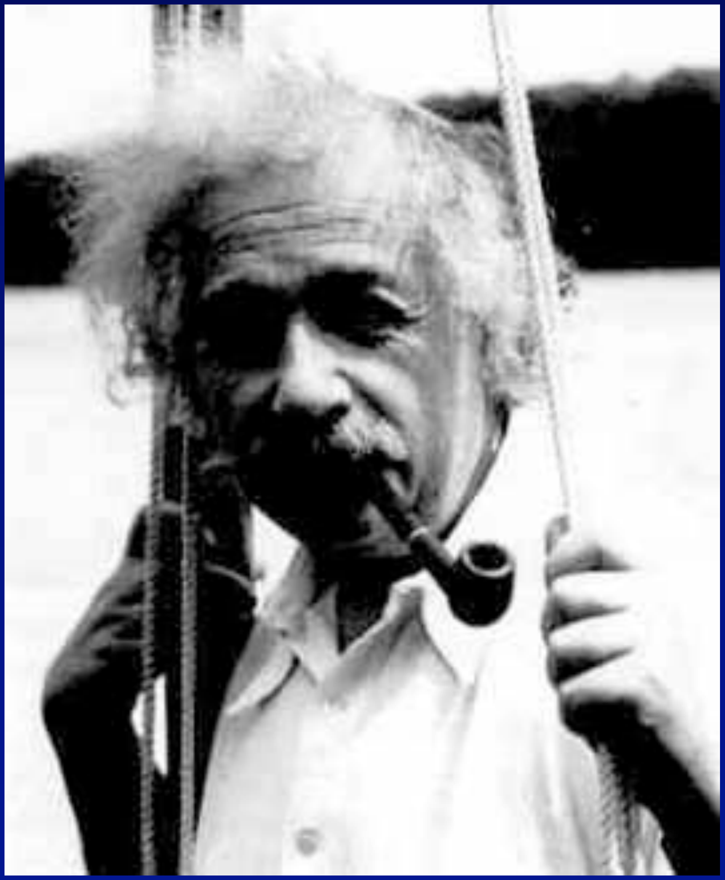
Chapter 37
Relativity

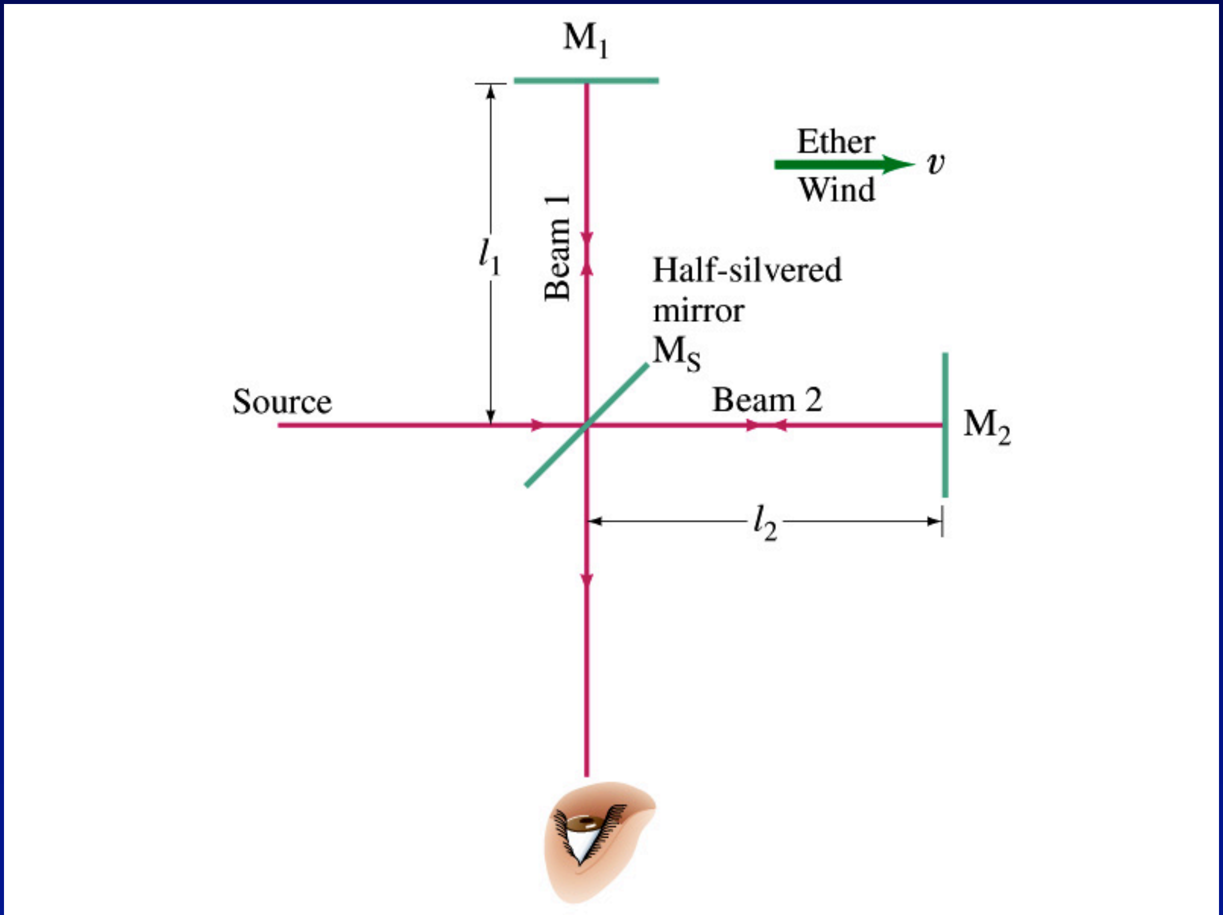
- Two postulates
- Relativity of simultaneity
- Time dilation; length contraction
- Lorentz transformations
- Doppler effect
- Relativistic kinematics



Two postulates

New Topic

Michelson-Morley experiment



Null result: There is no "ether" (no preferred frame) and all observers see the same speed.

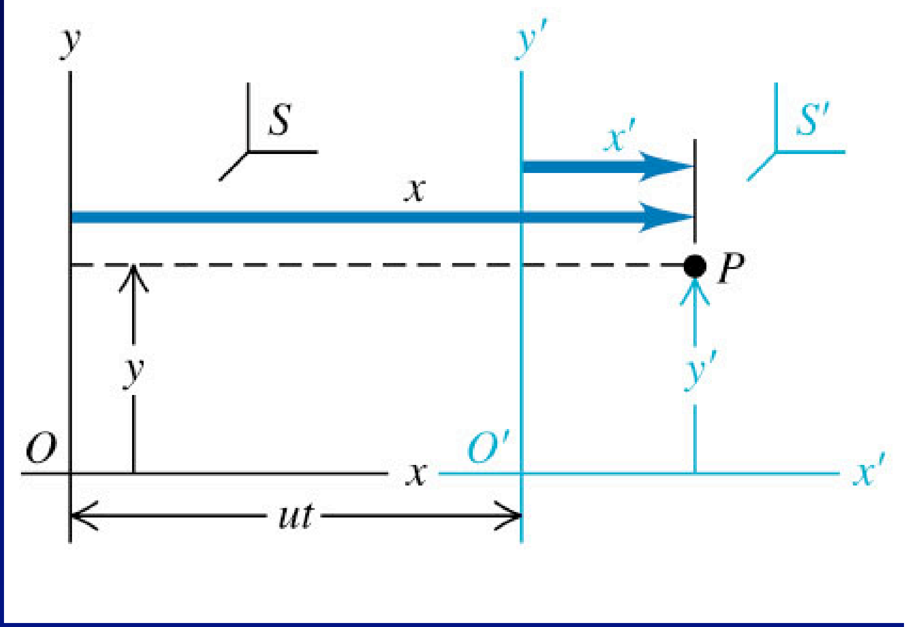
Galilean relativity

Inertial frames: Non-accelerating reference frame (ignoring gravity)

We know how coordinates and velocities behave in two such frames:

$$x = x' + ut$$

$$\frac{dx}{dt} = \frac{dx'}{dt} + u$$



Applied to a light beam in vacuum we get: c = c'± u.

This contradicts experiment !! What's wrong??

Two postulates

- The laws of physics are the same in every inertial frame of reference. (covariance)
  - Should be true for Maxwell's equations. It is!
- The speed of light is the same in all inertial reference frames and is independent of the motion of the source. (invariance)
  - What's wrong? Our implicit assumption that $t = t'$.

These postulates form the crux of the Special Theory of Relativity. There is also a General theory of Relativity, which deals only with gravitation, which we won't cover here.

We will assume that special relativity must reduce to Galilean relativity for $v \ll c$. This is called the "correspondence principle".

ConcepTest 37.1 Postulates

- Which statements follow from the two postulates of relativity?

(1) A spherical wave front in one inertial frame of reference appears as a spherical wave front in any other.

(2) One can not "catch up" with a light beam

(3) Experiments in different inertial frames behave the same

(4) All of the above

(5) Only (2) and (3) are correct.

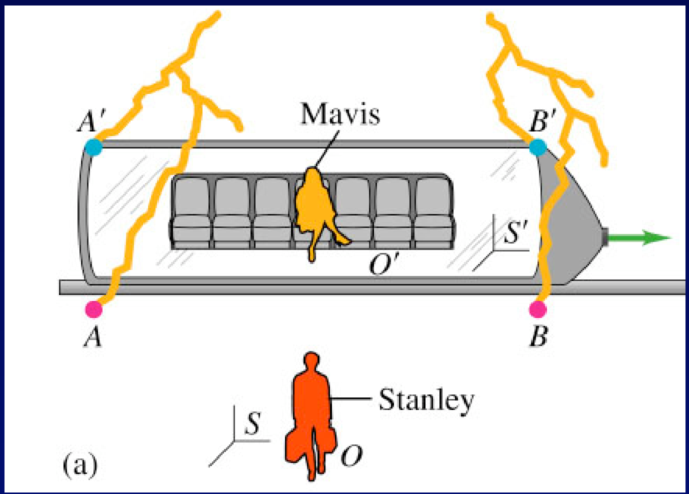
I. The laws of physics are the same in every inertial frame of reference.

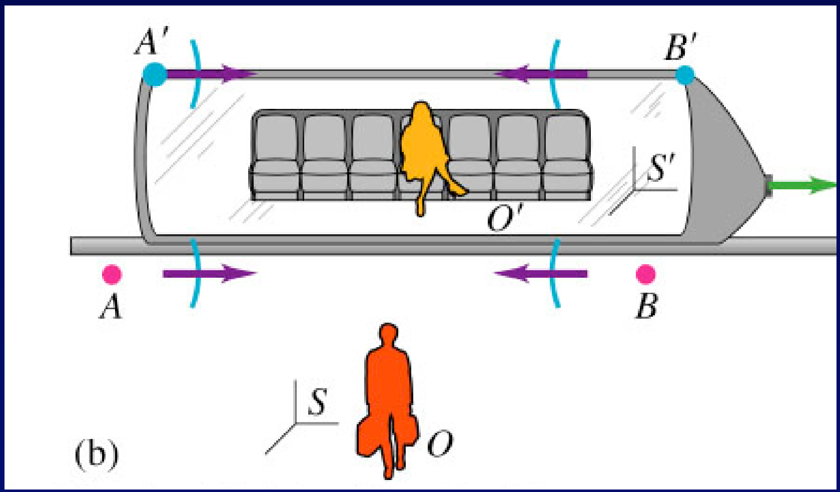
II. The speed of light is the same in all inertial reference frames and is independent of the motion of the source. (invariance)

Relativity of Simultaneity

New Topic

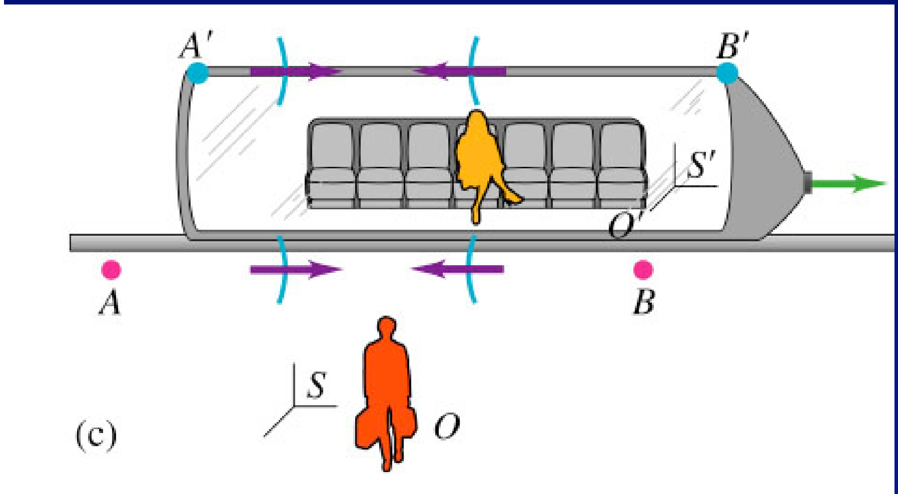
Simultaneity: Stanley and Mavis

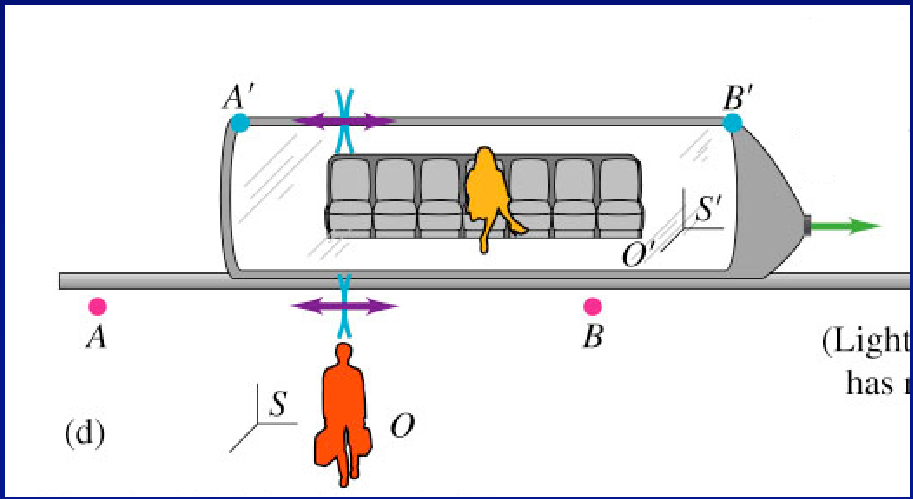




A' and B' are simultaneous to Mavis

Light start to spread out (Stanley POV)





Light from B' reaches Mavis first

Events are simultaneous for Stanley

Moral: Simultaneity is relative!

ConcepTest 37.2 Simultaneity

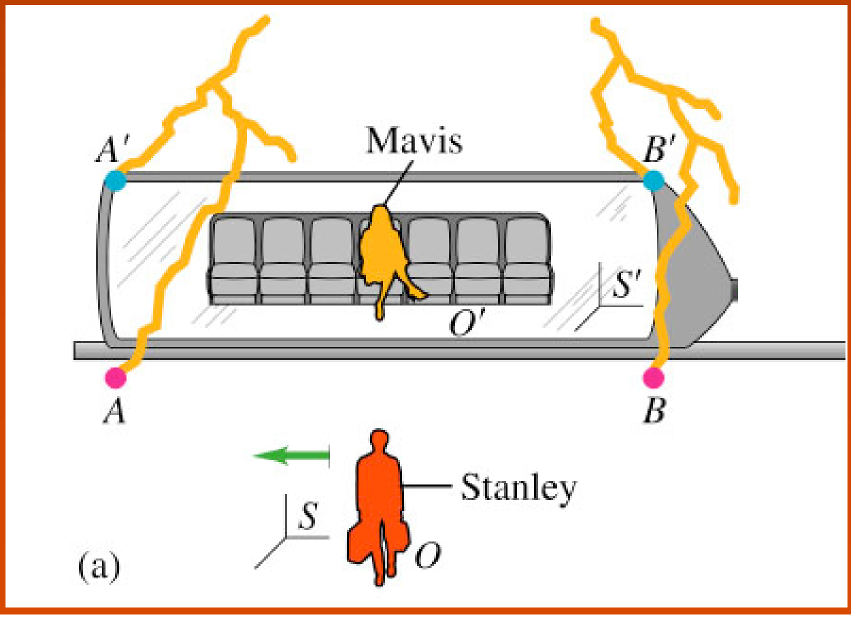
- Suppose the two lightning bolts shown are simultaneous to an observer on the train (Mavis). Which lightning strike does the ground observer (Stanley) measure to come first?

(1) The one at A

(2) The one at B

(3) They are still simultaneous

(4) Relativity can not answer this



Time dilation; length contraction

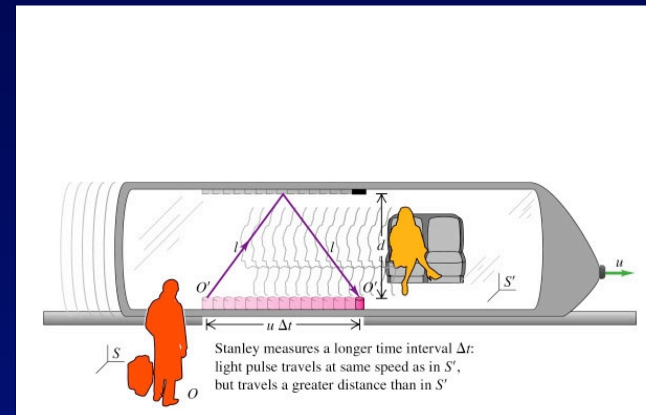
New Topic

Time dilation equations

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

$$\Delta t = \gamma\, \Delta t_0$$



Proper time (Δt_0): Time measured by an observer at rest with respect to two events.

Example: A particle known as a muon is generated high in the earth's atmosphere with a speed of 0.96c relative to the earth. The muon's average lifetime, measured at rest, is 2.2 x 10-6 sec. How far does such a muon travel through the Earth's atmosphere before decaying?

Click here for "Twin Paradox"

Length contraction equation

Twin paradox redux: If the traveling twin knows 1) the distance to the star is 9.5 light years away and 2) that he/she is traveling at a speed of 0.95c with respect to the Earth/star system, how can he/she possibly agree that it takes only 6.24 years to complete the journey??

Ans: Length contraction!

$$l = l_0\sqrt{1 - \frac{u^2}{c^2}} = \frac{l_0}{\gamma}$$

Contraction is only along the direction of motion, but distortions appear because of the finite speed of light.

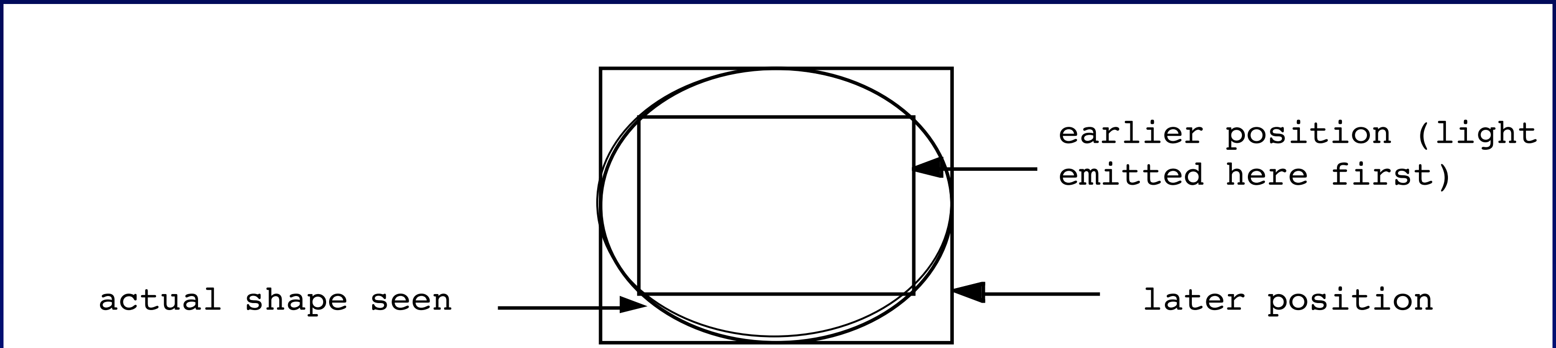
$$\frac{1}{\gamma} = \sqrt{1-(.95)^2} = .3122 \text{ !!}$$

The earth/star distance is contracted !!

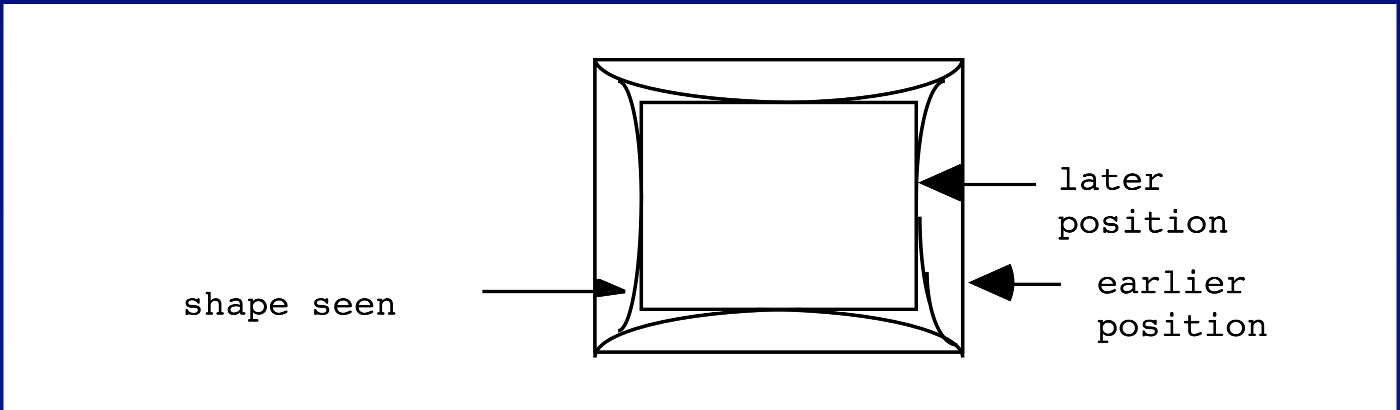
Proper length (l_0): Length measured by an observer at rest with respect to the object.

How things look

Coming toward you



Going away from you



Weirdness: A sphere still appears as a sphere, but rotated!!

ConcepTest 37.3 Relativity of time

- Both Samir and Maria start their stopwatches at the instant that Maria flies past him in her spaceship at a speed of 0.6c. What does each conclude about each other's stopwatches?

(1) Both agree Maria's stopwatch is running slowly

(2) Both agree Samir's stopwatch is running slowly

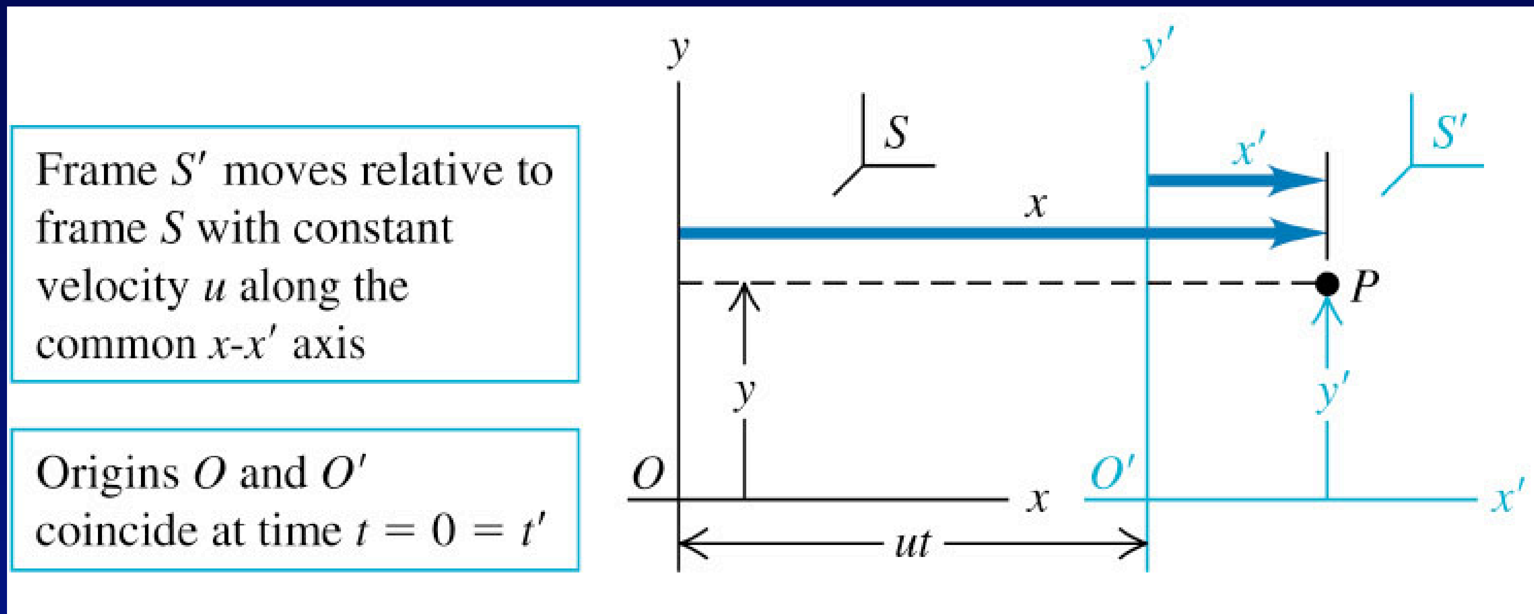
(3) Both agree that the other's stopwatch is running slowly

(4) Relativity can not answer this.

Lorentz transformations

New Topic

Transformation equations



$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}} = \gamma(x - ut)$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}} = \gamma(t - ux/c^2)$$

"Galilean":

x' = x - ut,

y' = y,

z' = z,

t' = t

Velocity addition law

$$v'_x = \frac{v_x - u}{1 - uv_x/c^2}$$

"Galilean":

$v_x' = v_x - u$

$$v_x = \frac{v'_x + u}{1 + uv'_x/c^2}$$

Caution: The Lorentz transformation equations assume that S' is moving in the positive x-direction with velocity u with respect to S.

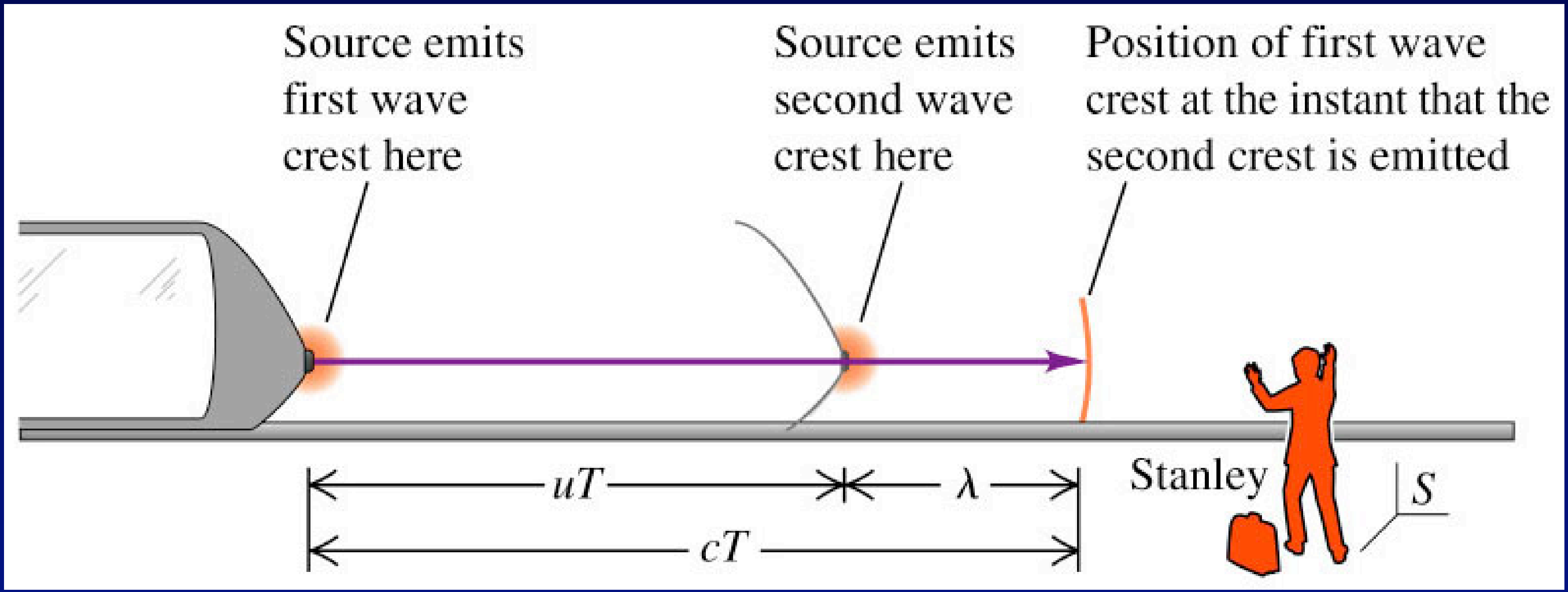
Example: $u = 0.95c$, $v_x' = 0.95c$:

$$v_{12} = \frac{.95c + .95c}{1 + \frac{(.95c)^2}{c^2}} = .99868c$$

Doppler effect

New Topic

Doppler effect



$$f = \sqrt{\frac{c+u}{c-u}} f_0$$

Note: When the source moves away from the observer, change the sign of u.

Toward: $u > 0$ => $f > f_0$ "higher frequency"

Away: $u < 0$ => $f < f_0$ "lower frequency"

What if u << c ?? Then we obtain

$$f \approx (1 + \frac{u}{c}) f_0$$

$$\Delta f \approx \frac{u}{c}$$

ConcepTest 37.4 Doppler effect

- A policeman's radar gun measures the frequency of an electromagnetic carrier wave of frequency f_0, which has bounced off of an oncoming car. It's velocity is u>0. The frequency measured by the gun, f, is:

(1) Equal to the original frequency, f_0.

(2) Has a positive frequency shift, $\Delta f = u/c$.

(3) Has a positive frequency shift, $\Delta f = 2u/c$.

(4) Has a negative frequency shift, $\Delta f = - u/c$.

(5) Has a negative frequency shift, $\Delta f = - 2u/c$.

Relativistic kinematics

New Topic

Relativistic momentum, work, etc.

Energy and momentum still conserved, but their expressions change!

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$$

$$\vec{p} = \gamma m\vec{v}$$

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = (\gamma - 1)mc^2$$

$$E = K + mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = \gamma mc^2$$

$$E^2 = (mc^2)^2 + (pc)^2$$

(old connection :

$$E = \frac{\vec{p}^2}{2m}.$$

)

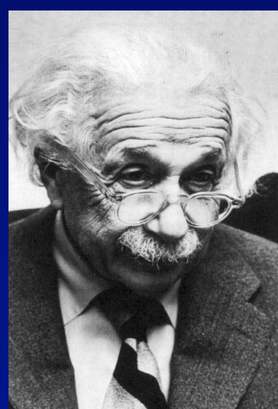
"Photon": (m=0)

$$E = pc$$

Most famous equation of all !

$$E_0 = mc^2 \quad (rest\ mass)$$

=>There is no "conservation of mass".



Einstein ("The Meaning of Relativity," p.47): "Mass and energy are therefore essentially alike; they are only different expressions of the same thing."