

1. HOMEWORK 1; JAN 22; DUE JAN 29

(1) Let  $M$  be an  $n \times n$  matrix with  $a$  on the diagonal and  $b$  everywhere else. Compute  $\det M$ .

(2) The citizens of Oddtown change their mind and set a new set of rules. Now every club must have an even number of members and the intersection of any two different clubs must have be odd. How many clubs can they form ? (Of course we assume the town has  $n$  citizens.)

(3) A new sheriff comes to Oddtown and now everything changes again. The citizens can form  $m$  Republican and  $m$  Democratic clubs with the following rules. Let  $R_1, \dots, R_m$  be the Republican clubs and  $D_1, \dots, D_m$  Democratic clubs then  $|R_i \cap B_i|$  must be odd for all  $i$  and  $|R_i \cap B_j|$  must be even for all  $i \neq j$ . How big can  $m$  be ?

(4) You get fed up with Oddtown and move out to...,well, town modulo 4. (Not an optimal choice, as you will see, but real estate is really down and interest is low.) Not so surprisingly, the rules here are that the size of each club is not divisible by 4 and the intersection of any two different clubs is divisible by 4. How many clubs can you form ?

*Warning. This is not as easy as it sounds, as congruents mod 4 do not form a field ! Too bad, you already bought your house.*

(5) Construct an  $n \times n$  matrix with entries either 0 or 1 and determinant 15. Try to make  $n \leq 15$ .