

Assignment 2

Yale Math 077

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Due: Thursday February 1, 2018 at 9:00am
—*Submit electronically via canvas*—

“I did poorly in math for a couple of years in middle school; I was just not interested in thinking about it. I can see that without being excited, mathematics can look pointless and cold. The beauty of mathematics only shows itself to more patient followers.” —Maryam Mirzakhani (1977-2017). In 2014, Professor Mirzakhani was awarded the Fields Medal for her work in “the dynamics and geometry of Riemann surfaces and their moduli spaces” [the first woman (or Iranian) to have won the award]. The Fields Medal is the highest honor in mathematics. They are awarded every four years to mathematicians under the age of forty who have made the most significant contributions to mathematics research. Professor Mirzakhani died of breast cancer in 2017.

“I don’t have any particular recipe [for finding new proofs] ... It is like being lost in a jungle and trying to use all the knowledge that you can gather to come up with some new tricks, and with some luck, you might find a way out.”

Required

As before, I have split the problems into required and optional.

- **Read** *If Mathematics Could be Painted*, which is available on canvas in files. This was written by Henry (goes by Hal) Reichard in a first-year English class. Hal is an undergrad math major here, and he and I collaborated on a summer research project ([click here for link](#)), which was recently accepted for publication [1].

- **Read** the first five pages of Lockhart's Lament (here's a link to that, and there's also a link in the canvas assignment 2 description).
- **Write** some reflection on the above readings (minimum one page). Please upload this as a separate file.

Here are required mathy-math-math problems (no length requirement):

- (1) Consider the *misère* variant of the (1, 2, 3)-subtraction game (i.e., in this game, the player who takes the last bean *loses*). As an example, say Alice and Bob play this game with 7 beans, and Alice goes first.
 - Alice first takes 1 bean [leaving 6 in the pile].
 - Bob then takes 3 beans [leaving 3 in the pile].
 - Alice then takes 2 beans [leaving 1 in the pile].
 - Bob is then forced to take the last bean, so Alice is the winner.

With 7 beans, would you prefer to go first or second? Why? In the example game, did either player make any mistakes? If so, what should they have done instead?

- (2) Consider the game Nim. (There are piles of beans. Each turn, you pick a pile and from that pile, you can remove as many beans as you like [must remove at least one, and you can't remove beans from separate piles on your turn]. The winner takes the last bean.) If there is only one pile of beans, then whoever's turn it is should win (namely just take all the beans from that pile!). **Analyze this game if there are two piles of beans.**
- (3) Pick (at least) one of the optional problems below and do it.

Optional

Feel free to jump in and do whatever you like with these. As always, off-the-wall generalizations or ideas you have are welcome, and feel free to explore whatever strikes your fancy.

- (a) Pick some subtraction-type game other than the (1,2)-subtraction game [or make up your own strategy game]. Play people in this game and try to beat them. What happened? Can you win even if you let them go first? How long would they agree to play with you before giving up in frustration (or until they figure out the strategy of how to win)?

- (b) Play and analyze a bunch of Chomp games with various initial sizes.
- (c) Analyze Nim if the game starts with three piles of sizes 1, 2, and 3.
- (d) Generalize (c).
- (e) Compare chess to tic-tac-toe.
- (f) Generalize the paper clip game from the first day.
- (g) Fully analyze any of the subtraction games we talked about in class or that you can think of.
- (h) Make up a game (in the same spirit as the ones we've been considering) and analyze it. [Last time I assigned this, one student (a psych major) submitted such an interesting (and elegant) game that she was able to get it published in a mathematics journal.]
- (i) Two players take turns putting quarters on a rectangular piece of paper, which initially has nothing on it. Each turn, a player must pick somewhere to place a quarter so that (1) it fits completely on the piece of paper, and (2) it doesn't overlap any of the other quarters already put down. [Players cannot move any quarters that are already placed.] The winner is the last person who's able to put down a quarter. Analyze this game.

References

- [1] David Brandfonbrener, Pat Devlin, Netanel Friedenberg, Yuxuan Ke, Steffen Marcus, Henry Reichard, and Ethan Sciamma. Two-vertex generators of Jacobians of graphs. *arXiv preprint arXiv:1708.03069*, 2017.