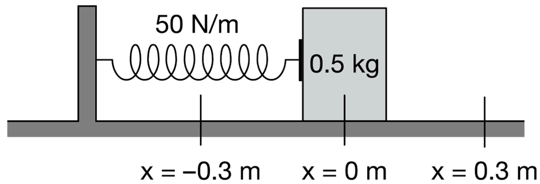
Springs and Simple Harmonic Motion

1.



A block of mass $0.5\ \text{kg}$ on a horizontal surface is attached to a horizontal spring of negligible mass and spring constant $50\ \text{N/m}$. The other end of the spring is attached to a wall, and there is negligible friction between the block and the horizontal surface. When the spring is unstretched, the block is located at $x = 0\ \text{m}$. The block is then pulled to $x = 0.3\ \text{m}$ and released from rest so that the block-spring system oscillates between $x = -0.3\ \text{m}$ and $x = 0.3\ \text{m}$. What is the magnitude of the acceleration of the block and the direction of the net force exerted on the block when it is located at $x = 0.3\ \text{m}$?

(A)

Magnitude of Acceleration	Direction of Net Force
$30\ \text{m/s}^2$	Positive

(B)

Magnitude of Acceleration	Direction of Net Force
$30\ \text{m/s}^2$	Negative

(C)

Magnitude of Acceleration	Direction of Net Force
$0\ \text{m/s}^2$	Positive

(D)

Magnitude of Acceleration	Direction of Net Force
$0\ \text{m/s}^2$	Negative

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2. This question is a long free-response question. Show your work for each part of the question.

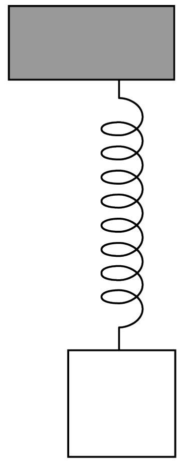

Figure 1

A group of students must determine the spring constant k of a spring that hangs vertically from a ring stand. The students attach a block to the bottom of the vertical spring, as shown in Figure 1. The students then pull the block-spring system $0.5\ \mathrm{m}$ below the system's equilibrium position and release the system so that it oscillates. The students record the mass M of the block and the period of oscillation T for several trials, as shown in the table.

M (kg)	T (s)		
0.3	0.77		
0.6	1.07		
0.9	1.29		
1.2	1.50		

(a) The students want to determine the spring constant k of the spring.

i. Indicate below which quantities could be graphed to determine the spring constant k of the spring by using a best-fit line. You may use the remaining columns in the table above, as needed, to record any quantities (including units) that are not already in the table.

Vertical Axis:____________________ Horizontal Axis:____________________

ii. On the grid below, plot the appropriate quantities to determine the spring constant of the spring by using a best-fit line. Clearly scale and label all axes, including units as appropriate.

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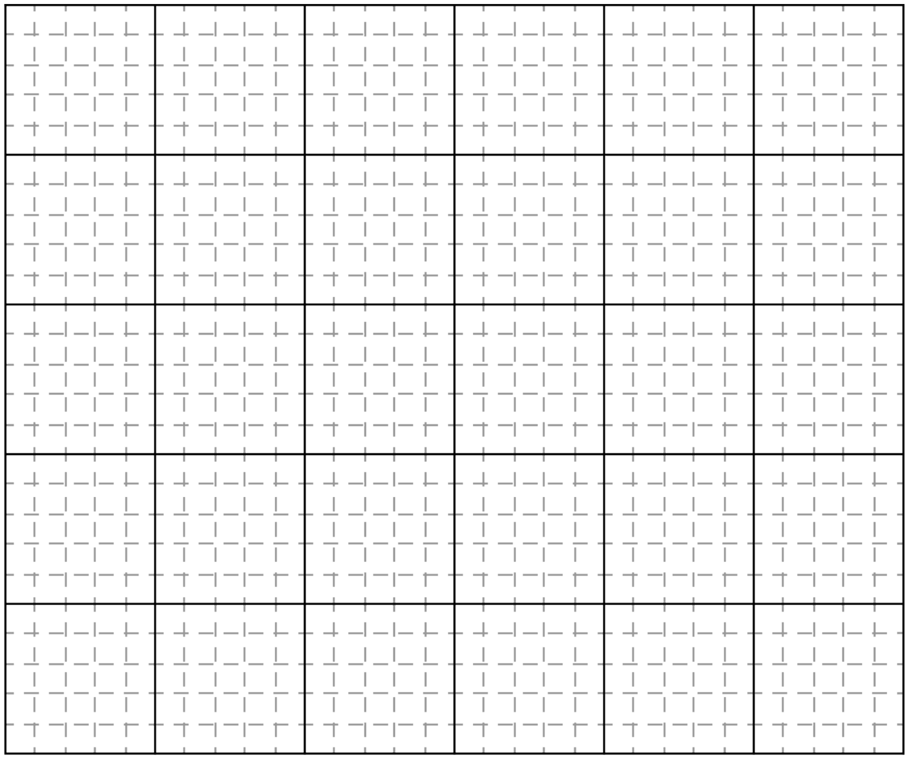

iii. Use the graph above to estimate a value for the spring constant k of the spring.

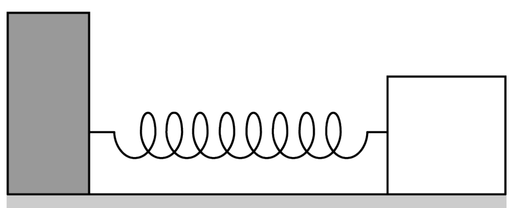

Figure 2

Figure 2

(b) Another group of students performs a similar experiment in which the same block-spring system from trial 1 oscillates, but in the horizontal direction, as shown in Figure 2. Frictional forces between the block and the surface are considered to be negligible. Furthermore, the students pull the block-spring system 0.8 m from the system's equilibrium before they release the system so that it oscillates. Will this change in the experiment affect the value of the period of oscillation T for the block-spring system compared to the situation from part (a)?

_____ Yes _____ No

Briefly state your reasoning.

(c) Describe an experimental procedure to determine whether or not the period of oscillation T for the block-

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spring system is different compared to the situation from part (a). Include any steps necessary to reduce experimental uncertainty. Give enough detail so that another student could replicate the experiment. As needed, include a diagram of the experimental setup. Assume equipment usually found in a school physics laboratory is available.

Part (a)i

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1 ✓

Student response accurately includes one of the following criteria.

- [ ] 1 point is earned for choosing two quantities that, when graphed together, can be used to determine the resistance.

Example Response:

T vs. $\sqrt{M}$

T vs. $2\pi\sqrt{M}$

$\frac{T}{2\pi}$ vs. $\sqrt{M}$

T^2 vs. M

T^2 vs. $4\pi^2 M$

$\frac{T^2}{4\pi^2}$ vs. M

Part (a)ii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

Student response accurately includes all of the following criteria.

Note: The following points can be earned only if the graphed points correspond to the quantities chosen in (a)i.

- 1 point is earned for a linear scale where plotted data uses at least half the grid.
- 1 point is earned for labeling both axes with units as appropriate.

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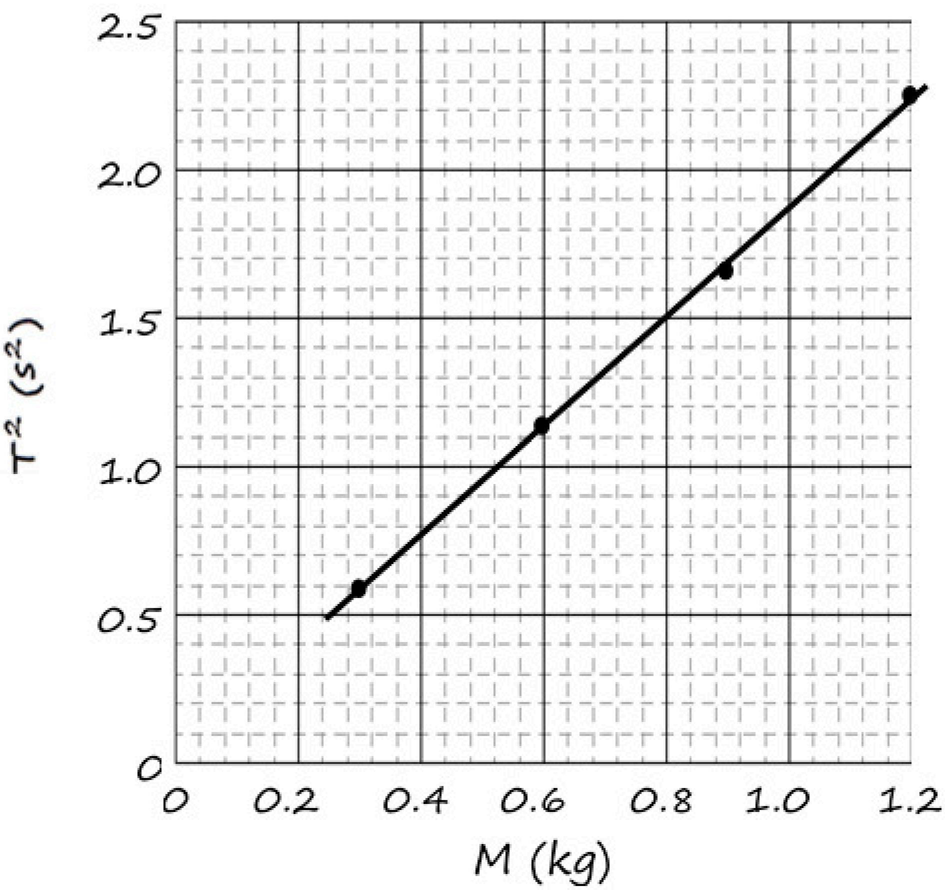
- [ ] 1 point is earned for data points plotted that represent the appropriate trend based on chosen quantities.

Note: The appropriate trend depends on the choice of graphed quantities and may be e.g., linear, inversely proportional, etc. although the prompt states that the graph should contain a best fit line.

Example Response:



Part (a)iii

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2

Student response accurately includes all of the following criteria.

- [ ] 1 point is earned for correctly using the slope of the graph to determine the spring constant *k* of a spring or stating that a calculator was used to determine the appropriate value from the graph.
- [ ] 1 point is earned for a correct value of the spring constant $k = 20\ \mathrm{N/m}\ (\pm 2.0)$ calculated from the best-fit line.

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Note: Data points should not be used to calculate the slope, unless the points lie on the best-fit line.

Part (b)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes one of the following criteria.

- ☐ 1 point is earned for stating that the period of oscillation for this situation will not be different from the period of oscillation from part (a).
- ☐ 1 point is earned for stating that the period of oscillation only depends on the mass of the object connected to the spring and the spring constant of the spring.

Part (c)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3	4 ✓

Student response accurately includes all of the following criteria.

- ☐ 1 point is earned for stating that a meterstick should be used to measure the horizontal displacement of the block-spring system a distance of 0.8m from its equilibrium position.
- ☐ 1 point is earned for stating that a stopwatch should be used to measure the time it takes for the object to complete a certain number of oscillations to determine the period.
- ☐ 1 point is earned for stating that period of oscillation for the situation in part (c) should be compared to the period of oscillation for trial 1 in part (a).
- ☐ 1 point is earned for stating that the experiment should be repeated for multiple trials to reduce experimental uncertainty.

Example Response:

1. Use a meterstick to measure the horizontal displacement of 0.8m of the center of mass of the block from the block-spring system's equilibrium position.

2. Release the system from rest at a horizontal position of 0.8m from the block-spring system's equilibrium position.

3. Use a stopwatch to measure the amount of time it takes the block-spring system to oscillate 10 times.

4. Repeat the experiment three times to reduce experimental uncertainty.

5. Use the data from step 3 and step 4 to determine the average amount of time it takes for the block-spring system to

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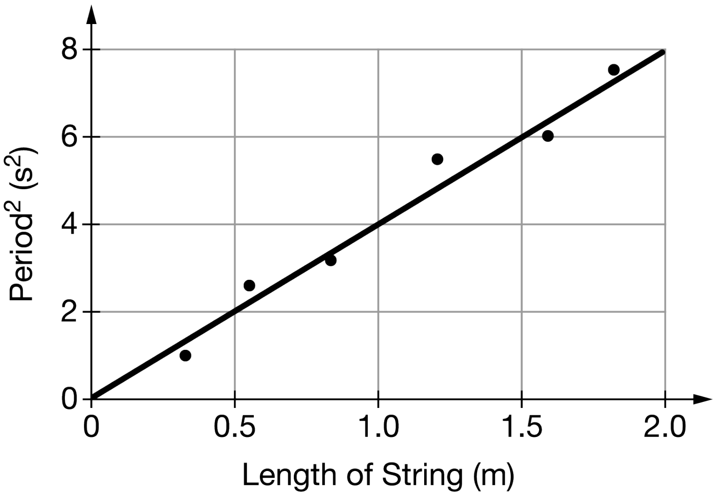
complete one oscillation.

6. Compare the period of oscillation for this experiment with the period of oscillation for trial of from the experiment described in part (a).

3.



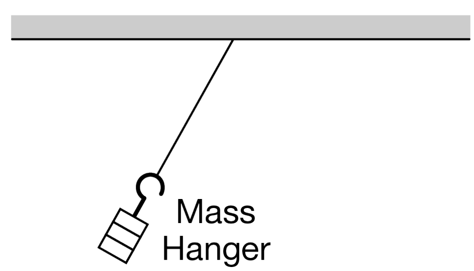
In an experiment, one end of a string is attached to an object of mass M, and the other end of the string is secured so that the object is at rest as it hangs from the string. When the object is raised to a height above its lowest point, the object undergoes simple harmonic motion. Data collected from the experiment is graphed and linearized, as shown. What information from the graph can be used to determine the acceleration due to gravity for an object that is released from rest near Earth's surface and allowed to fall to the ground?

(A) The slope of the graph represents $\frac{1}{\sqrt{g}}$.

(B) The slope of the graph represents the quantity $\frac{2\pi}{\sqrt{g}}$.

(C) The slope of the graph represents the quantity $\frac{4\pi^2}{g}$. ✓

(D) The graph cannot be used to determine the acceleration due to gravity for an object that is released from rest and allowed to fall to the ground.



A group of students must study the oscillatory motion of a pendulum. One end of a light string is attached to the ceiling, and the other end of the string is attached to a mass hanger so that small disks of various masses may be stacked on the hanger, as shown in the figure.

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4. Students are provided with data in which an experiment was conducted to determine the relationship between the length of the pendulum and the period of oscillation. The data include a pendulum of length $0.5\ \text{m}$, for which it took $81\ \text{s}$ for the pendulum bob to oscillate 10 times. However, the experiment was conducted at a location that is not near Earth's surface. The gravitational field strength where the experiment was conducted is most nearly

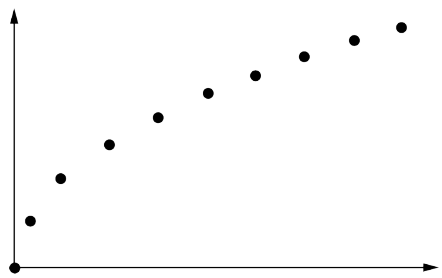
(A) $0.003\ \text{N/kg}$

(B) $0.024\ \text{N/kg}$

(C) $0.30\ \text{N/kg}$ ✓

(D) $2.40\ \text{N/kg}$

5.



The students perform four experiments, as described.

Experiment 1: Determine the relationship between the mass of the pendulum and the period of oscillation.

Experiment 2: Determine the relationship between the displacement of the pendulum and the period of oscillation.

Experiment 3: Determine the relationship between the period of oscillation and the length of the pendulum.

Experiment 4: Determine the relationship between the vertical release height of the hanger-disk system and its speed at the lowest point of its arc.

The students collect data to create the graph that is shown, but the horizontal and vertical axes are not labeled. Which experiments could be represented by the graph?

(A) Experiment 1 and Experiment 2

(B) Experiment 1 and Experiment 3

(C) Experiment 2 and Experiment 3

(D) Experiment 3 and Experiment 4 ✓

6. The students want to determine how the length of the pendulum affects the period of the pendulum. Which of the following procedures should the students use to conduct the experiment?

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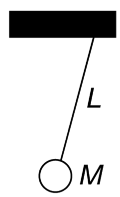
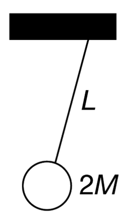
(A) Measure the length of the string with a meterstick. Place two disks on the hanger. Raise the hanger-disk system to a vertical position above its lowest point. Release the system from rest. Use a stopwatch to determine how long it takes the system to make ten oscillations. Repeat the experiment for different vertical heights.

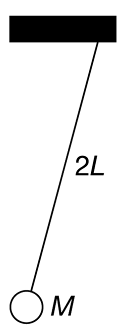
(B) Measure the length of the string with a meterstick. Place two disks on the hanger. Raise the hanger-disk system to a vertical position above its lowest point. Release the system from rest. Use a stopwatch to determine how long it takes the system to make ten oscillations. Repeat the experiment for different string lengths. ✓

(C) Measure the length of the string with a meterstick. Place two disks on the hanger. Raise the hanger-disk system to a vertical position above its lowest point. Release the system from rest. Use a stopwatch to determine how long it takes the system to make ten oscillations. Repeat the experiment for different vertical heights and string lengths.

(D) Measure the length of the string with a meterstick. Place two disks on the hanger. Raise the hanger-disk system to a vertical position above its lowest point. Release the system from rest. Use a stopwatch to determine how long it takes the system to make ten oscillations. Repeat the experiment for different string lengths and number of disks attached to the hanger.







Three different experiments are conducted that pertain to the oscillatory motion of a pendulum. For each experiment, the length of the pendulum and the mass of the pendulum are indicated. In all experiments, the pendulum is released from the same angle with respect to the vertical.

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7. If the students collect data about the kinetic energy of the pendulum as a function of time for each experiment, which of the following claims is true?

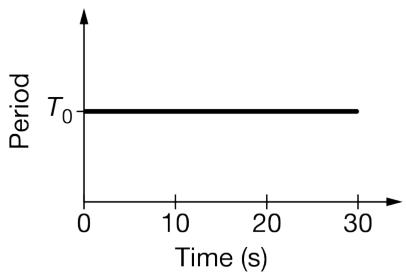
(A) The data collected from Experiment 1 will be the same as the data collected from Experiment 2.

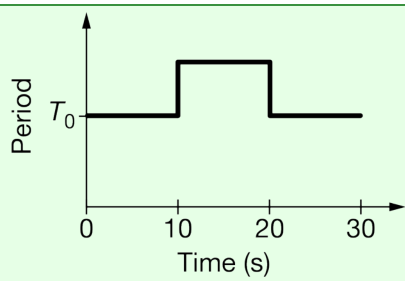
(B) The data collected from Experiment 1 will be the same as the data collected from Experiment 3.

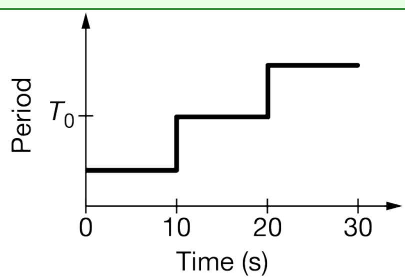
(C) The data collected from Experiment 2 will be the same as the data collected from Experiment 3.

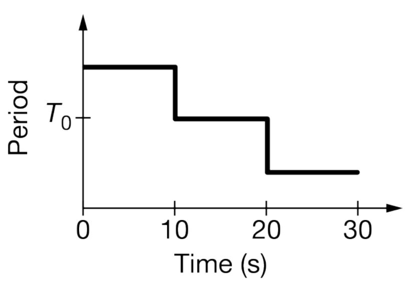
(D) The data collected from each experiment will be different.

8. The pendulum from Experiment 1 has been experimentally determined to have a period of T_0 when released from an angle θ_0. The pendulum is then attached to the ceiling of an elevator that travels upward. The pendulum is released from rest at an angle θ_0 at $t = 0$ s. From $t = 0$ s to $t = 10$ s, the elevator travels with a constant speed. From $t = 10$ s to $t = 20$ s, the elevator decreases its speed with a constant acceleration. From $t = 20$ s to $t = 30$ s, the elevator remains at rest. Which of the following graphs represents the period of the pendulum as a function of time?

(A) 

(B) 

(C) 

(D) 

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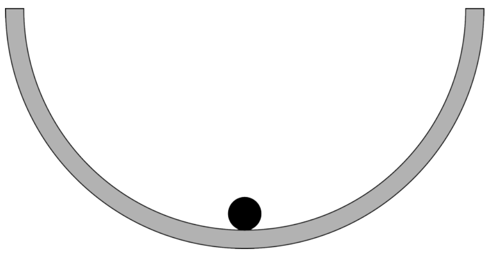
9.



A marble is placed at the bottom of a semi-spherical bowl, as shown in the figure. The marble is then displaced from the bottom of the bowl to a position about halfway from the top of the bowl. The marble is then released from rest such that the marble always remains in contact with the bowl. Students observe that the marble rolls back and forth as it oscillates about the bottom of the bowl. Which of the following statements best explains why the marble undergoes oscillatory motion?

(A) The sides of the bowl become steeper at positions farther from the bottom of the bowl.

(B) The net force exerted on the marble always has a component directed toward the bottom of the bowl. ✓

(C) The normal force exerted on the marble decreases with increasing distance from the bottom of the bowl.

(D) The gravitational force exerted on the marble is constant in magnitude and direction.

10. A student must determine how the mass of a block affects the period of oscillation when the block is attached to a vertical spring. The value of the spring constant is known. The student writes the following experimental procedure.

1. Use an electronic balance to measure the mass of the block.
2. Attach the block to the vertical spring.
3. Displace the block from the system's equilibrium position to a new vertical position.
4. Release the block from rest.
5. Use a meterstick to measure the vertical displacement of the center of mass of the block from the system's equilibrium position to its maximum vertical position above the equilibrium position.
6. Use a stopwatch to measure the time it takes for the system to make ten complete oscillations.
7. Repeat the experiment for different vertical displacements and block masses.

Which of the following steps of the procedure should the student revise to make the determination? Justify your selection.

(A) Step 3, because the student must specify whether the new vertical position should be above or below the system's equilibrium position.

(B) Step 5, because the meterstick should be used to measure total displacement of the system from its lowest vertical position to its highest vertical position.

(C) Step 6, because the stopwatch should be used only to measure the time it takes for the system to make 1 complete oscillation.

(D) Step 7, because the experiment should not be repeated for different vertical displacements and block masses. ✓

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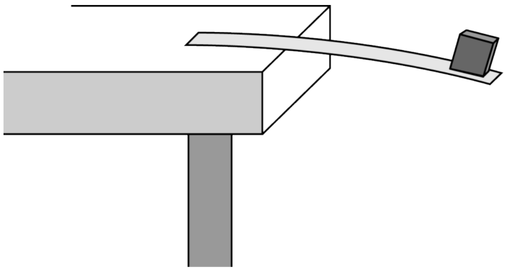
11.



Students attach a thin strip of metal to a table so that the strip is horizontal in relation to the ground. A section of the strip hangs off the edge of the table. A mass is secured to the end of the hanging section of the strip and is then displaced so that the mass-strip system oscillates, as shown in the figure. Students make various measurements of the net force F exerted on the mass as a result of the force due to gravity and the normal force from the strip, the vertical position y of the mass above and below its equilibrium position y, and the period of oscillation T when the mass is displaced by different amplitudes A. Which of the following explanations is correct about the evidence required to conclude that the mass undergoes simple harmonic motion?

(A) The period T of oscillation depends on the amplitude A of the mass, because the students can directly change this value during the experiment.

(B) The net force F exerted on the mass must be directly proportional to the vertical position y, because the net force exerted on the mass is the restoring force. ✓

(C) The mass's acceleration is proportional to the square of the vertical position y, because the elastic potential energy of the mass-strip system can be modeled by the equation for spring potential energy.

(D) The motion of the mass repeats after a specific time interval, because total mechanical energy is considered to be conserved in simple harmonic motion.

12. One end of a string is attached to an object of mass M, and the other end of the string is secured so that the object is at rest as it hangs from the string. When the object is raised to a height above its lowest point and released from rest, the object undergoes simple harmonic motion with a frequency f_0. In a second scenario, the length of the string is cut in half before the object undergoes simple harmonic motion again. What is the new frequency of oscillation of the object in terms of f_0?

(A) $\frac{1}{2} f_0$

(B) $\frac{1}{\sqrt{2}} f_0$

(C) $\sqrt{2} f_0$ ✓

(D) $2 f_0$

13. A student performs an experiment that involves the motion of a pendulum. The student attaches one end of a string to an object of mass M and secures the other end of the string so that the object is at rest as it hangs from the string. When the student raises the object to a height above its lowest point and releases it from rest, the object undergoes simple harmonic motion. As the student collects data about the time it takes for the pendulum to undergo one oscillation, the student observes that the time for one swing significantly changes after each oscillation. The student wants to conduct the experiment a second time. Which two of the following procedures should the student consider when conducting the second experiment? Select two answers.

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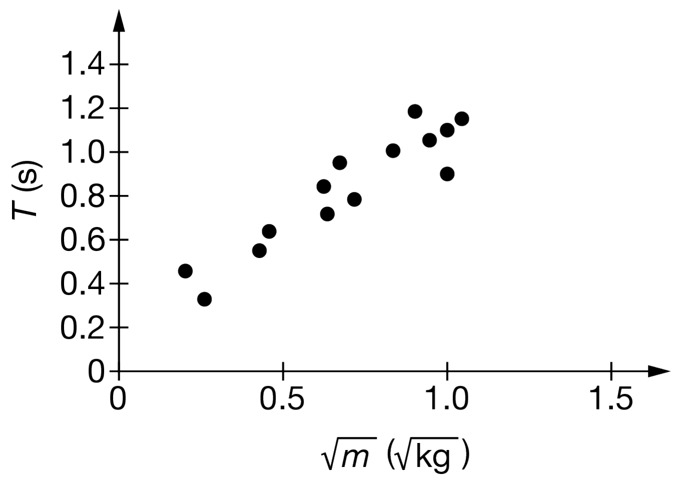
- A Make sure that the length of the string is not too long.
- B Make sure that the mass of the pendulum is not too large.
- C Make sure that the difference in height between the pendulum's release position and rest position is not too large. ✓
- D Make sure that the experiment is conducted in an environment that has minimal wind resistance. ✓

14.



A student attaches a block to a vertical spring of unknown spring constant k_0 so that the block-spring system will oscillate if the block-spring system is released from rest at a vertical position that is not the system's equilibrium position. The student varies the object's mass and uses a stopwatch to determine the time it takes the object to make one oscillation. The student creates the graph that is shown. The slope of the line of best fit is equal to which of the following quantities?

(A) $2\pi k_0$

(B) $\frac{2\pi}{k_0}$

(C) $2\pi\sqrt{k_0}$

(D) $\frac{2\pi}{\sqrt{k_0}}$ ✓

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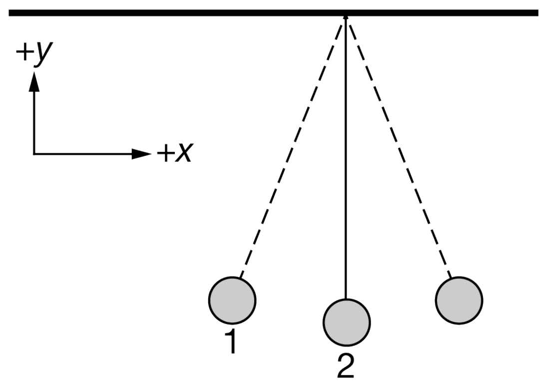
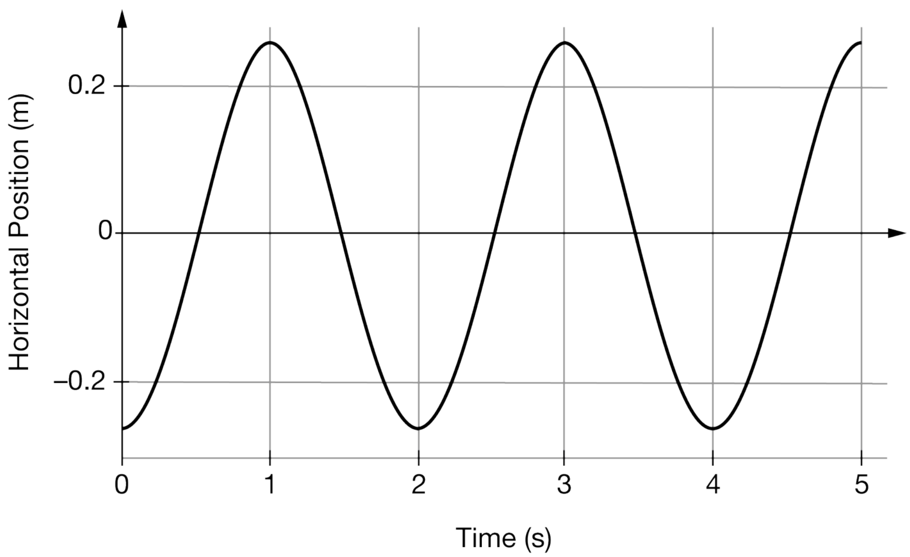
15. This question is a short free-response question. Show your work for each part of the question.





One end of a string is secured to the ceiling of a classroom, and the other end of the string is attached to a sphere of mass $0.2\ \mathrm{kg}$ so that a pendulum is established. The sphere is raised at an angle of 15° above the sphere's equilibrium position and then released from rest so that the pendulum oscillates, as shown in the figure. Location 1 is shown in the figure. A graph of the pendulum's horizontal position as a function of time is shown in the graph.

(a) The dot below represents the sphere at location 2. Draw a free-body diagram that shows and labels the forces (not components) exerted on the sphere at the location. Draw the relative lengths of all vectors to reflect the relative magnitudes of all forces.

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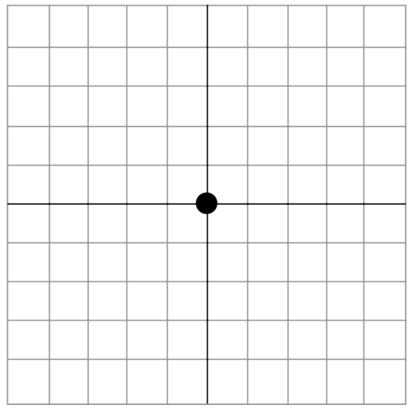
Location 2



(b) Calculate the maximum kinetic energy of the sphere as it oscillates.

The sphere is raised to a new angle above the sphere's equilibrium position that is less than the angle described for part (a). The sphere is released from rest so that the pendulum oscillates.

(c) Explain how the motion of the sphere changes, if at all, based on the appropriate evidence.

Part (a)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2

Student response accurately includes both of the following criteria.

- [ ] 1 point is earned for drawing an upward tension force and a downward force due to gravity
- [ ] 1 point is earned for drawing an upward tension force that has a larger magnitude than the force due to gravity

Example Response:

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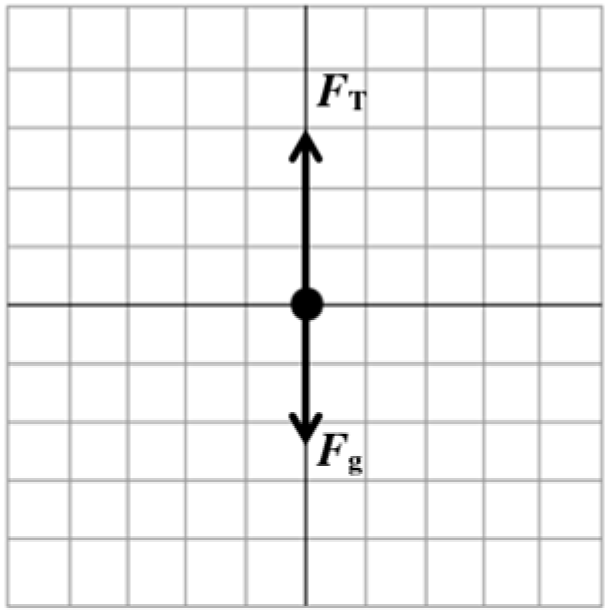

Part (b)

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✓

Student response accurately includes one of the following criteria.

Correct Answer: 0.068 J

☐ 1 point is earned for using the period of oscillation from the graph to determine the length of the string,

Example Response:

$$T_p = 2\pi\sqrt{\frac{\text{�}}{g}}$$

$$\frac{T_p}{2\pi} = \sqrt{\frac{\text{�}}{g}}$$

$$\frac{T_p{}^2}{4\pi^2} = \frac{\text{�}}{g}$$

$$\text{�} = \frac{gT_p{}^2}{4\pi^2}$$

$$\text{�} = \frac{(10\,\text{N/kg})(2.0\,\text{s})^2}{4\pi^2}$$

$$\text{�} \approx 1.0\,\text{m}$$

☐ 1 point is earned for using the correct trigonometric relationship to determine the maximum height of the sphere from its lowest position.

Example Response:

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$$\Delta y = \text{◈} - \text{◈} \cos(\theta)$$
$$\Delta y = (1.0\,\text{m}) - (1.0\,\text{m})\cos(15^{\circ})$$
$$\Delta y \approx 0.034\,\text{m}$$

- [ ] 1 point is earned for correctly applying the conservation of energy to obtain an expression for the maximum kinetic energy of the sphere.

Example Response:

$$E_0 = E_f$$
$$U_{g0} + U_{s0} + K_0 + W_{\text{External}} = U_{gf} + U_{sf} + K_f$$
$$U_{g0} = K_f$$
$$K_f = mgy_0$$
$$K_f = (0.2\,\text{kg})(10\,\text{N/kg})(0.034\,\text{m})$$
$$K_f \approx 0.068\,\text{J}$$

Part (c)

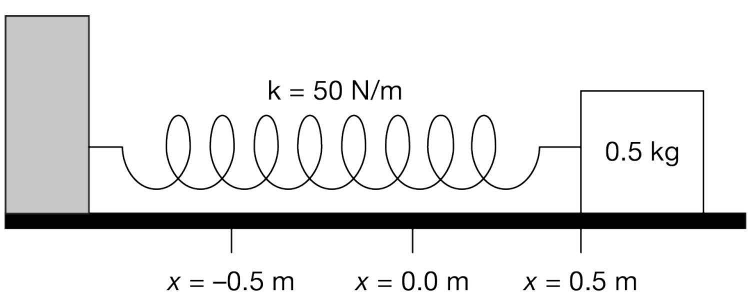
Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✓

Student response accurately includes both of the following criteria.

- [ ] 1 point is earned for stating that the period of oscillation will remain the same.
- [ ] 1 point is earned for stating that the period of oscillation for a pendulum only depends on the gravitational field strength (acceleration due to gravity) of the sphere at its location and the length of the pendulum.

16.



A block of mass $0.5\ \text{kg}$ on a horizontal surface is attached to a horizontal spring of negligible mass and spring constant $50\ \text{N}/\text{m}$. The other end of the spring is attached to a wall, and there is negligible friction between the block and the horizontal surface. When the spring is unstretched, the block is located at $x = 0\ \text{m}$. The block is then pulled to $x = 0.5\ \text{m}$ and released from rest so that the block-spring system oscillates between $x = -0.5\ \text{m}$ and $x = 0.5\ \text{m}$, as shown in the figure. Which of the following claims is correct about the block's period of oscillation?

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(A) The period would increase if the block were released from rest at $x = 0.8\ \text{m}$.

(B) The period would increase if the block had a mass of $1.2\ \text{kg}$.

(C) The period would increase if the spring had a spring constant of $75\ \text{N/m}$.

(D) The period would increase if the block-spring system was oriented vertically so that the block-spring system oscillates between $y = -0.5\ \text{m}$ and $y = 0.5\ \text{m}$, the mass of the block is $0.5\ \text{kg}$, and the spring constant is $50\ \text{N/m}$.

17. A student attaches a block to a vertical spring so that the block-spring system will oscillate if the block-spring system is released from rest at a vertical position that is not the system's equilibrium position. The system oscillates near Earth's surface. The system is then taken to the Moon's surface, where the gravitational field strength is nearly $\frac{1}{6}$ that of the gravitational field strength near Earth's surface. Which of the following claims is correct about the period of oscillation for the system?

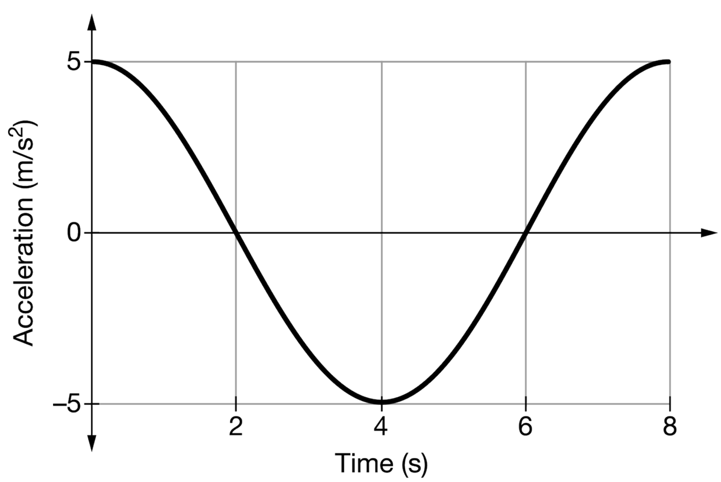
(A) The system has a longer period on Earth than on the Moon.

(B) The system has a shorter period on Earth than on the Moon.

(C) The system has the same period on Earth as the Moon.

(D) The period of oscillation cannot be determined without knowledge of the spring constant, the mass of the block, and the exact gravitational field strength near Earth's surface and the Moon's surface.

18.



A student attaches a block to a horizontal spring so that the block-spring system will oscillate if the block-spring system is released from rest at a horizontal position that is not the system's equilibrium position. The oscillatory motion of the system is described by the graph of the block's acceleration as a function of time. An identical experiment is conducted except that a new spring is used that has a spring constant that is twice the value of that of the original spring. Which two of the following predictions are correct about the system when it undergoes simple harmonic motion? Select two answers.

A The amplitude of oscillation will be increased by a factor of 2.

B The maximum acceleration of the system will be $10\ \text{m/s}^2$.

C The period of oscillation for the system will be nearly $5.7\ \text{s}$.

D The maximum spring potential energy of the system will be increased by a factor of 4.

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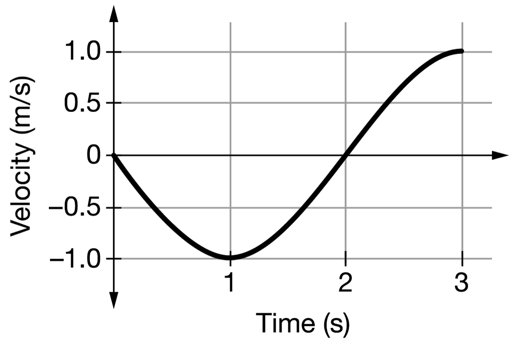
19.



A block that is connected to a horizontal spring is allowed to oscillate, and a graph of its velocity as a function of time is shown in the figure. At $t = 7$ s, the block detaches from the spring. All frictional forces are considered to be negligible. What distance will the block travel from $t = 7$ s to $t = 10$ s?

(A) 0.0 m

(B) 1.5 m

(C) 3.0 m ✓

(D) 10.0 m

20. A block of mass M on a horizontal surface is attached to a horizontal spring of negligible mass. The other end of the spring is attached to a wall, and there is negligible friction between the block and the horizontal surface. The object is displaced from the object-spring's equilibrium position and released from rest so that the system oscillates. A student collects data about the object's horizontal displacement x, velocity v_x, acceleration a_x, and the net force F_x exerted on the object as the object oscillates. Which of the following graphs, with appropriate evidence, could the student be able to create from the data to explain that the object undergoes simple harmonic motion?

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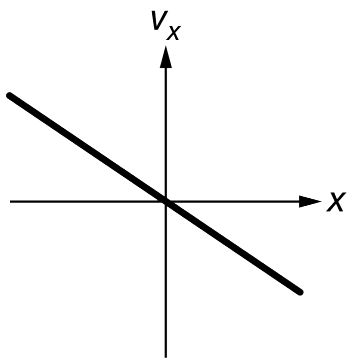
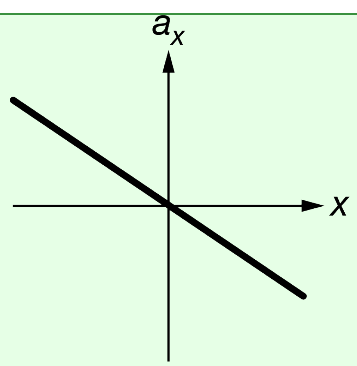
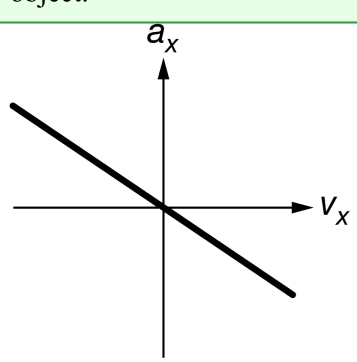
(A) 

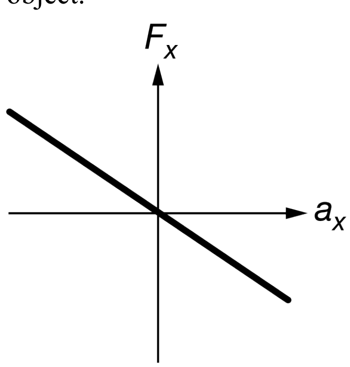
The kinetic energy and the velocity of the object are directly proportional to the displacement of the object.

(B) 

The restoring force and the acceleration of the object are directly proportional to the displacement of the object.

(C) 

The restoring force and the acceleration of the object are directly proportional to the velocity of the object.

(D) 

Springs and Simple Harmonic Motion

The restoring force is directly proportional to the acceleration of the object.