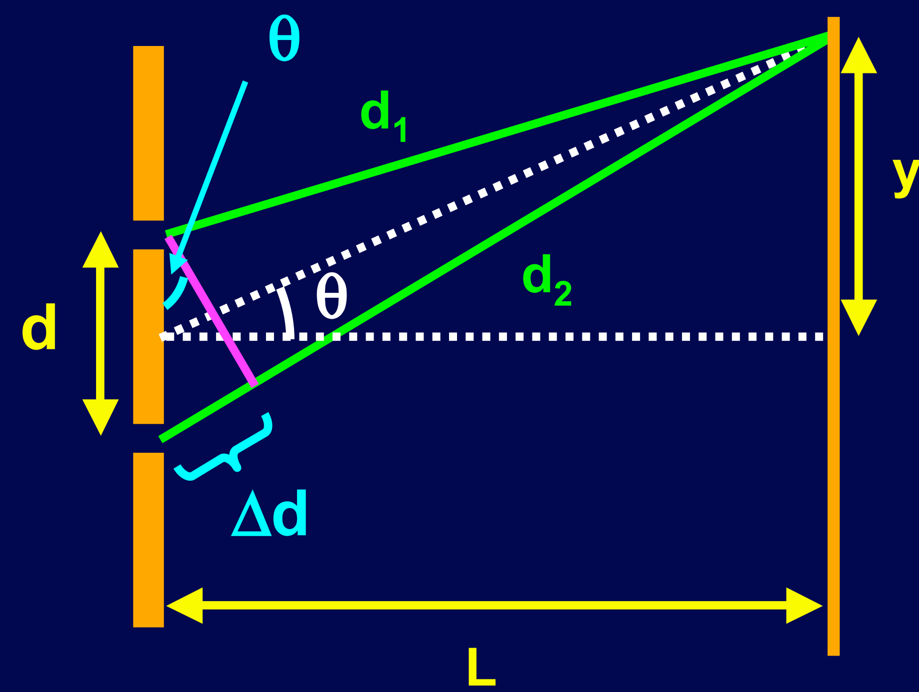
Interference of the two coherent waves when they meet at a point: constructive or destructive?

Determined by the path difference traveled.

Double-Slit Interference: The Math

Path difference Δ between the two waves is:

$$\Delta = d_2 - d_1 = d \sin\theta$$



For Destructive Interference

$$\Delta = {}^1/_2\lambda,\ {}^3/_2\lambda,\ {}^5/_2\lambda,\ {}^7/_2\lambda,\ \ldots$$
$$= (m + {}^1/_2)\,\lambda$$

$$d \sin\theta = (m + {}^1/_2)\,\lambda$$

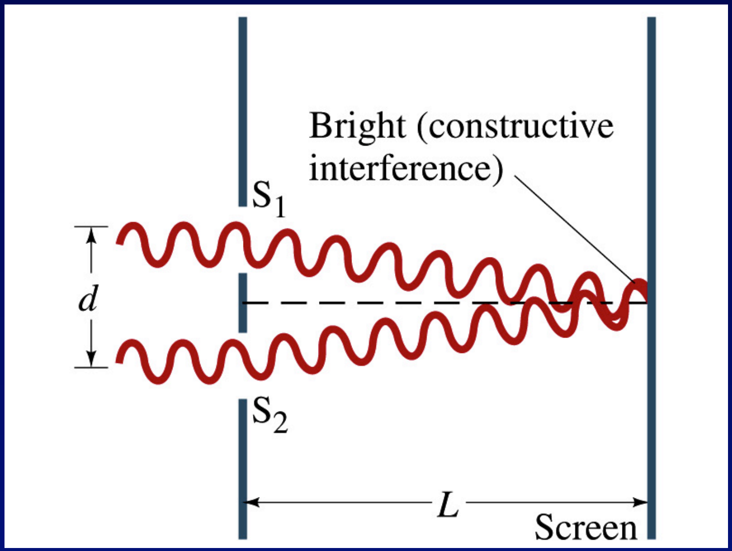
For Constructive Interference

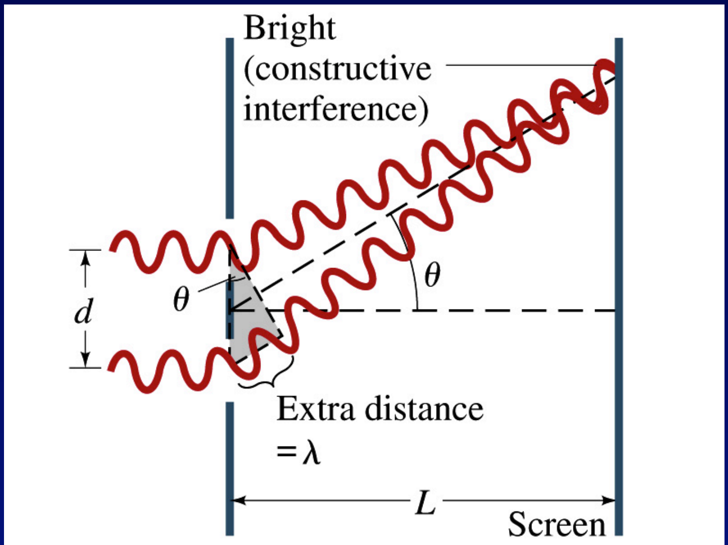
$$\Delta = 0, 1\lambda,\ 2\,\lambda,\ 3\,\lambda,\ 4\,\lambda,\ \ldots$$
$$= m\,\lambda$$

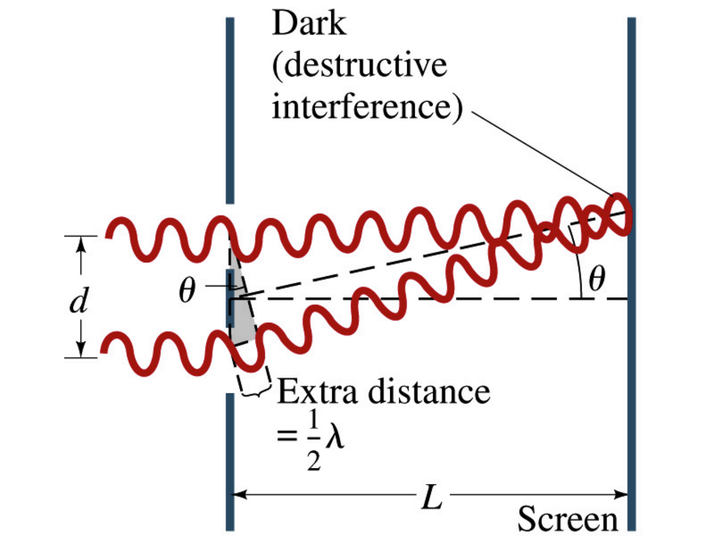
$$d \sin\theta = m\,\lambda$$

m is an integer: m = 0, ± 1, ± 2, ...

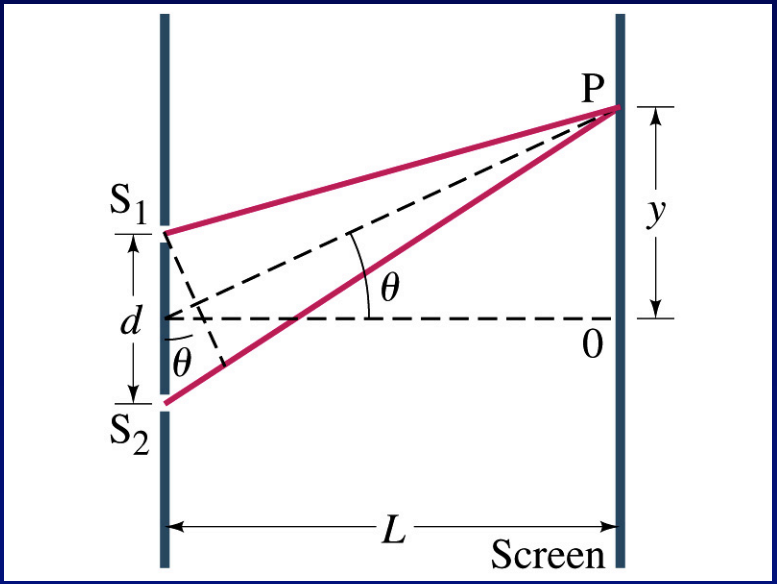
Double-slit Interference







Double-Slit Interference



Calculate the distance of the bright fringes from the axis:

Note that $\tan\theta = y / L$.

Usually, $L >> y$, so

$\tan\theta \approx \sin\theta$ to a good approximation.

$$\sin\theta = \frac{m\lambda}{d} = \frac{y}{L}$$

So the bright fringes will be at:

therefore:

$$y_{bright} = \frac{m\lambda L}{d}$$

So the bright fringes are evenly spaced a distance $\Delta y = \lambda L/d$ apart.

What about the dark fringes ?

$$y_{dark} = \frac{(m + 1/2)\lambda L}{d}$$

Two-source interference: Intensity

$$E_1 = E_0 \cos(\omega t + \phi)$$

$$E_2 = E_0 \cos \omega t$$

Intensity ∝ amplitude²

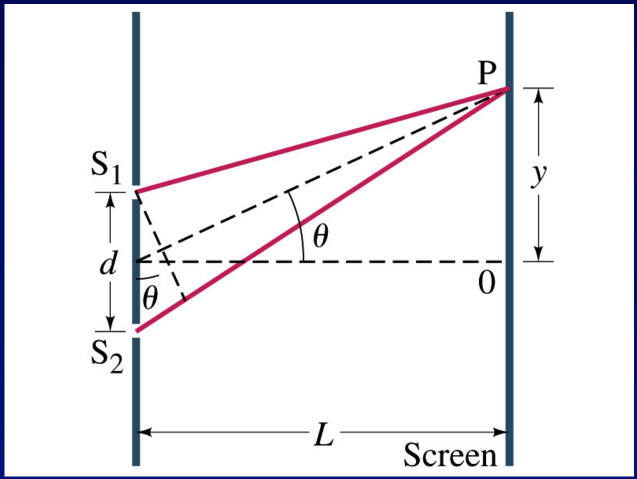
Incoherent light:

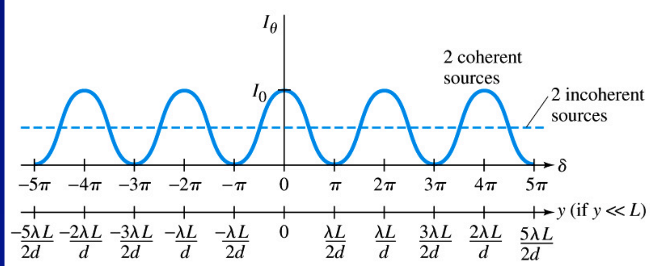
$$I \propto E_1^2 + E_2^2 = 2E_0^2$$

Coherent light:

$$I \propto (E_1 + E_2)^2 = 4E_0^2 \cos^2\left(\frac{\phi}{2}\right)$$

But $\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = \frac{2\pi}{\lambda} d \sin\theta$





$$d \sin\theta = m\lambda, \text{ bright}$$

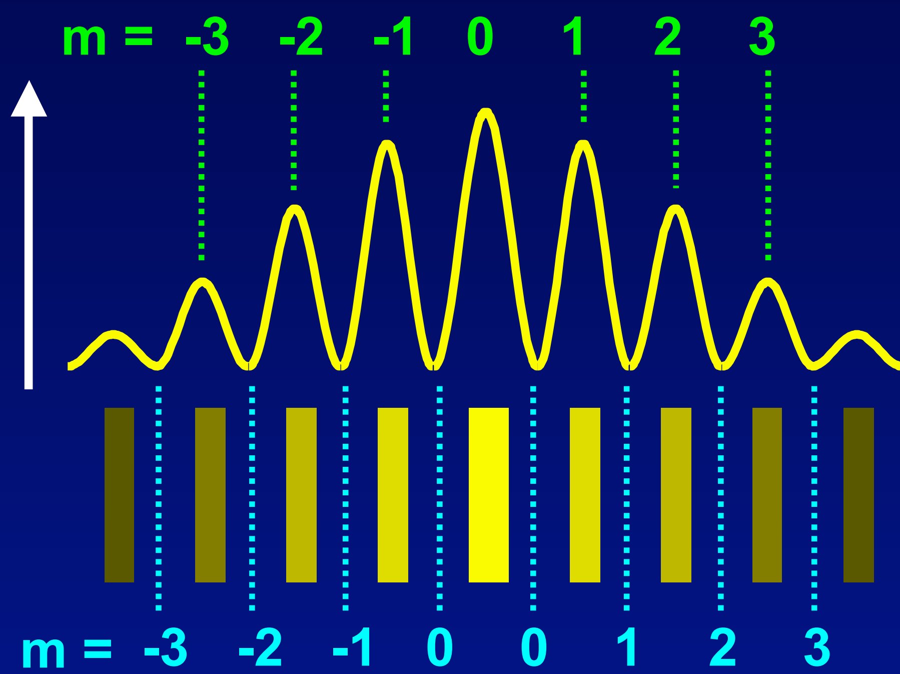
$$d \sin\theta = (m + 1/2)\lambda, \text{ dark}$$

Double-slit: Intensity of Fringes

Constructive Interference

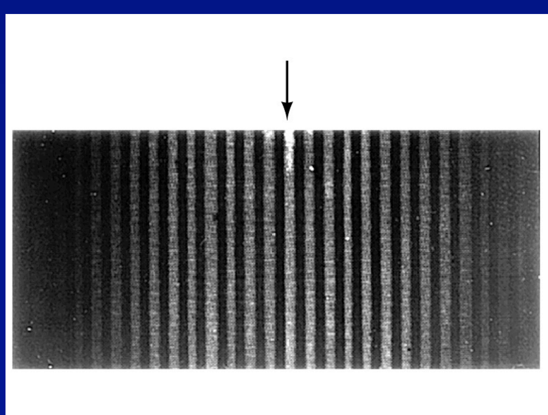
$$d \sin\theta = m\lambda$$

Light Intensity



Destructive Interference

$$d \sin\theta = (m + 1/2)\lambda$$



One needs interference + diffraction to explain this intensity pattern.

ConcepTest 35.1 interference

- Two coherent sources emit waves of wavelength 1 m which are in phase. They meet at a distant point. Wave 1 traveled 20 m to reach the point, and wave 2 traveled 30 m to reach the same point. At this point, there is

(1) constructive interference

(2) destructive interference

(3) in between

ConcepTest 35.2 Interference

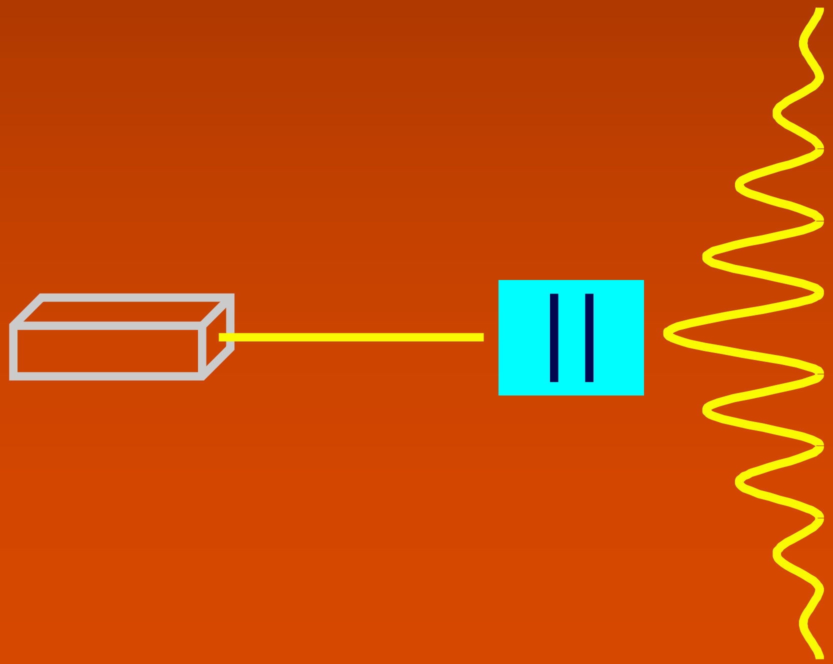
- If Young's double-slit experiment were submerged in water, how would the fringe pattern change?

(1) spreads out

(2) stays the same

(3) shrinks together

(4) disappears



ConcepTest 35.3 Interference

- In a double-slit experiment, What is the *path difference* in distance the waves from each slit traveled at the indicated position?

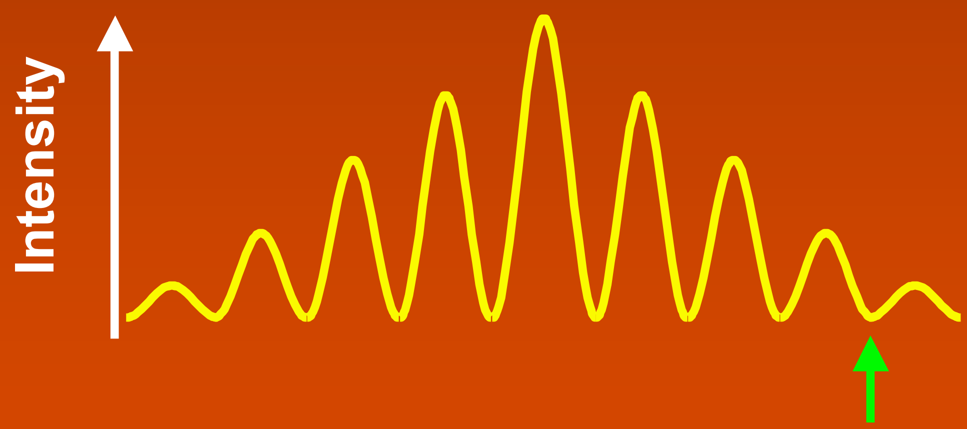
(1) the is no difference

(2) half a wavelength

(3) one wavelength

(4) three wavelengths

(5) more than three wavelengths

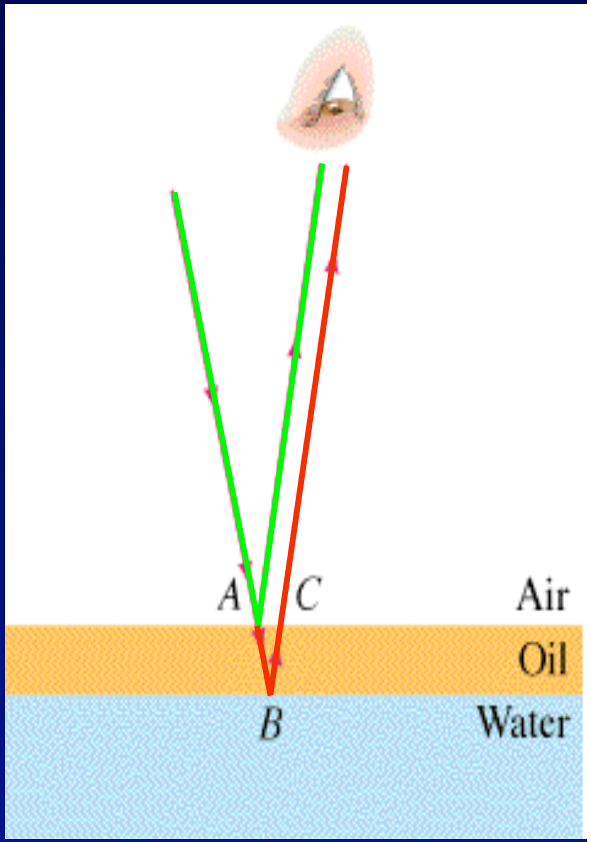


Interference in Thin Films

New Topic

Interference by Thin Films

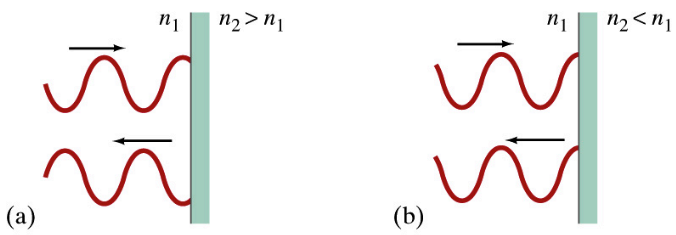
- **Example -- thin oil film on water:**
  - **Part of the incoming light is reflected off the top surface (point A), part at the lower surface (point B).**
  - **Light traveling through oil travels extra distance 2t (which is twice the thickness of the film).**
  - **If 2t is $\lambda, 2\lambda, 3\lambda, 4\lambda, \ldots$**
    - » ***constructive interference*!**
  - **If 2t is $\lambda/2, 3/2\lambda, 5/2\lambda, \ldots$**
    - » ***destructive interference*!**



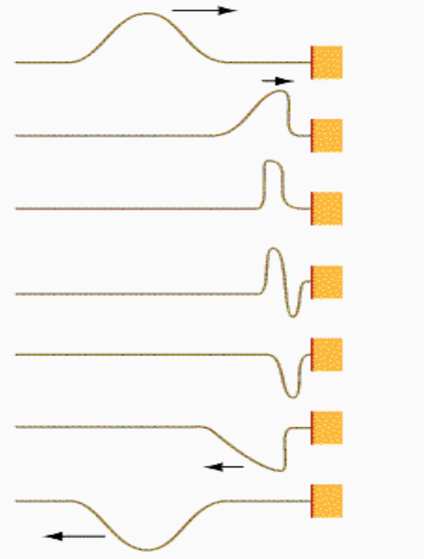
But watch out for possible phase changes at the boundaries.

Phase change at Interfaces

- **If a light wave is reflected by a material whose index of refraction is greater than that of the material it is going through, the wave changes phase by** π (or half wavelength).
  - **example: *air to oil, oil to water***
  - ***No phase change the other way around (oil to air, water to oil)***

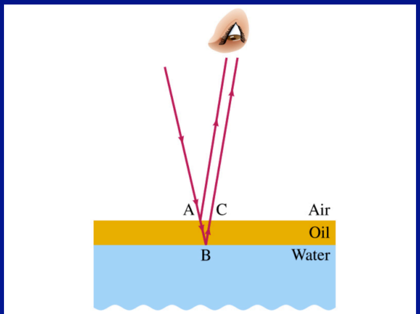


- **This is similar to a wave pulse traveling on a rope and being reflected with the end tied down.**
  - **The pulse flips over, the wave changes phase.**

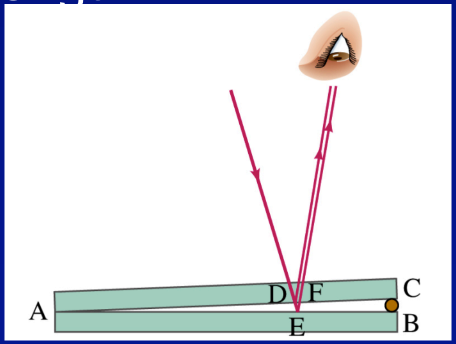


Thin-film Interference: summary

- The deciding factor is the total path difference between the two waves reflected from the two surfaces of the thin film of thickness t:
  - $2t + \delta$
  - where δ is the wavelength shift due to phase change from the reflections
- So if δ is zero, then
  - $2t = m\lambda_n$ gives constructive interference (bright)
  - $2t = (m+1/2)\lambda_n$ gives destructive interference (dark)
  - here λ_n is wavelength in the thin film: $\lambda_n = \lambda/n$
- If δ is half-wavelength, then the above situation is reversed.
- The phase change at the reflection depends on the situation at the surface:
  - small n to big n: half-wavelength
  - big n to small n: 0

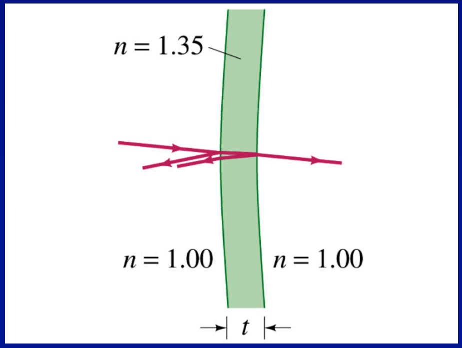


Thin film: oil layer: $\delta = 0$



Thin film: air gap

δ = half-wavelength - 0

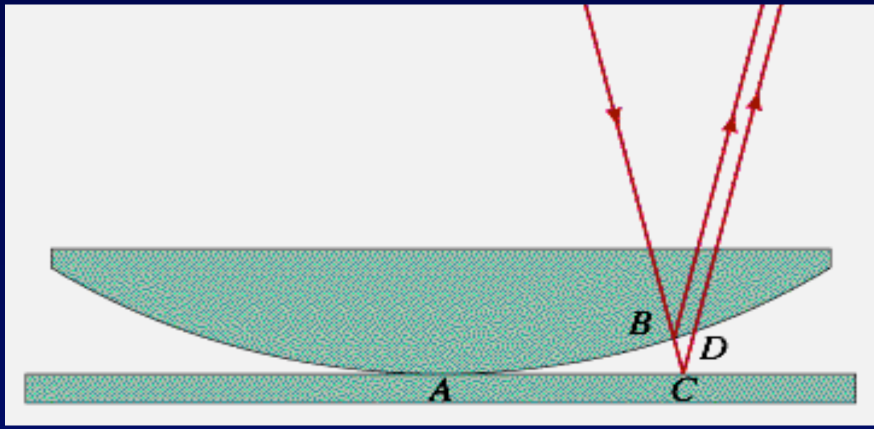


Thin film: soap bubble

$\delta = 0$ - half-wavelength

Newton's Rings

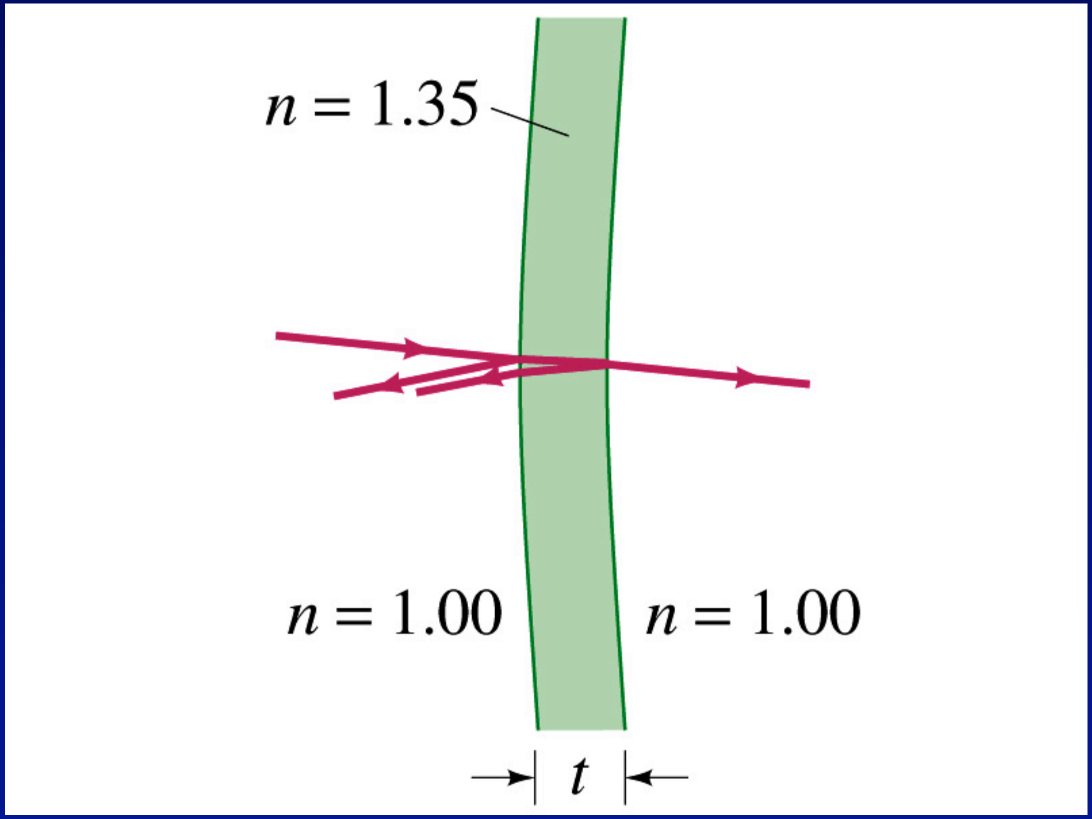
Click here for Newton's rings



- 'Thin film': air gap between two pieces of glass.
  - Path difference BCD (= 2 t) varies in the air gap
  - No phase change from B to C (glass to air)
  - half-wavelength phase change from C to D (air to glass)
- Total phase change is half wavelength, so
  - if $2t = \lambda/2, 3\lambda/2, 5\lambda/2, \ldots$: Bright (constructive)
  - If $2t = 0, \lambda, 2\lambda, 3\lambda, \ldots$: Dark (destructive)
  - The center point is dark (t=0)

Example: thickness of soap bubble skin

- A soap bubble of $n=1.35$ appears green ($\lambda=540$nm) at the point on its front surface nearest the viewer.
  - What is the minimum thickness?



ConcepTest 35.4 thin-film interference

- A laser shines on a pair of identical glass microscope slides that form a very narrow edge. The waves reflected from the top and the bottom slide interfere. What is the interference pattern from top view?