

Math 230: Vector Calculus and Linear Algebra I

Fall 2018 – Instructor: Pat Devlin

Last updated August 28, 2018

Instructor: Pat Devlin, he/him/his, Gibbs assistant professor [call me Pat]

Office hours: In LOM 222c (on the third floor)

Times to be determined by class vote; also available by appointment

Often in math lounge (Dunham lab, floor four) and happy to meet over lunch

Email: patrick.devlin@yale.edu

Course webpage: Use canvas for grades, assignments, resources, and announcements

Class Meetings: Monday, Wednesday, and Friday from 9:25am to 10:15am in LC 102.

Course Description: This class is a proof-based introduction to multivariable calculus and linear algebra, and it provides rigorous foundations to develop more advanced mathematics. It is meant to be a year-long course, divided into two parts (230 and 231). This two-course sequence is one of two routes to the math major—the other being (222 or 225) and 250. Those considering a math major should take one of these two routes (the difference being essentially personal preference with 230/231 consisting of a more concentrated time commitment).

Text: J.H. Hubbard, B.B. Hubbard *Vector Calculus, Linear Algebra, and Differential Forms* (any edition would suffice; same text to be used in 231)

Support Structure

This class was very intentionally designed to provide students with as broad a network of support as possible.

Instructor: Students are *very warmly encouraged* to come to my office hours to talk about the homework problems, the course material, or any other academic concern.

Graduate TA: Elijah Fromm (he/him/his) will hold weekly office hours, the time of which is to be determined. All students are welcome to attend.

Peer tutors: There are also three undergraduate peer tutors—Sanelma Heinonen (she/her/hers), Charlie Kenney (he/him/his), and Matt Larson (he/him/his)—who will each hold office hours for the class. This is to encourage further student collaboration and to provide students an opportunity to interact with a senior/junior math major to ask any questions about the material, the major, or Yale more generally. You are explicitly encouraged to meet with any peer tutors you want.

Diversity statement: It is important to me that this class be welcoming and inclusive for students of all identities, backgrounds, and experiences. I am especially committed to increasing the representation of populations that have been historically excluded from the study of mathematics. In order to encourage camaraderie and collaboration, this course is not graded on a curve.

I expect that students will join me in this effort to create and sustain an environment that supports all of their peers. Please let me know at any time if you have any suggestions for how the class could be improved either for you personally or for other students.

Improvement: Contrary to some beliefs, success in mathematics is not a result of inherent “brilliance” or of any particular upbringing. Instead, educational research has shown that higher mathematical thinking is a *skill* that can be acquired and dramatically improved simply through effort, time, and practice. Learning how to write mathematics and argue rigorously takes time, but it is one of the primary goals of this class. If you don’t get something right away, don’t get discouraged! Everyone feels that way.

Impostor syndrome: This is an intense course designed to make you work in a way that may be new to you; at some point, you will find yourself continuing to struggle with something despite trying your hardest. You might believe that you’re the only person who doesn’t

understand a topic or that the material just comes more naturally to everyone else. You may even get the feeling that you don't actually belong in this class, and you may feel like you're the only one who's faking it. But these feelings will not be grounded in reality.

This is a well-studied and very common phenomenon known as *impostor syndrome*, and according to psychological research as well as personal and pedagogical experience, it happens all the time. Every semester, I speak to dozens (and *dozens*) of students going through this, and even I still feel like an impostor lots of times. For me, it helps to talk about it and realize how common it is. It helps me to think of all the brilliant people who came before me feeling the exact same way. And it helps me to remember that struggling is how we grow and that perseverance in the face of difficulty is something to be proud of.

Grading

Homework: There will be weekly homework collected at the beginning of class. Please staple it.

You are encouraged to collaborate with your peers, but each student needs to submit their own work. Moreover, you must indicate on each problem with whom (if anyone) you collaborated. You should certainly feel comfortable looking up any concepts related to homework, and this is expected. But you will not look up *solutions* online, nor will you directly copy work from others. When in doubt, ask.

Quizzes: There will be several short quizzes/surveys distributed in class or through canvas.

Exams: There will be two midterms and a final exam. Midterms will take place *outside of class* starting at 7pm on October 8 and November 14. The final is December 14 starting at 2pm.

Contact me as soon as possible if you have a conflict with any of these dates.

Course grade: The course grade will be based on the grades of your quizzes (1%), weekly assignments (25%), two midterms (22% each), and the final exam (30%). In order to foster a collaborative and inclusive learning community, the course will *not* be graded on a curve.

Topics

Math 230 will focus on linear algebra, point-set topology, and differentiation. Math 231 will focus on integration, differential forms, and vector calculus culminating in the generalized Stokes's theorem. The material below essentially follows the text and will be presented roughly in this order. However, some things may be moved to 231, and some topics will be added at the instructor's discretion.

The language of mathematics. Rational, real, and complex numbers. Sets and functions. Proof techniques. Fields and spaces.

Linear transformations. Vectors and matrices. Linear transformations. Vector subspaces of \mathbb{R}^n . Dot and cross products. Triangle inequality and Cauchy-Schwarz inequality.

Introduction to topology. Closed, open and compact sets. Sequences in \mathbb{R}^n and of functions. Convergence and limits. Continuous functions. Fundamental theorem of algebra.

Calculus in \mathbb{R}^n . Differentiable functions in \mathbb{R} and \mathbb{R}^n . Derivatives in \mathbb{R}^n . The mean value theorem. Maybe Taylor series.

Linear algebra. Row reduction. Span, linear independence, and bases. Orthonormal bases. Rank of a matrix. The rank-nullity theorem. Abstract vector spaces. Diagonalizability.

Non-linear transformations in \mathbb{R}^n , The inverse function theorem. The implicit function theorem. Maybe the fixed point principle.

Manifolds. Tangent spaces. Lagrange multipliers.

Eigenvalues and eigenvectors. The spectral theorem.