Review of Economic Studies (2001) **68**, 593–612 0034-6527/01/00250593$02.00

Excess Worker Reallocation

GIUSEPPE MOSCARINI
Yale University

First version received November 1998; *final version accepted October* 2000 (*Eds.*)

Workers face a trade-off between *macroeconomic* and *individual incentives* to work in different occupations/industries; namely, between search frictions and personal comparative advantages. Workers endowed with heterogeneous multi-dimensional skills search for jobs that require different skill combinations. In equilibrium, specialized individuals contact few, selected types of vacancies, where they are likely to be hired; those with weak comparative advantages are seldom chosen among competing applicants, thus seek any job type. In a tight labour market, comparative advantages dominate waiting costs: offsetting labour mobility across industries/occupations—Excess Worker Reallocation—is lower and matches are more successful, consistently with direct and indirect evidence.

1. INTRODUCTION

The function of a labour market is to assign the right person to the right job. The evolution of labour market institutions in developed countries has considerably eased this task. But the increasing specialization and diversification in the range of available jobs, as well as in the required combinations of worker skills, operate in the opposite direction. As a consequence, the allocation process in labour markets is still a formidable drain on resources. Indeed, large flows of workers occur simultaneously among employment, unemployment, and non-participation (Blanchard and Diamond (1990) for the U.S.; Burda and Wyplosz (1994) for Europe), as well as across U.S. two-digit industries. "Most moves between sectors cancel out" (Jovanovic and Moffitt (1990, p. 828)). On the firm side, contemporaneous job creation and destruction of surprising dimensions typically occur within two-digit industries in many countries (Davis *et al.* (1996), and references therein).

This "churn" has recently gained considerable interest among macroeconomists, who have started to investigate its causes and properties at different frequencies (*e.g.* see Merz (1999)). Relatively little attention has been paid to one of its most interesting aspects, which emerges from a diverse but consistent range of macro and microeconometric findings. Depressed labour markets perform less well and produce "noisier" allocations: *ceteris paribus*, worker–firm matches formed at times of high unemployment are of relatively low quality. Indeed, a high unemployment rate predicts: infrequent job-to-job quits, reflecting few opportunities for better matches; a low expected tenure on the same job of newly hired workers, with entry wages moving accordingly to internalize in part this effect (Bowlus (1995)); a wide and nonselective menu of job search strategies adopted by jobless workers (Osberg (1993); Belizil (1996)); large simultaneous and offsetting worker flows across industries and occupations (Murphy and Topel (1987); Jovanovic and Moffitt (1990)). Similarly, on the firm side, U.S. manufacturing jobs created in expansions last longer than identical jobs created at times of stagnation (Davis *et al.* (1996, p. 26)).

In an accounting sense, the level of unemployment may affect "noise" in labour markets via three main channels. First, on the *labour demand* side, we learned from the work of Davis *et al.* (1996) that the individual fortunes of firms within industries vary more when overall economic activity is lower, for reasons that we are yet to fully comprehend. More severe job-idiosyncratic heterogeneity may then produce larger offsetting worker movements across industries or occupations. Second, on the *extensive margin of labour supply*, the proportion of participating workers with low industry or occupation attachment may be higher in a depressed labour market. However, all the available evidence in this respect (surveyed in Moscarini (1996)) points in the opposite direction: "marginal"—namely younger, less experienced, less educated—workers should be more likely to move randomly across jobs, and yet they experience much more cyclical participation and employment rates, hence they are less represented in the employment pool when unemployment is high. Third, on the *intensive margin of labour supply*, the need for a paycheck may overwhelm individual preferences over job types when competition for jobs is strong.

In this paper I focus on the last channel. I analyse the trade-off that job hunters face between *individual incentives*, comparative advantages to work in specific types of jobs (industries, occupations), and *macroeconomic incentives*, the cost of waiting and staying either unemployed or on the current, unsatisfactory job. The former underlie the self-selection tradition pioneered in Labour economics by Roy (1951); the latter are central to the equilibrium search literature and the "Flow Approach" to labour markets. Grafting these two traditions produces a "comparative advantage version" of the search and matching model of the labour market. Workers endowed with different bundles of skills apply to a variety of jobs that combine differently their abilities. A vacant firm observes the characteristics of each applicant—formal education, experience, probability of quitting to alternative jobs—combines them with equilibrium wages, and hires the candidate who "best fits the job", namely who produces more profits. In response to this optimal recruitment policy, workers with specialized skills search selectively and contact few vacancies, where they have very high chances of beating competing applicants. The other workers search more randomly and apply to any vacancy they hear of, driven by their need for a job and by the low acceptance rate they anticipate; hence they produce offsetting movements across job types, both job-to-job (quits) and through a phase of unemployment, that I call *Excess Worker Reallocation*.

Changes in labour market conditions, due to both sectoral and aggregate productivity or demand shocks, shift the trade-off between individual and macroeconomic considerations. A higher cost of waiting, due to low job creation and/or intense competition for jobs by more qualified workers, raises a worker's willingness to accept an offer that does not provide her with the highest possible value in the market. In turn, this weakens the worker's position in wage bargaining, so that each firm can extract a larger portion of individual productivity from its candidate employee, and thus is also willing to settle for a "second-best" worker rather than wait for a new pool of applicants. The model predicts high Excess Worker Reallocation in a depressed labour market.

The increasing dispersion in U.S. wages and European unemployment across educational groups has recently shifted the attention towards search and matching models of the labour market that incorporate heterogeneous worker skills (*e.g.* Sattinger (1995); Acemoglu (1996), Shimer (1999)) or irreversible investments by firms (Acemoglu and Shimer (1999)), implying absolute advantages to work in different jobs. Comparative advantages have been a staple of competitive labour market analysis, beginning with Roy (1951)'s seminal theory of self-selection and income distribution. Bils and McLaughlin (1992) show how far this idea goes in explaining inter-industry worker flows and wage differentials. In striking contrast, equilibrium analyses of multi-sector or occupation dynamic economies with imperfect labour mobility are rare. In Dixit and Rob (1994)'s two-sector economy, ongoing sectoral shocks hit identical workers who face exogenous mobility costs, and are then indifferent in equilibrium between moving or not. In Mortensen and Pissarides (1999) and Acemoglu (1999), a firm specifies *ex ante* a skill requirement for its vacancy and workers differ in their attitudes to fill vacancies with different requirements. This "specificity" choice is still a scalar and ordered variable, reminiscent of absolute advantages. Closer to this paper are the approaches chosen by Marimon and Zilibotti (1999) and Barlevy (1999) to address "mismatching" in a multi-dimensional setting. The comparison is taken up after illustrating our main results (see Section 4).

The paper is organized as follows. Section 2 introduces the model, Section 3 characterizes its stationary general equilibrium, Section 4 presents comparative statics predictions through numerical exercises and compares them to the results of related investigations, Section 5 illustrates the robustness of the results to alternative, noncooperative settings of the hiring game, Section 6 concludes, an Appendix contains the proofs.

2. A FRICTIONAL ROY ECONOMY

Endowments, Preferences and Technology. The economy has two sectors $i=0, 1$. There is a continuum of firms of each type i, ensuring free entry in both sectors, and a continuum of workers, with two-dimensional skill types $\mathbf{x}=(x_0, x_1)\in X\subset\Re^2_+$ distributed cross-sectionally by a *given* and known measure, which has bi-variate density or p.d.f. ψ.[1] Consumption good i is produced in 1:1 matches ("jobs") by a firm of type i and a worker $\mathbf{x}$. The match has flow output $p_i x_i$ where p_i is the price of good $i=0, 1$.[2] Worker $\mathbf{y}$ has a *comparative advantage* in job 1 over job 0 if $p_1 y_1 > p_0 y_0$, a *relative comparative advantage* w.r. to worker $\mathbf{x}$ in job 1 over job 0 if $y_1/y_0 > x_1/x_0$, and is *more skilled* than worker $\mathbf{x}$ if $\mathbf{y}\geqq\mathbf{x}$, *i.e.* $y_i\geqq x_i$ with at least one strict inequality. Figure 1 depicts an example of iso-density lines of the skill distribution and representative isoquants for the two types of job, clearly parallel to the axes.

Exogenous idiosyncratic productivity shocks force the closure of active production units in sector i at Poisson rate $\delta_i > 0$.

When jobless, worker $\mathbf{x}$ enjoys a flow value of leisure $b(\mathbf{x})$.

Assumption 1. *$x_i - b(x_0, x_1)$ is increasing in x_i for any given x_{1-i}.*

More skilled workers have a larger wedge of productivity over the opportunity cost of labour, thus a less elastic labour supply. Moscarini (1996) shows that, in a variety of wage-setting environments, this seemingly innocuous assumption is necessary and sufficient for more productive workers to be weakly more attractive to firms in equilibrium, consistently with the evidence that high-wage and skilled workers exit faster from unemployment. Knowing this, to simplify the algebra I assume $b(\mathbf{x})=0$ independent of types.

Workers and firms have risk-neutral preferences over the two consumption goods and maximize expected utility discounted at rate $r>0$. By risk neutrality, the equilibrium

1. In order to illustrate graphically the main arguments I restrict attention to the simplest case: two types of jobs (sectors, occupations) and two types of worker skills. So workers are represented by points in a plane, the coordinates being the levels of the two types of sector- or occupation-specific skills.

2. More generally, we may posit a production function f_i such that a worker with skill profile $\mathbf{a}$ produces a flow output $p_i f_i(\mathbf{a})$ of good i. Then, just relabel $x_i = f_i(\mathbf{a})$ and re-define the distribution of skills accordingly. Hence skills and job types can be taken in equal number without loss in generality.

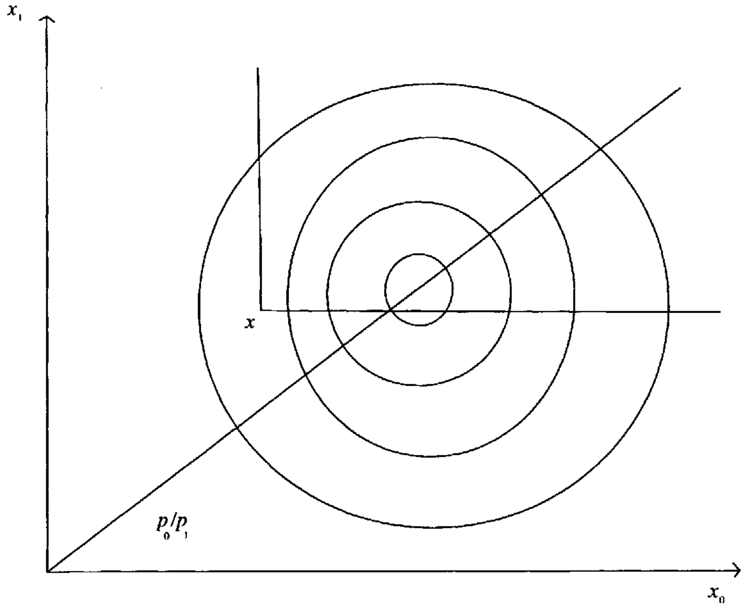

FIGURE 1

relative price of the two goods p_1/p_0 must equal their relative weight in preferences, as long as both goods are produced in positive amounts.

The matching process: Application and hiring. Matching of workers with firms (jobs) occurs in two sequential phases: first, workers receive news about open vacancies and send a job application; second, the firm selects one of the possibly multiple applicants who respond to its vacancy announcement.

An idle firm may post one i-vacancy at flow cost $\kappa_i > 0$; the measure of posted i-vacancies is denoted by v_i. Workers face no explicit search costs. A worker locates a specific open i-vacancy at probability rate s_i, when jobless, and at rate $\alpha_{1-i}s_i$, $\alpha_{1-i} \in [0, 1)$, while employed in sector $1-i$: employed search is feasible but less effective. If strictly profitable, she applies to that vacancy, revealing her skills to the firm. As we will see, there is nothing to gain from applying to another job in the same sector. Given the Poisson process for the arrival of news, there is zero chance that a worker simultaneously locates and applies to more than one vacancy. The arrival rate of news s_i about the v_i open vacancies is a continuous and increasing function of v_i, $s_i = s(v_i)$, independent of the mass of potential applicants, because knowledge about open vacancies is a non-rival good.

Congestion emerges in the subsequent hiring phase. Each open i-vacancy receives with some probability a (not necessarily representative) sample from the pool of applications to i-jobs, and only one applicant can be chosen to fill the job. Let $h_i(\mathbf{x})$ denote the *conditional acceptance rate*, the probability for worker $\mathbf{x}$ of being hired by a i-firm, conditional on successfully contacting an open i-vacancy. Firm i rationally selects the applicant $\mathbf{x}$ who guarantees it the highest rents (PDV of profits) $J_i(\mathbf{x})$. Let $z_i(\cdot)$ denote the unnormalized skill density of unemployed and employed in sector $1-i$ who actually apply to i-jobs. Then $h_i(\mathbf{x})$ is the probability that an open i-vacancy receives no applications from the pool of workers who are more profitable than worker $\mathbf{x}$. The measure of this pool, per open vacancy, is:

$$\sigma_i(\mathbf{x}) \equiv \frac{s(v_i)}{v_i} \int_{\{\xi_0, \xi_1 : J_i(\xi_0, \xi_1) \geqq J_i(\mathbf{x})\}} z_i(\xi_0, \xi_1) d\xi_0 d\xi_1 .$$

The conditional hiring rate for worker $\mathbf{x}$ is necessarily a *decreasing* function $H: \Re_+ \to [0, 1]$ of this measure $\sigma_i(\mathbf{x})$:

$$h_i(\mathbf{x}) = H(\sigma_i(\mathbf{x})), \tag{2.1}$$

because a worker $\mathbf{x}$ who is more profitable to i-firms than $\mathbf{y}$ $(J_i(\mathbf{x}) \geqq J_i(\mathbf{y}))$ expects less competition for the job she applies to $(\sigma_i(\mathbf{x}) \leqq \sigma_i(\mathbf{y}))$ and is more likely to be hired first $(h_i(\mathbf{x}) \geqq h_i(\mathbf{y}))$. For example, $H(t) = e^{-t}$ in Butters (1979)'s Large Number version of the urn-ball scheme. I also assume H continuous. When indifferent between hiring at all or not, a firm hires.

Combining the two stages of matching, the exit rate from unemployment to i-jobs $e_i(\mathbf{x})$ equals the application rate $s(v_i)$ times the conditional acceptance rate $h_i(\mathbf{x})$. For employed workers the analogous quitting rate from $1-i$ to i jobs is simply $\alpha_{1-i} e_i(\mathbf{x})$.

Wage setting. Due to search frictions, the match develops rents that the parties, unable by assumption to commit to an *ex ante* agreement, split through the wage according to a generalized Nash bargaining rule. In addition, parties cannot make lump-sum side-payments at the hiring stage, prior to bargaining. In Section 5 both restrictions are relaxed, and it is shown that they simplify the analysis without affecting qualitatively the equilibrium allocation.

3. EQUILIBRIUM

Absence of search frictions can be defined as a situation in which exit rates to i jobs both from unemployment, $e_i(\mathbf{x}) = s_i h_i(\mathbf{x})$, and from employment in jobs $1-i$, $\alpha_{1-i} e_i(\mathbf{x})$, are infinite for every worker $\mathbf{x}$. In this scenario, we obtain Roy (1951)'s celebrated model of self-selection and income distribution. The frictionless competitive equilibrium exists, is unique and easily described. Firms are the long side of the market, and workers sort according to comparative advantages: for each type of job the equilibrium wage exhausts output $w_i^*(\mathbf{x}) = p_i x_i$, and worker $\mathbf{x}$ chooses the job that maximizes her wage. The ray with slope p_1/p_2 in Figure 1 illustrates the worker self-selection policy. This allocation is a very instructive benchmark for the stationary equilibrium of the frictional economy. First, I characterize individual optimal policies for job search, tenure, and creation, given exit rates; second, aggregate conditions pin down the skill distribution of exit rates and (un)employment as a function of these policies.

3.1. *Bellman equations*

Let $U(\mathbf{x})$ denote the value to a worker $\mathbf{x}$ of being unemployed and searching, $\mathscr{W}_i(\mathbf{x})$ the total discounted returns from holding a job i, and $W_i(\mathbf{x}) = \max\langle \mathscr{W}_i(\mathbf{x}), U(\mathbf{x})\rangle$ the corresponding Bellman value. Let $\gamma_i(\mathbf{x}) \in [0, 1]$ denote worker $\mathbf{x}$ (mixed) strategy of applying to a vacancy i located while searching on the job. The rental cost of being on this job $(r\mathscr{W}_i(\mathbf{x}))$ equals the sum of: the flow wage $w_i(\mathbf{x})$, the capital loss $U(\mathbf{x}) - W_i(\mathbf{x}) \leqq 0$ following exogen ous separation at rate δ_i, the capital gain $\max\langle W_{1-i}(\mathbf{x})-\mathscr{W}_i(\mathbf{x}),0\rangle$ from quitting to sector $1-i$, which accrues at rate $\alpha_i\gamma_i(\mathbf{x})e_{1-i}(\mathbf{x})$

$$r\mathscr{W}_i(\mathbf{x})=w_i(\mathbf{x})+\delta_i[U(\mathbf{x})-\mathscr{W}_i(\mathbf{x})]+\alpha_i\gamma_i(\mathbf{x})e_{1-i}(\mathbf{x})\max\langle W_{1-i}(\mathbf{x})-\mathscr{W}_i(\mathbf{x}),0\rangle. \quad (3.1)$$

Clearly $\gamma_i(\mathbf{x})=1$ if $\mathscr{W}_i(\mathbf{x})<W_{1-i}(\mathbf{x})$. The value of unemployed job search solves

$$rU(\mathbf{x})=e_0(\mathbf{x})[W_0(\mathbf{x})-U(\mathbf{x})]+e_1(\mathbf{x})[W_1(\mathbf{x})-U(\mathbf{x})]. \quad (3.2)$$

Similarly, let $\mathscr{J}_i(\mathbf{x})$ denote total expected discounted profits accruing to a firm i employing a worker $\mathbf{x}$, V_i the value of the vacancy, and $J_i(\mathbf{x})=\max\langle V_i,\mathscr{J}_i(\mathbf{x})\rangle$ the Bellman value of the job. The rental cost to the firm of this job $r\mathscr{J}_i(\mathbf{x})$ equals the sum of flow output p_ix_i net of the wage $w_i(\mathbf{x})$ and of the capital loss $V_i-\mathscr{J}_i(\mathbf{x})\leqq 0$ due to a separation. The latter may be either exogenous, at rate δ_i, or endogenous, as the worker quits to the other sector at probability rate $\alpha_i\gamma_i(\mathbf{x})e_{1-i}(\mathbf{x})$ whenever profitable:

$$r\mathscr{J}_i(\mathbf{x})=p_ix_i-w_i(\mathbf{x})+[\delta_i+\alpha_i\gamma_i(\mathbf{x})e_{1-i}(\mathbf{x})\Im_{W_i(\mathbf{x})\leqq W_{1-i}(\mathbf{x})}](V_i-\mathscr{J}_i(\mathbf{x})) \quad (3.3)$$

where $\Im_A$ is the indicator function of a set or event A. Free entry of firms in both sectors ensures at all times $V_i=0$ for $i=0,1$.

Summing up: the worker searches in market i iff $\mathscr{W}_i(\mathbf{x})=W_i(\mathbf{x})>U(\mathbf{x})$, is hired there at rate $e_i(\mathbf{x})$, and quits to the other sector as soon as possible iff $W_i(\mathbf{x})<W_{1-i}(\mathbf{x})$. The firm employs worker $\mathbf{x}$ as long as $\mathscr{J}_i(\mathbf{x})$ exceeds the zero value of search, so that $J_i(\mathbf{x})=\mathscr{J}_i(\mathbf{x})$ solves (3.3); otherwise, it rejects worker $\mathbf{x}$ and posts a new vacancy.

The generalized Nash bargaining solution for the flow wage yields the worker a fraction $\beta\in[0,1]$ of the joint match surplus:

$$J_i(\mathbf{x})=\frac{1-\beta}{\beta}[W_i(\mathbf{x})-U(\mathbf{x})]. \quad (3.4)$$

After some algebra, (3.4) and the arbitrage equations (3.1), (3.2) and (3.3) reveal that the flow wage equals a fraction β of the flow outcome of production (p_ix_i) plus the expected returns from job search, the worker's threat point:

$$W_i(\mathbf{x})=\beta[p_ix_i+\textstyle\sum_{k=0,1}e_k(\mathbf{x})J_k(\mathbf{x})-\gamma_i(\mathbf{x})\alpha_ie_{1-i}(\mathbf{x})J_{1-i}(\mathbf{x})\Im_{J_i(\mathbf{x})<J_{1-i}(\mathbf{x})}]. \quad (3.5)$$

Interestingly, if $\mathbf{x}$ prefers the other type of job $1-i$ to her current job i, then she keeps searching on the job $(\gamma_i(\mathbf{x})>0)$ and compensates her current employer of type i for this opportunity through a wage reduction $-\beta\gamma_i(\mathbf{x})\alpha_ie_{1-i}(\mathbf{x})J_{1-i}(\mathbf{x})$.

Finally, I can use (3.5) to replace the wage out of (3.3) and link the rents produced by the worker in the two sectors:

$$\mathscr{J}_i(\mathbf{x})=\frac{(1-\beta)p_ix_i-\beta e_{1-i}(\mathbf{x})(1-\alpha_i\gamma_i(\mathbf{x})\Im_{\mathscr{J}_i(\mathbf{x})\leqq\mathscr{J}_{1-i}(\mathbf{x})})\max\langle\mathscr{J}_{1-i}(\mathbf{x}),0\rangle}{r+\delta_i+\beta e_i(\mathbf{x})+e_{1-i}(\mathbf{x})\alpha_i\gamma_i(\mathbf{x})\Im_{\mathscr{J}_i(\mathbf{x})\leqq\mathscr{J}_{1-i}(\mathbf{x})}}. \quad (3.6)$$

Notice that the first term in the numerator is always positive, so the only reason for not accepting a job i is that the other sector looks much better: $\mathscr{J}_i(\mathbf{x})\leqq 0$ implies $\mathscr{J}_{1-i}(\mathbf{x})$ positive and large enough. Therefore, each worker seeks at least one type of job: $\mathscr{J}_i(\mathbf{x})>0$ for at least one $i=0,1$.

As usual under the axiomatic solution (3.4) separation is consensual when the worker quits to unemployment: $J_i(\mathbf{x})=V_i=0\Leftrightarrow W_i(\mathbf{x})=U(\mathbf{x})$. A new implication instead obtains when a worker quits a profitable job i $(W_i(\mathbf{x})-U(\mathbf{x})>0)$ for an even better one $1-i$ and obtains a gain $W_{1-i}(\mathbf{x})-W_i(\mathbf{x})>0$. In fact the quit is not consensual: now the current employer loses $J_i(\mathbf{x}) \propto W_i(\mathbf{x}) - U(\mathbf{x}) > 0$. However the Nash solution (3.4) *does* imply that the new employer obtains higher profits: $W_{1-i}(\mathbf{x}) > W_i(\mathbf{x}) \Rightarrow J_{1-i}(\mathbf{x}) > J_i(\mathbf{x})$ and therefore would always win a bidding war. Yet the current i-employer could make anyway a counteroffer to her quitting employee $\mathbf{x}$ to help her obtain a higher wage from the new firm $1-i$, for example a signing bonus, and then split the extra surplus. Such a collusive behaviour cannot arise because, by assumption, the worker cannot commit to transfer *ex post* some of the extra surplus to the old employer. Then the latter has no *ex post* incentives (and an *ex ante* strict disincentive) to make a counteroffer that would just raise the worker's returns to quit, the last value in the numerator of (3.6), and erode own rents $\mathcal{J}_i(\mathbf{x})$. By the same logic, the worker may hope to gain nothing from searching in the same sector of employment.

3.2. *Who searches for whom?*

Equations (3.6) for $i=0,1$ form a non-linear system that can be solved by a guess-and-verify method for *given values of the exit rates* $e_i(\mathbf{x})$. Namely, WLOG let $\mathcal{J}_i(\mathbf{x}) \leqq J_{1-i}(\mathbf{x})$: since $J_i(\mathbf{x}) + J_{1-i}(\mathbf{x}) > 0$, assume $\mathcal{J}_i(\mathbf{x}) \leqq 0 < \mathcal{J}_{1-i}(\mathbf{x})$ to obtain in (3.6) a linear system for these two unknown values, solve it and check *ex post* that the solution satisfies the assumed inequality. Else, assume $0 < \mathcal{J}_i(\mathbf{x}) \leqq \mathcal{J}_{1-i}(\mathbf{x})$, solve the resulting new linear system and again check the guess *ex post*. We infer that the system may have only two types of solutions and corresponding search behaviours, and states explicitly the condition that a skill profile must satisfy to follow one behaviour or the other.

Lemma 1 (Cutoff Search Policies). *For any* $\mathbf{x} \in X$, $i=0,1$, *let*:

$$g_i(\mathbf{x}) \equiv r + \delta_i + \beta e_i(\mathbf{x}); \qquad \lambda_i(\mathbf{x}) \equiv \frac{g_i(x)}{\beta e_i(\mathbf{x})(1-\alpha_{1-i})} \in (1, \infty).$$

Suppose that worker $\mathbf{x}$ *chooses the search strategy* $\{\gamma_i(\mathbf{x})\}_{i=0,1}$ *that maximizes her expected value of search* $r^{-1}\sum_{i=0}^{1} e_i(\mathbf{x})[W_i(\mathbf{x}) - U(\mathbf{x})]$. *Then there are two possibilities.*

1. Selective search in sector *i*. *For* $p_i x_i / p_{1-i} x_{1-i} > \lambda_i(\mathbf{x})$ *the system* (3.6) *has a unique solution*: $\gamma_i(\mathbf{x}) = \gamma_{1-i}(\mathbf{x}) = 0$,

$$J_i^{s_i}(\mathbf{x}) = (1-\beta)\frac{p_i x_i}{g_i(\mathbf{x})} > 0,$$

$$\mathcal{J}_{1-i}^{s_i}(\mathbf{x}) = \frac{(1-\beta)p_{1-i}x_{1-i} - \beta e_i(\mathbf{x})(1-\alpha_{1-i})J_i^{s_i}(\mathbf{x})}{g_{1-i}(\mathbf{x}) + e_i(\mathbf{x})\alpha_{1-i}} \leqq 0.$$

Therefore worker $\mathbf{x}$ *seeks only i-jobs from unemployment and, when employed, she stops searching (the superscript* s_i *stands for "Selective in sector i").*

2. Random search with stop-gap in sector $1-i$. *For* $p_i x_i / p_{1-i} x_{1-i} \in (\rho_i(\mathbf{x}), \lambda_i(\mathbf{x}))$, *a non-empty interval,* (3.6) *has a unique solution*: $\gamma_i(\mathbf{x}) = 0, \gamma_{1-i}(\mathbf{x}) = 1$,

$$J_i^{q_i}(\mathbf{x}) = (1-\beta)\frac{p_i x_i[g_{1-i}(\mathbf{x}) + e_i(\mathbf{x})\alpha_{1-i}] - \beta e_{1-i}(\mathbf{x})p_{1-i}x_{1-i}}{g_i(\mathbf{x})[g_{1-i}(\mathbf{x}) + e_i(\mathbf{x})\alpha_{1-i}] - \beta^2 e_1(\mathbf{x})e_0(\mathbf{x})(1-\alpha_{1-i})} > 0$$

$$J_{1-i}^{q_i}(\mathbf{x}) = (1-\beta)\frac{p_{1-i}x_{1-i}g_i(\mathbf{x}) - \beta e_i(\mathbf{x})(1-\alpha_{1-i})p_i x_i}{g_i(\mathbf{x})[g_{1-i}(\mathbf{x}) + e_i(\mathbf{x})\alpha_{1-i}] - \beta^2 e_1(\mathbf{x})e_0(\mathbf{x})(1-\alpha_{1-i})} \in (0, J_i^{q}(\mathbf{x}))$$

where $\rho_i(\mathbf{x}) = 1/\rho_{1-i}(\mathbf{x})$ *is the unique cutoff such that* $p_i x_i = p_{1-i} x_{1-i} \rho_i(\mathbf{x})$ *solves* $\sum e_k(\mathbf{x}) J_k^{q_i}(\mathbf{x}) = \sum e_k(\mathbf{x}) J_k^{q_{1-i}}(\mathbf{x})$ *as an identity in* $\mathbf{x}$. *Hence, such a jobless worker* $\mathbf{x}$ *seeks both types of jobs and, if hired in sector* $1-i$, *keeps searching for i-jobs, but not vice versa (the superscript* q_i *stands for "Quit to sector i").*

The proof illustrates also the knife-edge case of types who meet exactly the threshold test $p_i x_i = \lambda_i(\mathbf{x}) p_{1-i} x_{1-i}$, and then are indifferent and randomize ($\gamma_i(\mathbf{x}) \in (0, 1)$).

Two remarks are in order. From the definition of $\rho_i(\mathbf{x})$ it is easy to verify that $\rho_i(\mathbf{x}) \neq 1$, because destruction rates δ_i, on-the-job search rates α_i and exit rates $e_i(\mathbf{x})$ differ between sectors. Hence the frictional "indifference locus" $p_i x_i = p_{1-i} x_{1-i} \rho_i(\mathbf{x})$ does not coincide with the Walrasian–Roy one $p_0 x_0 = p_1 x_1$. However, as expected, the two loci do coincide if the frictional economy becomes frictionless for $e_i \to \infty$. Second, the inequalities $1/\lambda_{1-i}(\mathbf{x}) < 1 < \lambda_i(\mathbf{x})$ and $1/\lambda_{1-i}(\mathbf{x}) < \rho_i(\mathbf{x}) < \lambda_i(\mathbf{x})$ valid for all $\alpha_i \in [0, 1)$ together prove the following:

Proposition 1 (Mismatch). *Whenever job-finding rates* $e_i(\mathbf{x})$ *are finite for all workers* $\mathbf{x} \in X$, *the set of skills who search in both sectors is non-empty and contains the frictionless (Roy) indifference locus* $p_0 x_0 = p_1 x_1$. *A positive mass of workers are hired in the sector where they do not produce the highest rents and wage available in the market, and then search on the job in the other sector: they are on the "wrong side" of the frictionless line.*

This is just a partial characterization, because the solution for the values $J_i(\mathbf{x})$ in Lemma 1 depend on the endogenous cutoffs $\lambda_i(\mathbf{x})$ and $\rho_i(\mathbf{x})$, which depend on the exit rates $e_i(\mathbf{x}) = s(v_i) h_i(\mathbf{x})$ which in turn depend on the values $J_i(\mathbf{x})$ via (2.1). The final step is to resolve this circularity and ask which types of workers satisfy each condition in Lemma 1 and therefore how firms rank applicants by their exogenous skill characteristics.

The answer depends on the relative supply of skills. Even in a world of one-dimensional worker heterogeneity Blanchard and Diamond (1994) show that in equilibrium firms may deem equally profitable two workers that they strictly order by productivity. Their analysis sheds some light on the present one, of which it is a special case. In the present notation they have only one sector, say 1, one-dimensional types $\mathbf{x} = x_1$ and rents

$$J_1(x_1) = \frac{(1-\beta) p_1 x_1}{r + \delta_1 + \beta e_1(x_1)}.$$

By a simple contradiction argument the chance of finding a job $e_1(x_1)$, weakly increasing in $J_1(x_1)$ by the firms' hiring strategy, cannot be declining in productivity x_1. However, if this chance $e_1(\cdot)$ rises across skills sufficiently fast due to the relative scarcity of good workers, then the outside option of some $y_1 > x_1$ can be so high to make y_1 no more attractive than x_1 for the vacant 1-firm. In this case the firm randomizes and the equilibrium entails "Stochastic Ranking", otherwise "Strict Ranking".

In this bi-dimensional world skills are not ordered and the definition of Ranking needs to be adapted. A worker who is more productive in a job can be ranked second by the firm if a much stronger comparative advantage to work in the other sector sustains her outside option and wage.

Lemma 2 (Monotonicity of Values in Skills). *An increase in i-productivity* x_i *for a given* x_{1-i} *raises weakly the value* $J_i(\mathbf{x})$ *produced by the worker in sector i and makes her more likely to search there, both from unemployment and from employment in sector* $1-i$. *An increase in* x_{1-i} *for a given* x_i *has the opposite effects.*

By continuity of the value J_i in skills, an obvious consequence of (2.1) and Lemma 1, this claim holds even if one simultaneously raises x_i and reduces x_{1-i} slightly: *comparative*, not absolute advantages determine the direction of job search and the ranking of job applicants. Following the literature, I will say that the equilibrium features Strict Ranking if *strict* monotonicity of values in skills obtains in Lemma 2. In this case

Proposition 2 (Specialization of skills and search behaviour with strict ranking). *In any equilibrium with Strict Ranking, the worker population partitions into three non-empty connected regions: selective searchers in each of the two sectors, who have strong comparative advantages as defined by the thresholds $\lambda_i(\mathbf{x})$, and random searchers, who have weak comparative advantages and include the frictionless line $p_1x_1 = p_0x_0$. In turn, each random searcher who finds a job in the "wrong" market, namely a job $1-i$ when $x_i > \rho_i(\mathbf{x})x_{1-i}$, keeps seeking from employment a job i for which she has a stronger comparative advantage.*

The possibility that the equilibrium does indeed feature Strict Ranking depends on the parameters of the economy. The next section illustrates one such case.

Propositions 1 and 2 provide a fairly complete cross-sectional characterization of individual search policies, ranking at the hiring stage, exit rates and wages, as illustrated in Figure 2. For example, $\mathbf{y}$ is mismatched if she finds a 1-job first, and accepts it, which is rational from an individual point of view. If Strict Ranking does not hold, then the sets of indifferent workers are no longer one-dimensional curves as in Figure 2 but rather two-dimensional regions; other than that, from Lemma 1 the allocation is the same as in the Strict Ranking case.

It is interesting to notice the *self-reinforcing* nature of job search selection, originating from forward-looking behaviour and job competition. Suppose a worker may credibly

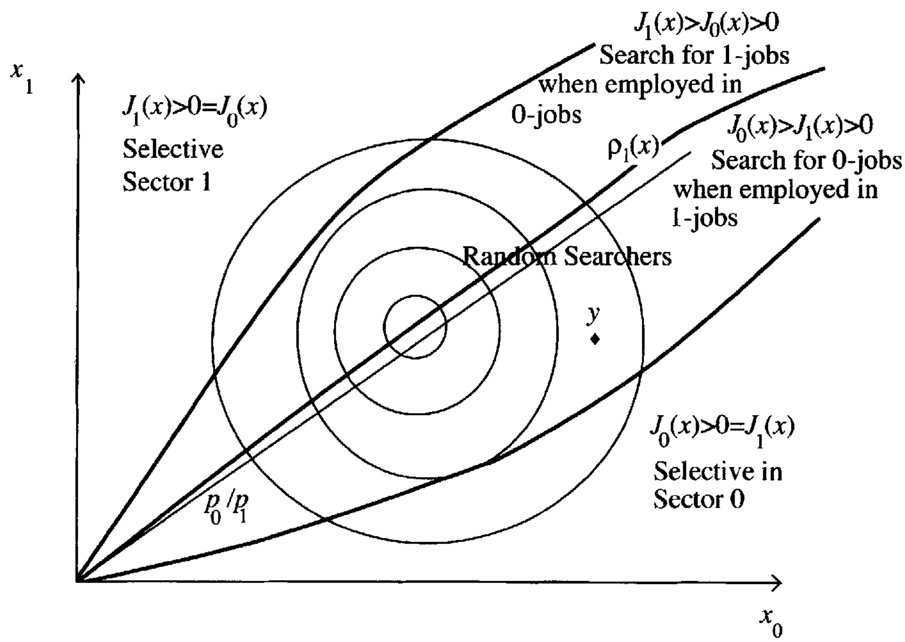

FIGURE 2

Selective and non-selective searchers

commit to be selective and to restrict her job search to sector 1 only. This commitment restricts her range of employment chances, weakens her outside option, and makes her very attractive to firms in sector 1. This fact in turn raises her chance of beating the competition for 1-jobs, thus the incentives to take only those jobs and to commit to search selectively in the first place. Conversely, a non selective search strategy erodes the match surplus by raising the outside option, making it harder to beat the competition for any job; this effect *per se* provides incentives not to be selective.

3.3. *Wages and search outcomes*

In practice individual skills are partially unobservable to the econometrician, so the previous results cannot be directly tested. The same observation applies to other variables, such as search effort and skill specificity, emphasized by other authors. In this framework skills can be mapped into wages to obtain a prediction on the correlation between wages and search behaviour. The only detailed evidence in this regard is provided by Thomas (1998), who finds in a sample of British workers that a lower wage worker contacts a significantly larger number of vacancies and yet experiences a smaller exit rate from unemployment, implying an even smaller success rate of her job applications. Hence "wait unemployment", due to an excess selectivity in job search, should not be a source of concern because observed mostly at the upper end of the wage distribution. This is exactly what the present model predicts.

Proposition 3 (Wages and search behaviour). *In each sector i wages paid to employed workers are increasing in productivity x_i for any given x_{1-i}. Therefore wages are correlated negatively with job contact rates and positively with job acceptance rates.*

The proof is simple. If we do not use (3.2), recalling $\gamma_i(\mathbf{x}) \in \{0, 1\}$ with Strict Ranking, the wage equation (3.5) reads

$$w_i(\mathbf{x}) = \beta p_i x_i + (1-\beta) r U(\mathbf{x}) - (1-\beta)\alpha_i \gamma_i(\mathbf{x}) e_{1-i}(\mathbf{x}) W_{1-i}(\mathbf{x}) \mathfrak{I}_{W_i(\mathbf{x}) < W_{1-i}(\mathbf{x})}.$$

The wage rises in x_i for a given x_{1-i} through three channels, corresponding to the three terms on the RHS. First, directly through output sharing; second, through the outside option $U(\mathbf{x})$, which increases in each x_i by (3.2), (3.4) and Lemma 2; third, if the worker is mismatched and searches on the job ($\gamma_i(\mathbf{x}) = 1$), through the reduction in the expected value of employment in the other sector, namely $e_{1-i}(\mathbf{x}) W_{1-i}(\mathbf{x})$, due to the comparative advantage effect (Lemma 2 again).[3]

3.4. *Aggregation and general equilibrium*

We may finally aggregate the optimal individual policies derived so far and impose market-clearing and stationarity to close the general equilibrium of this economy. Let $m_i(\cdot)$

3. Evidence on employed versus unemployed job search (Pissarides and Wadsworth (1994) for the U.K., Blau and Robins (1990) for the U.S., Belizil (1996) for Canada) consistently indicates that the former produces fewer contacts but more job offers, and is more effective for high-wage, mature, long-tenured workers. Also, employed search takes place mostly through personal contacts, typically a selective search strategy, while the unemployed rely more upon job centres and public agencies, which are less concerned with targeting specific types of jobs.

denote the unnormalized density of workers employed in sector i. By Large Numbers, the skill density of applicants to i jobs defined in Section 2 equals

$$z_i(\mathbf{x}) = [\psi(\mathbf{x}) - m_0(\mathbf{x}) - m_1(\mathbf{x})]\Im_{p_i x_i \lambda_{1-i}(\mathbf{x})/p_{1-i}x_{1-i} > 1} + m_{1-i}(\mathbf{x})\alpha_{1-i}\gamma_{1-i}(\mathbf{x})\Im_{\rho_i(\mathbf{x}) < p_i x_i/p_{1-i}x_{1-i} < \lambda_i(\mathbf{x})}, \tag{3.7}$$

because of the $\psi(\mathbf{x}) - m_0(\mathbf{x}) - m_1(\mathbf{x})$ unemployed by Lemma 1 only those with $p_i x_i/p_{1-i}x_{1-i} > 1/\lambda_{1-i}(\mathbf{x})$ produce positive rents $J_i(\mathbf{x}) > 0$ and then participate to market i; and, of the $m_{1-i}(\mathbf{x})$ employed in sector $1-i$, only those satisfying $\rho_i(\mathbf{x}) < p_i x_i/p_{1-i}x_{1-i} < \lambda_i(\mathbf{x})$ produce $J_i(\mathbf{x}) > J_{1-i}(\mathbf{x}) > 0$ and thus keep searching for i jobs. If we denote by $z_i \equiv \int_X z_i(\mathbf{y})d\mathbf{y}$ the total mass of effective searchers for i jobs, $z_0 + z_1$ exceeds total unemployment both due to employed search and to simultaneous search in both markets by a fraction of the unemployed.

For each skill type $\mathbf{x}$ the flow of workers hired by i firms, either from unemployment or from the other sector $1-i$, equals the flow of exogenous separations and quits to i:

$$z_i(\mathbf{x})e_i(\mathbf{x}) = [\delta_i + \alpha_i\gamma_i(\mathbf{x})e_{1-i}(\mathbf{x})\Im_{p_i x_i/p_{1-i}x_{1-i} < \rho_i(\mathbf{x})}]m_i(\mathbf{x}). \tag{3.8}$$

The model is closed by free entry conditions in each sector. The unnormalized skill density of applications that a i-vacancy expects to receive is $z_i(\mathbf{x})s(v_i)/v_i$, because the $z_i(\mathbf{x})s(v_i)$ applications sent by $\mathbf{x}$ workers land randomly on v_i identical open i-vacancies. The total mass of applications per unit vacancy is therefore $z_i s(v_i)/v_i$, and this is also the rate at which applications accrue to an open i-vacancy. By free entry the flow cost κ_i of posting the vacancy equals this chance times the expected value of the filled job

$$\kappa_i = \frac{s(v_i)z_i}{v_i}\int_X J_i(\mathbf{x}) \cdot \frac{z_i(\mathbf{x})s(v_i)/v_i}{z_i s(v_i)/v_i} \cdot h_i(\mathbf{x})d\mathbf{x}.$$

$$= \frac{s(v_i)}{v_i}\int_X J_i(\mathbf{x})z_i(\mathbf{x})h_i(\mathbf{x})d\mathbf{x}. \tag{3.9}$$

A stationary Rational Expectations Equilibrium of this economy is a pair (for $i = 0, 1$) of scalars $v_i \in \Re_+$ and vector functions $\{z_i, m_i, J_i, e_i, h_i, \gamma_i\}$: $X \to \Re_+^5 \times [0, 1]$ solving $e_i(\cdot) = s(v_i)h_i(\cdot)$, (2.1), (3.6), (3.7), (3.8) and (3.9).

4. LABOUR MARKET TIGHTNESS AND MISMATCH

The analysis has been conducted so far at some level of generality, with unspecified skill density ψ, news function s, and matching function H, in order to establish a few robust equilibrium properties of this economy, as illustrated in Propositions 1–3. In this section a numerical exercise illustrates the comparative statics effects of changes in relative sector productivities on the steady state equilibrium.

A cursory look at the thresholds in Lemma 1 reveals that an increase in the success rate of job search, $e_i(\mathbf{x})$, makes worker $\mathbf{x}$ more willing to search selectively, and reduces the potential for mismatch. Intuitively, in a tighter labour market where job search duration is lower, so are the cost of waiting for better offers and the incentive to take stop-gap jobs, thus individual comparative advantages tend to prevail.

This phenomenon, mentioned in the Introduction, is illustrated in general equilibrium by the following numerical example. The reference period is one quarter. The economy has four types of workers, $\mathbf{x} = A, B, C, D$, of measure normalized to one and distribution $\psi(A) = \psi(B) = 0{\cdot}1$, $\psi(C) = \psi(D) = 0{\cdot}4$. Skill endowments are $A = (1, 12)$, $B = (12, 1)$, $C =$

(2·5, 3), and $D = (3, 2.5)$. The strong specialization of A and B as opposed to C and D is needed to obtain a Strict Ranking equilibrium. The interest rate is 1%, the bargaining share $\beta = 1/2$, job destruction rates are $\delta_0 = \delta_1 = 0{\cdot}1$ so a job lasts on average 10 quarters. For lack of any guide in the literature I choose an isoelastic news function, $s(v) = s \cdot \sqrt{v}$. To obtain empirically plausible magnitudes for aggregate observables I set $s = 12$, on-the-job search effectiveness $\alpha_1 = \alpha_0 = 0{\cdot}11$, and job creation costs $\kappa_0 = \kappa_1 = 2{\cdot}5$, which are in between total expected profits obtained from specialized and non-specialized employees. The hiring function is the exponential $h_i(\mathbf{x}) = e^{-\sigma_i(\mathbf{x})}$ derived from the urn-ball scheme of Butters (1979).

In the resulting stationary equilibrium type A specialize search and employment in sector 1, B in sector 0, C are indifferent between searching selectively in sector 1 and randomly, and *vice versa* for D. A fraction of unemployed $C(D)$ workers search randomly, and when they happen to get a job in sector 0(1) they keep searching for 1 (0) jobs, creating congestions for random $D(C)$ searchers. In sector 1 firms prefer the specialized type A to the less specialized type C, which in turn prevails over mismatched type D. The resulting system of equations is solved by GAUSS Nonlinear Equations module.

TABLE 1

(*All numbers are in* /100)

		$p_0 = 1$, $p_1 = 1$	$p_0 = 1$, $p_1 = 1{\cdot}01$	$p_0 = 1{\cdot}01$, $p_1 = 1{\cdot}01$
Employment in sector 0	A	0	0	0
by worker skill group	B	9·829	9·826	9·831
mismatched →	C	2·271	0·187	1·690
	D	36·278	34·056	36·970
Employment in sector 1	A	9·829	9·832	9·831
by worker skill group	B	0	0	0
	C	36·278	37·888	36·970
mismatched →	D	2·271	5·060	1·690
Unemployment rate	A	1·71	1·68	1·69
	B	1·71	1·74	1·69
	C	3·62	4·81	3·35
	D	3·62	2·25	3·35
	Total	10·66	10·48	10·08
Percentage of the unemployed	C	11·765	9·034	11·029
who search in both sectors	D	11·765	19·744	11·029
Job-to-job quits	0 to 1	0·52	0·06	0·39
	1 to 0	0·52	1·89	0·39

Table 1 illustrates the effects of changes in the p_i's—which have been specified as taste parameters, but could equally well capture sector-wide technology level—given a fixed value of leisure. The comparison of columns 1 and 3 in Table 1 illustrates the effects of a 1% aggregate change in productivity, while the comparison of columns 1 and 2 those of a reallocative shock, raising productivity by 1% in sector 1 only. The size of the mismatched employed labour force shrinks as productivity rises; but the reallocative shocks moves even more type D workers towards the "wrong" sector 1. A reallocative shock in sector 1 makes type-1 jobs more attractive, so relatively 1-specialized workers of type C find it optimal to wait longer for them, and their unemployment rises; conversely, 0-specialized workers of type D accept more often mismatched 1-jobs that are no longer so bad, until they create enough congestion to restore indifference between job types, and their unemployment falls. Quits are lower in a tight labour market and higher in an asymmetric one (after the reallocative shock). This is not at odds with the evidence on procyclical quits because we are comparing steady states. As the measure of mismatched workers declines from the first to the third column, it must be the case that in the transition a two-sided reallocation of employment takes place, in part through quits. The table shows that, once the economy has settled on a stationary equilibrium with higher labour demand, there are fewer mismatched workers and therefore fewer quits.

It is instructive to confront these predictions with a few related results. On the theoretical side, two recent papers by Marimon and Zilibotti (1999) and Barlevy (1999) also address the issue of "mismatching" between supply and demand of *ex ante* heterogeneous and multidimensional worker skills. In both contributions workers and firms have symmetric characteristics located on a circle and comparative advantages to join closer partners, every worker is relatively specialized in some type of job, precluding a distinction between "weak" and "strong" comparative advantages. Furthermore, jobless workers and firms meet randomly, without simultaneous applications to the same vacancy, hence all unemployed face the same labour market prospects, independently of skills. Marimon and Zilibotti show that unemployment insurance may raise in equilibrium average matching quality, through the reduction in the cost of "wait" unemployment. More closely related is Barlevy's focus on the effects of labour market tightness on mismatching. Unemployed workers (mis)match randomly and, once employed, spend on-the-job search effort to find a new match and maximize flow output. As in the previous simulations, the extent of mismatching co-moves with unemployment, but for different reasons: in a depressed labour market, employed workers search *less hard* for a better job, so the outflow from mismatching declines relative to the inflow. Here, unemployed and employed workers alike become less choosy and search in a *wider range of markets*, sacrificing their individual specialization in order to land some job, so the inflow rises relative to the outflow.

The scant available evidence on the relationship between labour market tightness and excess worker reallocation is uniformly consistent with the predictions of the numerical exercise. Jovanovic and Moffitt (1990) compute from the National Longitudinal Survey of Young Men at two-year intervals (1966–1981) a measure of gross mobility G_t, the fraction of employed individuals who change a broadly defined sector of the economy in a given year, controlling for aging, and a measure of net mobility $N_t \equiv \sum_i(|\Delta n_{it}|/2n_{it})$, where n_{it} is employment in industry i at time t. Their results show that the G_t/N_t ratio has co-moved substantially with unemployment both in the 1971 and 1974–76 slumps and in the following recoveries. Fallick (1993) documents that *displaced* workers tend to remain attached to the industry of last employment, but also search more intensively in industries that exhibit above-average recent employment growth. Since the dispersion of industry employment growth rates is countercyclical (Lilien (1982)), the foregone wage cost of continuing job search relative to the sacrifice of comparative advantages must increase in a recession. Osberg (1993) reports that the number of search methods adopted by jobless workers in a Canadian sample is countercyclical, expressing harsher competition for jobs at times of high unemployment (1983) as compared to times of lower unemployment (1981 and, especially, 1986). More importantly, according to the same evidence, direct job search methods, such as direct applications and personal contacts, are procyclical, while the resort to public and private employment agencies, suggesting more random strategies, is *strongly* countercyclical.

5. BIDDING FOR JOBS

The Nash bargaining solution for wages captures many important feedbacks among labour market variables while maintaining tractability of the model, and is in fact common in the equilibrium search literature. This cooperative solution may appear problematic at the hiring stage of this model, where more than two heterogeneous agents may happen to interact: one or two firms (in case of job-to-job quits) and two or more applicants. When losing a job i to competing applicants, worker $\mathbf{x}$ loses the foregone net value of the job $W_i(\mathbf{x}) - U(\mathbf{x}) > 0$, hence one would expect that she would attempt to underbid more skilled or specialized applicants. Similarly, a firm suffers a discrete loss when its employee quits to another job. This section illustrates the robustness of previous results to two noncooperative solutions of the hiring game. The basic intuition behind this robustness is compelling and originates from Assumption 1: more specialized workers have more to offer to their "elective" employer and less to lose from giving up search in the other sector, thus always prevail in the hiring competition, however this is framed by the firm.

Shimer (1999) investigates competition for a vacancy among multiple applicants endowed with private and heterogeneous one-dimensional skills. The firm sets up a "Job Auction" with a credible reserve bid, the value of the vacancy, and assigns the job to the highest bidder. In equilibrium, the most skilled applicant always wins, "buys the firm" and enjoys the output stream net of the bid, but her "wage" is affected by the potential bids of other applicants. We will see that this result breaks down when specialization, rather than ability *per se* matters: a worker $\mathbf{x}$ may lose the i-job auction to a less i-productive competitor $\mathbf{y}$ if $\mathbf{x}$'s comparative advantage to work in the other sector relative to $\mathbf{y}$ is so strong to deter $\mathbf{x}$ from bidding enough for a job i. A standard assumption in the auction literature is the commitment of all parties to the auction outcome; this becomes less compelling when the relationship between seller (firm) and buyer (worker) continues over time beyond the auction. I illustrate two different outcomes that obtain in this framework depending on the extent of commitment. To simplify the algebra I dispose of employed search, which plays the same role as before.

First, maintain the usual commitment (or enforceability) assumption. I assume that the firm offers to job applicants a second-price auction with a $V_i = 0$ minimum bid, a robust mechanism as bidding the own valuation $W_i(\cdot) - U(\cdot)$ is a dominant strategy whether or not competitors' skills are private information.[4] Facing this mechanism in both markets, worker $\mathbf{x}$ expects to pay for the two jobs the expected second-highest valuations, say $\tau_0(\mathbf{x})$ and $\tau_1(\mathbf{x})$, which are interdependent: τ_i affects the continuation value of search and therefore the value of a job $1-i$ and in turn τ_{1-i}, and *vice versa*. The chance $h_i(\tau_i)$ of winning the job and being hired increases in the valuation for the job, thus in the payment τ_i. As before denote the exit rate of unemployment to sector i by $e_i(\mathbf{x}) = s(v_i)h_i(\tau_i(\mathbf{x})) = s(v_i)h_i(\mathbf{x})$, which is now the rate at which the worker participates to and wins an i-job auction. The values of employment and unemployment solve the system

$$\begin{aligned} rU(\mathbf{x}) &= \textstyle\sum_{i=0}^{1} e_i(\mathbf{x}) \max\langle W_i(\mathbf{x}) - \tau_i(\mathbf{x}) - U(\mathbf{x}), 0\rangle \\ rW_i(\mathbf{x}) &= p_i x_i + \delta_i[U(\mathbf{x}) - W_i(\mathbf{x})]. \end{aligned} \tag{5.1}$$

This has a unique solution, of one of three possible types, corresponding to: selective search in sector i, for $i = 0, 1$, and random search.[5]

4. The auction environment is non-standard for two reasons: valuations for jobs and distribution of participating types are endogenous, and applicants may ignore the *number* as well as the identity of the competitors. The optimality of a second-price auction for the seller in these circumstances goes beyond the scope of this paper.

5. A first-price auction yields the same equations, where now $\tau_i(\mathbf{x})$ is the optimal bid maximizing the expected net surplus from the job times the chance of winning.

For i-selective behaviour we require $W_{1-i}(\mathbf{x}) < U(\mathbf{x}) < W_i(\mathbf{x}) - \tau_i(\mathbf{x})$, which is self-explanatory. Using these inequalities in (5.1) yields an explicit solution for $U(\mathbf{x})$, which can be used to check *ex post* the guess $W_{1-i}(\mathbf{x}) < U(\mathbf{x})$. This is confirmed iff

$$p_{1-i}x_{1-i} < rU(\mathbf{x}) = e_i(\mathbf{x})\frac{p_i x_i - (r+\delta_i)\tau_i(\mathbf{x})}{r+\delta_i+e_i(\mathbf{x})}.$$

This is the new "endogenous" cutoff for selective search, corresponding to $\lambda_i(\mathbf{x})$ in Lemma 1. By simple algebra this inequality can hold at most for one $i = 0, 1$; otherwise a worker would be selectively searching in both markets, an oxymoron. If this inequality fails for both $i = 0, 1$ then search is random, $W_i(\mathbf{x}) - \tau_i(\mathbf{x}) - U(\mathbf{x}) > 0$, and from (5.1) I can solve for the corresponding equilibrium values.

In order to map skills into equilibrium values and expected payments observe that, first, a more skilled worker always enjoys a larger value of unemployment, because she could always choose the bids of a less skilled type and produce the same output by free disposal: $\mathbf{y} \geqq \mathbf{x}$ implies $U(\mathbf{y}) \geqq U(\mathbf{x})$. Second, this fact and (5.1), rearranged to read $W_i(\mathbf{x}) = [p_i x_i + \delta_i U(\mathbf{x})](r+\delta_i)^{-1}$, together imply that a more skilled worker produces a higher gross value in *both* sectors: $\mathbf{y} \geqq \mathbf{x}$ implies $W_i(\mathbf{y}) \geqq W_i(\mathbf{x})$. The ordering of net values $W_i(\cdot) - U(\cdot)$ along the lines of Lemma 2 and Proposition 2 is therefore nontrivial, and proven in the Appendix.

Proposition 4 (Equilibrium with job auctions). *In any equilibrium with job auctions the worker population partitions into three non-empty connected regions: selective searchers in each of the two sectors, encompassing skills with very strong comparative advantages, and random searchers, who have weak comparative advantages and include the frictionless indifference line $p_1 x_1 = p_0 x_0$. Payments and exit rates for a job i $(1-i)$ are increasing (resp. decreasing) in skill i for every given skill $1-i$.*

The second case of interest concerns a job auction without commitment: the winning bid pays only for the right to form a match and start bargaining with the employer on the resulting surplus, divided either by Nash bargaining or in an alternating offer game (see Osborne and Rubinstein 1994, Chapter 15 for the equivalence). It is easy to see how the cooperative allocation of Section 3 may survive to this type of competition at the hiring stage. Assume that there is no recall, and impose a "trembling hand" refinement, to introduce an arbitrarily small cost of participating to the auction and break ties: payment of the winning bid takes $T > 0$ time, and $T \downarrow 0$. Due to the costly delay $T > 0$ and to the winning bidder's credible threat to pay and get the job, losing bidders have no incentive to stick around, and go back to job search right away. Without commitment the winner after T reneges on her bid and proposes to work for and bargain with the firm. The firm has no reason to reject this proposal nor the winning bid, if this gets paid to eliminate other competitors who deviate and stick around. This sequential equilibrium of the auction/bargaining game generates simple Bellman equations; straightforward but tedious (and thus omitted) algebra shows that, as the delay T vanishes, this outcome converges to the cooperative allocation of Section 3.4. In summary, without commitment to upfront promises, as soon as the match is formed parties are shielded from outside competition by search frictions, and revert to bargaining.[6]

6. With commitment, the winner would pay the bid after $T > 0$, and as $T \downarrow 0$ we would obtain back Proposition 4. The comparison between the two cases reveals that no bidder, winner or loser, has any *ex ante* incentive to commit to its bid, because this would just reduce her equilibrium payoffs.

6. CONCLUDING REMARKS

Ex ante comparative advantages in individual worker skills add a fundamental dimension to the search-matching models of the labour market. This paper illustrates a formal positive analysis along these lines. Workers face a trade-off between macroeconomic incentives and individual comparative advantages to search for different types of jobs. More specialized workers tend to be more selective in their job search, because it is relatively costlier to accept a mismatch and sacrifice search for better jobs. In addition, this choosy attitude is self-reinforcing, as it entails giving up to alternatives in other sectors and thus becoming very attractive to the "elective" employers. Conversely, workers with weak comparative advantages rush to accept any job; this strategy enhances their outside option and makes them less likely to be hired anywhere, thus reinforcing the rationale for accepting any job offer. At times of low unemployment the cost of waiting for the "perfect" job declines and this trade-off shifts in favour of individual comparative advantages, consistently with a wealth of empirical evidence on mismatch and worker mobility.

The analysis calls for further investigation on (at least) two dimensions. First, it is performed on purely positive grounds and neglects normative implications of equilibrium mismatching. Second, it abstracts from *ex post* heterogeneity in the form of accumulated (on-the-job) firm-specific human capital. A substantial part of mobility costs is indeed represented by the loss of on-the-job training or knowledge about match success. More generally, this analysis has ignored the effects of comparative advantages on *entry* into unemployment, traditionally related to firm-specific factors.

APPENDIX

Proof of Lemma 1. The system (3.6) for $i=0,1$ may have three types of solution.

1. $J_i(\mathbf{x})>0=J_{1-i}(\mathbf{x})\geqq\mathscr{J}_{1-i}(\mathbf{x})$. This is true when $J_i(\mathbf{x})=\mathscr{J}_i(\mathbf{x})=J_i^{s_i}(\mathbf{x})$ and

$$0\geqq\mathscr{J}_{1-i}(\mathbf{x})=\frac{(1-\beta)p_{1-i}x_{1-i}-\beta e_i(\mathbf{x})(1-\alpha_{1-i})J_i^{s_i}(\mathbf{x})}{r+\delta_{1-i}+\beta e_{1-i}(\mathbf{x})+e_i(\mathbf{x})\alpha_{1-i}},$$

namely $\beta e_i(\mathbf{x})(1-\alpha_{1-i})J_i^{s_i}(\mathbf{x})\geqq(1-\beta)p_{1-i}x_{1-i}$, which is equivalent to $p_ix_i/p_{1-i}x_{1-i}>\lambda_i(\mathbf{x})$ as claimed.

2. $J_i(\mathbf{x})>J_{1-i}(\mathbf{x})=\mathscr{J}_{1-i}(\mathbf{x})>0$, *i.e.*

$$J_i(\mathbf{x})=\frac{(1-\beta)p_ix_i-\beta e_{1-i}(\mathbf{x})J_{1-i}(\mathbf{x})}{r+\delta_i+\beta e_i(\mathbf{x})}, \tag{A.1}$$

$$\begin{aligned}J_{1-i}(\mathbf{x})&=\frac{(1-\beta)p_{1-i}x_{1-i}}{g_{1-i}(\mathbf{x})+e_i(\mathbf{x})\alpha_{1-i}}-\frac{\beta e_i(\mathbf{x})(1-\alpha_{1-i})}{g_{1-i}(\mathbf{x})+e_i(\mathbf{x})\alpha_{1-i}}J_i(\mathbf{x})\\&=\frac{(1-\beta)p_{1-i}x_{1-i}}{g_{1-i}(\mathbf{x})+e_i(\mathbf{x})\alpha_{1-i}}-\frac{\beta e_i(\mathbf{x})(1-\alpha_{1-i})}{g_{1-i}(\mathbf{x})+e_i(\mathbf{x})\alpha_{1-i}}\left[\frac{(1-\beta)p_ix_i-\beta e_{1-i}(\mathbf{x})J_{1-i}(\mathbf{x})}{g_i(\mathbf{x})}\right]\\&=(1-\beta)\frac{p_{1-i}x_{1-i}g_i(\mathbf{x})-\beta e_i(\mathbf{x})(1-\alpha_{1-i})p_ix_i}{[g_{1-i}(\mathbf{x})+e_i(\mathbf{x})\alpha_{1-i}]g_i(\mathbf{x})-\beta^2e_i(\mathbf{x})e_{1-i}(\mathbf{x})(1-\alpha_{1-i})},\end{aligned}$$

and then substituting back into (A.1)

$$\begin{aligned}J_i(\mathbf{x})&=(1-\beta)\frac{p_ix_i}{g_i(\mathbf{x})}\left[1+\frac{\beta^2e_i(\mathbf{x})e_{1-i}(\mathbf{x})(1-\alpha_{1-i})}{[g_{1-i}(\mathbf{x})+e_i(\mathbf{x})\alpha_{1-i}]g_i(\mathbf{x})-\beta^2e_i(\mathbf{x})e_{1-i}(\mathbf{x})(1-\alpha_{1-i})}\right]\\&\quad-(1-\beta)\frac{\beta e_{1-i}(\mathbf{x})p_{1-i}x_{1-i}}{[g_{1-i}(\mathbf{x})+e_i(\mathbf{x})\alpha_{1-i}]g_i(\mathbf{x})-\beta^2e_i(\mathbf{x})e_{1-i}(1-\alpha_{1-i})(\mathbf{x})}\\&=(1-\beta)\frac{p_ix_i[g_{1-i}(\mathbf{x})+e_i(\mathbf{x})\alpha_{1-i}]-\beta e_{1-i}(\mathbf{x})p_{1-i}x_{1-i}}{[g_{1-i}(\mathbf{x})+e_i(\mathbf{x})\alpha_{1-i}]g_i(\mathbf{x})-\beta^2e_i(\mathbf{x})e_{1-i}(\mathbf{x})(1-\alpha_{1-i})}.\end{aligned}$$

The premise $J_i > J_{1-i}$ is true if

$$p_i x_i[g_{1-i}(\mathbf{x}) + e_i(\mathbf{x})\alpha_{1-i}] - \beta e_{1-i}(\mathbf{x})p_{1-i}x_{1-i} > p_{1-i}x_{1-i}g_i(\mathbf{x}) - \beta e_i(\mathbf{x})(1-\alpha_{1-i})p_i x_i$$

$$p_i x_i[g_{1-i}(\mathbf{x}) + e_i(\mathbf{x})\alpha_{1-i} + \beta e_i(\mathbf{x})(1-\alpha_{1-i})] > p_{1-i}x_{1-i}[\beta e_{1-i}(\mathbf{x}) + g_i(\mathbf{x})]$$

$$\Leftrightarrow \frac{p_i x_i}{p_{1-i}x_{1-i}} > \frac{\beta e_{1-i}(\mathbf{x}) + g_i(\mathbf{x})}{g_{1-i}(\mathbf{x}) + e_i(\mathbf{x})\alpha_{1-i} + \beta e_i(\mathbf{x})(1-\alpha_{1-i})} \equiv \mu_i(\mathbf{x}).$$

3. $J_i(\mathbf{x}) = J_{1-i}(\mathbf{x}) > 0$ *i.e.* for $i = 0, 1$

$$J_i(\mathbf{x}) = \frac{(1-\beta)p_i x_i - \beta e_{1-i}(\mathbf{x})(1-\alpha_i\gamma_i(\mathbf{x}))J_{1-i}(\mathbf{x})}{r + \delta_i + \beta e_i(\mathbf{x}) + e_{1-i}(\mathbf{x})\alpha_i\gamma_i(\mathbf{x})}.$$

Solving for

$$\begin{aligned}
J_i(\mathbf{x}) &= \frac{(1-\beta)p_i x_i}{g_i(\mathbf{x}) + e_{1-i}(\mathbf{x})\alpha_i\gamma_i(\mathbf{x})} - \frac{\beta e_{1-i}(\mathbf{x})(1-\alpha_i\gamma_i(\mathbf{x}))}{g_i(\mathbf{x}) + e_{1-i}(\mathbf{x})\alpha_i\gamma_i(\mathbf{x})} J_{1-i}(\mathbf{x}) \\
&= \frac{(1-\beta)p_i x_i}{g_i(\mathbf{x}) + e_{1-i}(\mathbf{x})\alpha_i\gamma_i(\mathbf{x})} - \frac{\beta e_{1-i}(\mathbf{x})(1-\alpha_i\gamma_i(\mathbf{x}))}{g_i(\mathbf{x}) + e_{1-i}(\mathbf{x})\alpha_i\gamma_i(\mathbf{x})} \cdot \frac{(1-\beta)p_{1-i}x_{1-i}}{g_{1-i}(\mathbf{x}) + e_i(\mathbf{x})\alpha_{1-i}\gamma_{1-i}(\mathbf{x})} \\
&\quad + \frac{\beta e_{1-i}(\mathbf{x})(1-\alpha_i\gamma_i(\mathbf{x}))}{g_i(\mathbf{x}) + e_{1-i}(\mathbf{x})\alpha_i\gamma_i(\mathbf{x})} \cdot \frac{\beta e_i(\mathbf{x})(1-\alpha_{1-i}\gamma_{1-i}(\mathbf{x}))}{g_{1-i}(\mathbf{x}) + e_i(\mathbf{x})\alpha_{1-i}\gamma_{1-i}(\mathbf{x})} J_i(\mathbf{x}) \\
&= \frac{(1-\beta)p_i x_i[g_{1-i}(\mathbf{x}) + e_i(\mathbf{x})\alpha_{1-i}\gamma_{1-i}(\mathbf{x})] - (1-\beta)p_{1-i}x_{1-i}\beta e_{1-i}(\mathbf{x})(1-\alpha_i\gamma_i(\mathbf{x}))}{\prod_{i=0}^{1}[g_i(\mathbf{x}) + e_{1-i}(\mathbf{x})\alpha_i\gamma_i(\mathbf{x})] - \beta^2\prod_{i=0}^{1} e_i(\mathbf{x})(1-\alpha_{1-i}\gamma_{1-i}(\mathbf{x}))},
\end{aligned}$$

and symmetrically for $J_{1-i}(\mathbf{x})$. Since they have the same denominator, $J_i(\mathbf{x}) = J_{1-i}(\mathbf{x})$ when the numerators are equal, namely after some algebra

$$\frac{p_i x_i}{p_{1-i}x_{1-i}} = \frac{g_i(\mathbf{x}) + e_{1-i}(\mathbf{x})\alpha_i\gamma_i(\mathbf{x}) + \beta e_{1-i}(\mathbf{x})(1-\alpha_i\gamma_i(\mathbf{x}))}{g_{1-i}(\mathbf{x}) + e_i(\mathbf{x})\alpha_{1-i}\gamma_{1-i}(\mathbf{x}) + \beta e_i(\mathbf{x})(1-\alpha_{1-i}\gamma_{1-i}(\mathbf{x}))}$$

The RHS is increasing in $\gamma_i(\mathbf{x}) \in [0,1]$ and decreasing in $\gamma_{1-i}(\mathbf{x}) \in [0,1]$, so this may hold only if:

$$\frac{g_i(\mathbf{x}) + \beta e_{1-i}(\mathbf{x})}{g_{1-i}(\mathbf{x}) + e_i(\mathbf{x})\alpha_{1-i} + \beta e_i(\mathbf{x})(1-\alpha_{1-i})} \equiv \mu_i(\mathbf{x}) \leqq \frac{p_i x_i}{p_{1-i}x_{1-i}} \leqq \frac{1}{\mu_{1-i}(\mathbf{x})}.$$

It is easy to verify that $1/\mu_i(\mathbf{x}) < \lambda_i(\mathbf{x})$, so for every $\mathbf{x}$ satisfying this double inequality the system (3.6) has three solutions, the symmetric one just illustrated and two asymmetric solutions of the type sub (2) in this proof. Of those, a substantial amount of algebra shows that the one that maximizes expected human wealth $e_0(\mathbf{x})W_0(\mathbf{x}) + e_1(\mathbf{x})W_1(\mathbf{x})$ and then $e_0(\mathbf{x})J_0(\mathbf{x}) + e_1(\mathbf{x})J_1(\mathbf{x})$ is the asymmetric solution $J_k^{q_i}(\mathbf{x})$ defined in the statement of the lemma. The claimed defining properties of the cutoff $\rho_i(\mathbf{x})$ follow trivially. Finally, by construction $p_i x_i/p_{1-i}x_{1-i} \gtreqq \lambda_i(\mathbf{x})$ is necessary and sufficient for $\mathbf{x}$ to obtain a higher expected surplus to unemployed search only in sector i than in both sectors, or $e_i(\mathbf{x})J_i^{s_i}(\mathbf{x}) \gtreqq \Sigma e_k(\mathbf{x})J_k^{q_i}(\mathbf{x})$. ||

Proof of Lemma 2. The proof is divided in two parts.

(a) If $\mathbf{x}$ is 1-selective so is any $\mathbf{y} = (x_0, y_1)$ with $y_1 > x$, and values are ordered as follows

$$J_1(\mathbf{x}) = J_1^{s_1}(\mathbf{x}) > 0 = J_0(\mathbf{x}) = J_0^{s_1}(\mathbf{x}) \text{ implies } J_1(\mathbf{y}) = J_1^{s_1}(\mathbf{y}) > J_1^{s_1}(\mathbf{x}) > 0 = J_0^{s_1}(\mathbf{x}) = J_0^{s_1}(\mathbf{y}).$$

By contradiction: suppose $J_0(\mathbf{y}) > J_0(\mathbf{x}) = 0$. Then

$$\frac{g_1(\mathbf{x})}{\beta e_1(\mathbf{x})(1-\alpha_0)} < \frac{p_1 x_1}{p_0 x_0} < \frac{p_1 y_1}{p_0 x_0} \leqq \frac{g_1(\mathbf{y})}{\beta e_1(\mathbf{y})(1-\alpha_0)},$$

which implies $g_1(\mathbf{x})e_1(\mathbf{y}) < g_1(\mathbf{y})e_1(\mathbf{x})$ and then $e_1(\mathbf{y}) < e_1(\mathbf{x})$ and in turn $g_1(\mathbf{y}) < g_1(\mathbf{x})$. By the optimal hiring rule of 1-firms this also implies $J_1(\mathbf{y}) \leqq J_1^{s_1}(\mathbf{x}) = J_1(\mathbf{x})$. Now there are three possibilities. First,

$$J_1(\mathbf{y}) = J_1^{s_1}(\mathbf{y}) = \frac{(1-\beta)p_1 y_1}{g_1(\mathbf{y})} \leqq \frac{(1-\beta)p_1 x_1}{g_1(\mathbf{x})} = J_1^{s_1}(\mathbf{x}) = J_1(\mathbf{x}),$$

which is impossible as $y_1 > x_1$ and $g_1(\mathbf{y}) < g_1(\mathbf{x})$. Second $J_1(\mathbf{y}) = J_1^{q_1}(\mathbf{y}) \leqq J_1^{s_1}(\mathbf{x}) = J_1(\mathbf{x})$. Using the expressions for $J_1^{q_1}(\mathbf{y})$ and $J_1^{s_1}(\mathbf{x})$, some algebra, and $e_1(\mathbf{y}) < e_1(\mathbf{x})$:

$$\frac{p_1 x_1}{p_0 x_0} > \frac{g_1(\mathbf{y})}{\beta e_1(\mathbf{y})(1-\alpha_0)} > \frac{p_1 y_1}{p_0 x_0},$$

again contradicting $y_1 > x_1$. Third possibility, $J_1(\mathbf{y}) = J_1^{q_0}(\mathbf{y}) \leqq J_1^{s_1}(\mathbf{x}) = J_1(\mathbf{x})$, which is ruled out in the same manner. Therefore, $J_0(\mathbf{y}) = J_0(\mathbf{x}) = 0$ and both are selective searchers in sector 1. Thus only their 1-skill matters for profits and, as in the one-dimensional case discussed in Section 3, necessarily $J_1(\mathbf{y}) = J_1^{s_1}(\mathbf{y}) > J_1^{s_1}(\mathbf{x}) = J_1(\mathbf{x})$.

(b) If x is a random searcher with Stop-Gap in sector 0, then any $\mathbf{y} = (x_0, y_1)$ with $y_1 > x_1$ either adopts the same strategy or is 1-selective. In addition values are ordered as follows: $J_1^{q_1}(\mathbf{x}) > J_0^{q_1}(\mathbf{x}) > 0$ implies $J_1(\mathbf{y}) > J_1(\mathbf{x}) > J_0(\mathbf{x}) > J_0(\mathbf{y}) \geqq 0$.

The first claim is proven by contradiction. Suppose $J_1(\mathbf{y}) \leqq J_1(\mathbf{x}) = J_1^{q_1}(\mathbf{x})$ so $e_1(\mathbf{y}) \leqq e_1(\mathbf{x})$. There are two possibilities (the two symmetric possibilities $J_1(\mathbf{y}) = J_1^{s_0}(\mathbf{y}) = 0 < J_1^{q_1}(\mathbf{x})$ and $J_1(\mathbf{y}) = J_1^{q_0}(\mathbf{y}) < J_1^{q_1}(\mathbf{x})$ can be ruled out in a similar way). Either $J_1(\mathbf{y}) = J_1^{s_1}(\mathbf{y}) < J_1^{q_1}(\mathbf{x})$; since from the previous lemma $J_1^{s_1}(\mathbf{y}) > J_1^{s_1}(\mathbf{x})$, this implies $J_1^{s_1}(\mathbf{x}) < J_1^{q_1}(\mathbf{x})$ which is equivalent, after simple algebra, to $p_1 x_1 / p_0 x_0 > \lambda_1(\mathbf{x})$, contradicting $J_1(\mathbf{x}) = J_1^{q_1}(\mathbf{x})$. Or, $J_1(\mathbf{y}) = J_1^{q_1}(\mathbf{y}) < J_1^{q_1}(\mathbf{x})$. The assumption $J_1(\mathbf{y}) = J_1^{q_1}(\mathbf{y})$ implies that y is non-selective in her search, so $p_1 y_1 / p_0 x_0 < \lambda_1(\mathbf{y})$ and $J_1^{q_1}(\mathbf{y}) > J_1^{s_1}(\mathbf{y})$, and then *a fortiori* $J_1^{q_1}(\mathbf{x}) > J_1^{s_1}(\mathbf{x})$. Using the corresponding expressions and rearranging terms this inequality is equivalent to $p_1 y_1 / p_0 x_0 > \lambda_1(\mathbf{y})$, a contradiction. Therefore $J_1(\mathbf{y}) > J_1^{q_1}(\mathbf{x}) > 0$ and $e_1(\mathbf{y}) > e_1(\mathbf{x})$. All this also implies $J_0(\mathbf{y}) = J_0^{q_1}(\mathbf{y})$.

The claim on value ordering is proven similarly. Suppose $0 < J_0^{q_1}(\mathbf{x}) \leqq J_0^{q_1}(\mathbf{y})$. Then: $e_0(\mathbf{x}) \leqq e_0(\mathbf{y})$, which combined with $e_1(\mathbf{y}) > e_1(\mathbf{x})$ yields $g_0(\mathbf{y}) + e_1(\mathbf{y})\alpha_0 > g_0(\mathbf{x}) + e_1(\mathbf{x})\alpha_0$. Using the expressions for $J_0^{q_1}(\mathbf{x}) < J_0^{q_1}(\mathbf{y})$, rearranging and using $p_1 y_1 / p_0 x_0 < \lambda_1(\mathbf{y})$ we obtain $0 < x_0[g_0(\mathbf{y}) + e_1(\mathbf{y})\alpha_0 - g_0(\mathbf{x}) - e_1(\mathbf{x})\alpha_0] < \beta(1-\alpha_0) \times [e_1(\mathbf{x})x_1 - e_1(\mathbf{y})y_1] < 0$, the desired contradiction. ||

Proof of Proposition 4. To establish that the population partitions as claimed in three groups it suffices to prove the following: if $\mathbf{x} = (x_0, x_1)$ is (say) 1-selective so is any type who is more 1-skilled: $\mathbf{y} = (x_0, y_1)$ with $y_1 > x_1$. The proof is simple. As shown in the text: $\mathbf{x}$ is 1-selective iff $p_0 x_0 < rU(\mathbf{x})$, while $U(\mathbf{x}) < U(\mathbf{y})$, so $p_0 x_0 < rU(\mathbf{x}) < rU(\mathbf{y})$ as desired.

The next step is to order net valuations for jobs, thus payments and exit rates, across skill types. Consider again $\mathbf{x} = (x_0, x_1)$ and $\mathbf{y} = (x_0, y_1)$ with $y_1 > x_1$. If they are both 0-selective then they are observationally equivalent. Else, there are three possibilities.

First, both $\mathbf{x}$ and $\mathbf{y}$ are 1-selective. Observe that both workers can choose each other's optimal bids and enjoy each other's winning chances, although they weakly prefer not to. Therefore:

$$U(\mathbf{y}) = \frac{rW_1(\mathbf{y}) + e_1(\mathbf{y})\tau_1(\mathbf{y})}{r + e_1(\mathbf{y})} \geqq \frac{rW_1(\mathbf{y}) + e_1(\mathbf{x})\tau_1(\mathbf{x})}{r + e_1(\mathbf{x})}$$

$$U(\mathbf{x}) = \frac{rW_1(\mathbf{x}) + e_1(\mathbf{x})\tau_1(\mathbf{x})}{r + e_1(\mathbf{x})} \geqq \frac{rW_1(\mathbf{x}) + e_1(\mathbf{y})\tau_1(\mathbf{y})}{r + e_1(\mathbf{y})}.$$

Subtracting and rearranging:

$$\frac{e_1(\mathbf{x})[W_1(\mathbf{y}) - W_1(\mathbf{x})]}{r + e_1(\mathbf{x})} \leqq U(\mathbf{y}) - U(\mathbf{x}) \leqq \frac{e_1(\mathbf{y})[W_1(\mathbf{y}) - W_1(\mathbf{x})]}{r + e_1(\mathbf{y})}.$$

Since $W_1(\mathbf{y}) > W_1(\mathbf{x})$, as shown in the text, we obtain $e_1(\mathbf{y}) \geqq e_1(\mathbf{x})$, so that bids and surpluses are ordered in the same way.

Second, suppose both $\mathbf{x}$ and $\mathbf{y}$ are random searchers. They have values (after solving the corresponding system):

$$W_i(\mathbf{x}) = \frac{p_i x_i (r + e_i(\mathbf{x}) + e_{1-i}(\mathbf{x})(1 - \delta_{1-i}))}{(r+\delta_0)(r+\delta_1) - \delta_0 e_0(\mathbf{x})(r+\delta_1) - \delta_1 e_1(\mathbf{x})(r+\delta_0)}$$
$$+ \frac{\delta_i (e_{1-i}(\mathbf{x}) p_{1-i} x_{1-i} - (r + \delta_{1-i})(e_i(\mathbf{x})\tau_i(\mathbf{x}) + e_{1-i}(\mathbf{x})\tau_{1-i}(\mathbf{x})))}{(r+\delta_0)(r+\delta_1) - \delta_0 e_0(\mathbf{x})(r+\delta_1) - \delta_1 e_1(\mathbf{x})(r+\delta_0)}.$$

Immediately from the value equations (5.1) we get

$$W_0(\mathbf{x}) - U(\mathbf{x}) - [W_0(\mathbf{y}) - U(\mathbf{y})] = \frac{p_0 x_0 - rW_0(\mathbf{x}) - [p_0 x_0 - rW_0(\mathbf{y})]}{\delta_0} > 0,$$

because $W_0(\mathbf{x}) < W_0(\mathbf{y})$ as shown in the text. Hence, $\mathbf{y}$ bids less for 0-jobs than the less 1-skilled (and equally 0-skilled) $\mathbf{x}$ for the usual reason: the productivity on 0-jobs is the same, but $\mathbf{y}$ has a higher continuation value of search. Conversely, $\mathbf{y}$ bids more for and is always chosen with higher chance in 1-vacancies. In fact, as $\mathbf{x}$ may always bid $W_i(\mathbf{y}) - U(\mathbf{y})$, pay $\tau_i(\mathbf{y})$ and enjoy the winning chances $e_i(\mathbf{y})$, while both workers are random searchers:

$$\begin{aligned} rU(\mathbf{x}) &\geqq \textstyle\sum_{i=0}^{1} e_i(\mathbf{y})[W_i(\mathbf{x}) - U(\mathbf{x}) - \tau_i(\mathbf{y})] \\ rU(\mathbf{y}) &= \textstyle\sum_{i=0}^{1} e_i(\mathbf{y})[W_i(\mathbf{y}) - U(\mathbf{y}) - \tau_i(\mathbf{y})]. \end{aligned} \tag{A.2}$$

Subtracting and using $U(\mathbf{y}) > U(\mathbf{x})$ from the text:

$$0 < r[U(\mathbf{y}) - U(\mathbf{x})] = \sum_{i=0}^{1} e_i(\mathbf{y})[W_i(\mathbf{y}) - U(\mathbf{y}) - W_i(\mathbf{x}) + U(\mathbf{x})].$$

But we have just found $W_0(\mathbf{y}) - U(\mathbf{y}) - W_0(\mathbf{x}) + U(\mathbf{x}) < 0$; this inequality, together with $e_i(\mathbf{y}) > 0$ for $i = 0, 1$ (by the optimality of random search), finally leads to $W_1(\mathbf{y}) - U(\mathbf{y}) - W_1(\mathbf{x}) + U(\mathbf{x}) > 0$, as desired.

Third, and last, suppose $\mathbf{x}$ is a random searcher while $\mathbf{y}$ is 1-selective (the opposite cannot occur from the first part of the claim). Using (A.2) with $W_0(\mathbf{y}) - U(\mathbf{y}) = 0 < W_0(\mathbf{x}) - U(\mathbf{x})$, subtracting and using again $U(\mathbf{y}) > U(\mathbf{x})$ yields $W_1(\mathbf{y}) - U(\mathbf{y}) - W_1(\mathbf{x}) + U(\mathbf{x}) > 0$ as desired. ||

Acknowledgements. A previous version circulated with the title "Self Selection of Workers in Search Equilibrium", September 1996. I thank the Editor, Orazio Attanasio, and three anonymous referees for very constructive suggestions that greatly helped to improve this article, Ricardo Caballero and Robert Solow for invaluable advice, Boyan Jovanovic and Robert Shimer for helpful discussions, and participants to the 1996 CEPR Summer Symposium in Macroeconomics, Tarragona, and seminars at NYU, UCL and Yale for comments. I gratefully acknowledge the financial support of a Yale University Junior Faculty Fellowship and the kind hospitality of the Department of Economics at NYU while working on a revision.

REFERENCES

ACEMOGLU, D. (1996), "Good Jobs vs. Bad Jobs", *Journal of Labor Economics* (forthcoming).

ACEMOGLU, D. (1999), "Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence", *American Economic Review*, **89**, 1259–1278.

ACEMOGLU, D. and SHIMER, R. (1999), "Efficient Unemployment Insurance", *Journal of Political Economy*, **107**, 893–928.

BARLEVY, G. (1999), "The Sullying Effect of Recessions" (Mimeo, Northwestern University, Department of Economics).

BELIZIL, C. (1996), "Relative Efficiencies and Comparative Advantages in Job Search", *Journal of Labor Economics*, **14**, 154–173.

BILS, M. and McLAUGHLIN, K. (1992), "Inter-Industry Mobility and the Cyclical Upgrading of labor" (NBER Working Paper no. 4130, August).

BLANCHARD, O. J. and DIAMOND, P. A. (1990), "The Cyclical Behavior of the Gross Flows of U.S. Workers", *Brookings Papers on Economic Activity*, **2**, 85–155.

BLANCHARD, O. J. and DIAMOND, P. A. (1994), "Ranking, Unemployment Duration, and Wages", *Review of Economic Studies*; **61**, 417–434.

BLAU, D. and ROBINS, P. (1990), "Job Search Outcomes for the Employed and the Unemployed", *Journal of Political Economy*, **98**, 637–655.

BOWLUS, A. (1995), "Matching Workers and Jobs: Cyclical Fluctuations in Match Quality", *Journal of Labor Economics*, **13**, 335–350.

BURDA, M. and WYPLOSZ, C. (1994), "Gross Worker and Job Flows in Europe", *European Economic Review*, **38**, 1287–1315.

BUTTERS, G. (1979), "Equilibrium Distribution of Sales and Advertising Prices", *Review of Economic Studies*, **44**, 465–491.

DAVIS, S. J., HALTIWANGER, J. C. and SCHUH, S. (1996) *Job Creation and Destruction* (Cambridge: MIT Press).

DIXIT, A. and ROB, R. (1994), "Switching Costs and Sectoral Adjustments in General Equilibrium with Uninsured Risk", *Journal of Economic Theory*, **62**, 48–69.

FALLICK, B. (1993), "The Industrial Mobility of Displaced Workers", *Journal of Labor Economics*, **11**, 302–323.

JOVANOVIC, B. and MOFFITT, R. (1990), "An Estimate of a Sectoral Model of Labor Mobility", *Journal of Political Economy*, **98**, 1076–1107.

LILIEN, D. (1982), "Sectoral Shifts and Cyclical Unemployment", *Journal of Political Economy*, **90**, 777–794.

MARIMON, R. and ZILIBOTTI, F. (1999), "Unemployment vs. Mismatch of Talent: Reconsidering Unemployment Benefits", *Economic Journal*, **109**, 266–291.

MERZ, M. (1999), "Heterogeneous Job-Matches and the Cyclical Behavior of Labor Turnover", *Journal of Monetary Economics*, **43**, 91–124.

MORTENSEN, D. and PISSARIDES, C. A. (1994), "Job Creation and Job Destruction in the Theory of Unemployment", *Review of Economic Studies*, **61**, 397–415.

MORTENSEN, D. and PISSARIDES, C. A. (1999), "Unemployment Responses to 'Skill-Biased' Technology Shocks: the Role of Labour Market Policy", *Economic Journal*, **109**, 242–265.

MOSCARINI, G. (1996), "Worker Heterogeneity and Job Search in the Flow Approach to Labor Markets: A Theoretical Analysis", (Unpublished Ph.D. dissertation, MIT, Department of Economics).

MURPHY, K. and TOPEL, R. (1987), "The Evolution of Unemployment in the United States: 1968-1985", in S. Fischer (ed.), *NBER Macroeconomic Annual 1987* (Cambridge: MIT Press).

OSBERG, L. (1993), "Fishing in Different Pools: Job-Search Strategies and Job-Finding Success in Canada in the Early 1980s", *Journal of Labor Economics*, **11**, 348–386.

OSBORNE, M. and RUBINSTEIN, A. (1994) *A Course in Game Theory* (Cambridge: MIT Press).

ROY, A. D. (1951), "Some Thoughts on the Distribution of Earnings", *Oxford Economic Papers*, **3** 135–146.

SATTINGER, M. (1995), "Search and the Efficient Assignment of Workers to Jobs", *International Economic Review*, **36**, 283–302.

SHIMER, R. (1999), "Job Auctions" (Mimeo, Princeton University, Department of Economics).

THOMAS, J. (1998), "The Role of Selective Job Search in U.K. Unemployment". *Economic Journal*, **108**, 646–664.