JOSEPHSON QUBIT CIRCUITS AND THEIR READOUT

Michel DEVORET, Applied Physics and Physics, Yale University

Sponsors:

W.M.
KECK

Final version of this presentation available at http://qulab.eng.yale.edu/archives.htm

2009 APS March Meeting Pittsburgh, PA

QUANTUM INFORMATION PROCESSING

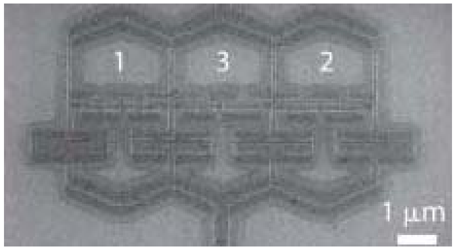
of atoms

micro

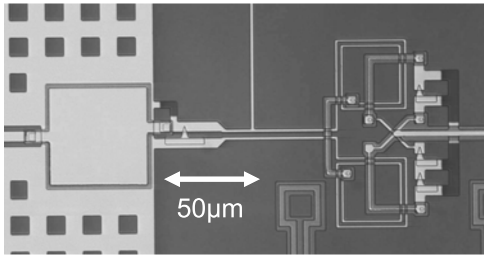
MACRO

- OPTICAL PHOTONS
- NUCLEAR SPINS
- IONS
- ATOMS
- MOLECULES
- QUANTUM DOTS
- SUPERCONDUCTING JOSEPHSON TUNNEL CIRCUITS

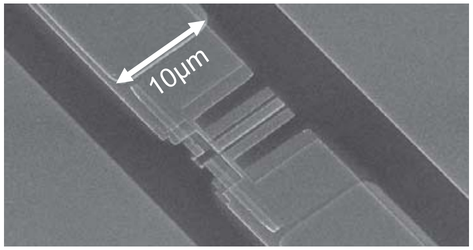
coupling
with env^{t}.



from A. O. Niskanen et al. Science 316, 723 (2007)

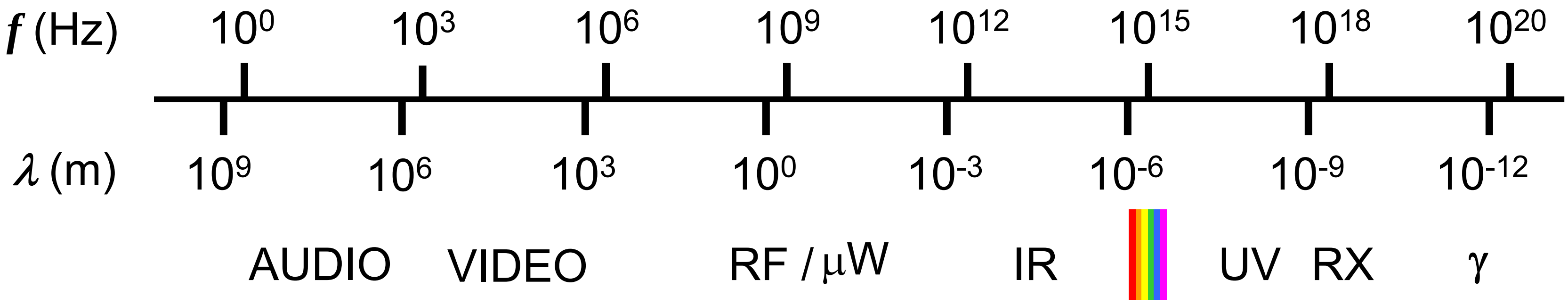


courtesy of J. Martinis, 2009



from Metcalfe et al., 2007

ELECTROMAGNETIC SPECTRUM



f (Hz) 10^0 10^3 10^6 10^9 10^{12} 10^{15} 10^{18} 10^{20}

λ (m) 10^9 10^6 10^3 10^0 10^{-3} 10^{-6} 10^{-9} 10^{-12}

AUDIO VIDEO RF / μW IR UV RX γ

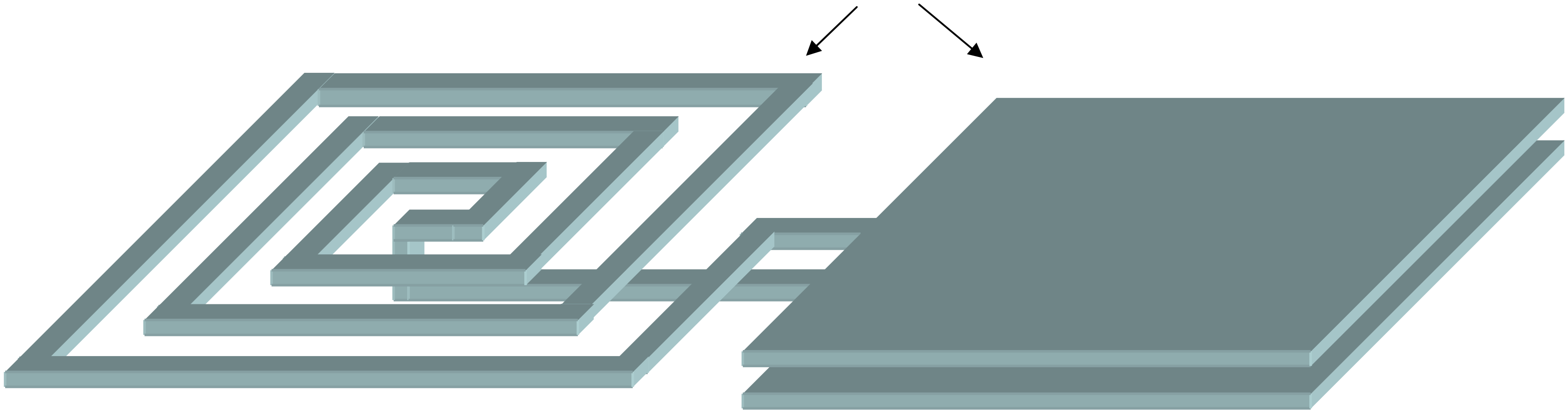
ELECTRICITY ← → *OPTICS*

1 bit = RF signal with 0/1 photon?

10 GHz ~ 0.5K

HOW CAN A SUPERCONDUCTING CIRCUIT BEHAVE LIKE AN ATOM?

SIMPLEST EXAMPLE: SUPERCONDUCTING **LC** OSCILLATOR CIRCUIT

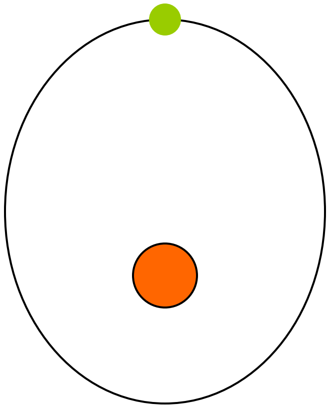


MICROFABRICATION → $L \sim 3nH$, $C \sim 10pF$, $\omega_r/2\pi \sim 2GHz$

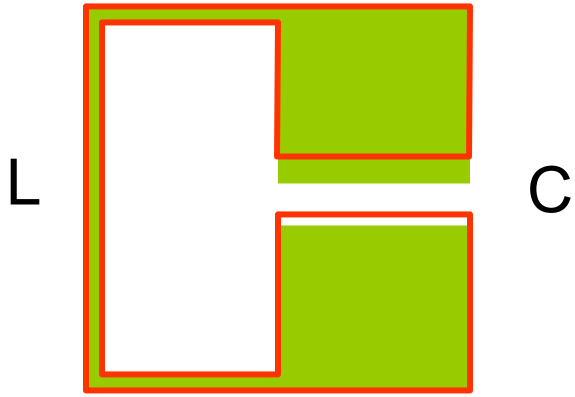
ELECTRONIC FLUID SLOSHES BACK AND FORTH
FROM ONE PLATE TO THE OTHER, INTERNAL MODES FROZEN,
BEHAVES AS A SINGLE CHARGE CARRIER

DEGREE OF FREEDOM IN ATOM vs CIRCUIT

Example of H atom with large principal quantum number



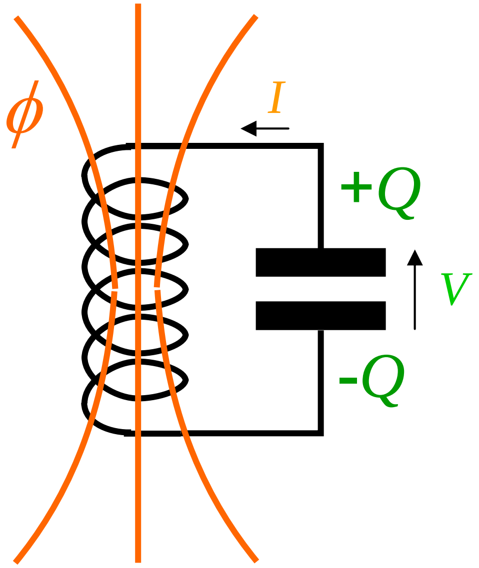
Superconducting LC oscillator



velocity of electron → voltage across capacitor

force on electron → current through inductor

FLUX AND CHARGE DO NOT COMMUTE

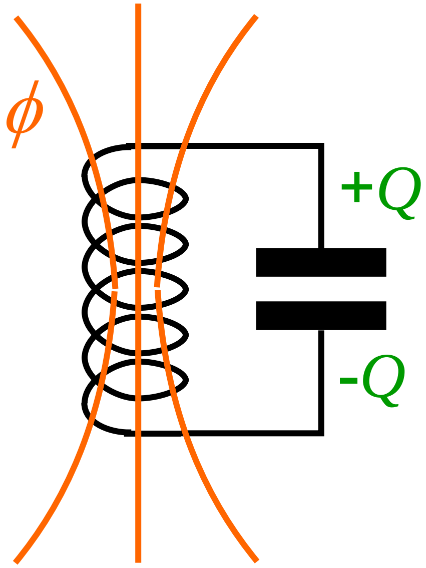


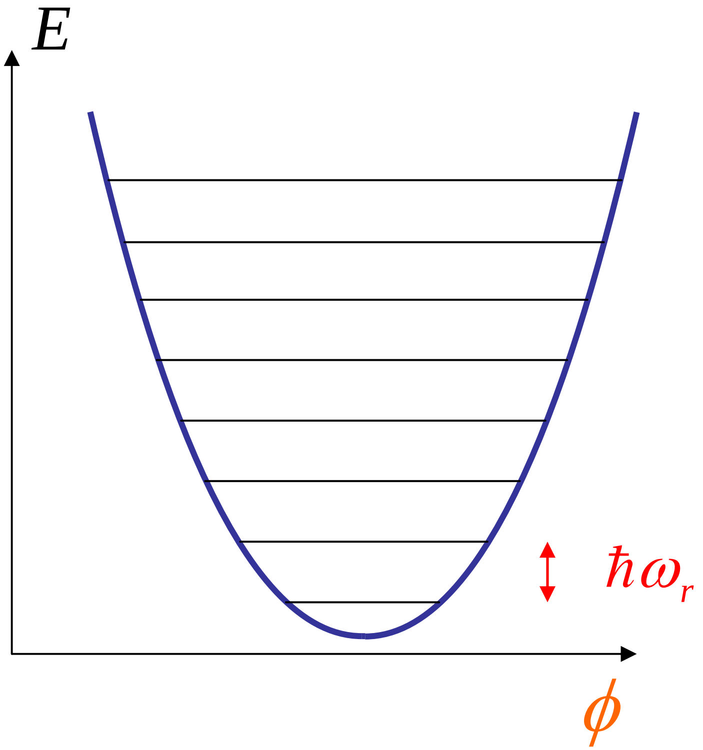
$$\phi = LI$$

$$Q = CV$$

$$\boxed{\left[\hat{\phi},\hat{Q}\right] = i\hbar}$$

LC CIRCUIT AS QUANTUM HARMONIC OSCILLATOR





$$\hat{H} = \hbar\omega_r\left(\hat{a}^\dagger\hat{a} + {}^{1}\!/_{2}\right)$$

$$\hat{a} = \frac{\hat{\phi}}{\phi_r} + i\frac{\hat{Q}}{Q_r}; \quad \hat{a}^\dagger = \frac{\hat{\phi}}{\phi_r} - i\frac{\hat{Q}}{Q_r}$$

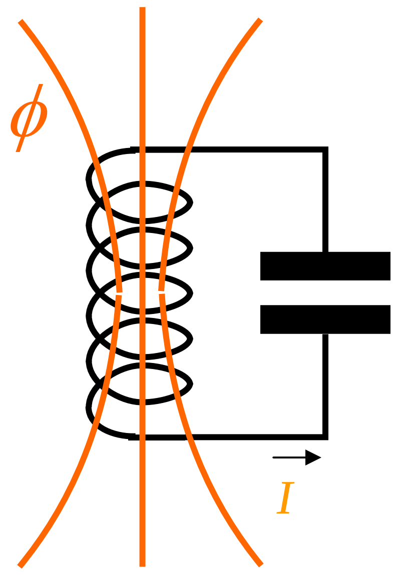
←

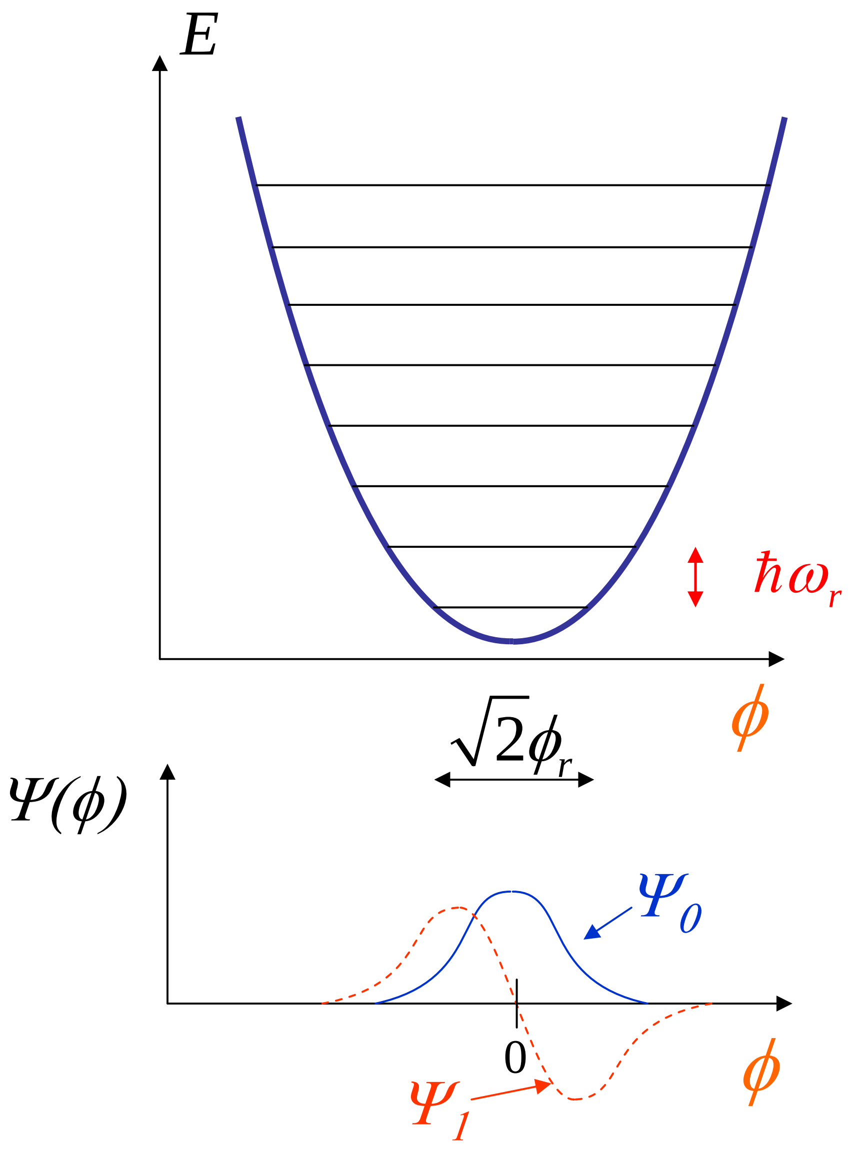
annihilation and creation operators for mesoscopic excitation of circuit

$$\phi_r = \sqrt{2\hbar\omega_r L}$$

$$Q_r = \sqrt{2\hbar\omega_r C}$$

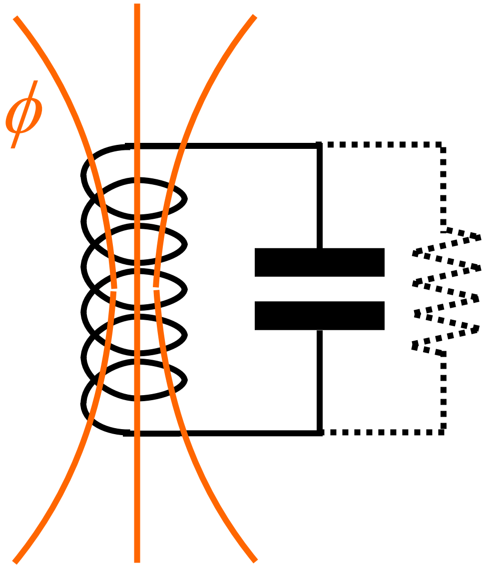
WAVEFUNCTIONS OF LC CIRCUIT

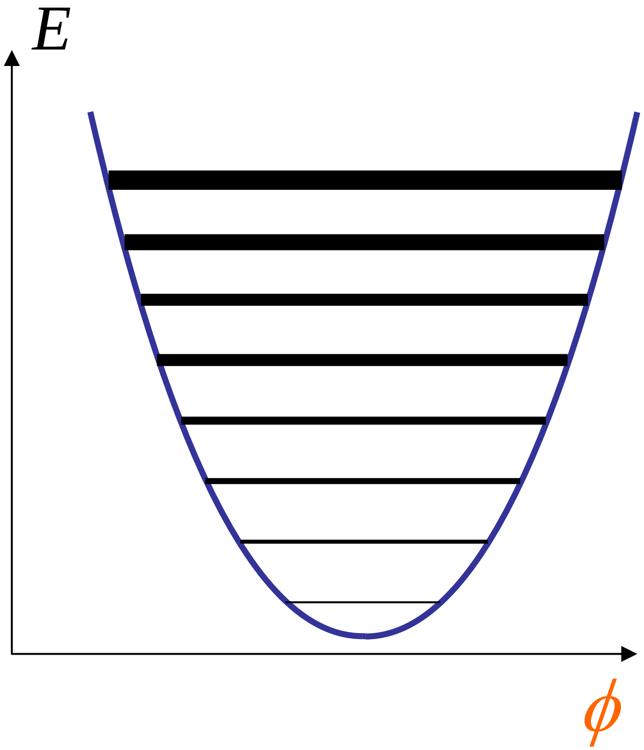




In every energy eigenstate,
(photon state)
current flows in opposite
directions simultaneously!

EFFECT OF DAMPING



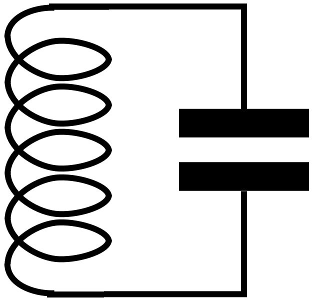


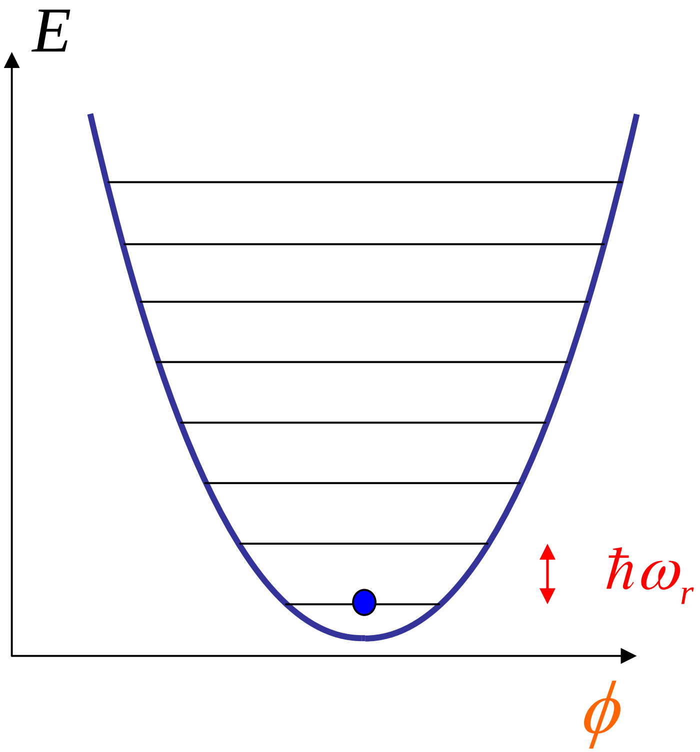
important: as little dissipation as possible

dissipation broadens energy levels

$$\left.\begin{aligned} E_n &= \hbar\omega_r\left[n\left(1+\frac{i}{2\mathcal{Q}}\right)+\frac{1}{2}\right] \\ \mathcal{Q} &= RC\omega_r \end{aligned}\right\}$$

CAN PLACE CIRCUIT IN ITS GROUND STATE





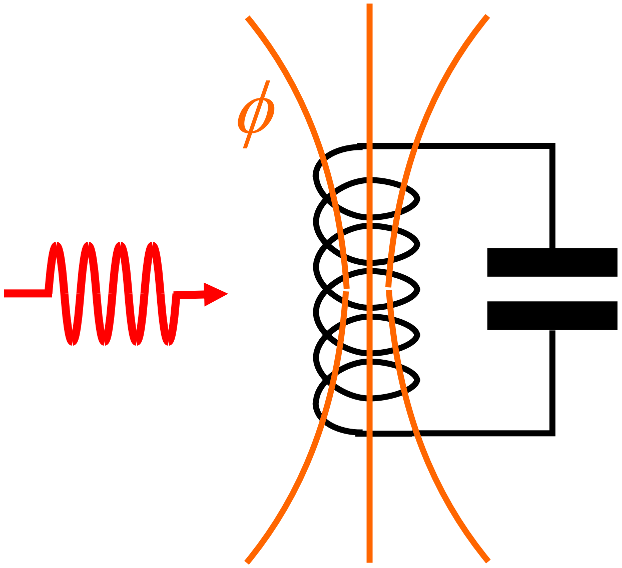
residual dissipation
provides
reset of circuit

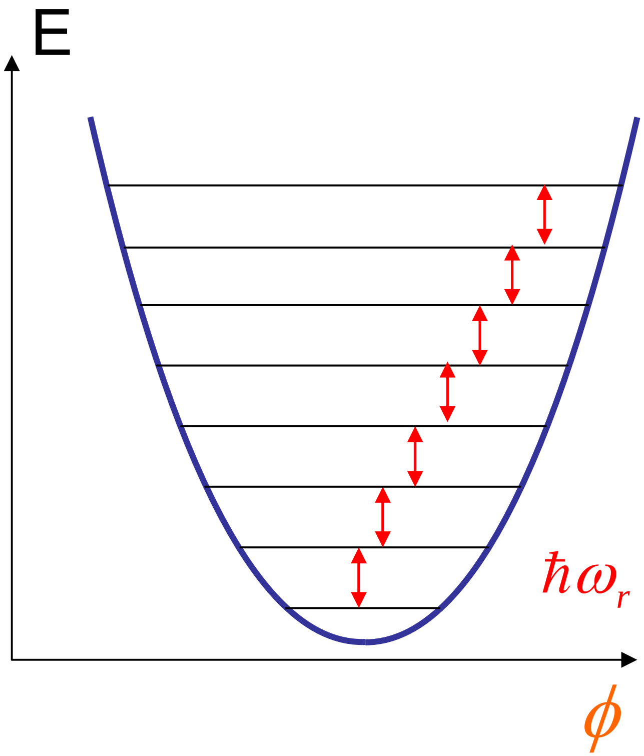
$$\hbar\omega_r \gg k_B T$$

10 GHz

25mK

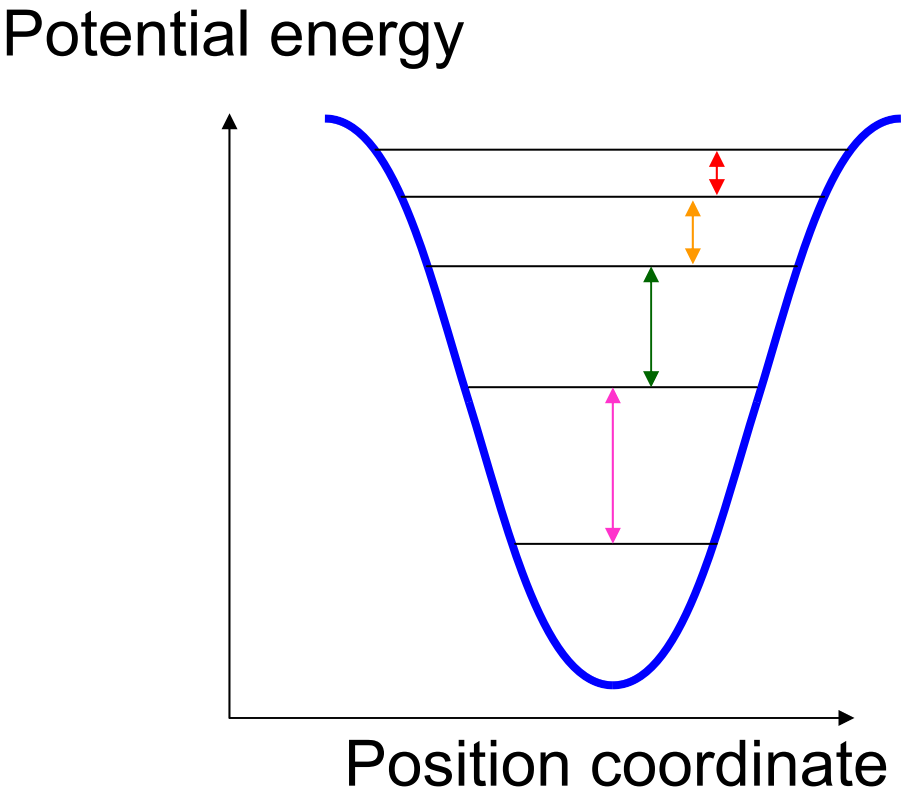
PB: ALL TRANSITIONS ARE DEGENERATE!



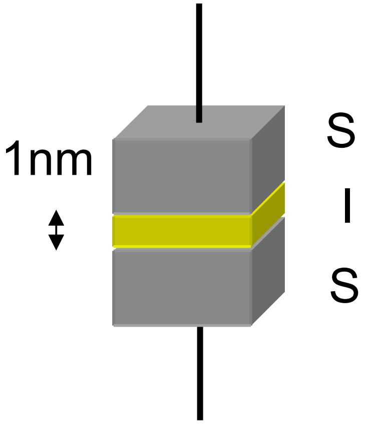


CANNOT STEER THE SYSTEM TO AN ARBITRARY STATE
IF PERFECTLY LINEAR

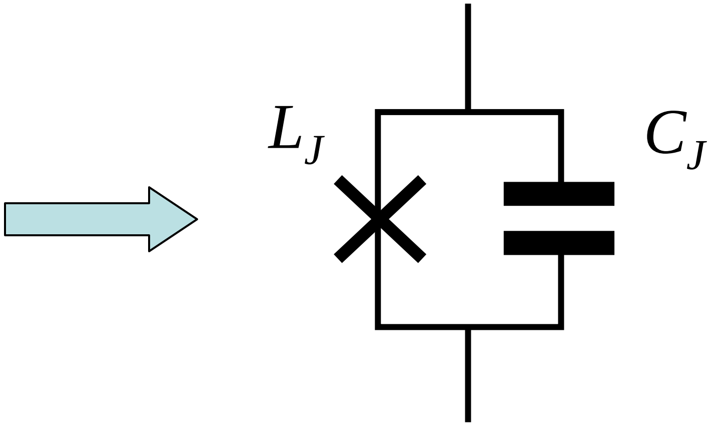
NEED NON-LINEARITY TO FULLY REVEAL QUANTUM MECHANICS



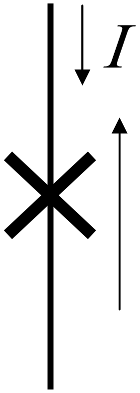
JOSEPHSON TUNNEL JUNCTION PROVIDES A NON-LINEAR INDUCTOR WITH NO DISSIPATION



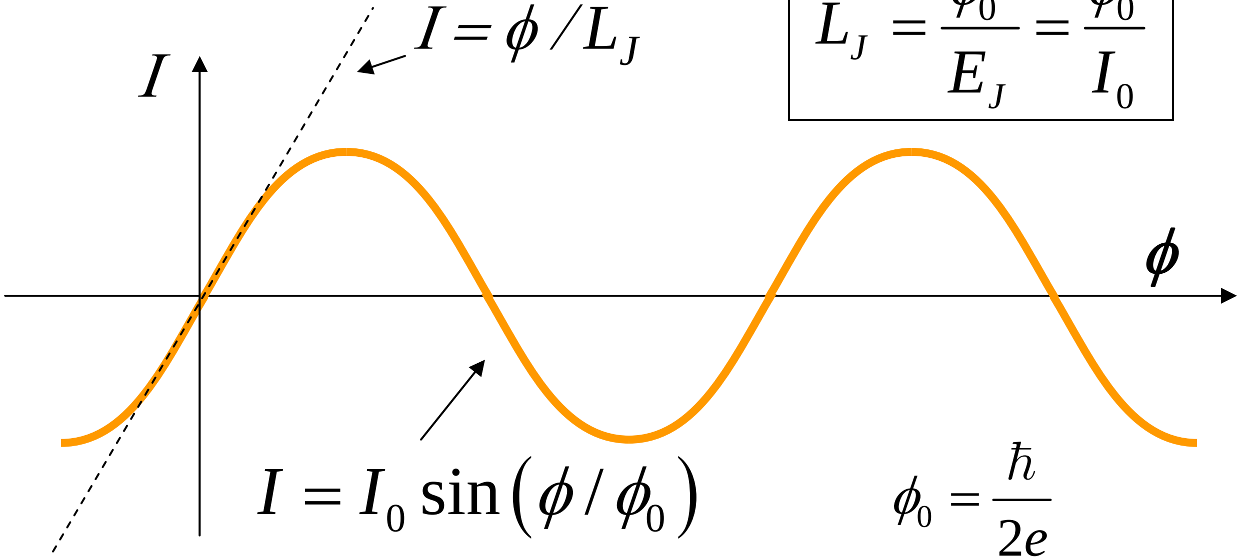
superconductor-
insulator-
superconductor
tunnel junction



$$L_J = \frac{\phi_0^2}{E_J} = \frac{\phi_0}{I_0}$$



$$\phi = \int_{-\infty}^{t} V(t')dt'$$

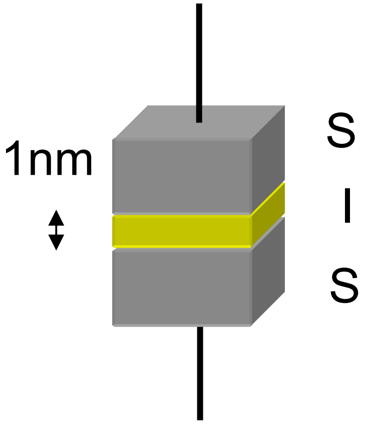


$$I = \phi / L_J$$

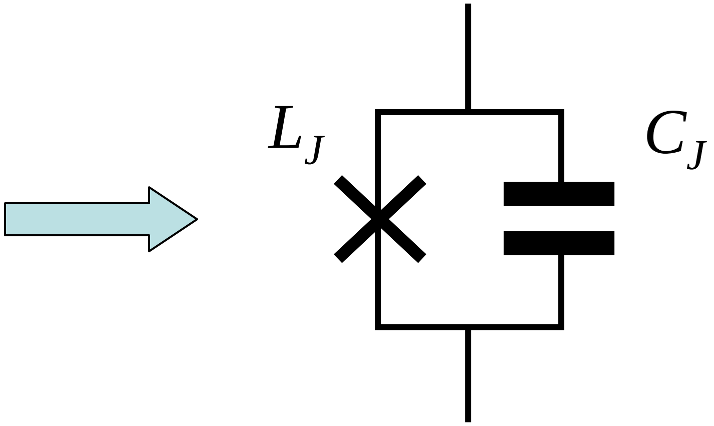
$$I = I_0 \sin\left(\phi / \phi_0\right)$$

$$\phi_0 = \frac{\hbar}{2e}$$

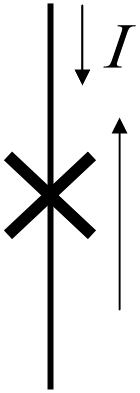
JOSEPHSON TUNNEL JUNCTION PROVIDES A NON-LINEAR INDUCTOR WITH NO DISSIPATION



superconductor-insulator-superconductor tunnel junction

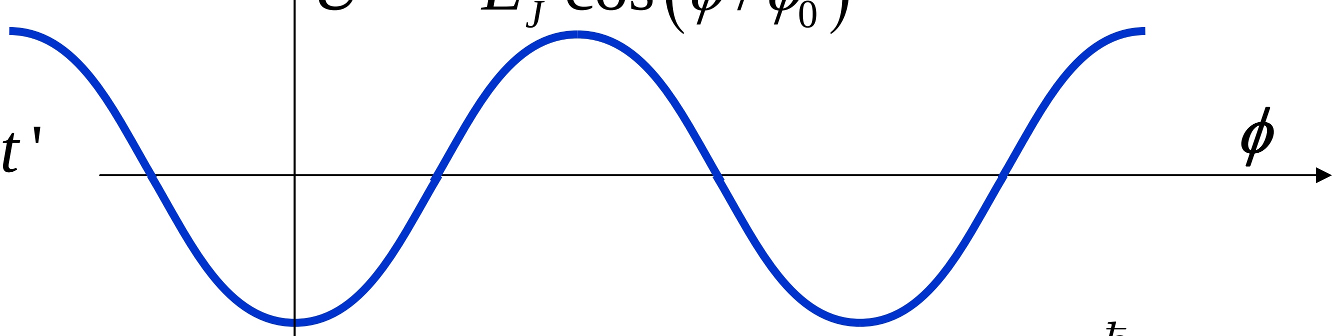


$$L_J = \frac{\phi_0^2}{E_J} = \frac{\phi_0}{I_0}$$



$$\phi = \int_{-\infty}^{t} V(t')dt'$$

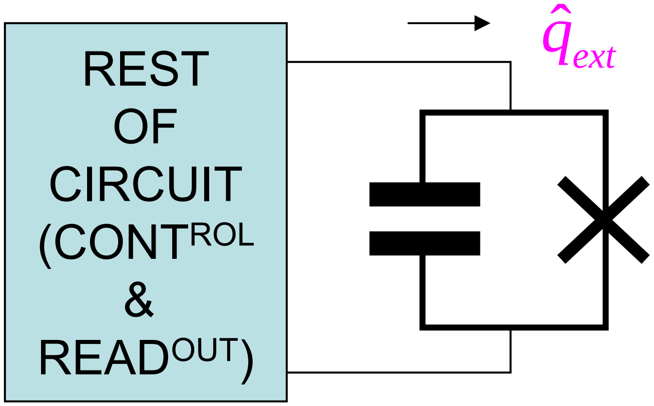
$$U = -E_J \cos(\phi / \phi_0)$$



bare Josephson potential

$$\phi_0 = \frac{\hbar}{2e}$$

COUPLING PARAMETERS
OF THE JOSEPHSON "ATOM"



Josephson tunnel junction
contribution to total circuit
Hamiltonian

THE Hamiltonian:
(we mean it!)

$$\widehat{H}_J = \overbrace{\frac{1}{2C_J}\left(\widehat{Q} - \hat{q}_{ext}\right)^2 - E_J \cos\frac{2e\widehat{\phi}}{\hbar}}$$

Comparable with lowest order
model for hydrogen atom →

$$\widehat{H} = \frac{1}{2m_e}\left(\hat{p} - \frac{e\hat{A}}{\hbar}\right)^2 - \frac{e^2}{4\pi\varepsilon_0}\frac{1}{\hat{r}}$$

TWO ENERGY SCALES

Two dimensionless variables:

$$\hat{\varphi} = \frac{2e\hat{\phi}}{\hbar}$$

$$\hat{N} = \frac{\hat{Q}}{2e}$$

$$\Longrightarrow \quad \left[\hat{\varphi}, \hat{N}\right] = i$$

Hamiltonian becomes :

$$\widehat{H}_J = 8E_C \frac{\left(\widehat{N} - N_{ext}\right)^2}{2} - E_J \cos\hat{\varphi}$$

Coulomb charging energy for 1e

Josephson energy

$$E_C = \frac{e^2}{2C_J}$$

reduced offset charge

$$N_{ext} = \frac{q_{ext}}{2e}$$

barrier transp$^{\text{cy}}$

$$E_J = \frac{1}{8}\mathcal{N}\mathcal{T}\Delta$$

gap

valid for opaque barrier

cond$^{\text{ion}}$ channels

HARMONIC APPROXIMATION

$$\widehat{H}_J = 8E_C \frac{\left(\widehat{N} - N_{ext}\right)^2}{2} - E_J \cos \hat{\varphi}$$

⇓

$$\widehat{H}_{J,h} = 8E_C \frac{\left(\widehat{N} - N_{ext}\right)^2}{2} + E_J \frac{\hat{\varphi}^2}{2}$$

Josephson
"plasma" frequency:

$$\omega_P = \frac{\sqrt{8E_C E_J}}{\hbar}$$

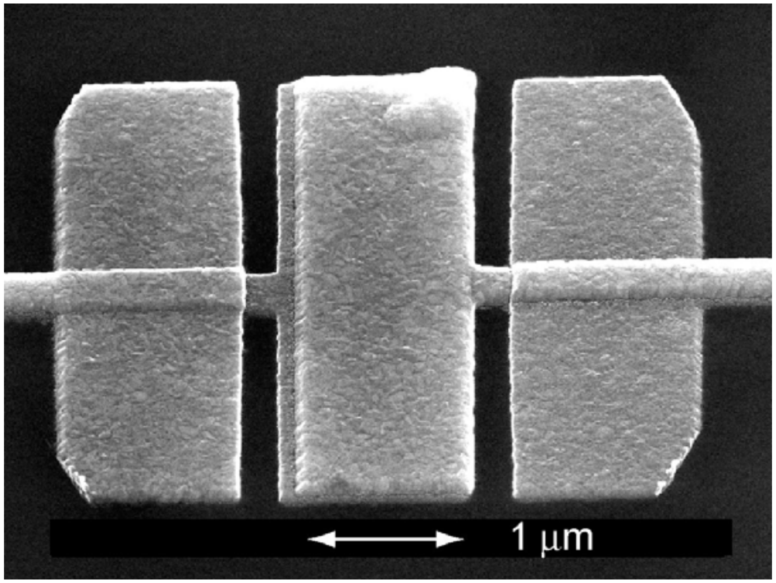
Josephson
RF impedance:

$$Z_J = \frac{\hbar}{\left(2e\right)^2} \sqrt{\frac{8E_C}{E_J}}$$

Spectrum independent of DC value of N_{ext}

SOME JOSEPHSON TUNNEL JUNCTIONS IN REAL LIFE

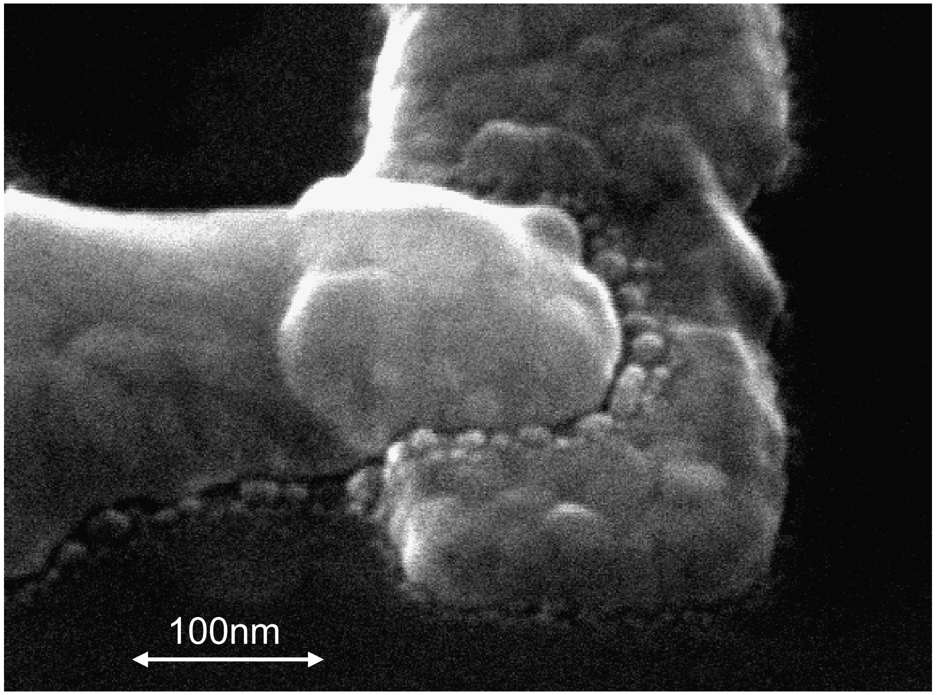
credit I. Siddiqi and F.Pierre



E_J ~ 50K

ω_p ~ 30-40GHz

credit L. Frunzio and D. Schuster



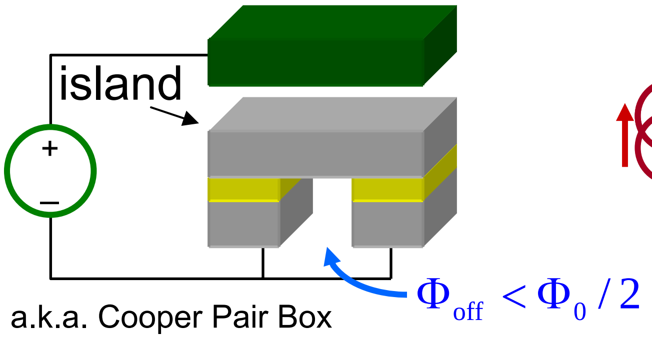
E_J ~ 0.5K

RF CONTROL & BIAS vs SENSITIVITY TO NOISE

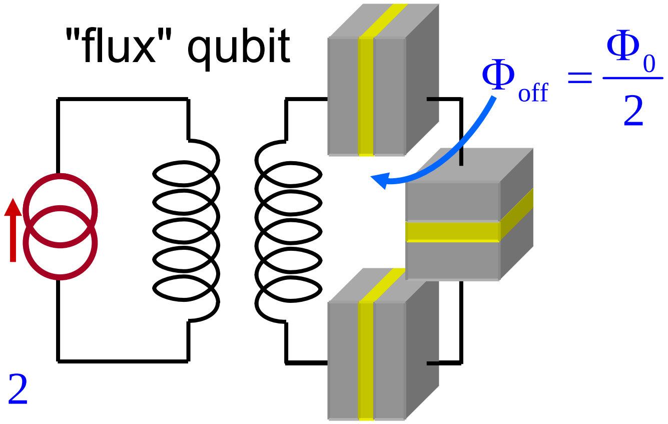
Devoret and Martinis, Quant. Inf. Proc. 2004

see R. McDermot's tutorial

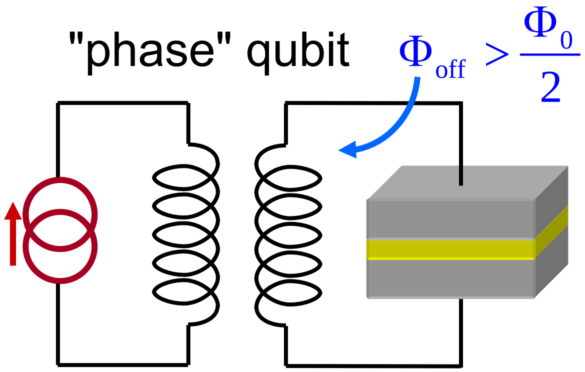
"charge" qubit



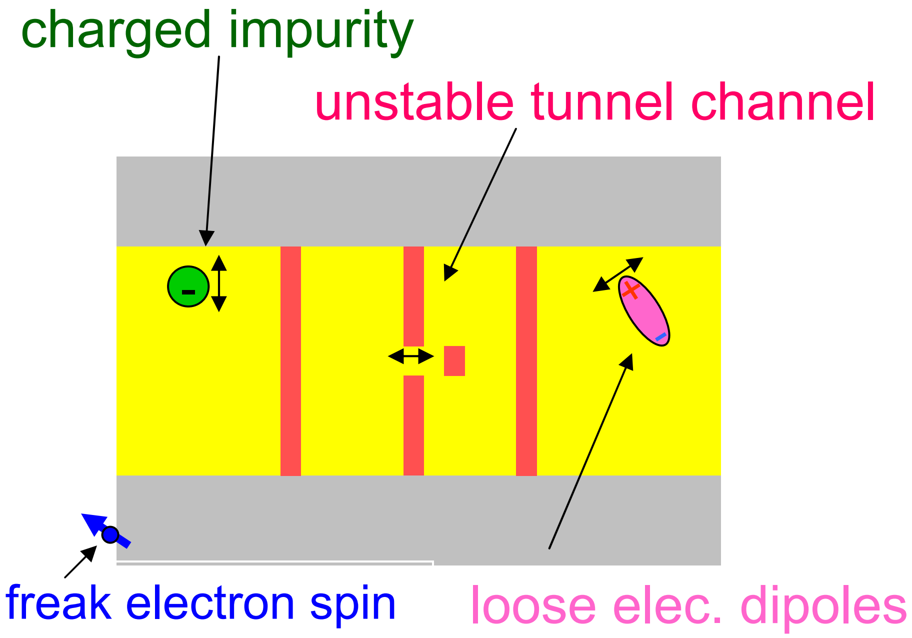
"flux" qubit



"phase" qubit



THE MONSTERS WE FEAR:

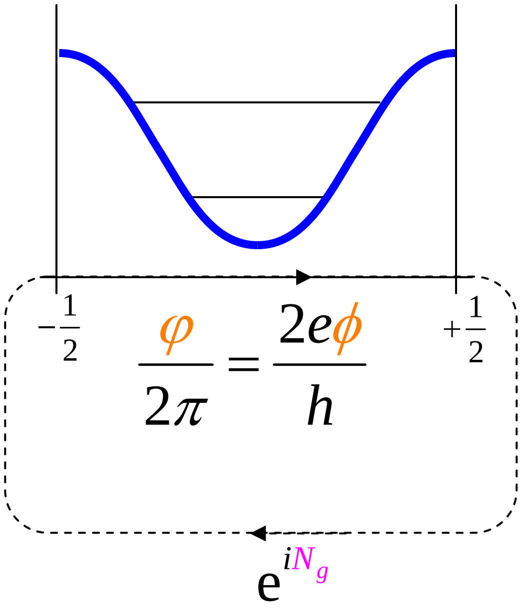


	noises		
flavors	ΔQ_{off}	$\Delta\Phi_{\mathrm{off}}$	ΔE_J
"charge"	⚡	⚡	⚡
"flux"		⚡⚡	⚡⚡⚡
"phase"		⚡⚡	⚡⚡

EFFECTIVE POTENTIAL OF 3 MAIN BIAS SCHEMES

see also proposals for
topologically protected
qubits, for example
Feigelman et al. PRL 92,
098301 (2004)

"charge" bias

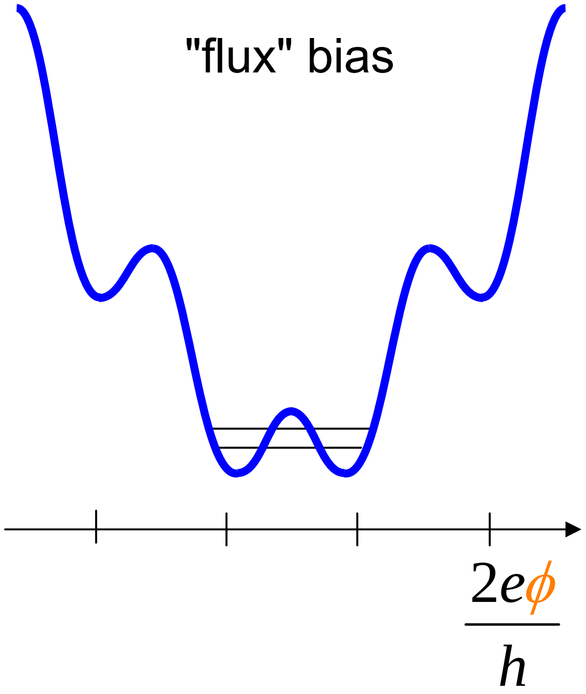


$$-\frac{1}{2} \qquad \frac{\varphi}{2\pi} = \frac{2e\phi}{h} \qquad +\frac{1}{2}$$

$$e^{iN_g}$$

CEA Saclay, NEC, Yale
Chalmers, JPL, ...

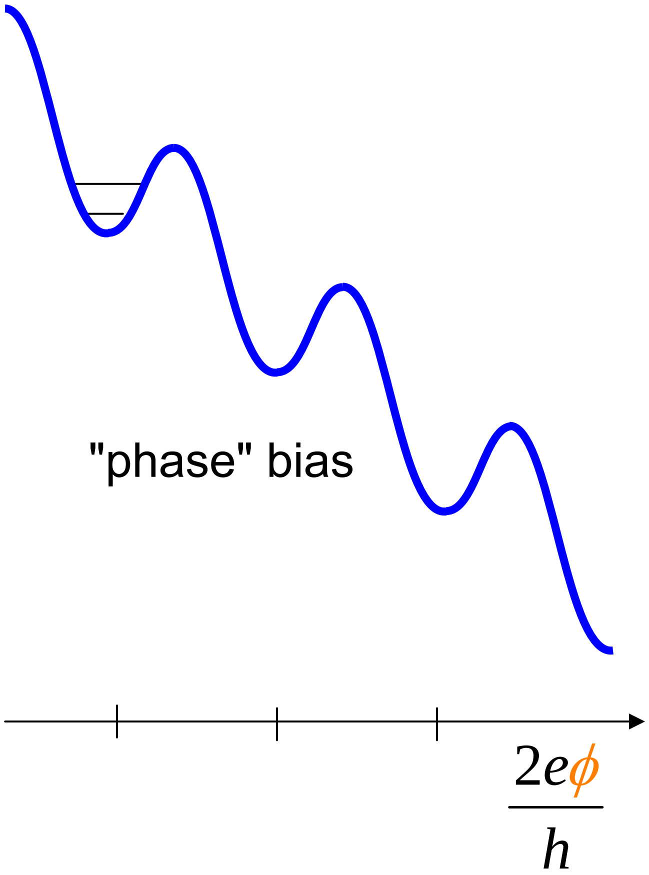
"flux" bias



$$\frac{2e\phi}{h}$$

TU Delft, NEC, NTT, IBM,
MIT, UC Berkeley, SUNY,
IPHT Jena ....

"phase" bias



$$\frac{2e\phi}{h}$$

UC Berkeley, NIST, UCSB,
U. Maryland, I. Neel Grenoble...

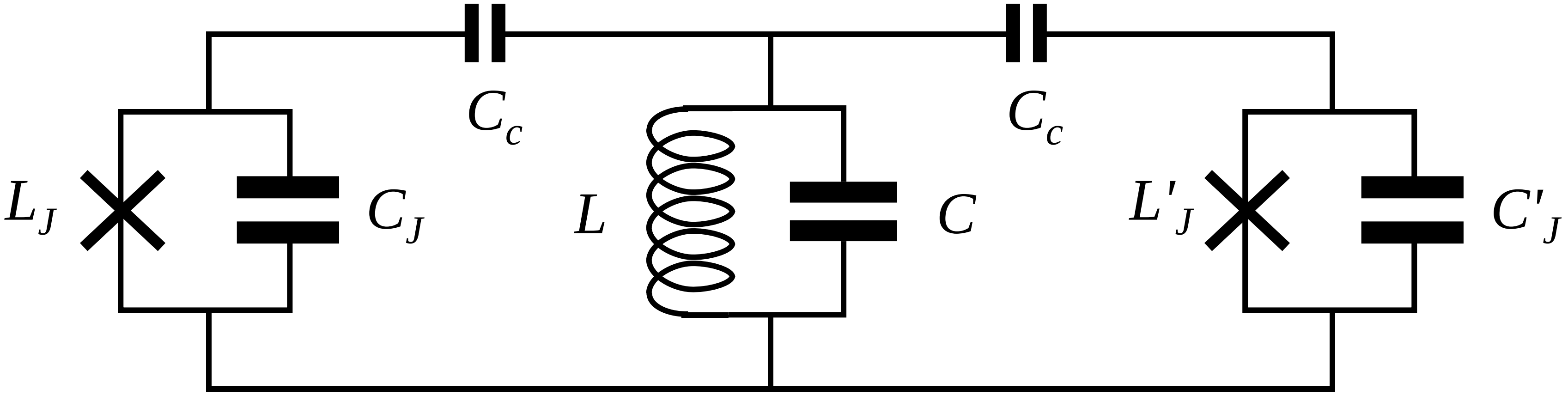
HOW DO WE FIND THE HAMILTONIAN OF AN ARBITRARY CIRCUIT?

Yurke B. and Denker J.S., Phys. Rev. A 29, 1419 (1984)

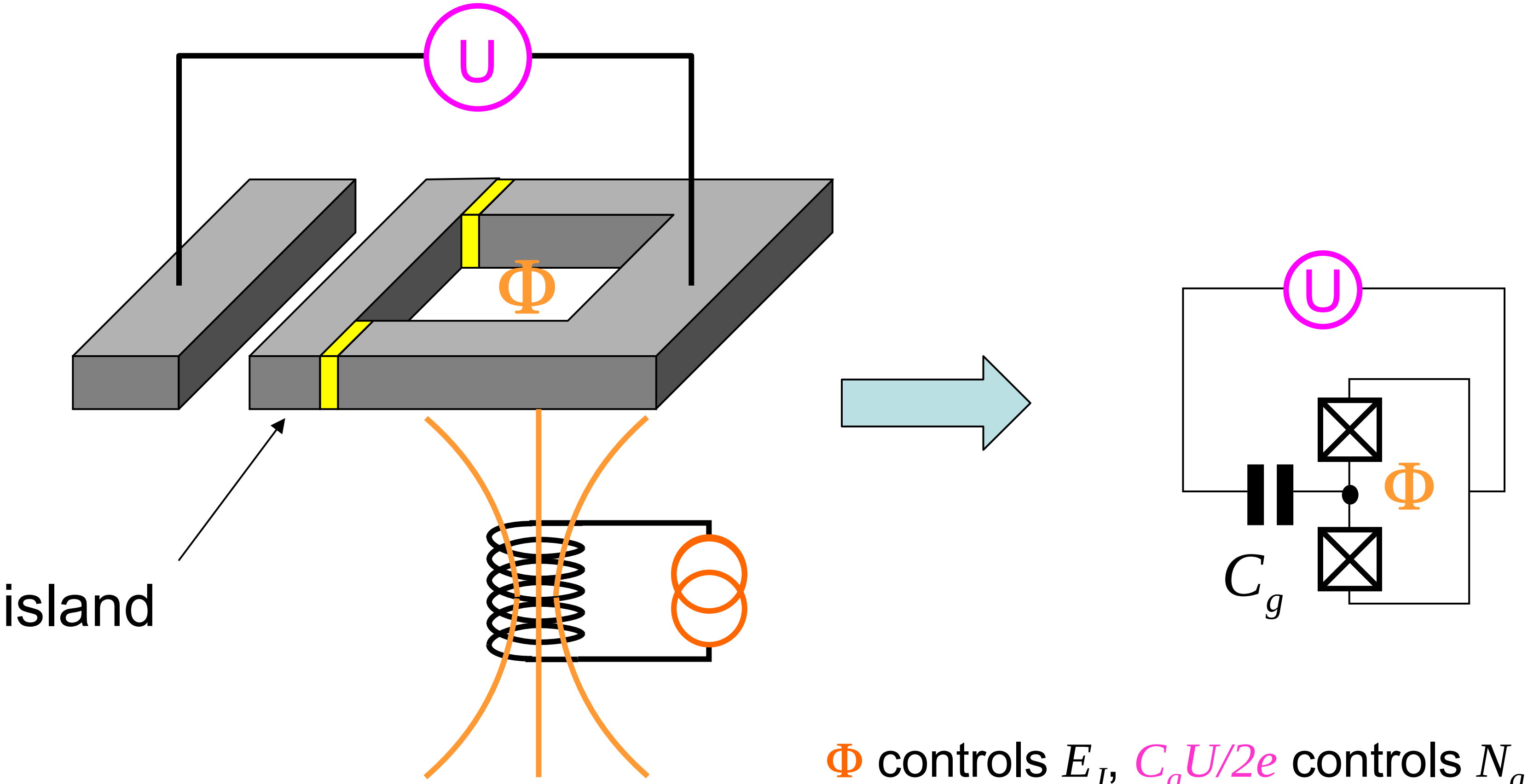
Devoret M. H. in "Quantum Fluctuations", S. Reynaud, E. Giacobino,
J. Zinn-Justin, Eds. (Elsevier, Amsterdam, 1997) p. 351-385

G. Burkard, R. H. Koch, and D. P. DiVincenzo, Phys. Rev. B 69, 064503 (2004)

Example: 2 Josephson junctions capacitively coupled to 1 resonator mode



COOPER PAIR "BOX"

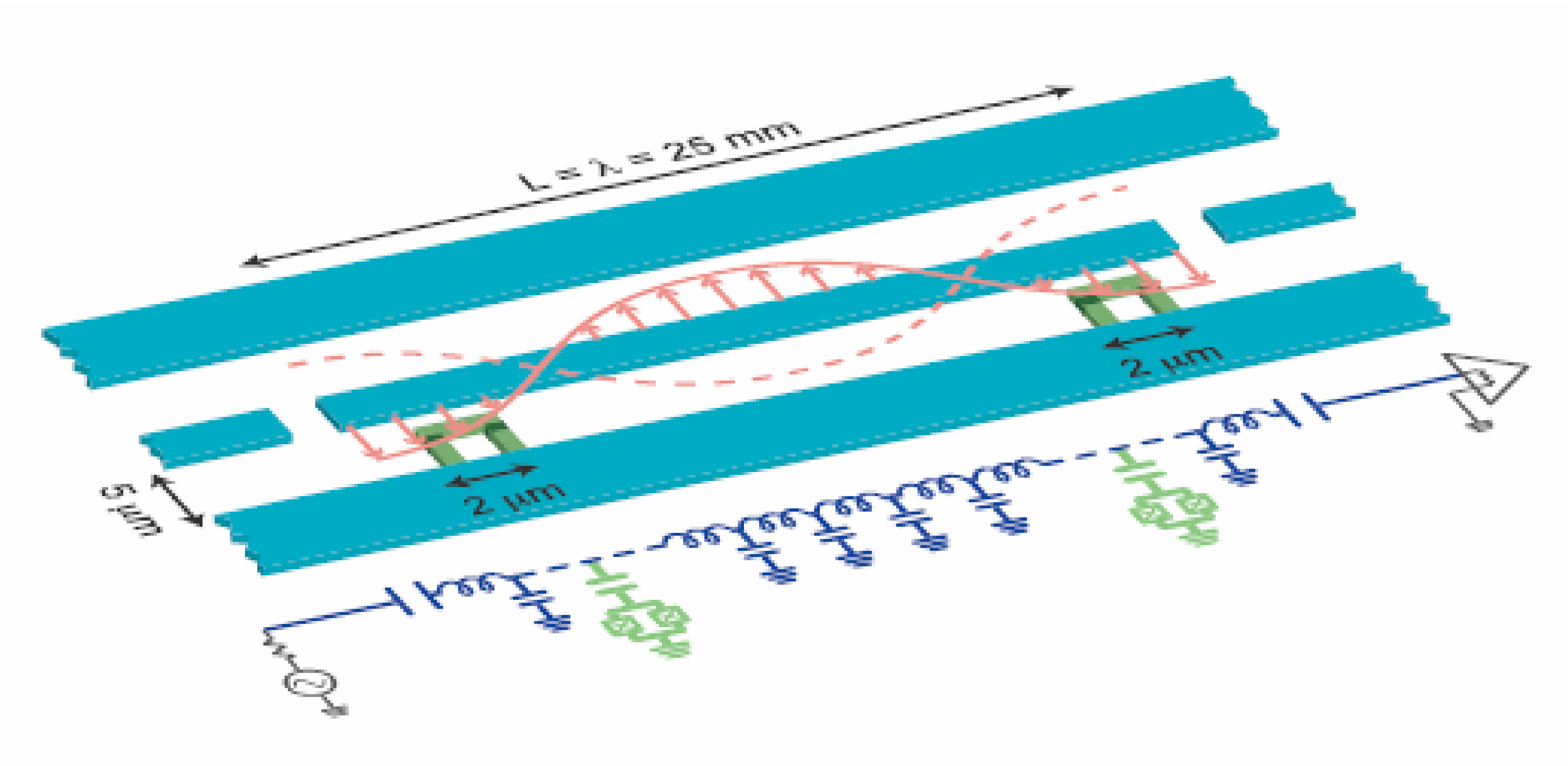


Φ controls E_J, $C_gU/2e$ controls N_g

qbit space = Hilbert space of 0 or 1 quanta in this non-linear oscillator

Bouchiat PhD Thesis 97, et al. Physica Scripta '98
Nakamura, Pashkin & Tsai, Nature '99

SCHEMATIC OF COOPER PAIR BOXES IN A MICROWAVE CAVITY

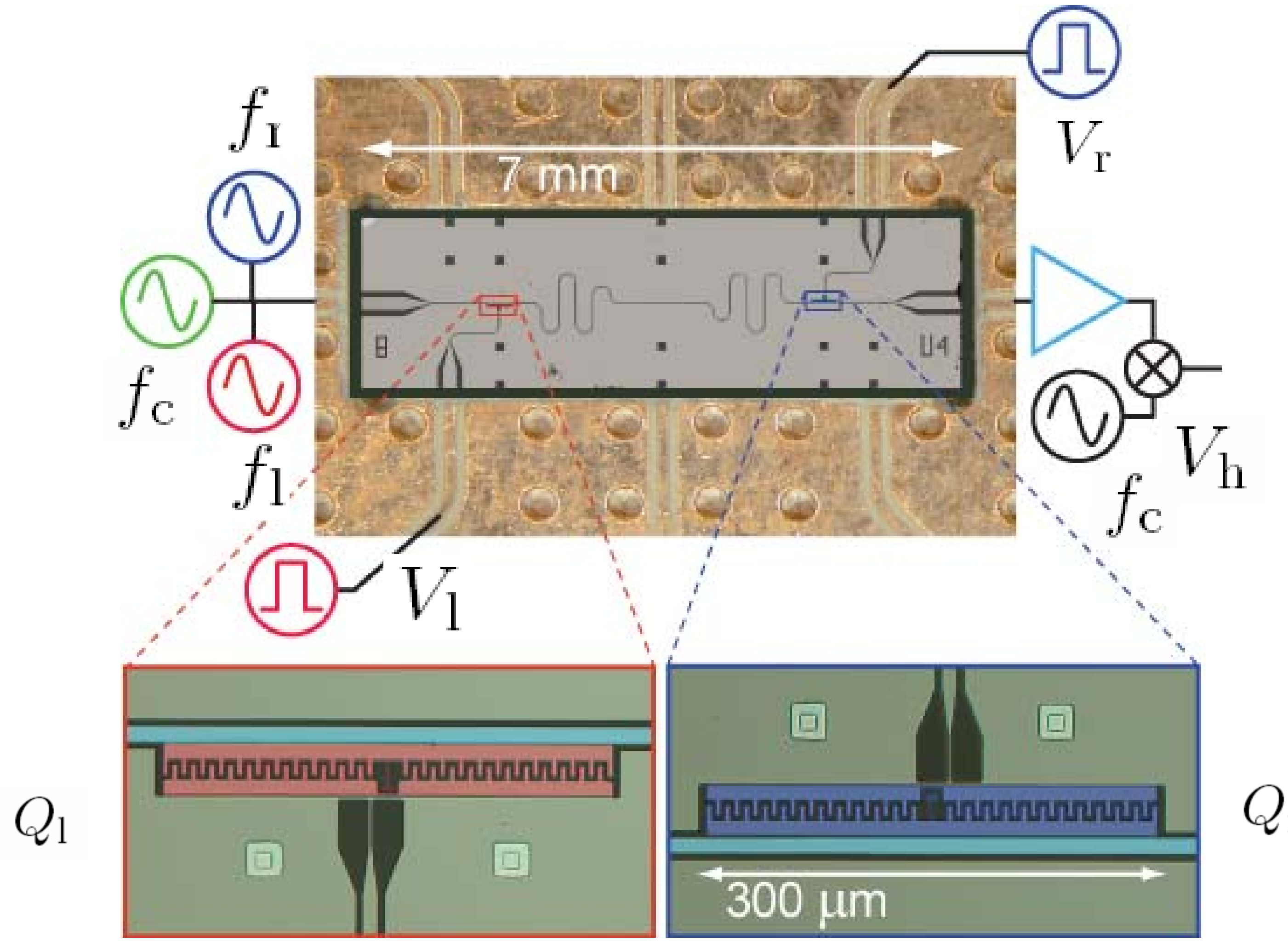


Review by Blais et al., Phys. Rev. A **75**, 032329 (2007)

TWO-QUBIT QUANTUM PROCESSOR

V17.6 Thu. March 19

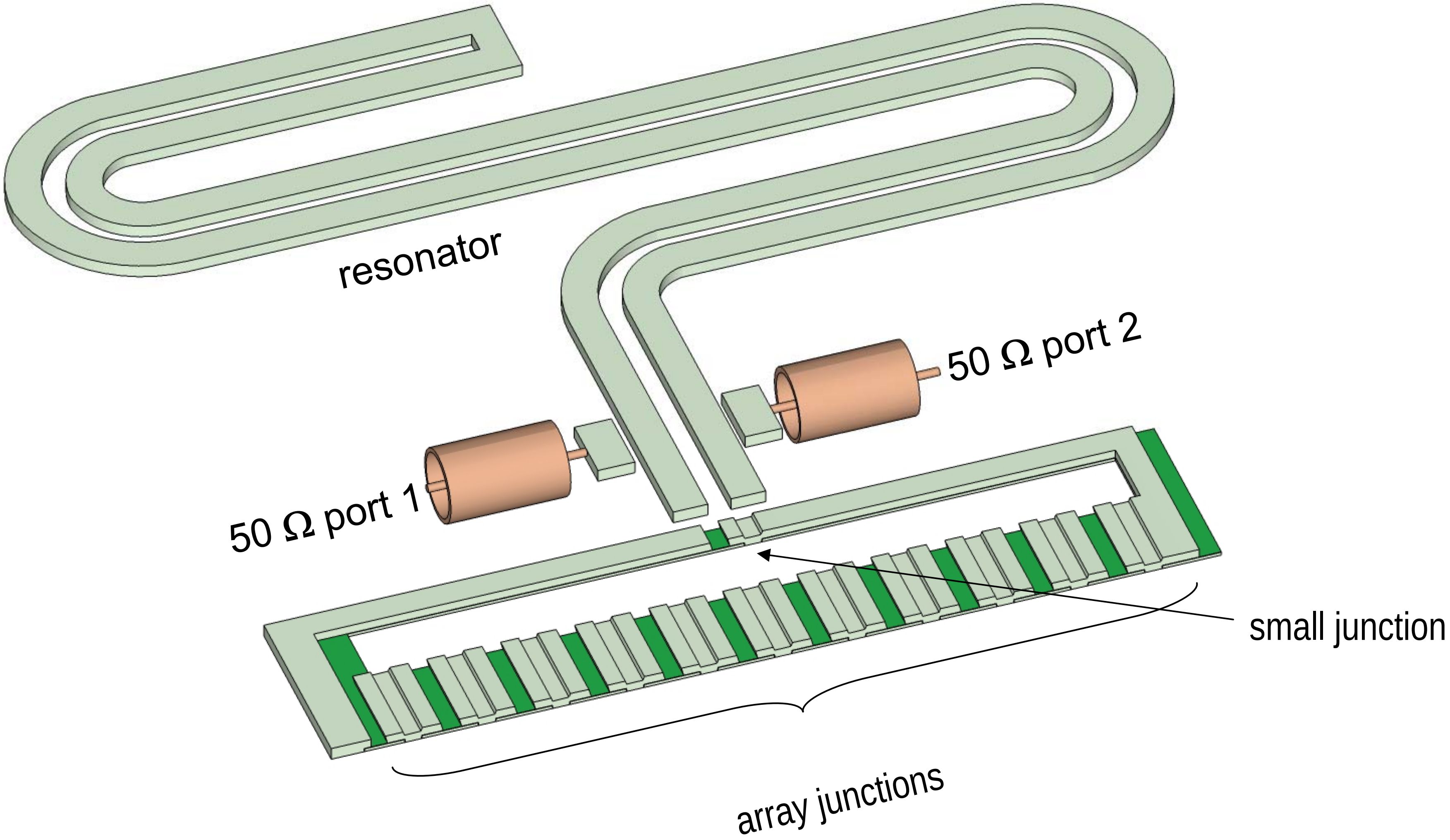
slide courtesy of
L. DiCarlo & Rob Schoelkopf



see also 1 qubit and 2 cavities: B. Johnson et al. V17.4

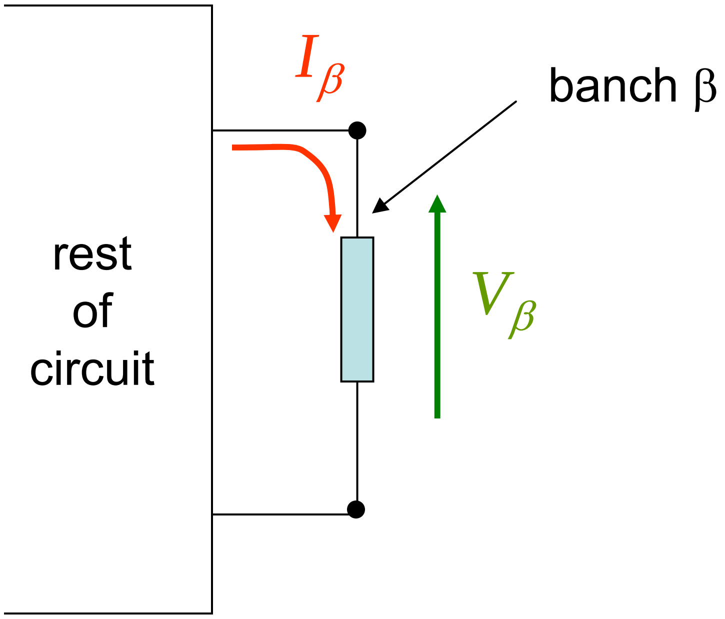
"FLUXONIUM QUBIT"

V. Manucharyan et al. **<u>Q17.5</u> Wednesday**



A FEW USEFUL IDEAS FOR CIRCUIT HAMILTONIANS.....

BRANCH VARIABLES



Introduce branch flux and charge

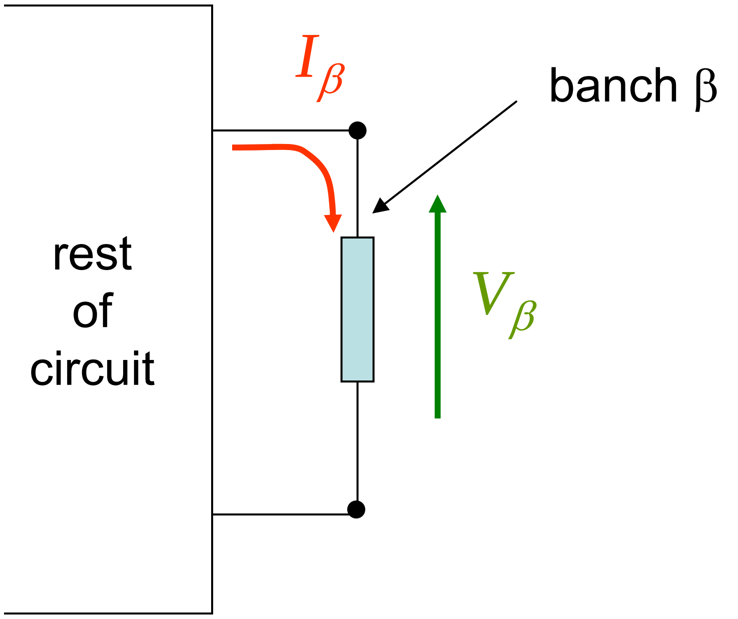
$$\phi_\beta(t) = \int_{-\infty}^{t} V_\beta(t')dt'$$

$$Q_\beta(t) = \int_{-\infty}^{t} I_\beta(t')dt'$$

position variable: $\phi \longleftrightarrow X$
momentum variable: $Q \longleftrightarrow P$

gener$^{\text{alized}}$ force : $I \longleftrightarrow f$
gener$^{\text{alized}}$ velocity: $V \longleftrightarrow V$

BRANCH VARIABLES



Introduce branch flux and charge

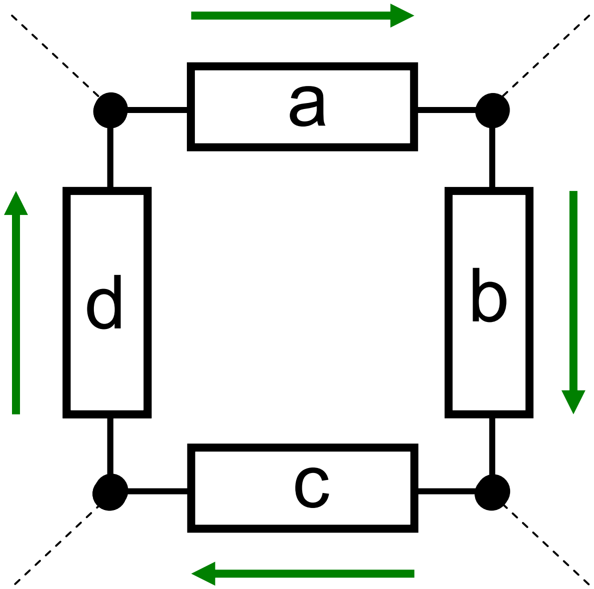
$$\phi_\beta(t) = \int_{-\infty}^{t} V_\beta(t')dt'$$

$$Q_\beta(t) = \int_{-\infty}^{t} I_\beta(t')dt'$$

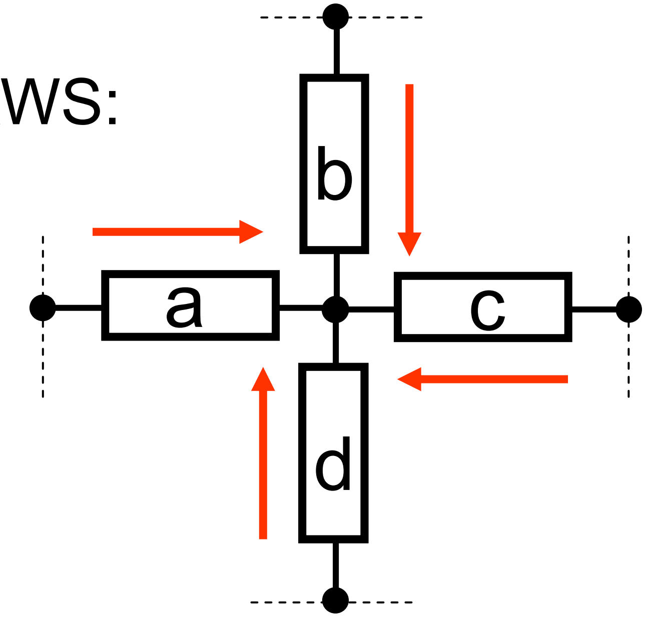
For every branch β in the circuit:

$$\boxed{\left[\hat{\phi}_\beta, \hat{Q}_\beta\right] = i\hbar}$$

PROBLEM: NOT ALL BRANCH VARIABLES ARE INDEPENDENT



KIRCHHOFF'S LAWS:

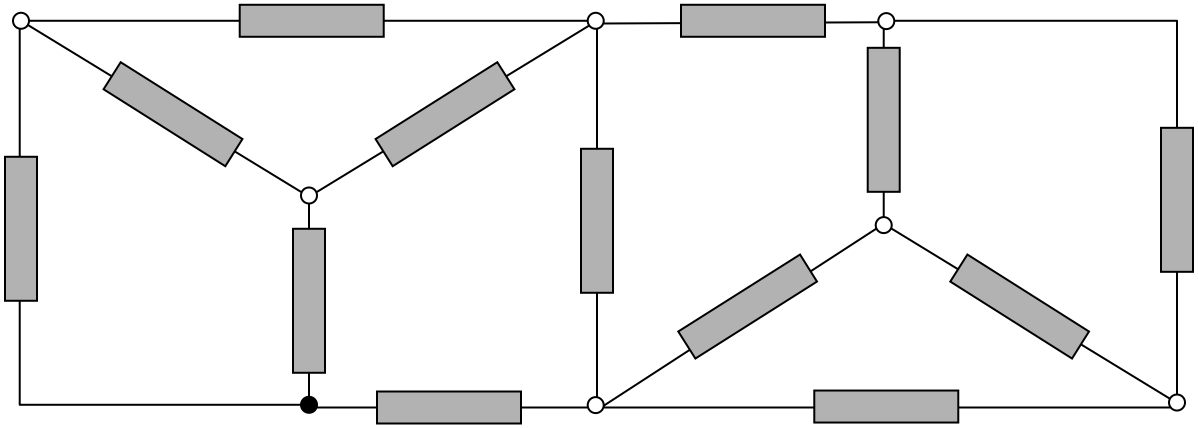


$$\sum_{\substack{\text{branches } \lambda \\ \text{around loop}}} V_\lambda = 0$$

$$\sum_{\substack{\text{branches } \nu \\ \text{tied to node}}} I_\nu = 0$$

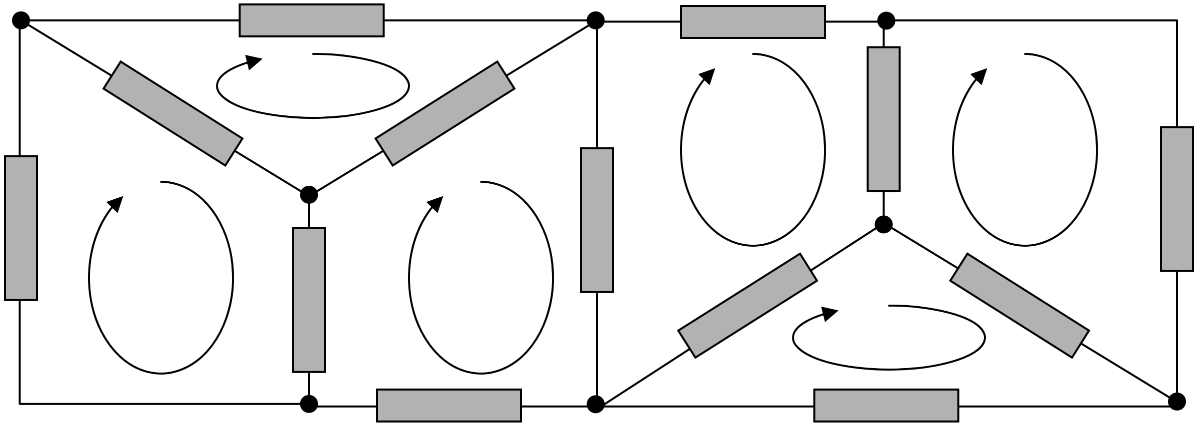
IMPOSE CONSTRAINTS ON BRANCH VARIABLES

TWO METHODS FOR DEFINING A COMPLETE SET OF INDEPENDENT VARIABLES



Method of nodes

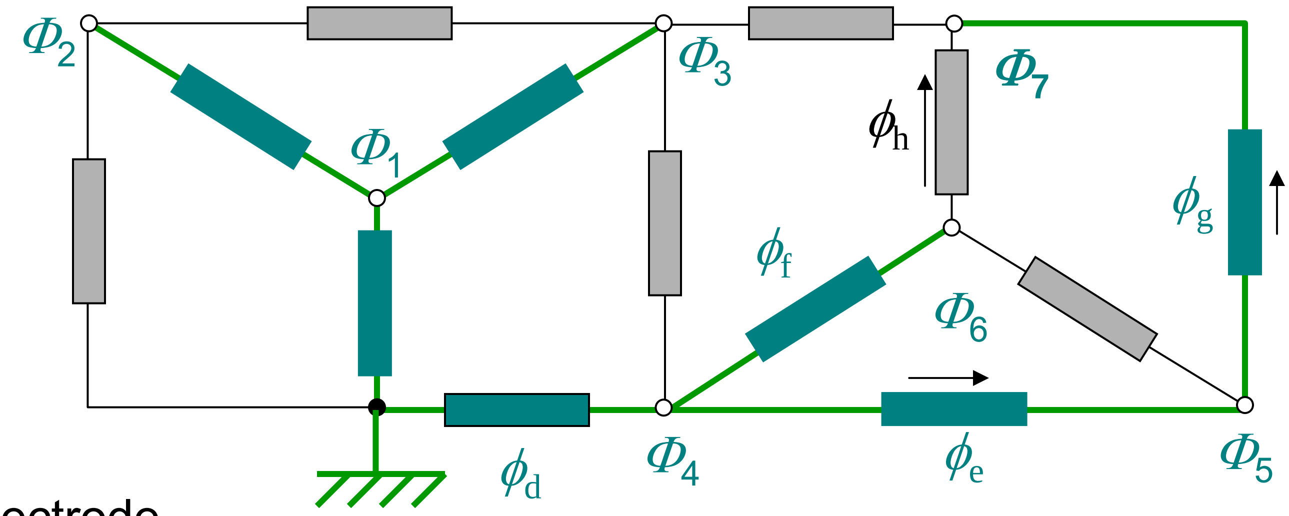
Defines node fluxes



Method of loops

Defines loop charges

EXAMPLE OF A COMPLETE SET OF INDEPENDENT VARIABLES



Method of nodes

1) Choose a reference electrode
(ground)

2) Choose a spanning tree
(accesses every node, no loop)

3) Select tree branch fluxes
(closure branches left out)

4) Node flux Φ_n is sum of branch fluxes to ground (closure branch fluxes are expressed as differences between node fluxes)

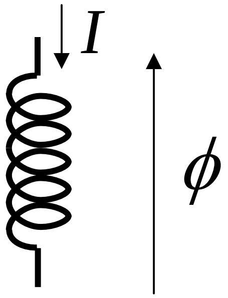
example: $\Phi_7 = \phi_d + \phi_e + \phi_g$

$\phi_h = \Phi_7 - \Phi_6 + \text{cst}$

$$\Phi_n = \sum_{\substack{\text{tree branches } \beta \\ \text{leading to } n}} \phi_\beta$$

$$\phi_\gamma = \Phi_{n_+(\gamma)} - \Phi_{n_-(\gamma)} + \text{cst}$$

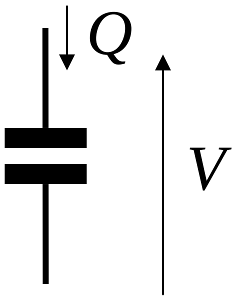
INDUCTIVE vs CAPACITIVE ELEMENTS



Inductance : current I function only of flux ϕ

$$E=\int_{-\infty}^{t} I \boxed{\cdot V dt'} = \int_{0}^{\phi} I\left(\phi'\right) d\phi'$$

Electrical equivalent of spring: $\phi \leftrightarrow X$; $I \leftrightarrow f$



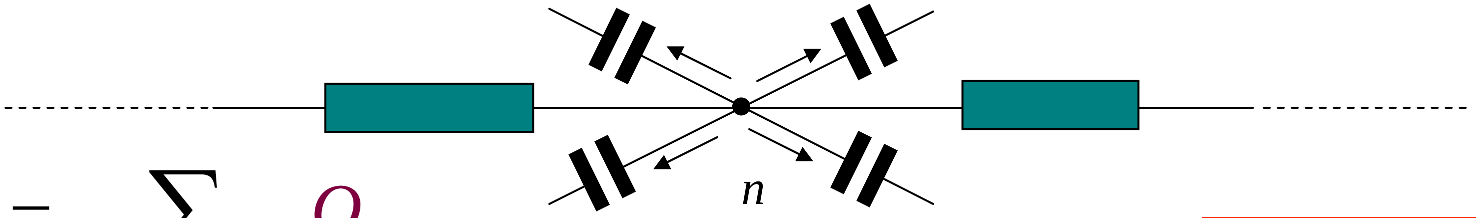
Capacitance : voltage V function only of charge Q

$$E=\int_{-\infty}^{t} V \boxed{\cdot I dt'} = \int_{0}^{Q} V\left(Q'\right) dQ'$$

Electrical equivalent of mass: $Q \leftrightarrow P$; $V \leftrightarrow V$

NODE CHARGES

The conjugate coordinates of node fluxes
are node charges: they are the sum of all the charges
going into capacitances linked to this node.



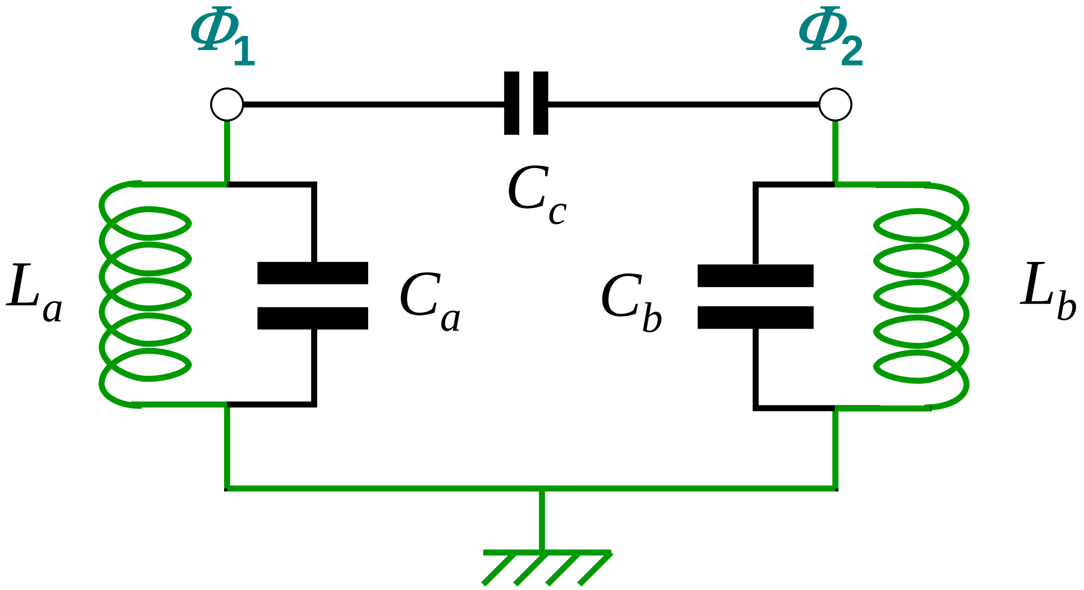
$$Q_n = \sum_{\substack{\text{capacitive}\\ \text{branches}\\ \text{exiting from } n}} Q_\beta$$

for each node: $\boxed{\left[\Phi_n, Q_n\right] = i\hbar}$

This can be demonstrated by writing the Lagrangian
of the circuit from its dynamical equations
and performing a Legendre transform to obtain the Hamiltonian

Dynamical equations → Lagrangian $L\left(x, \dot{x}, t\right)$ → conjugate coordinates $p = \partial L / \partial \dot{x}$ → Hamiltonian $H\left(x, p, t\right)$

HAMILTONIAN OF TWO CAPACITIVELY COUPLED RESONATOR MODES



Reduced capacitance matrix:

$$\mathbf{C} = \begin{bmatrix} C_a + C_c & -C_c \\ -C_c & C_b + C_c \end{bmatrix}$$

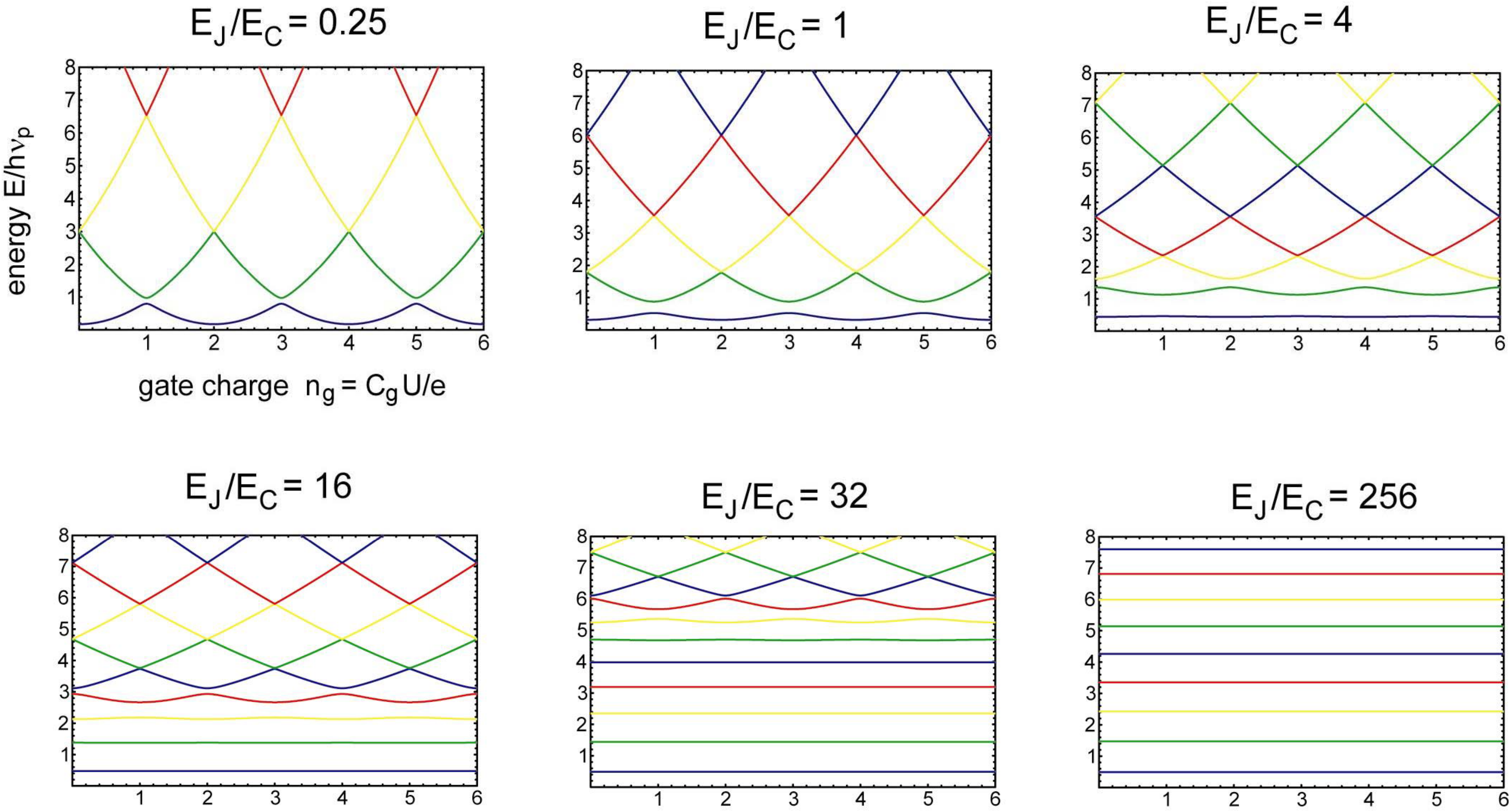
Inverse of reduced capacitance matrix:

$$\mathbf{C}^{-1} = \frac{1}{C_a C_b + C_a C_c + C_b C_c} \begin{bmatrix} C_b + C_c & C_c \\ C_c & C_a + C_c \end{bmatrix}$$

$$\hat{H}\left(\hat{\Phi}_1, \hat{Q}_1; \hat{\Phi}_2, \hat{Q}_2\right) = \frac{\hat{\Phi}_1^2}{2L_a} + \frac{\hat{\Phi}_2^2}{2L_b} + \frac{\hat{Q}_1^2}{2C_1} + \frac{\hat{Q}_2^2}{2C_2} + \boxed{\frac{\hat{Q}_1 \hat{Q}_2}{C_3}}$$

ANHARMONICITY vs CHARGE SENSITIVITY

Cooper pair box levels are "exactly soluble" (A. Cottet, PhD thesis, Orsay, 2002)



ANHARMONICITY vs CHARGE SENSITIVITY IN THE LIMIT $E_J/E_C >> 1$

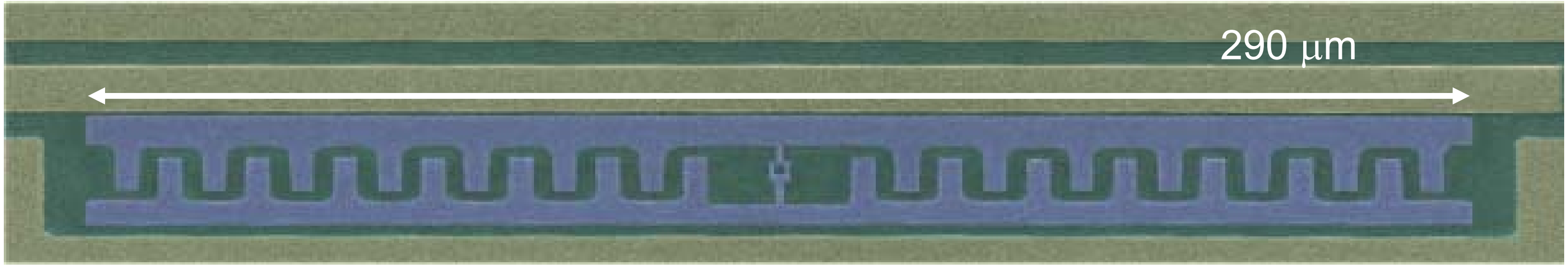
anharmonicity:
$$\frac{\omega_{12}-\omega_{01}}{(\omega_{12}+\omega_{01})/2} \to \sqrt{\frac{E_C}{8E_J}}$$

J. Koch et al. Phys. Rev. A '07

peak-to-peak charge modulation amplitude of level m:

$$\epsilon_m \to (-1)^m E_C \frac{2^{4m+5}}{m!} \sqrt{\frac{2}{\pi}} \left(\frac{E_J}{2E_C}\right)^{\frac{m}{2}+\frac{3}{4}} e^{-\sqrt{8E_J/E_C}}$$

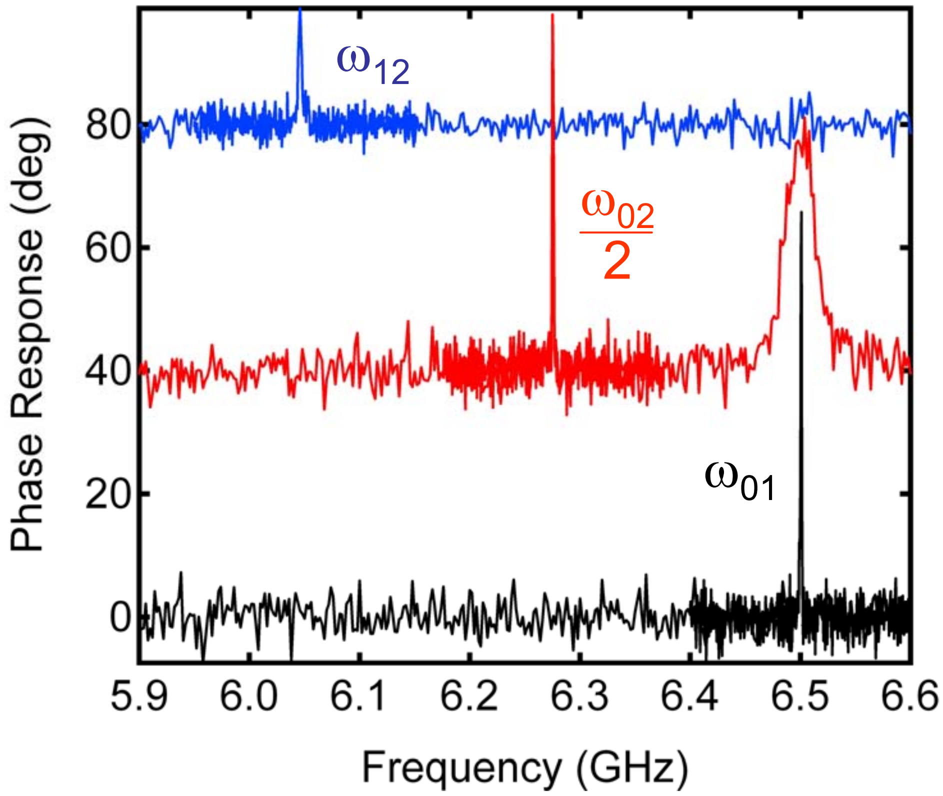
TRANSMON: SHUNT CPB JUNCTION WITH CAPACITANCE

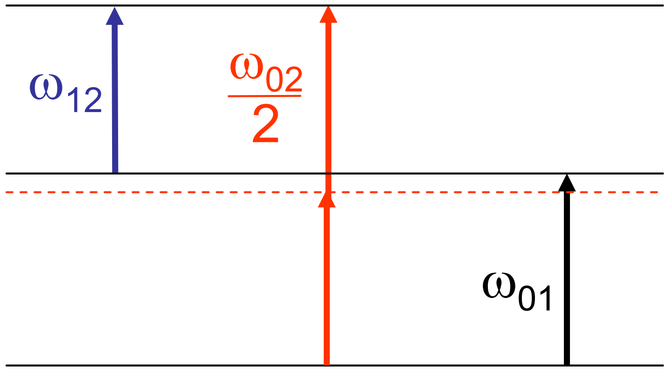


Courtesy of J. Schreier and R. Schoelkopf

SPECTROSCOPY OF A JOSEPHSON ATOM

J. Schreier et al., Phys. Rev. B '08





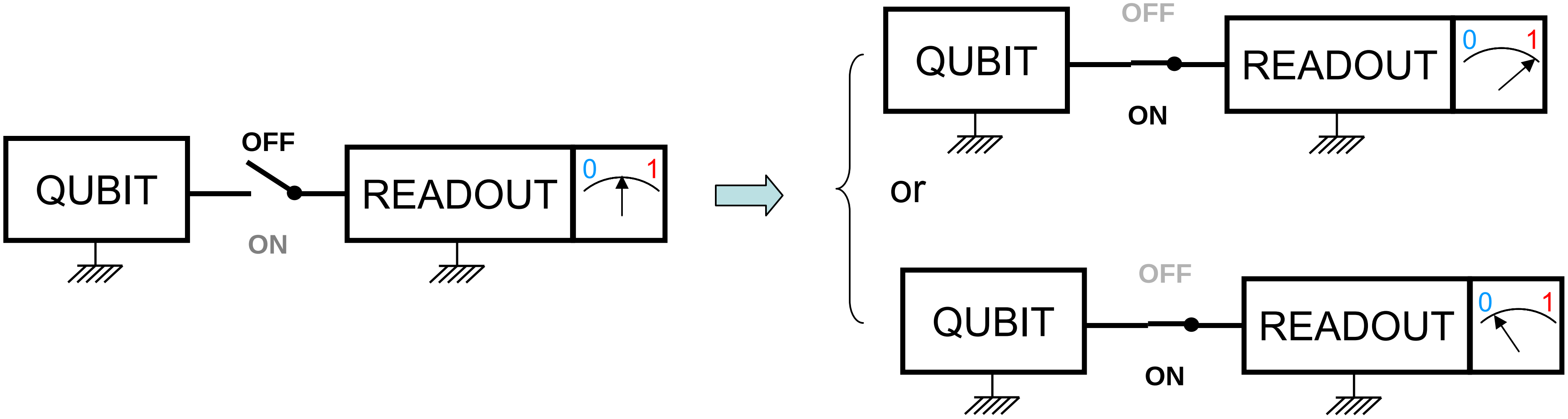
Anharmonicity:

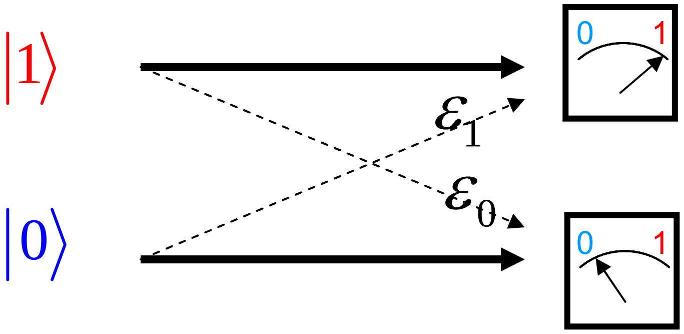
$$\omega_{01} - \omega_{12} = 455\text{MHz} \simeq E_C$$

Sufficient to control the artificial atom as a two level system: Qubit

Slide courtesy of J. Schreier and R. Schoelkopf

THE MEMORY READOUT PROBLEM





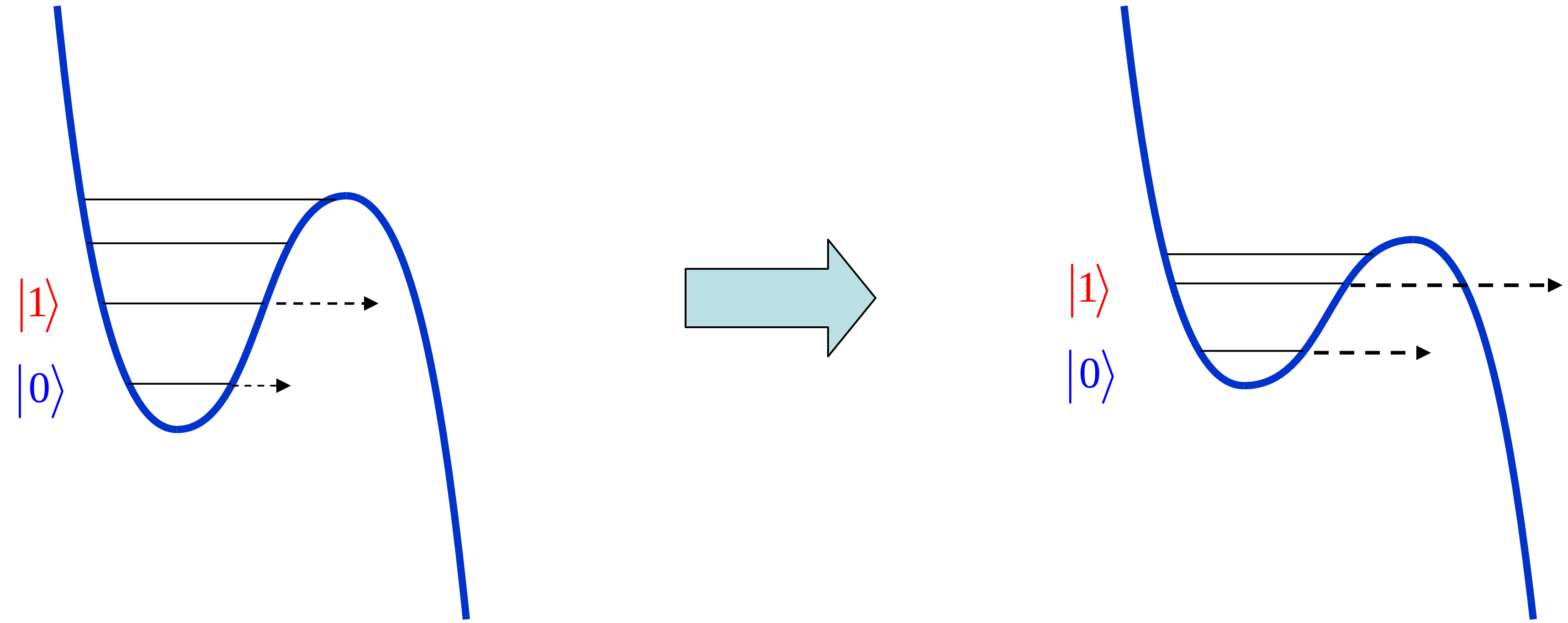
FIDELITY:

$$F = 1 - \varepsilon_0 - \varepsilon_1$$

WANT:

1) SWITCH WITH ON/OFF RATIO AS LARGE AS POSSIBLE
2) READOUT WITH ***F*** AS CLOSE TO 1 AS POSSIBLE

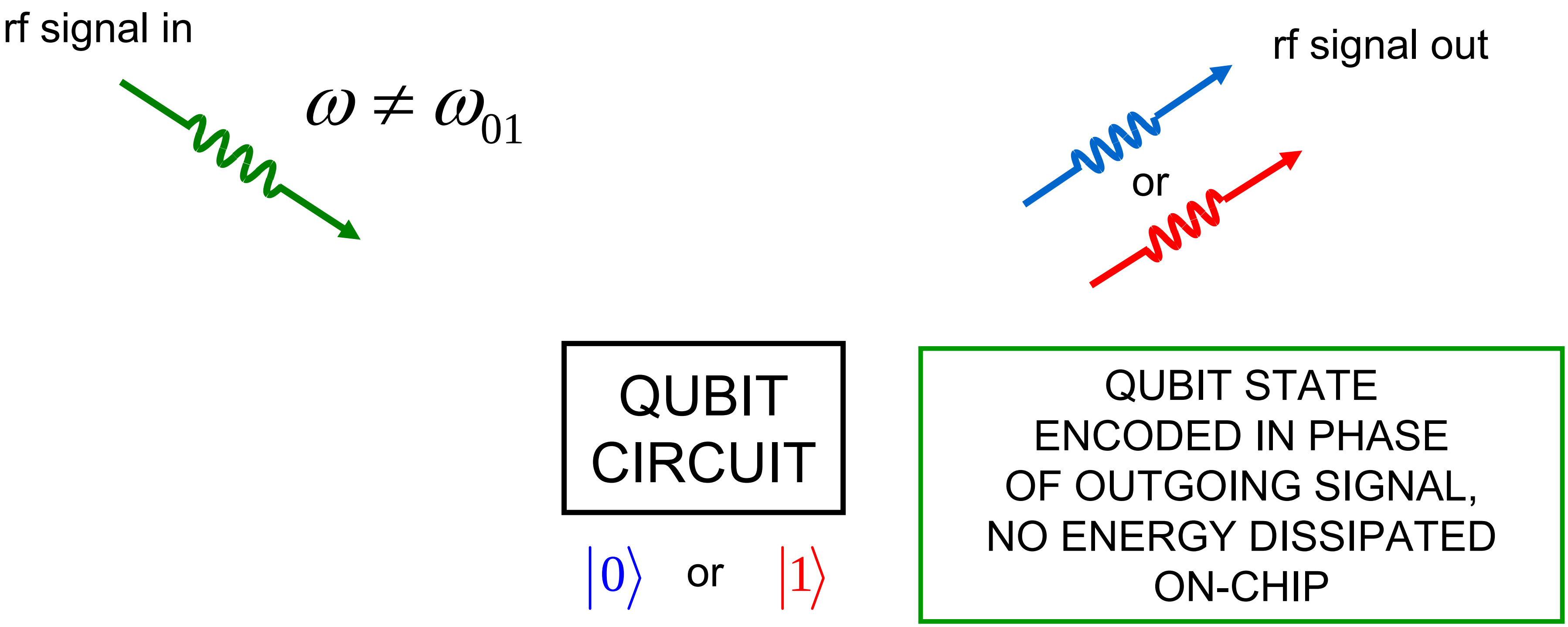
STATE DECAY STRATEGY



Martinis, Devoret and Clarke, PRL **55** (1985)
Martinis, Nam, Aumentado and Urbina, PRL **89** (2002)

DISPERSIVE READOUT STRATEGY

Blais et al. PRA 2004, Walraff et al., Nature 2004



rf signal in

$\omega \neq \omega_{01}$

rf signal out

or

QUBIT CIRCUIT

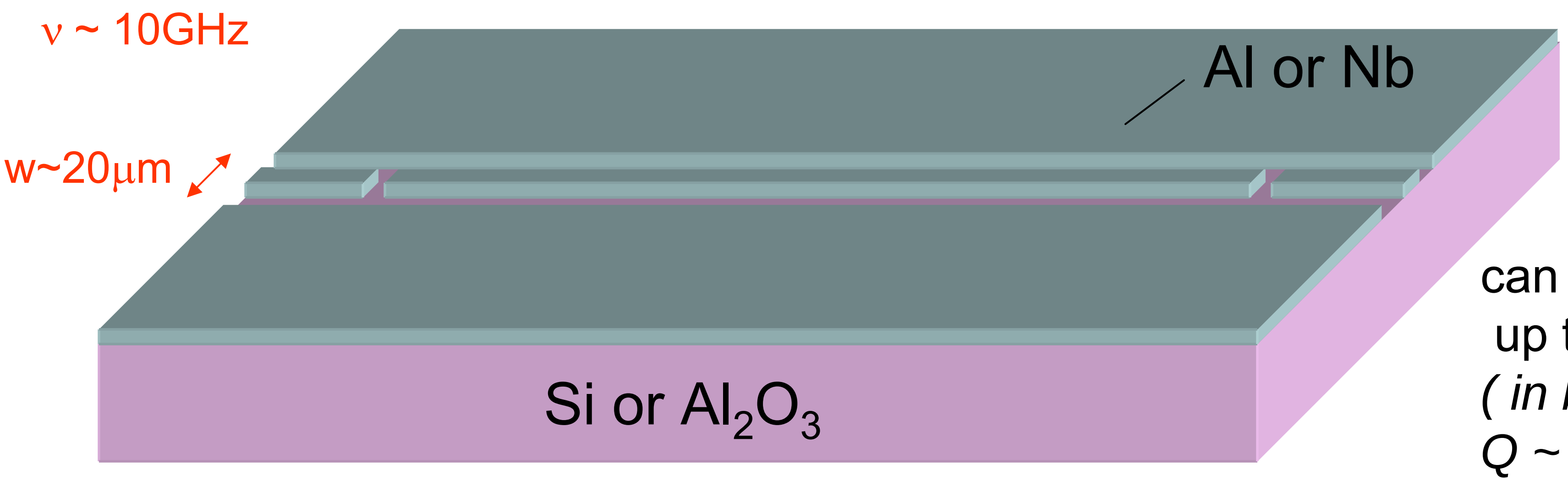
$|0\rangle$ or $|1\rangle$

QUBIT STATE
ENCODED IN PHASE
OF OUTGOING SIGNAL,
NO ENERGY DISSIPATED
ON-CHIP

A) FILTER OUT EVERYTHING ELSE THAN READOUT RF
B) REPEAT WITH ENOUGH PHOTONS TO BEAT NOISE : USE THE BEST AMPLIFIER AS POSSIBLE

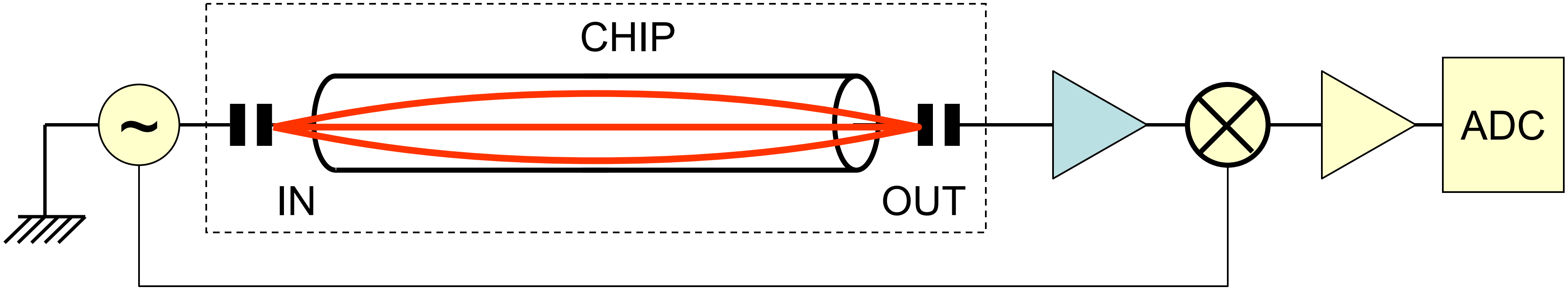
(see whole sessions J3 & L17 + Q17.6& Y34.4)

SUPERCONDUCTING MICROWAVE RESONATOR: ANALOG OF FABRY-PEROT CAVITY FILTER



can get Q
up to 10^6
(in bulk:
$Q \sim 10^{10}$)

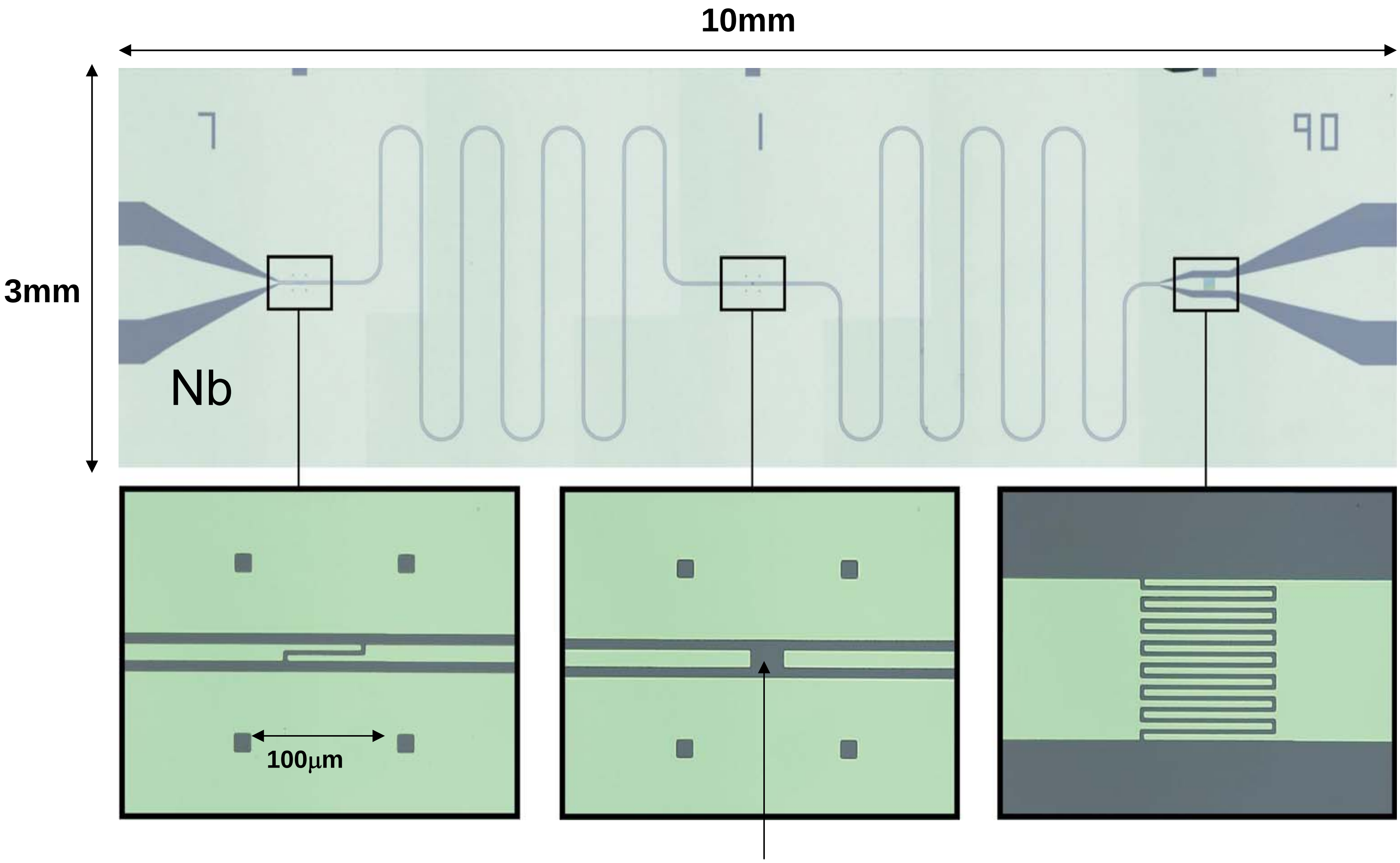
length ~ 1cm, but photon decay length ~ 10km!



VERY SMALL MODE VOLUME

1/2 photon: ~1nA
~100nV

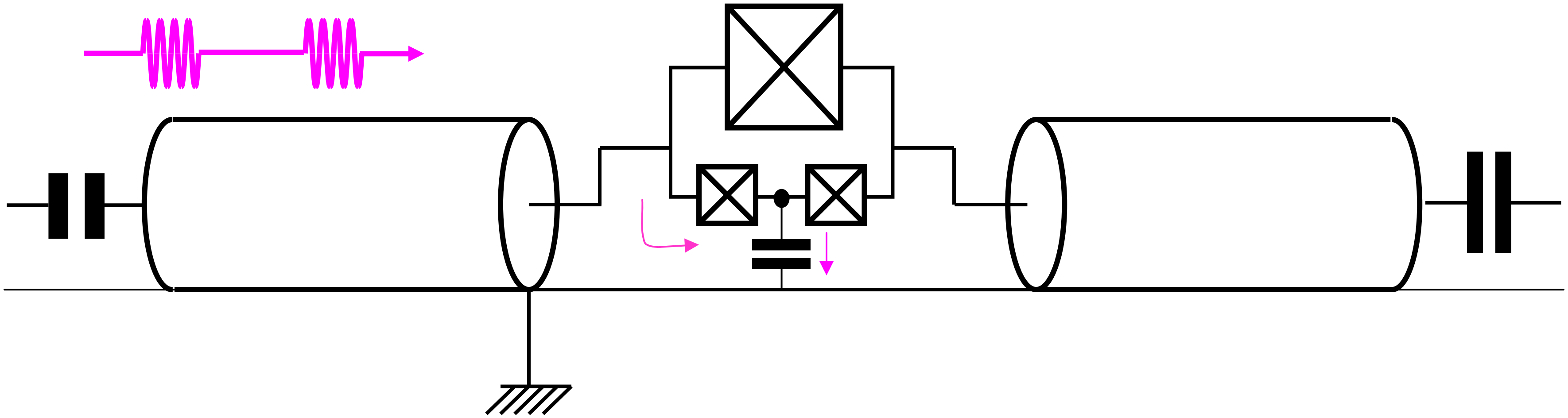
SUPERCONDUCTING CAVITY = FABRY-PEROT



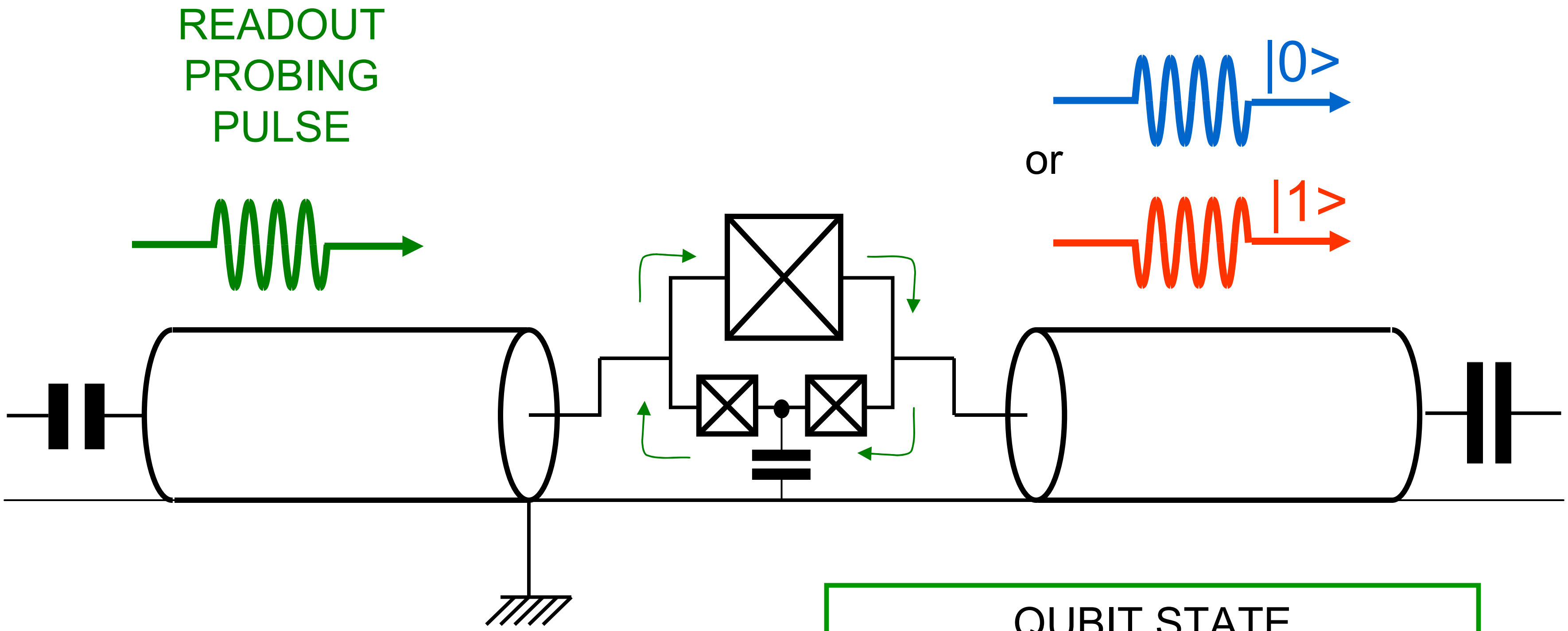
qubit goes here (see slide 2, lower right, in this talk)

EXAMPLE OF QUANTRONIUM IN MICROWAVE CAVITY

OFF-RESONANT NMR-TYPE
PULSE SEQUENCE
FOR QUBIT MANIPULATION



QUANTRONIUM IN MICROWAVE CAVITY



READOUT
PROBING
PULSE

|0>

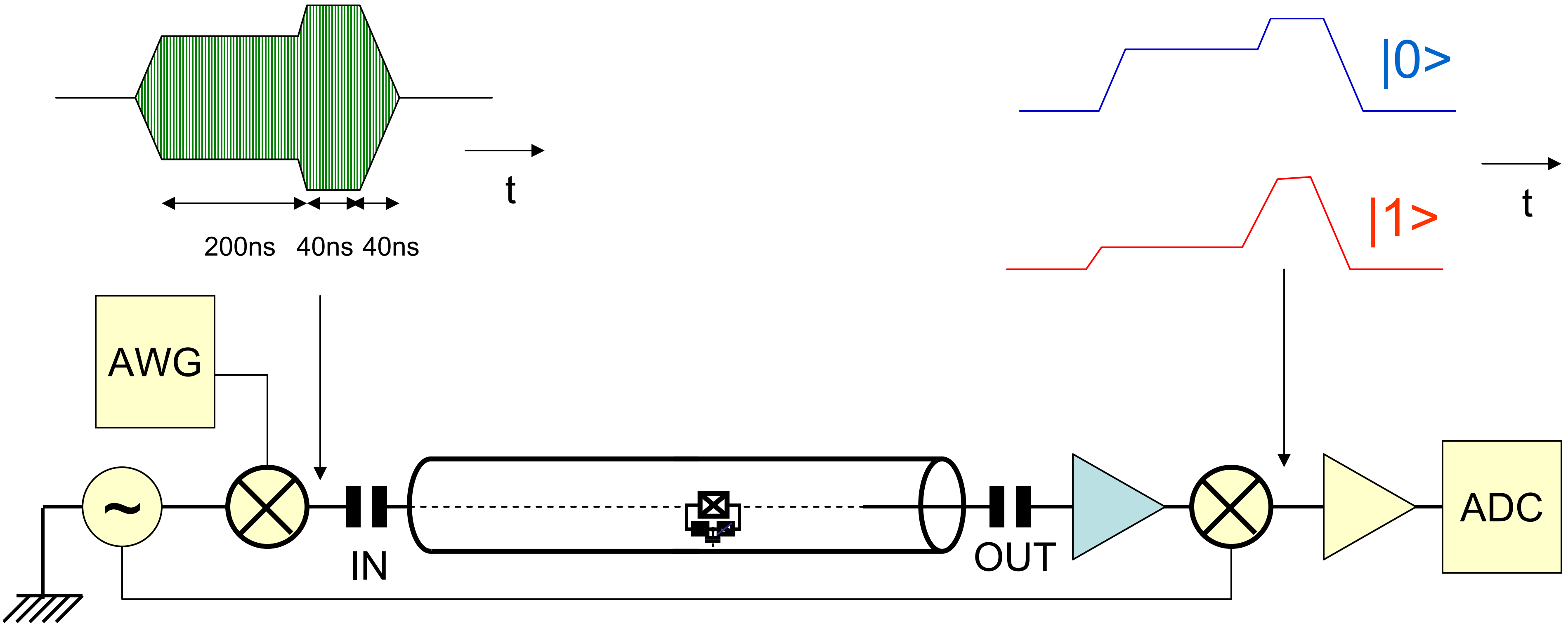
or

|1>

QUBIT STATE
ENCODED IN PHASE
OF TRANSMITTED PULSE

QUANTRONIUM IN MICROWAVE CAVITY

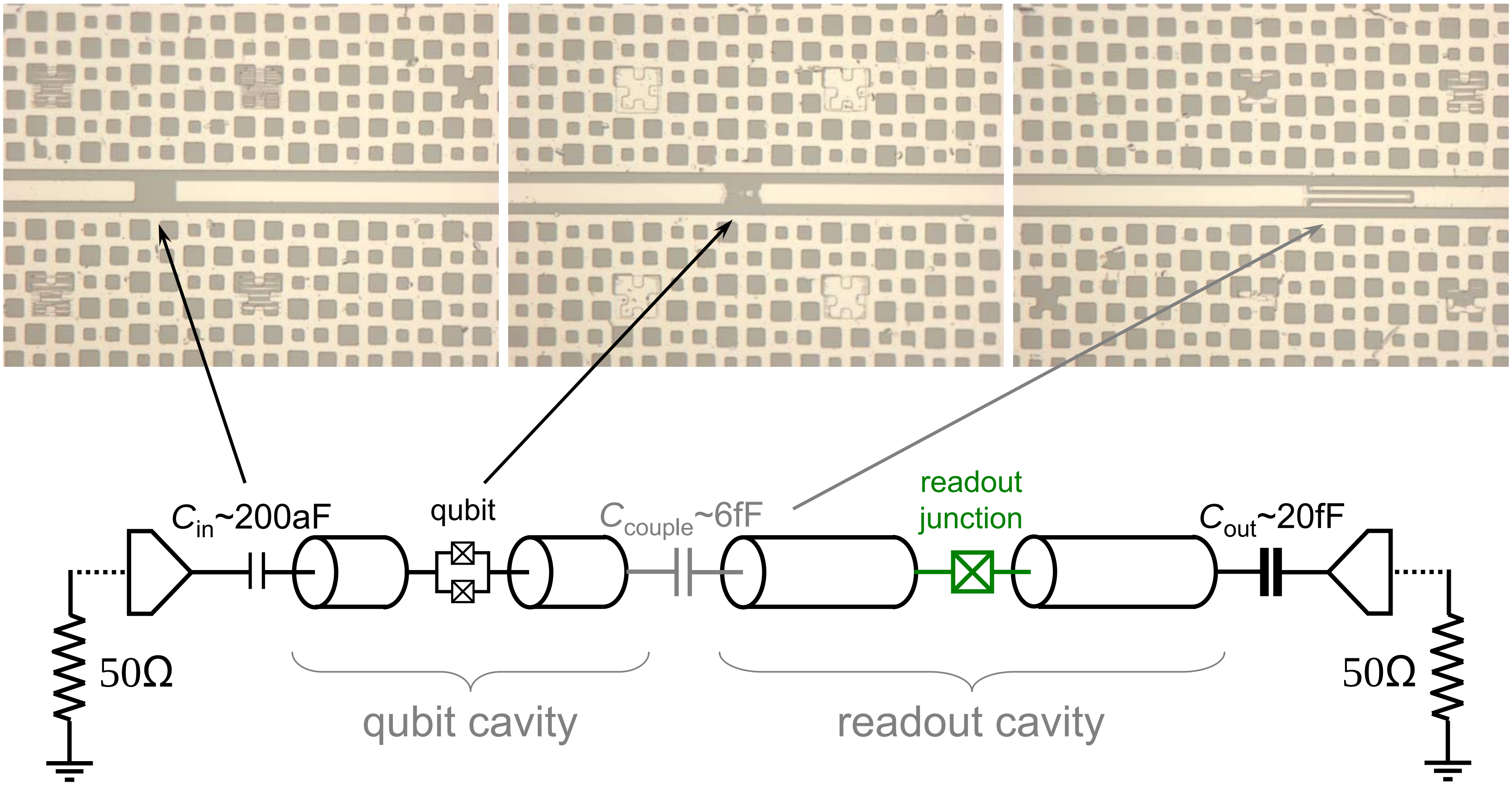
Metcalfe et al.
Phys. Rev. B6
174516 (2007)



latching effect
due to bifurcation
of cavity mode

IN-LINE TRANSMON WITH BIFURCATING MICROWAVE CAVITY READOUT

See **V17.6** M. Brink et al. Thursday morning, and also recent Saclay group results (in preparation)



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