Finding Coterminal Angles

To find an angle that is between 0 and 2π that is coterminal with any given angle, we need to find the "closest" multiple of 2π and subtract it (if $\theta > 0$) or add it (if $\theta < 0$). Given any angle $\theta = \frac{\pm a\pi}{b}$, use the following procedure to find an angle between 0 and 2π that is coterminal with θ:

- Identify a and b. Form $2b$.
- If $\theta > 0$, then find the **largest** multiple of $2b$ that is **smaller** than a, call it c. Then the angle between 0 and 2π that is coterminal with θ is $\frac{(a-c)\pi}{b}$.
- If $\theta < 0$, then find the **smallest** multiple of $2b$ that is **larger** than a, call it c. Then the angle between 0 and 2π that is coterminal with θ is $\frac{(c-a)\pi}{b}$.

Let's practice a few. For each of the following angles, find a coterminal angle between 0 and 2π:

$\theta = \frac{11\pi}{2}$ $a =$ _____, $b =$ _____, $2b =$ _____

Largest multiple of $2b$ that is smaller than a: _____ Coterminal angle:_____

$\theta = \frac{33\pi}{6}$ $a =$ _____, $b =$ _____, $2b =$ _____

Largest multiple of $2b$ that is smaller than a: _____ Coterminal angle:_____

$\theta = -\frac{19\pi}{3}$ $a =$ _____, $b =$ _____, $2b =$ _____

Smallest multiple of $2b$ that is larger than a: _____ Coterminal angle:_____

$\theta = \frac{26\pi}{4}$ $a =$ _____, $b =$ _____, $2b =$ _____

Largest multiple of $2b$ that is smaller than a: _____ Coterminal angle:_____

$\theta = -\frac{26\pi}{4}$ $a =$ _____, $b =$ _____, $2b =$ _____

Smallest multiple of $2b$ that is larger than a: _____ Coterminal angle:_____

$\theta = -\frac{14\pi}{6}$ $a =$ _____, $b =$ _____, $2b =$ _____

Smallest multiple of $2b$ that is larger than a: _____ Coterminal angle:_____

Find a coterminal angle between 0 and 2π:

$\theta = \frac{51\pi}{6}$ $a =$ _____, $b =$ _____, $2b =$ _____, $c =$ _____, Coterminal angle:_____

$\theta = -\frac{47\pi}{3}$ $a =$ _____, $b =$ _____, $2b =$ _____, $c =$ _____, Coterminal angle:_____

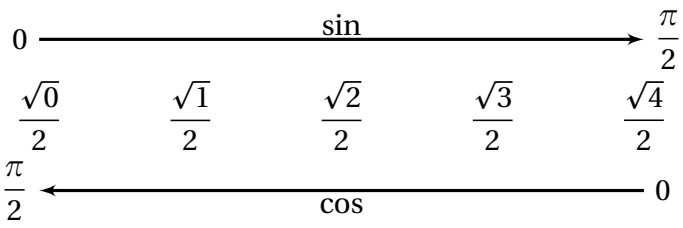
$\theta = -\frac{99\pi}{2}$ $a =$ _____, $b =$ _____, $2b =$ _____, $c =$ _____, Coterminal angle:_____

$\theta = \frac{163\pi}{4}$ $a =$ _____, $b =$ _____, $2b =$ _____, $c =$ _____, Coterminal angle:_____

Evaluating Trig Functions

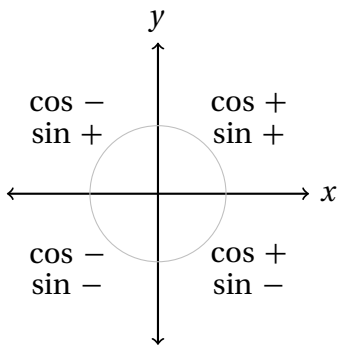
The critical piece of information that every trig student must learn is the value of sine and cosine at the "reference angles" 0, $\pi/6$, $\pi/4$, $\pi/3$, and $\pi/2$ (the angles in the first quadrant). Recognizing that $0 = \sqrt{0}$, $1 = \sqrt{1}$, and $2 = \sqrt{4}$, all you have to remember is that the value of sin or cos at the standard angles will be one of $\sqrt{k}/2$ where $k = 0, 1, 2, 3, 4$. For cosine, we go from 4 to 0 as α increases from 0 to 2π. For sine, we go from 0 to 4 as α increases from 0 to 2π.

α:	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\alpha$:	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\alpha$:	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0



The value of cosine and sine at any angle **are the same** as the values of cosine and sine at the corresponding reference angles. To find the reference angle for any angle between 0 and 2π, just make sure your fraction is in lowest terms, i.e., all common factors of a and b in $a\pi/b$ have been canceled. Then the reference angle is the one in the first quadrant with the denominator b (and remember that the denominator of 0 and π is 1).

As cosine and sine correspond to x and y values, respectively, of points on the unit circle, the sign (positive or negative) of the sine and cosine functions are easy to determine.



The key to finding the value of $\sin\theta$ and $\cos\theta$ when θ is any given multiple of $\pi/4$ and $\pi/6$ boils down to three separate tasks:

1. Find the angle between 0 and 2π that is coterminal with θ
2. Determine the sign of the trig function in the quadrant of the coterminal angle.
3. Determine which reference angle (between 0 and $\pi/2$) corresponds to the coterminal angle and evaluate the trig function at the reference angle.

Combine the results from steps 2 and 3 to find the trig function at θ.

Evaluate the following trig functions:

$\cos\left(\frac{7\pi}{2}\right) =$ _____ Coterminal angle:_____ Sign:_____ Reference angle:_____

$\sin\left(-\frac{7\pi}{4}\right) =$ _____ Coterminal angle:_____ Sign:_____ Reference angle:_____

$\cos\left(-\frac{43\pi}{6}\right) =$ _____ Coterminal angle:_____ Sign:_____ Reference angle:_____

$\sin\left(\frac{26\pi}{3}\right) =$ _____ Coterminal angle:_____ Sign:_____ Reference angle:_____

$\cos\left(-\frac{19\pi}{6}\right) =$ _____ Coterminal angle:_____ Sign:_____ Reference angle:_____

$\sin\left(\frac{275\pi}{4}\right) =$ _____ Coterminal angle:_____ Sign:_____ Reference angle:_____

$\cos\left(-\frac{21\pi}{3}\right) =$ _____ Coterminal angle:_____ Sign:_____ Reference angle:_____

$\sin\left(-\frac{39\pi}{6}\right) =$ _____ Coterminal angle:_____ Sign:_____ Reference angle:_____