APS 2010 March Meeting, Tutorial #3

Advances in Josephson Quantum Circuits

Instructors:

Michel Devoret, Yale University
"Introduction to superconducting quantum circuits"

Yasunobu Nakamura, NEC Japan
"Superconducting qubits coupled to a transmission line "

John Martinis, University of California, Santa Barbara
"Precision Control of Josephson Qubits"

Leo DiCarlo, Yale University
"Production and detection of entanglement in cQED processors"

Portland Convention Center
Sunday, March 14
8:30 a.m. - 12:30 p.m.

Introduction to superconducting quantum circuits

Outline

- Motivation: quantum information
- Why Josephson junctions?
- Main flavors of Josephson qubits
- Readout of qubits
- 1-qubit qnd 2-qubit gates

RECENT REVIEWS ON JOSEPHSON QUANTUM CIRCUITS

M.H. Devoret and J.M. Martinis, *Quantum Information Processing* **3**,163 (2004)

A. Blais et al., *Phys. Rev.* A **75**, 032329 (2007)

J. Clarke and F. Wilhelm, *Nature* **453**, 1031-1042 (2008)

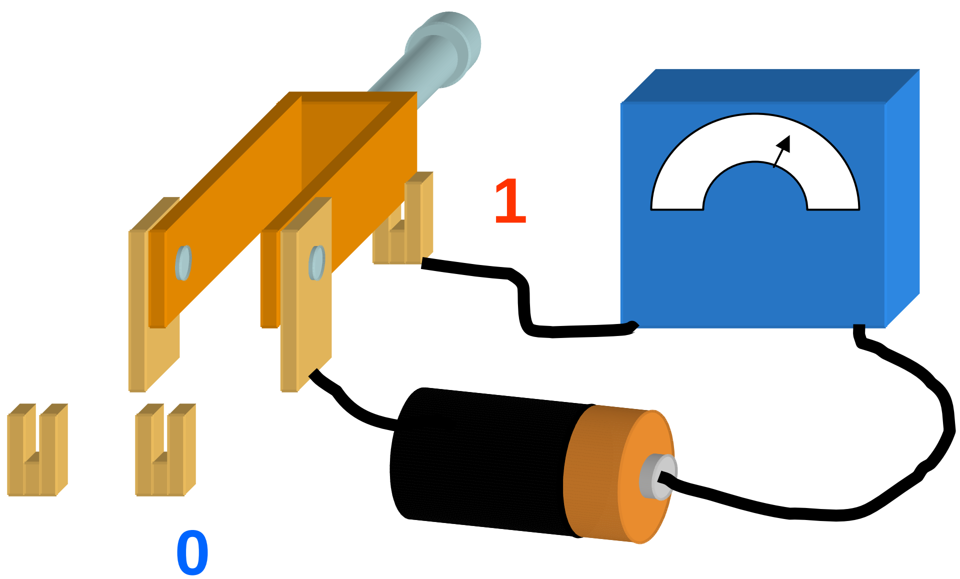
R. Schoelkopf and S. M. Girvin, *Nature* **451**, 664-669 (2008)

J.M. Martinis, *Quantum Information Processing* **8**, 81 (2009)

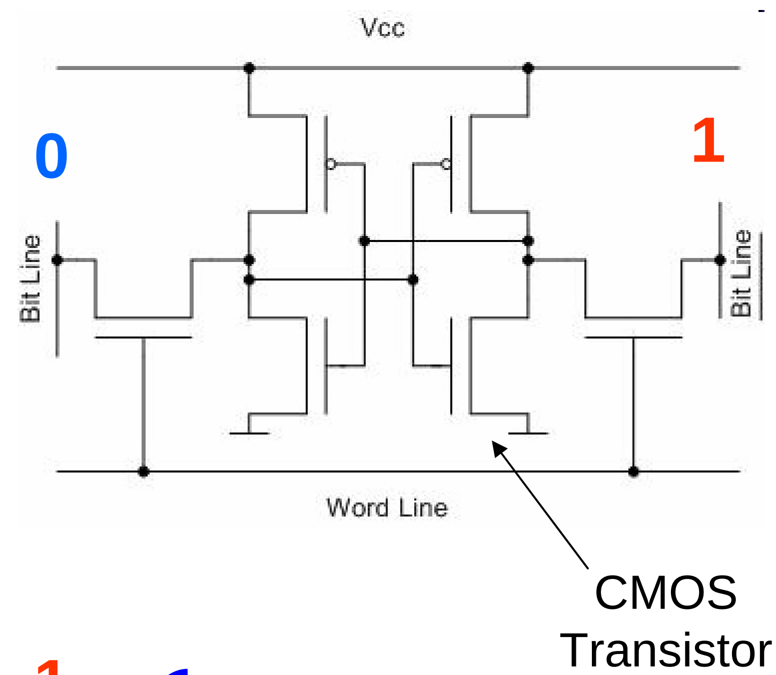
Final version of this presentation available at
http://qulab.eng.yale.edu/archives.htm (talks)

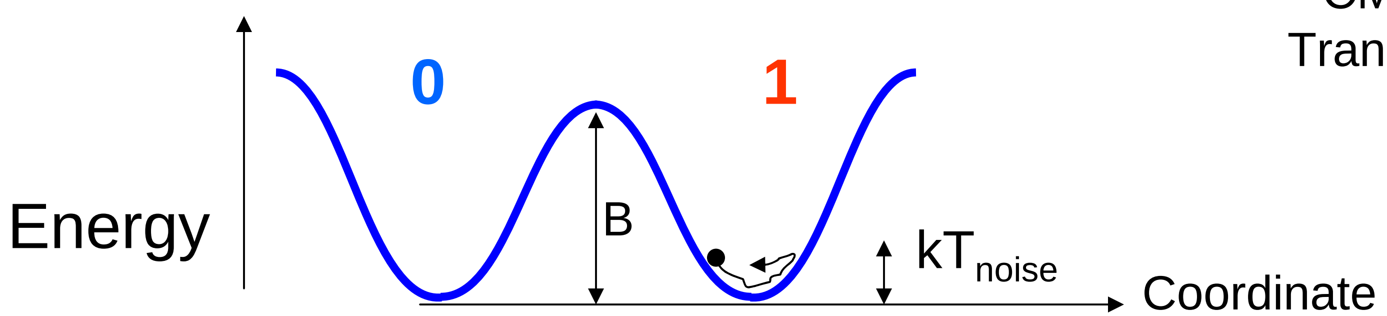
CLASSICAL BIT = SWITCH

Mechanical switch



Electrical switch

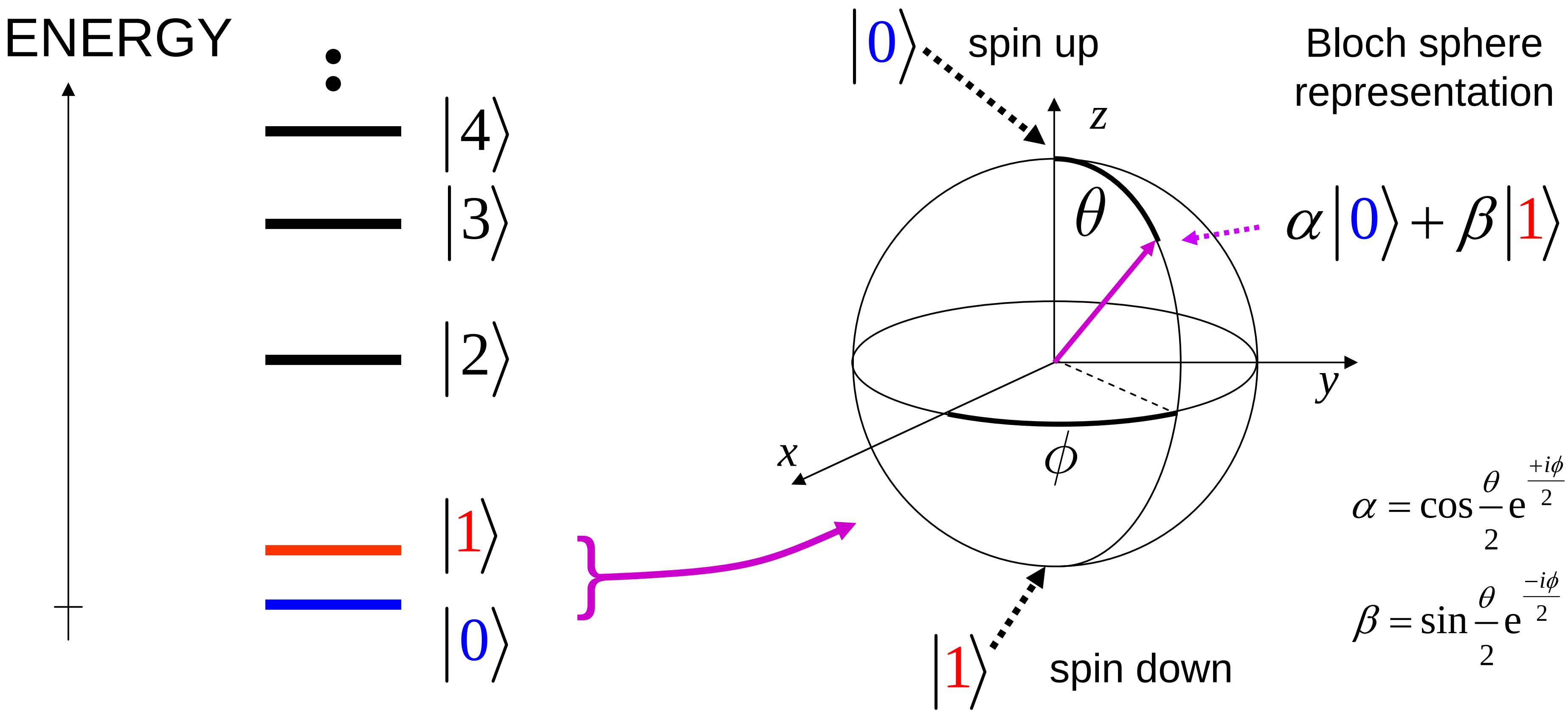




Bit state is either 0 or 1: 1) strong dissipation and 2) $kT_{noise} << B$

QUANTUM BIT: 2 LEVELS
FORMING EFFECTIVE SPIN 1/2

MOLECULE, ATOM, PARTICLE...

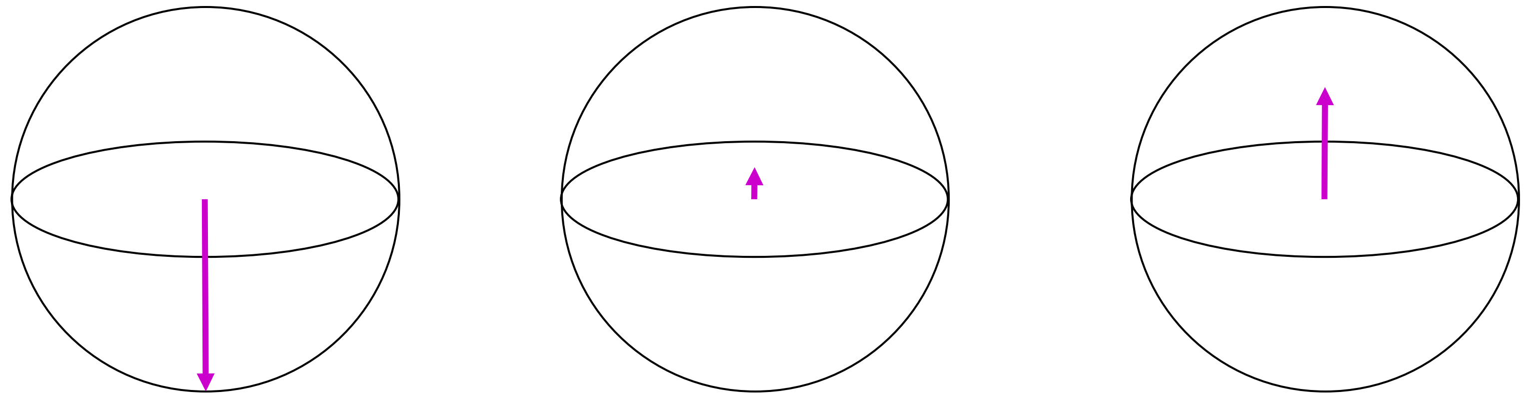


Qubit state can be 0 <u>and</u> 1: 1) no dissipation and 2) $kT_{noise} << \hbar\omega_{01}$

RELAXATION TIMES OF QUANTUM MEMORY

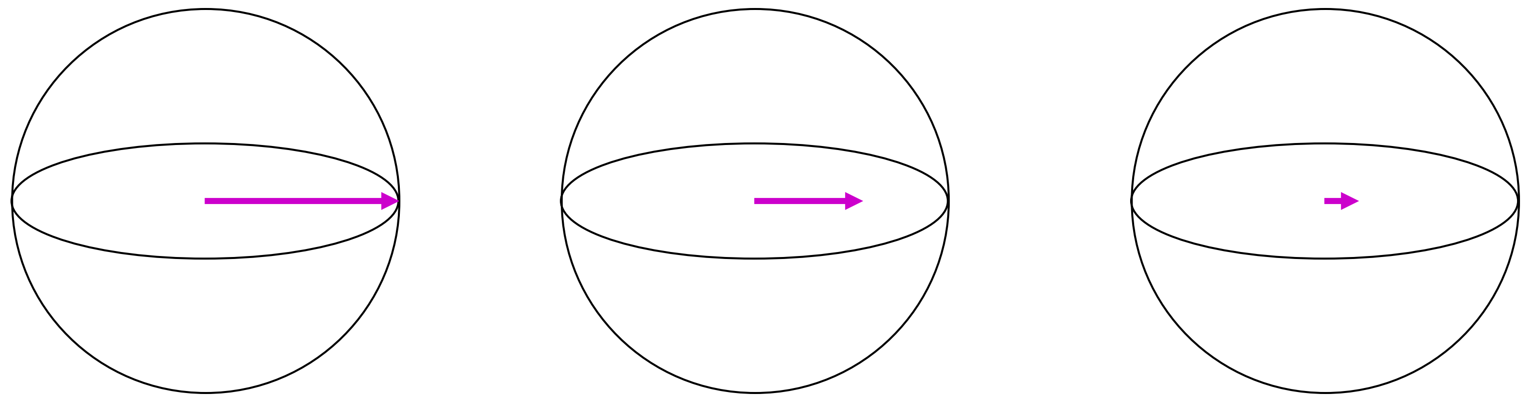
T_1 PROCESS

random fields
in x,y plane



T_ϕ PROCESS

random field
along z

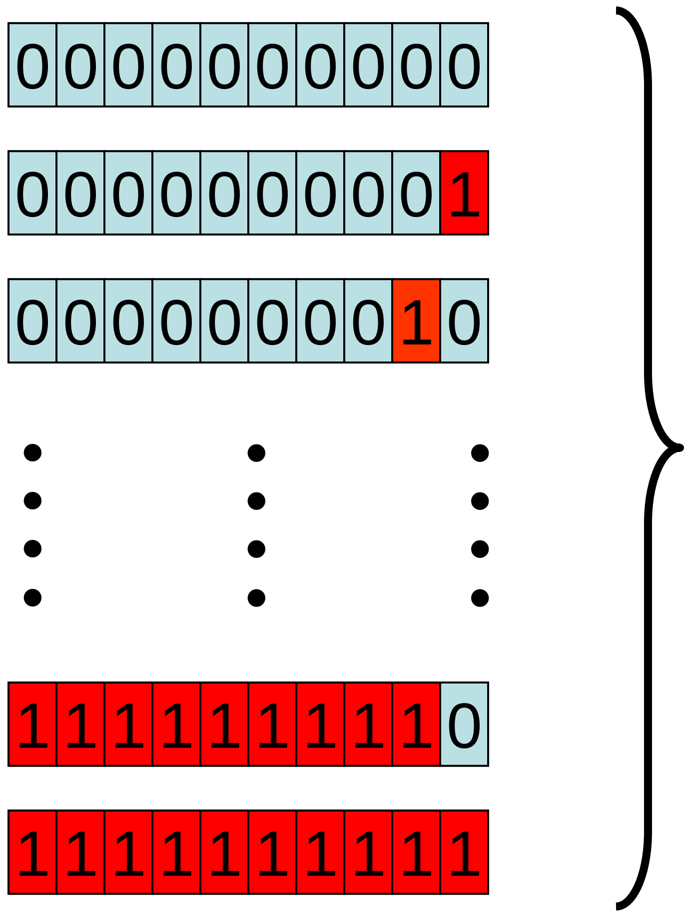


$$T_2 = \frac{1}{\frac{1}{2T_1} + \frac{1}{T_\phi}}$$ DECOHERENCE TIME

$$\omega_{01} T_2$$ DECOHERENCE QUALITY FACTOR

THE POWER OF QUANTUM SUPERPOSITION

REGISTER WITH N=10 BITS:

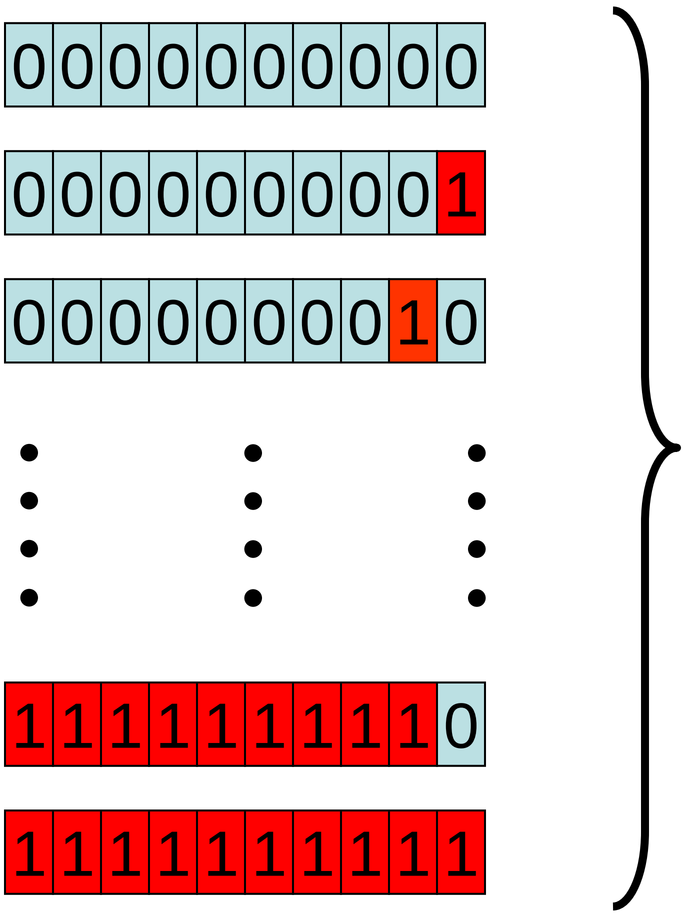


2^N = 1024 POSSIBLE CONFIGURATIONS

classically, can store and work only on one number between 0 et 1023

THE POWER OF QUANTUM SUPERPOSITION

REGISTER WITH N=10 BITS:



2^N = 1024 POSSIBLE CONFIGURATIONS

classically, can store and work only on one number between 0 et 1023

"quantally", can store and work on an arbitrary superposition of these numbers!

$$|\Psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle + ... + \alpha_{2^N-1} |2^N - 1\rangle$$

QUANTUM PARALLELISM

suppose a function f $\quad j \in \{0, 1023\} \rightarrow n = f(j) \in \{0, 1023\}$

Classically, need 1000 $\times$10-bit registers (10,000 bits) to store information about this function and to work on it.

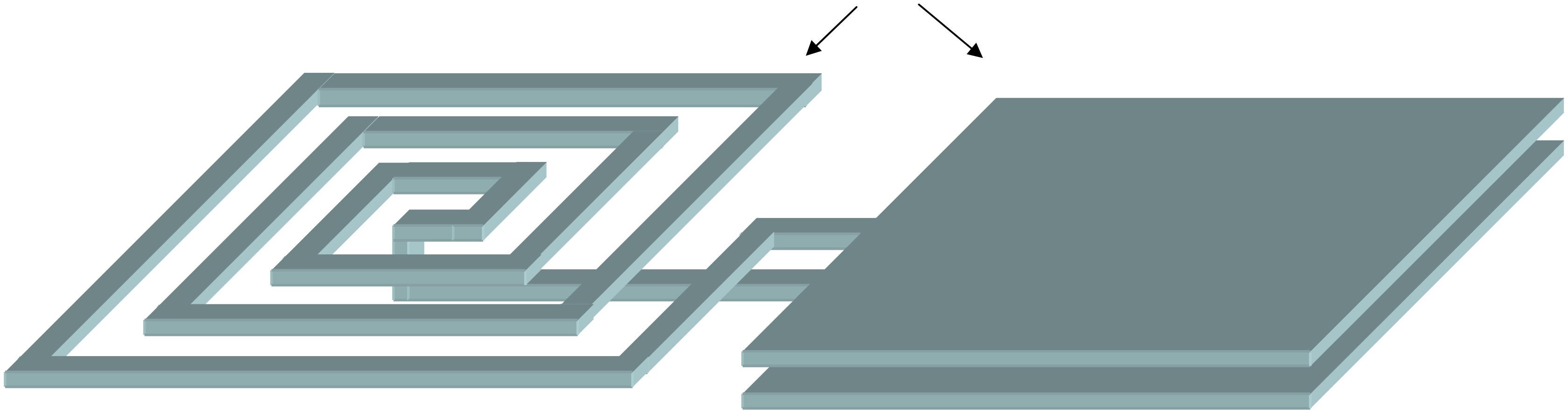
Quantum-mechanically, a 20-qubit register can suffice!

$$|\Psi\rangle = \frac{1}{2^{N/2}} \sum_{j=0}^{2^N - 1} |j\rangle |f(j)\rangle$$

Function encoded in a superposition of states of register

HOW CAN A SUPERCONDUCTING CIRCUIT BEHAVE LIKE AN ATOM?

SIMPLEST EXAMPLE: SUPERCONDUCTING **LC** OSCILLATOR CIRCUIT

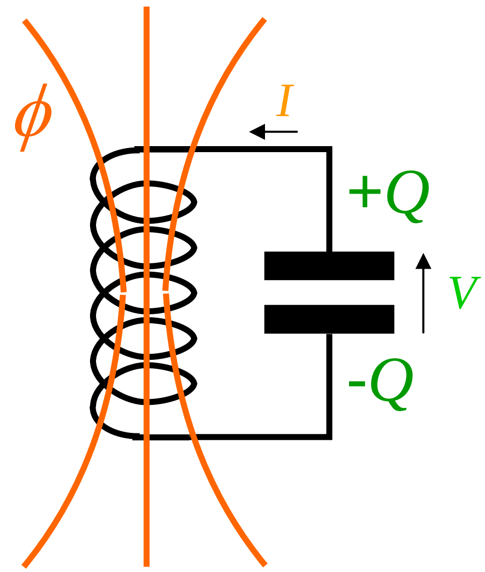


MICROFABRICATION ➡ L ~ 3nH, C ~ 1pF, $\omega_r/2\pi$ ~ 4GHz

ELECTRONIC FLUID FLOWS BACK AND FORTH BETWEEN PLATES:
ALL ELECTRONS BEHAVE AS A SINGLE CHARGED ENTITY

see practical LC superconducting resonators: Lindström *et al.,* PRB **80**, 132501 (2009)
Paik & Osborn, APL **96**, 072505 (2010)

QUANTUM CIRCUITS IN A NUTSHELL: FLUX AND CHARGE DO NOT COMMUTE

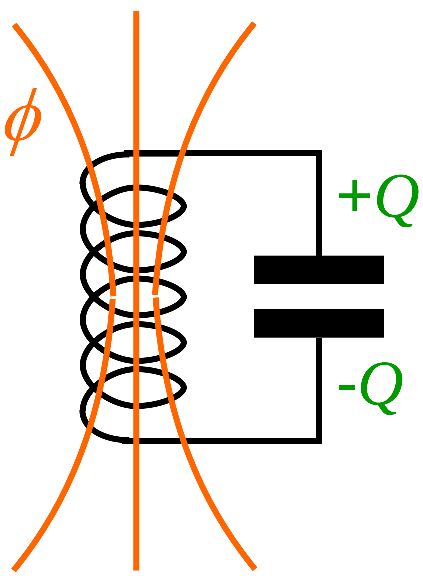


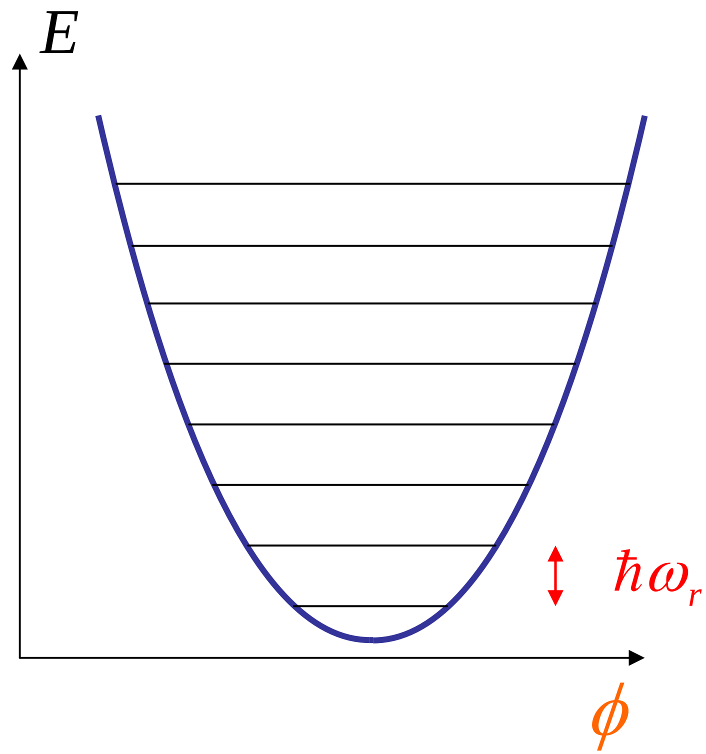
$$\left[\hat{\phi}, \hat{Q}\right] = i\hbar$$

$$\phi = LI$$

$$Q = CV$$

LC CIRCUIT AS QUANTUM HARMONIC OSCILLATOR





$$\hat{H} = \hbar\omega_r\left(\hat{a}^\dagger\hat{a} + \tfrac{1}{2}\right)$$

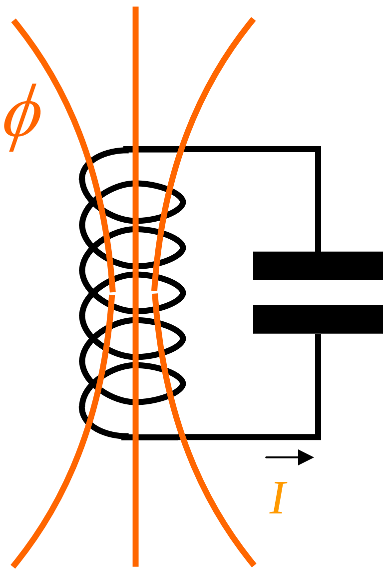
$$\hat{a} = \frac{\hat{\phi}}{\phi_r} + i\frac{\hat{Q}}{Q_r}; \quad \hat{a}^\dagger = \frac{\hat{\phi}}{\phi_r} - i\frac{\hat{Q}}{Q_r}$$

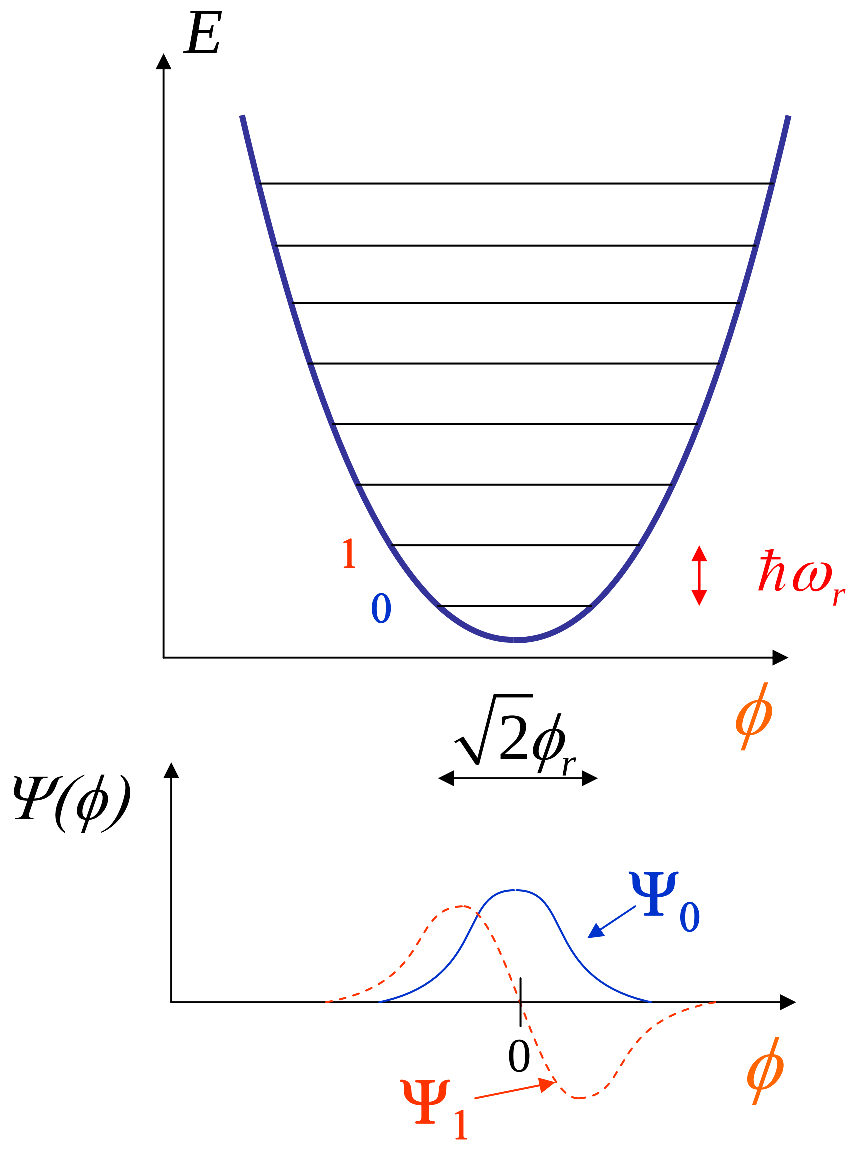
← annihilation and creation operators for excitation quanta of circuit (standing photons)

$$\phi_r = \sqrt{2\hbar\omega_r L}$$

$$Q_r = \sqrt{2\hbar\omega_r C}$$

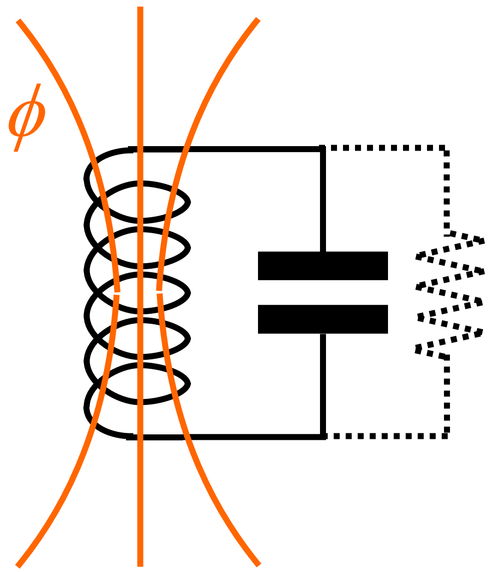
WAVEFUNCTIONS OF LC CIRCUIT

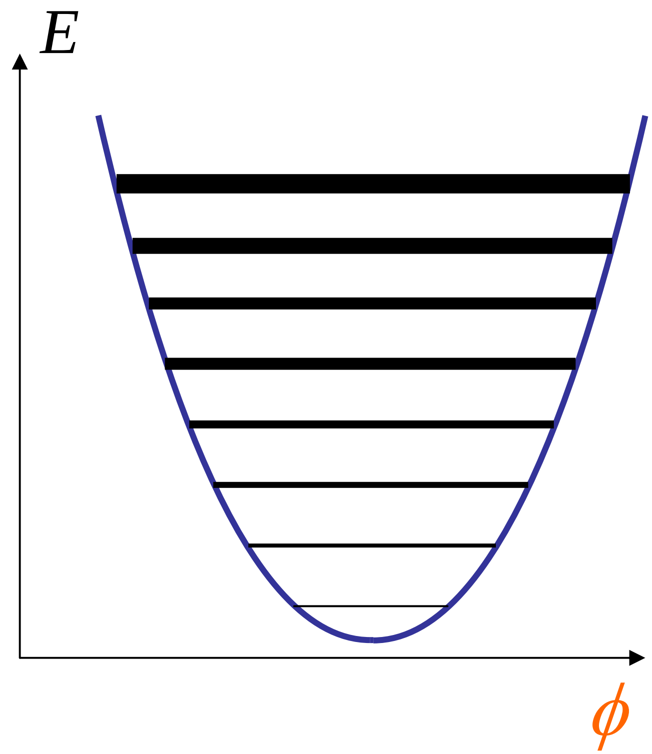




In every energy eigenstate,
(standing photon state)
current flows in opposite
directions simultaneously!

EFFECT OF DAMPING



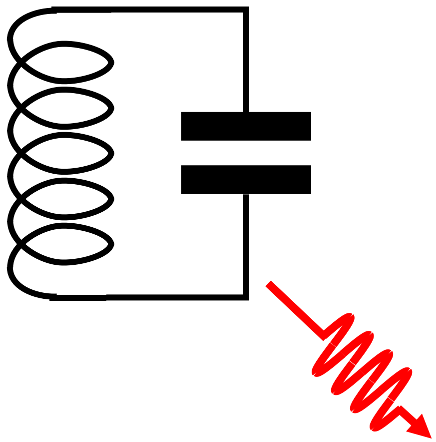


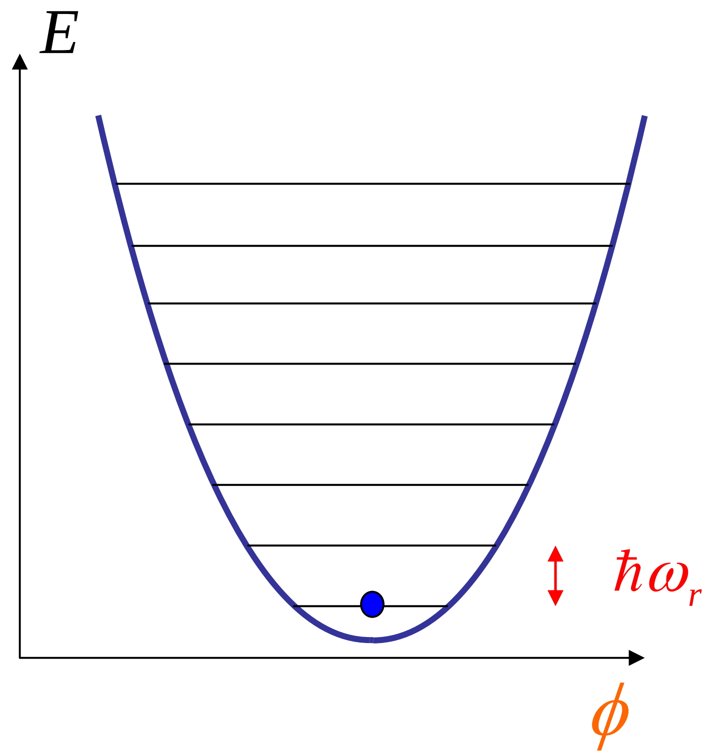
important: as little dissipation as possible

dissipation broadens energy levels

$$
\left.
\begin{aligned}
E_n &= \hbar\omega_r \left[n\left(1+\frac{i}{2\mathcal{Q}}\right)+\frac{1}{2} \right] \\
\mathcal{Q} &= RC\omega_r
\end{aligned}
\right\}
$$

CAN PLACE CIRCUIT IN ITS GROUND STATE



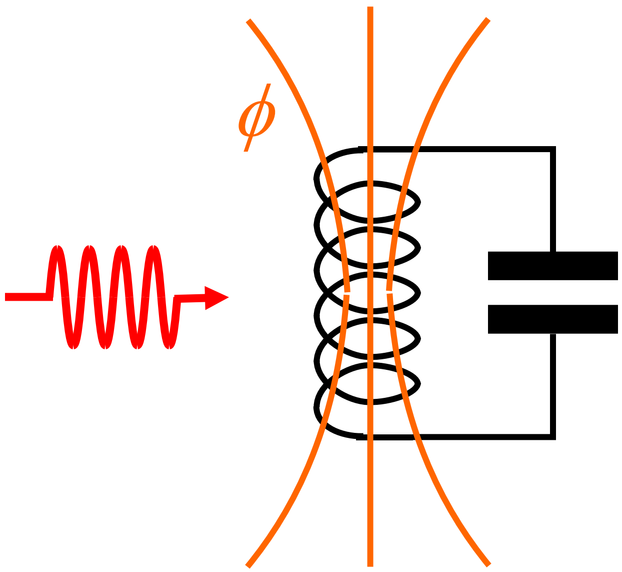


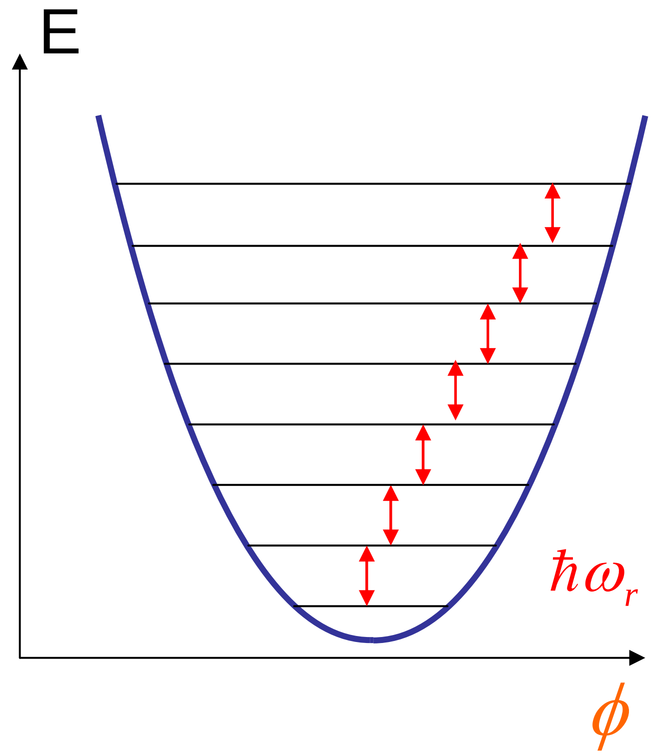
residual dissipation
provides
reset of circuit

$$\hbar\omega_r \gg k_B T$$

5 GHz
15 mK

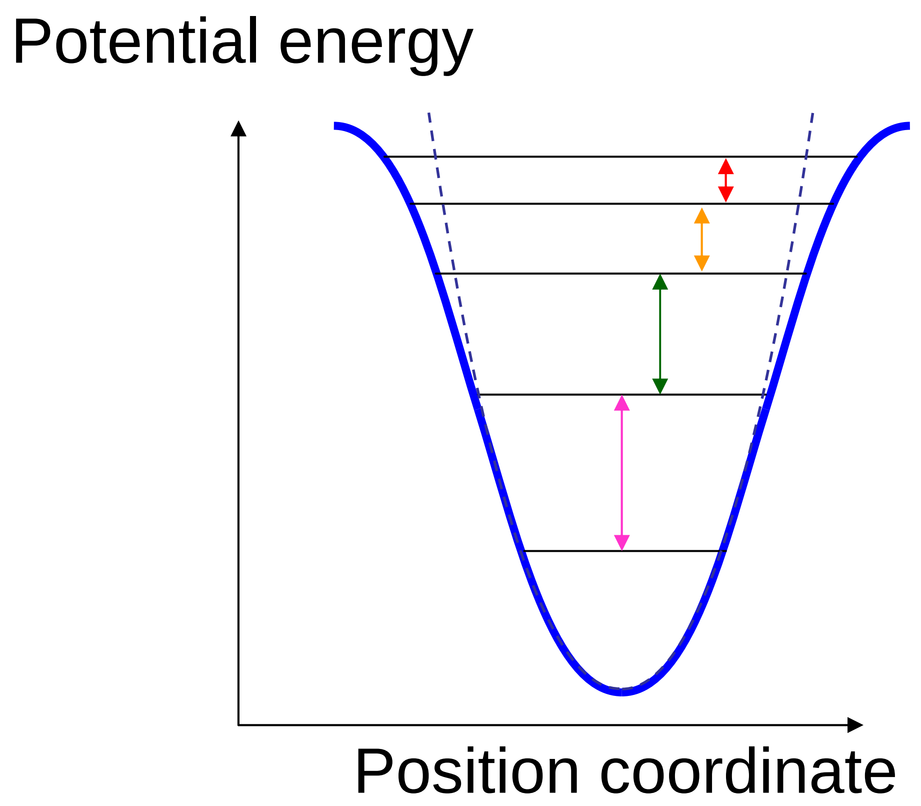
PB: ALL TRANSITIONS ARE DEGENERATE!

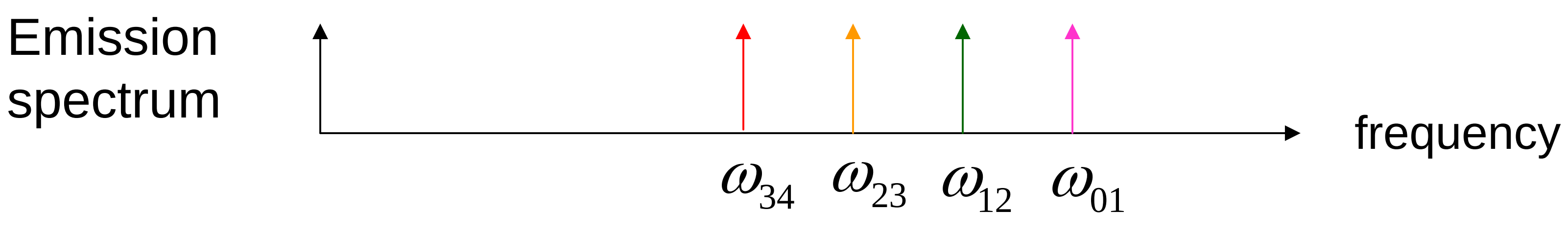




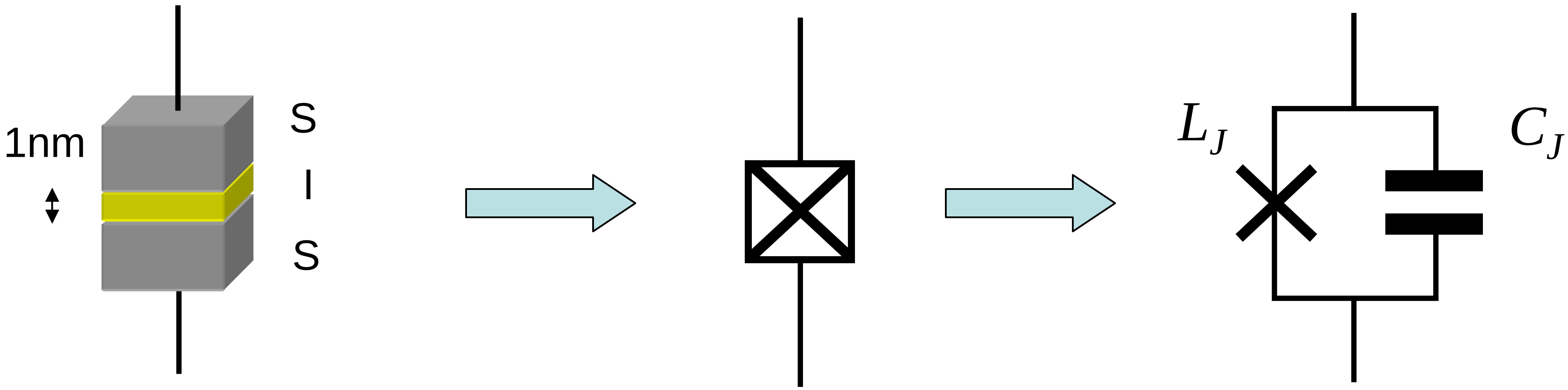
CANNOT STEER THE SYSTEM TO AN ARBITRARY STATE IF PERFECTLY LINEAR

NEED NON-LINEARITY TO FULLY REVEAL QUANTUM MECHANICS

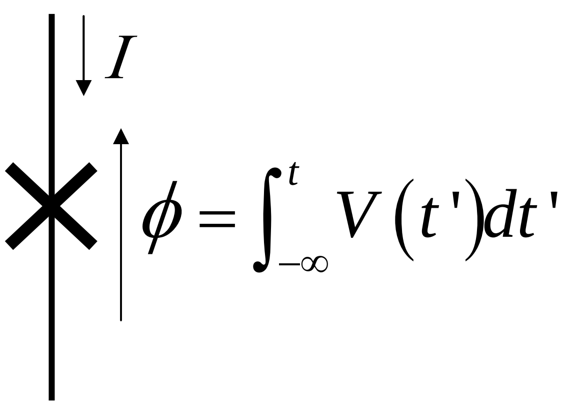


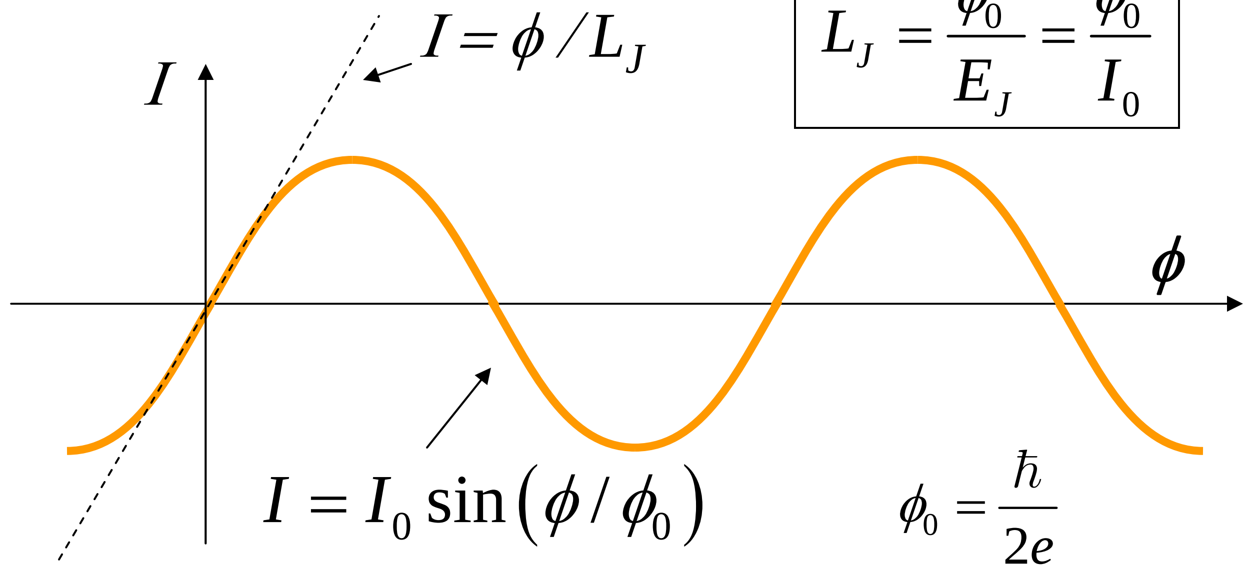


JOSEPHSON TUNNEL JUNCTION PROVIDES A NON-LINEAR INDUCTOR WITH NO DISSIPATION



$$L_J = \frac{\phi_0^2}{E_J} = \frac{\phi_0}{I_0}$$





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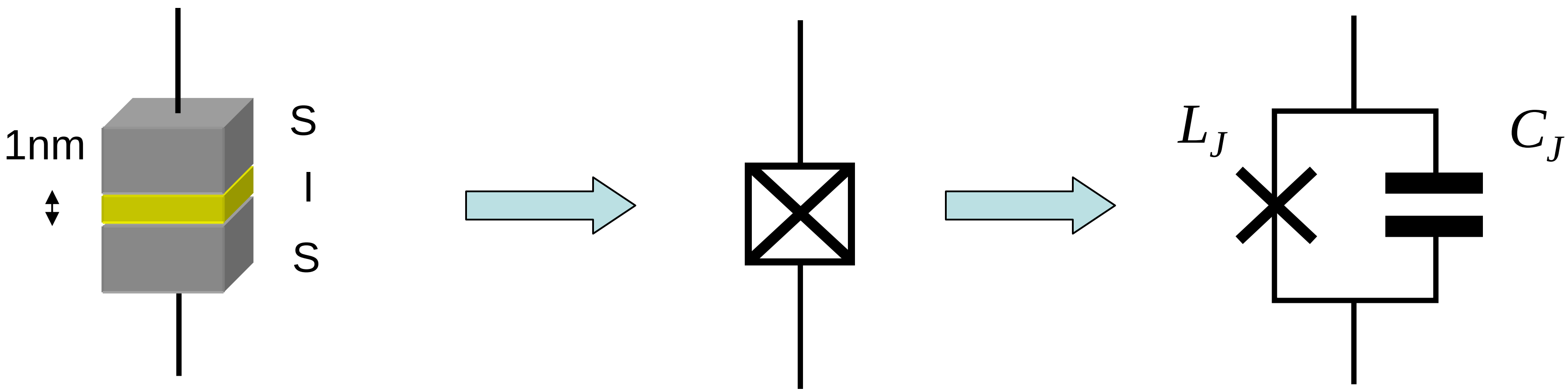

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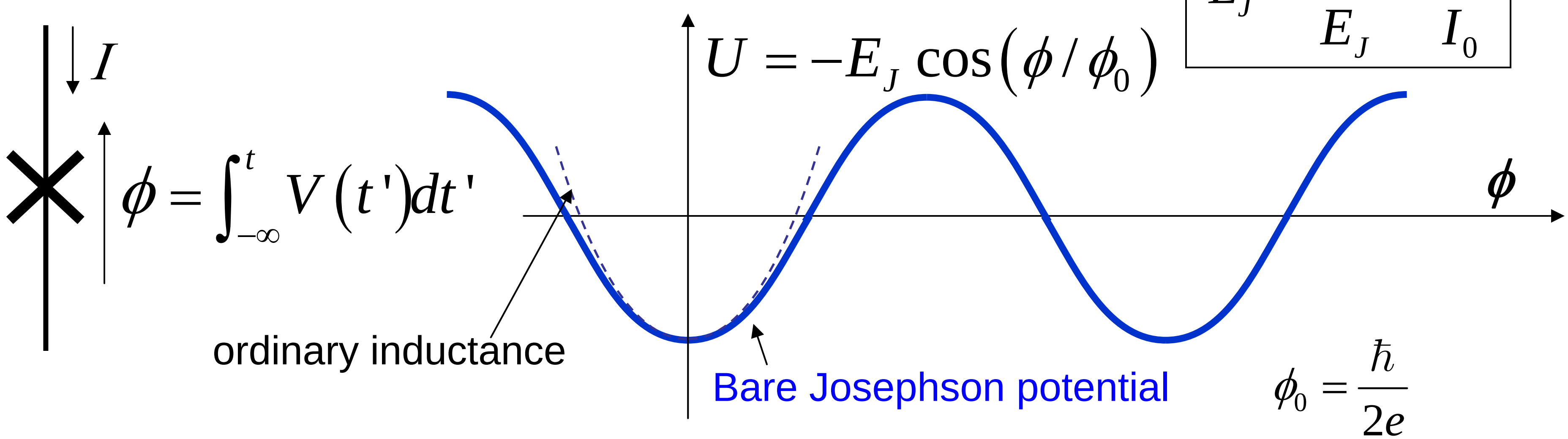
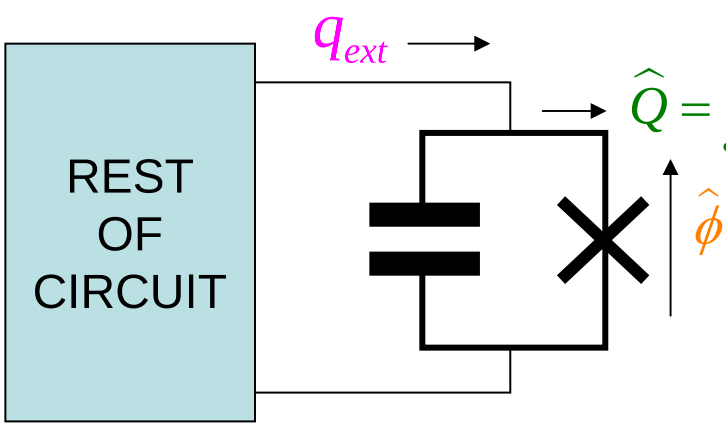


ENERGY SCALES
OF THE JOSEPHSON JUNCTION "ATOM"



$$\hat{Q} = \int_{-\infty}^{t} \hat{I}(t')dt'$$

$$\hat{\varphi} = \frac{2e\hat{\phi}}{\hbar}$$

$$\hat{N} = \frac{\hat{Q}}{2e}$$

$$\left[\hat{\varphi}, \hat{N}\right] = i$$

Hamiltonian: $$\hat{H}_J = 8E_C \frac{\left(\hat{N} - N_{ext}\right)^2}{2} - E_J \cos\hat{\varphi}$$

<u>Coulomb charging energy for 1e</u>

$$E_C = \frac{e^2}{2C_j}$$

reduced offset charge

$$N_{ext} = \frac{q_{ext}}{2e}$$

<u>Josephson energy</u>

$$E_J = \frac{1}{8}\mathcal{N}\mathcal{T}\Delta$$

barrier transp^{cy} ($\mathcal{T}$)

gap (Δ)

cond^{ion} channels ($\mathcal{N}$)

valid for opaque barrier

HARMONIC APPROXIMATION

$$\hat{H}_J = 8E_C \frac{\left(\widehat{N} - N_{ext}\right)^2}{2} - E_J \cos\hat{\varphi}$$

$E_C \ll E_J$, low energy

$$\widehat{H}_{J,h} = 8E_C \frac{\left(\widehat{N} - N_{ext}\right)^2}{2} + E_J \frac{\hat{\varphi}^2}{2}$$

Josephson "plasma" frequency:

$$\omega_P = \frac{\sqrt{8E_C E_J}}{\hbar}$$

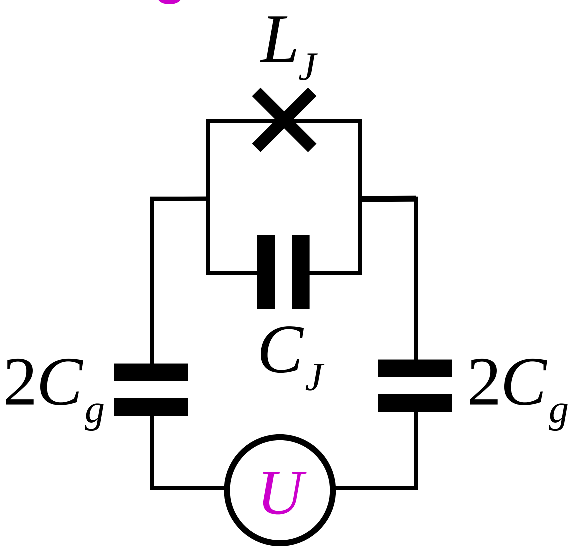
Josephson RF impedance:

$$Z_J = \frac{\hbar}{(2e)^2}\sqrt{\frac{8E_C}{E_J}}$$

Spectrum independent of DC value of N_{ext}

3 TYPES OF BIASES

charge



$$8E_C \frac{\left(\hat{N} - C_g U / 2\right)^2}{2} - E_J \cos\hat{\varphi}$$

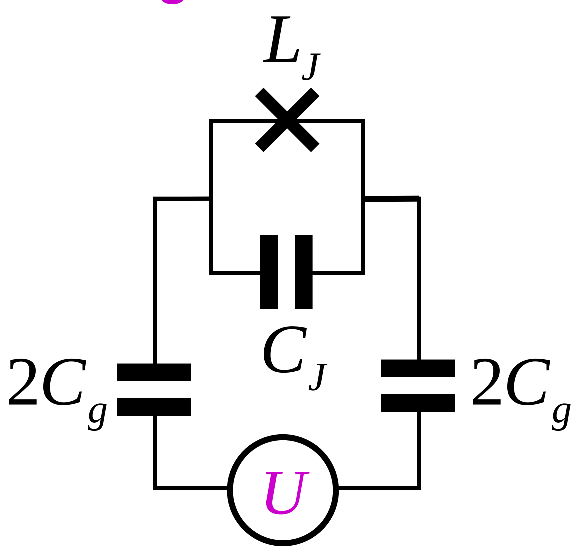
"Cooper pair box"

$\hat{\varphi}$ lives on circle

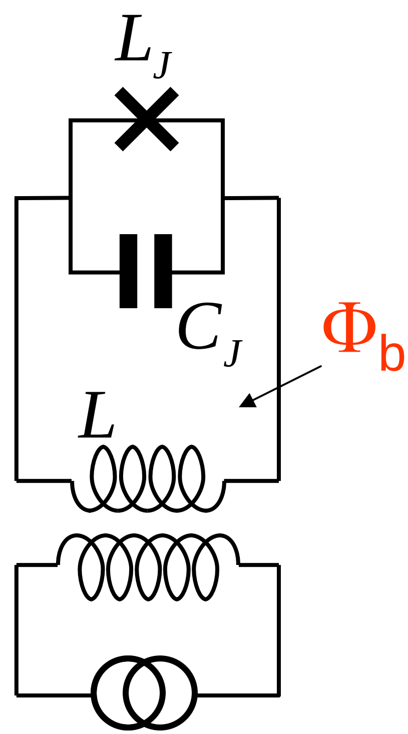
$\hat{N}$ integer

3 TYPES OF BIASES

charge



flux



$$8E_C \frac{\left(\hat{N} - C_g U / 2\right)^2}{2} - E_J \cos\hat{\varphi}$$

"Cooper pair box"

$\hat{\varphi}$ lives on circle

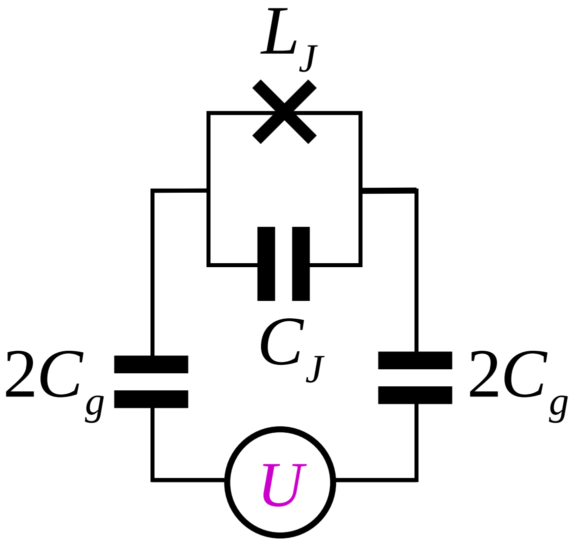
$\hat{N}$ integer

$$8E_C \frac{\hat{N}^2}{2} + E_L \frac{\left(\hat{\varphi} - \frac{2e\Phi_b}{\hbar}\right)^2}{2} - E_J \cos\hat{\varphi}$$

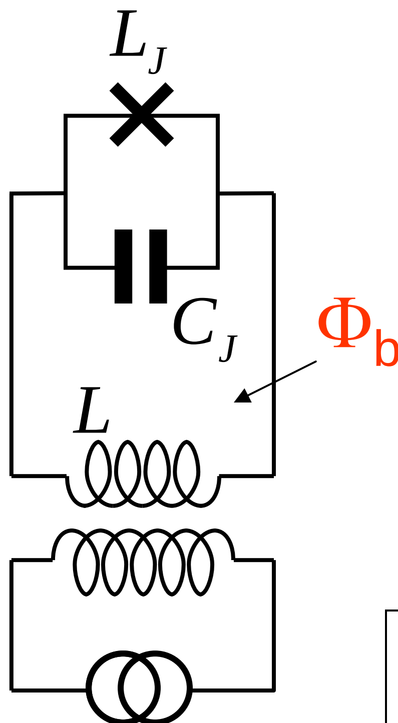
"RF-Squid", $\hat{\varphi}$ lives on line, $\hat{N}$ real number

3 TYPES OF BIASES

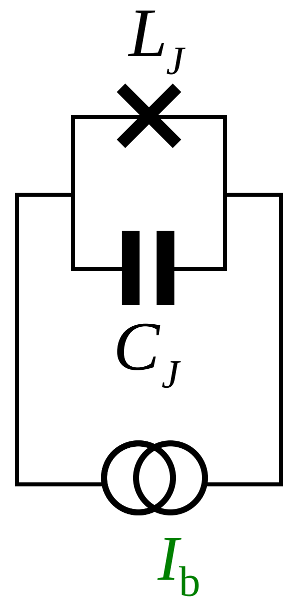
charge



flux



current



$$8E_C \frac{\left(\hat{N} - C_g U/2\right)^2}{2} - E_J \cos\hat{\varphi}$$

"Cooper pair box"

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$$8E_C \frac{\hat{N}^2}{2} + E_L \frac{\left(\hat{\varphi} - \frac{2e\Phi_b}{\hbar}\right)^2}{2} - E_J \cos\hat{\varphi}$$

"RF-Squid", $\hat{\varphi}$ lives on line, $\hat{N}$ real number

$$8E_C \frac{\hat{N}^2}{2} - E_J \left(\cos\hat{\varphi} - \frac{I_b}{I_0}\hat{\varphi}\right)$$

in the limit

$\Phi_b \to \infty$

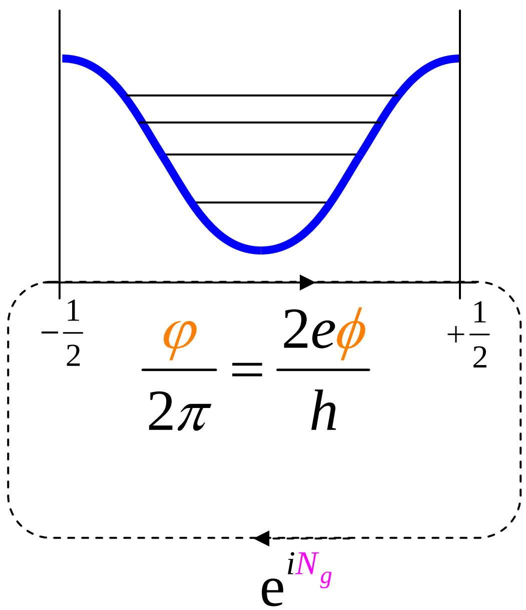
$L \to \infty$

$\frac{\Phi_b}{L} \to I_b$

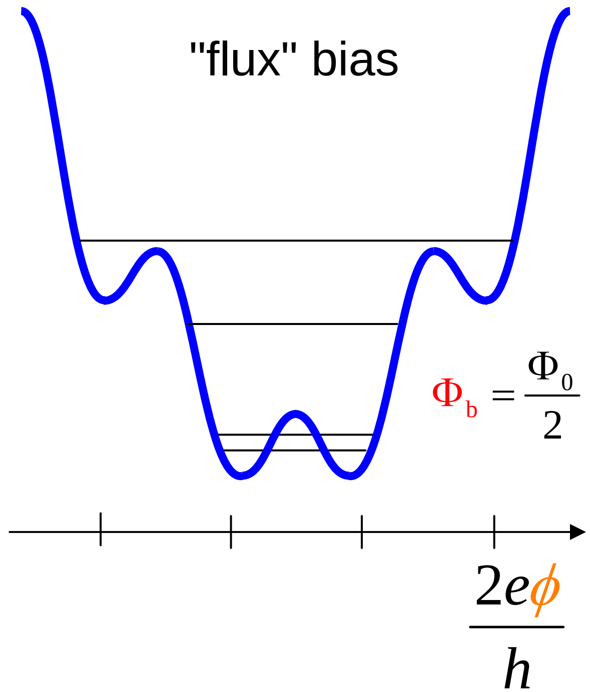
EFFECTIVE POTENTIAL OF 3 MAIN BIAS SCHEMES

see also proposals for topologically protected qubits, for example Feigelman et al. PRL 92, 098301 (2004)

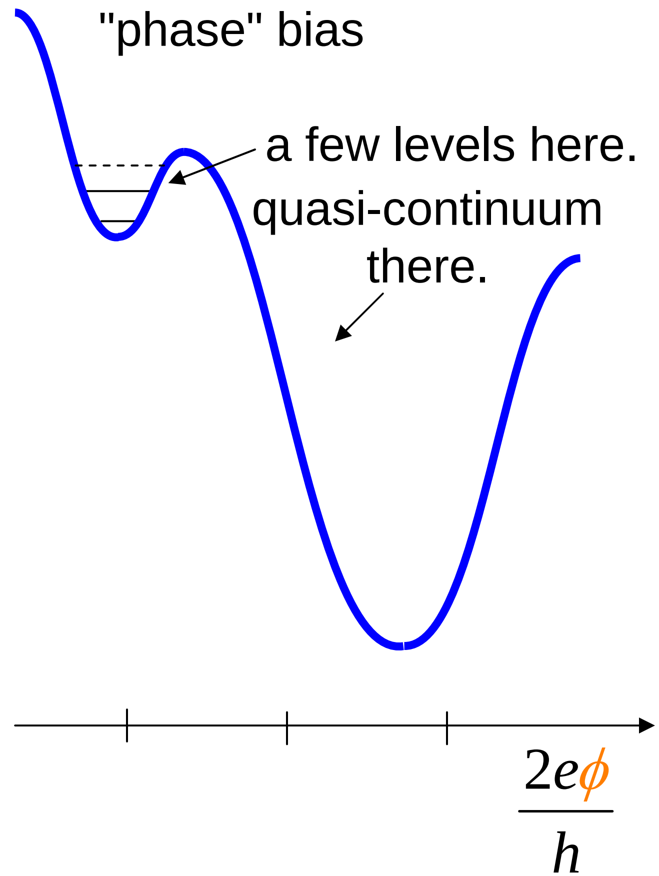
"charge" bias



CEA Saclay, NEC, Yale
Chalmers, JPL, ...

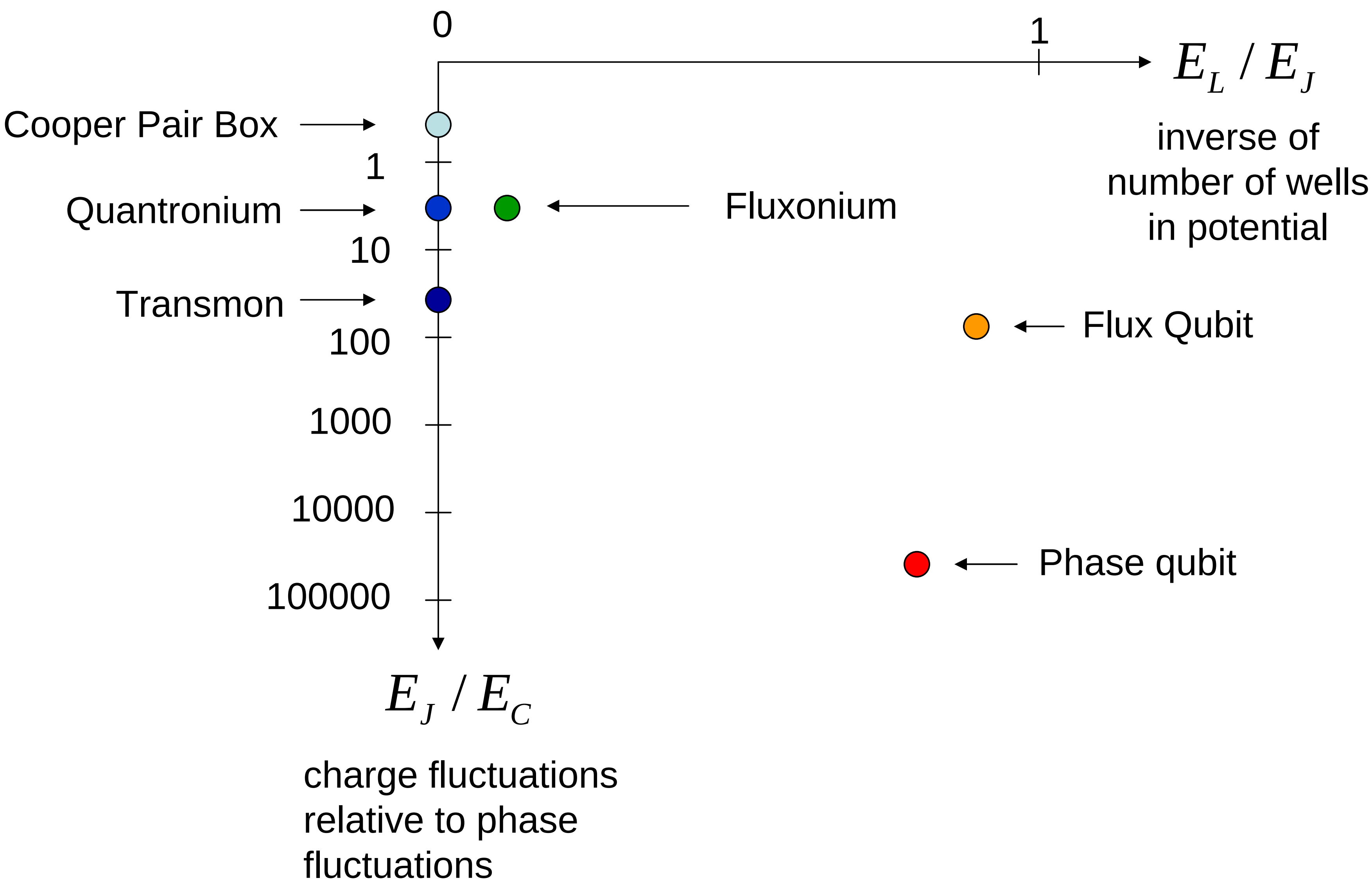


TU Delft, NEC, NTT, IBM,
MIT, UC Berkeley, SUNY,
IPHT Jena ....

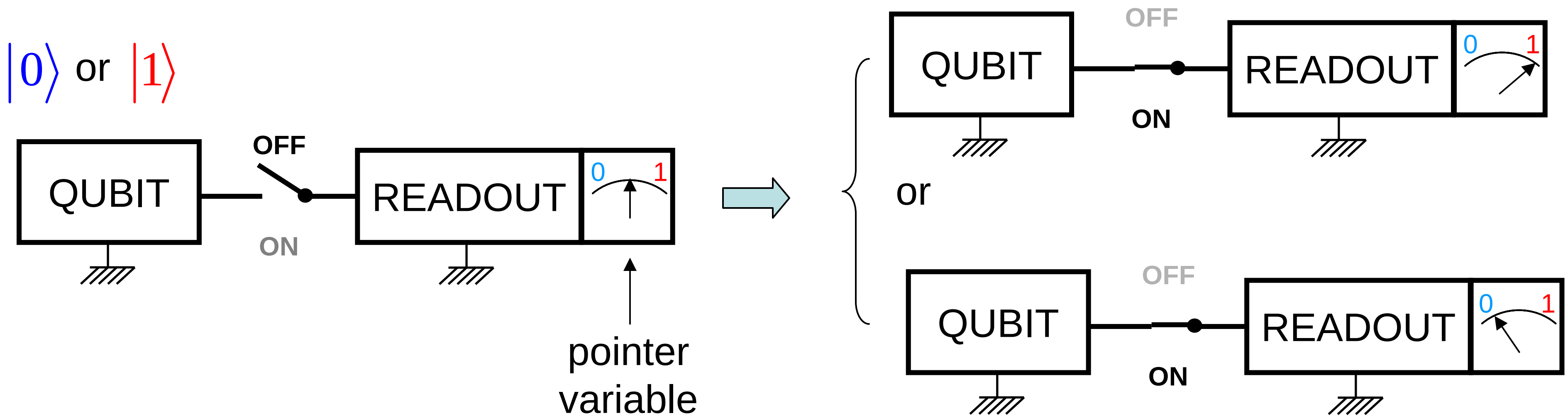


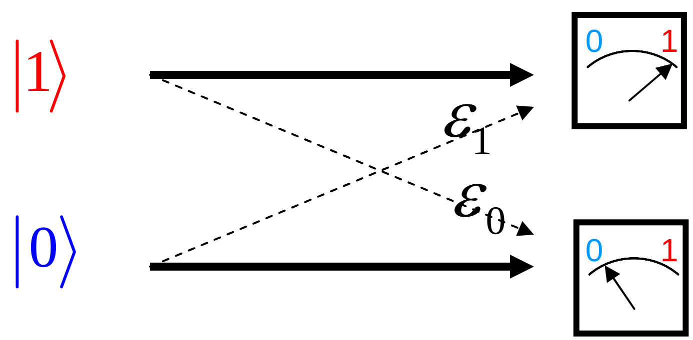
NIST, UCSB,
U. Maryland, I. Neel Grenoble...

SUPERCONDUCTING ARTIFICIAL ATOMS "MENDELEEV" TABLE



THE MEMORY READOUT PROBLEM





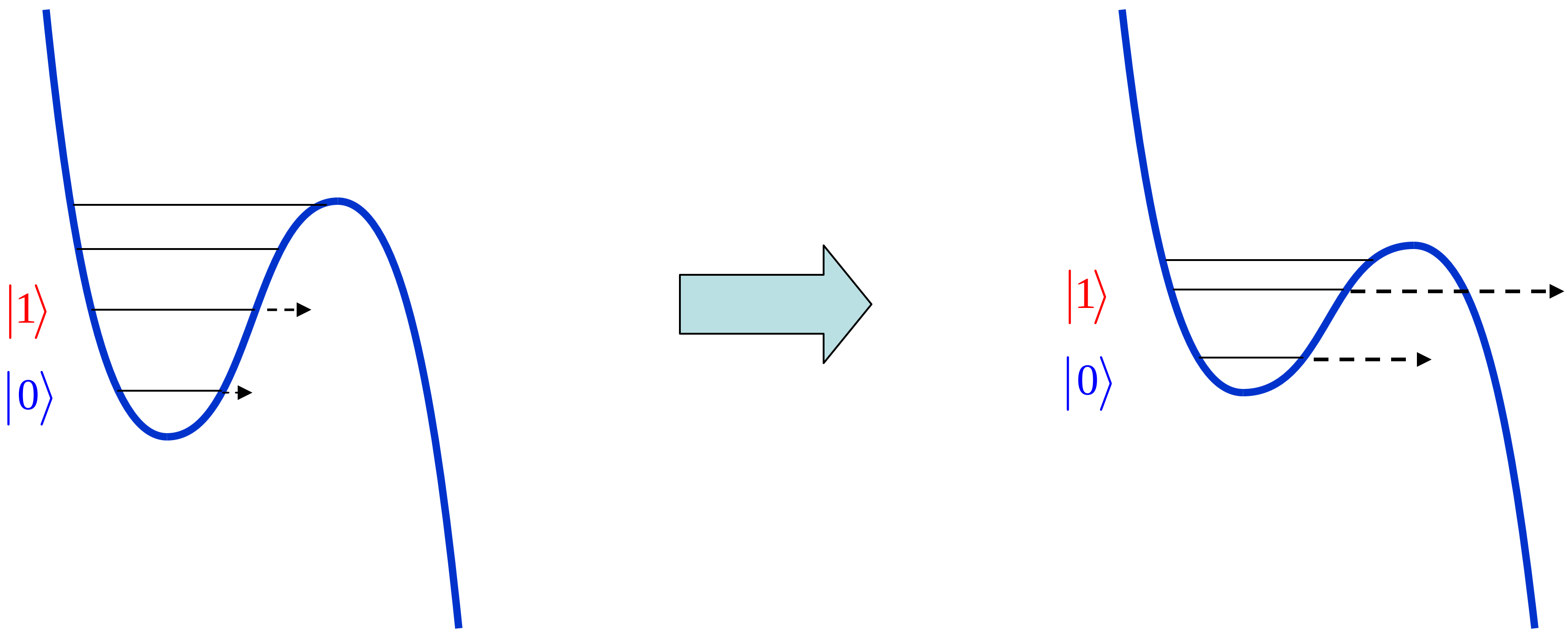
FIDELITY:

$$F = 1 - \varepsilon_0 - \varepsilon_1$$

WANT:

1) SWITCH WITH ON/OFF RATIO AS LARGE AS POSSIBLE
2) READOUT WITH $\boldsymbol{F}$ AS CLOSE TO 1 AS POSSIBLE
3) FAST, 4) PRESERVE STATE (QND)

STATE DECAY STRATEGY



Martinis, Devoret and Clarke, PRL **55** (1985)
Martinis, Nam, Aumentado and Urbina, PRL **89** (2002)

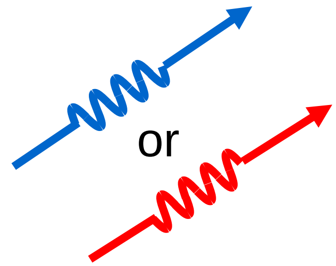
DISPERSIVE READOUT STRATEGY

Blais et al. PRA 2004, Walraff et al., Nature 2004

rf signal in

$\omega \neq \omega_{01}$

rf signal out



QUBIT CIRCUIT

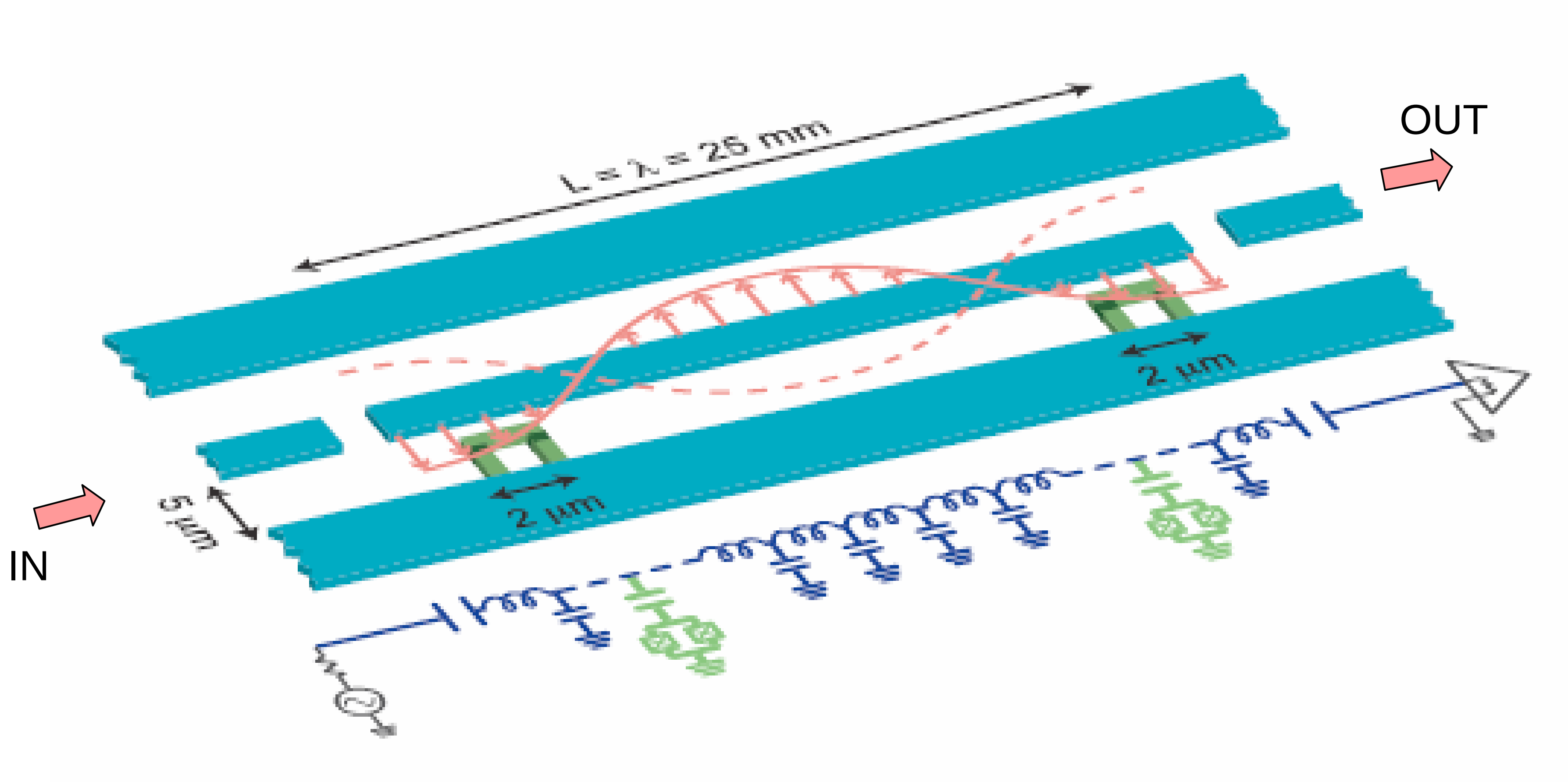
$|0\rangle$ or $|1\rangle$

QUBIT STATE
ENCODED IN PHASE
OF OUTGOING SIGNAL,
NO ENERGY DISSIPATED
ON-CHIP

A) FILTER OUT EVERYTHING ELSE THAN READOUT RF
B) REPEAT WITH ENOUGH PHOTONS TO BEAT
NOISE : USE THE BEST AMPLIFIER AS POSSIBLE

(see session V26)

SCHEMATIC OF COOPER PAIR BOXES IN A MICROWAVE RESONATOR (CAVITY)



Roles of cavity: 1) Filter, 2) Dispersive measurement, 3) Quantum bus

"Circuit QED": Review by Blais et al., Phys. Rev. A **75**, 032329 (2007)

PAULI SPIN MATRICES AND ROTATIONS

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = Z$$

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = X$$

$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = Y$$

$$\sigma_w = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

useful notation of Pauli spin matrices

HERMITIAN (MEASUREMENT)

$$[Z] = -i\sigma_z = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix} \rightarrow R_z(\pi)$$

$$[X] = -i\sigma_x = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} \rightarrow R_x(\pi)$$

$$[Y] = -i\sigma_y = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \rightarrow R_y(\pi)$$

$$I = \sigma_w = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow Identity$$

UNITARY (GATE)

ELEMENTARY GATES ARE π/2 ROTATIONS

In lab frame:

$$\frac{\hat{H}}{\hbar} = \frac{\omega_z}{2}\sigma_z + \omega_x(t)\cos\left[\omega_z t + \phi(t)\right]\sigma_x$$

transverse osc. field amplitude

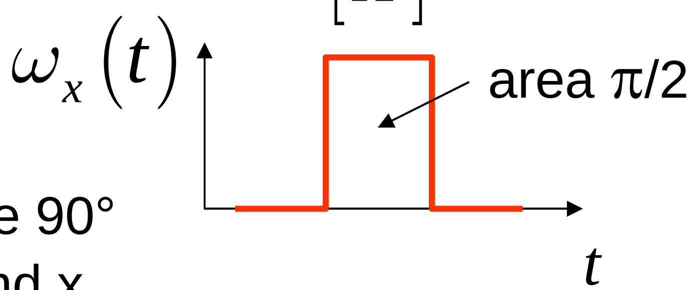
Do rotating wave approximation

In rotating frame at Larmor freq.:

$$\frac{\hat{H}}{\hbar} = \omega_x(t)\cos\left[\phi(t)\right]\frac{\sigma_x}{2} + \omega_x(t)\sin\left[\phi(t)\right]\frac{\sigma_y}{2}$$

$[Z]^{1/2}$: shift Zeeman field

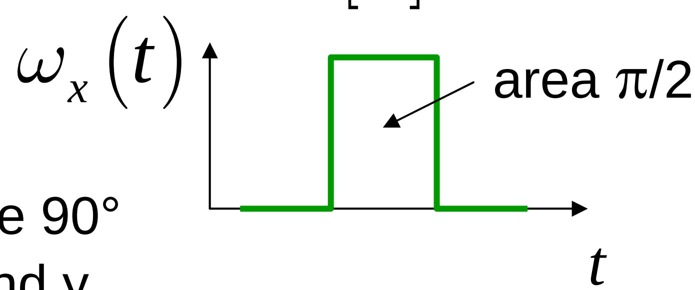
$[X]^{1/2}$



Pulse 90° around x

$\phi = 0$

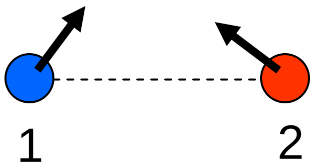
$[Y]^{1/2}$



Pulse 90° around y

$\phi = \pi/2$

NATURAL ENTANGLING OPERATIONS



$$\hat{U}(\tau) = \exp\left(-i\hat{H}_{\text{int}}\tau/\hbar\right)$$

Secular interaction: $$\hat{H}_{\text{int}} = g_{\parallel}\sigma_{1z}\sigma_{2z} \longrightarrow [ZZ]^{1/2}$$

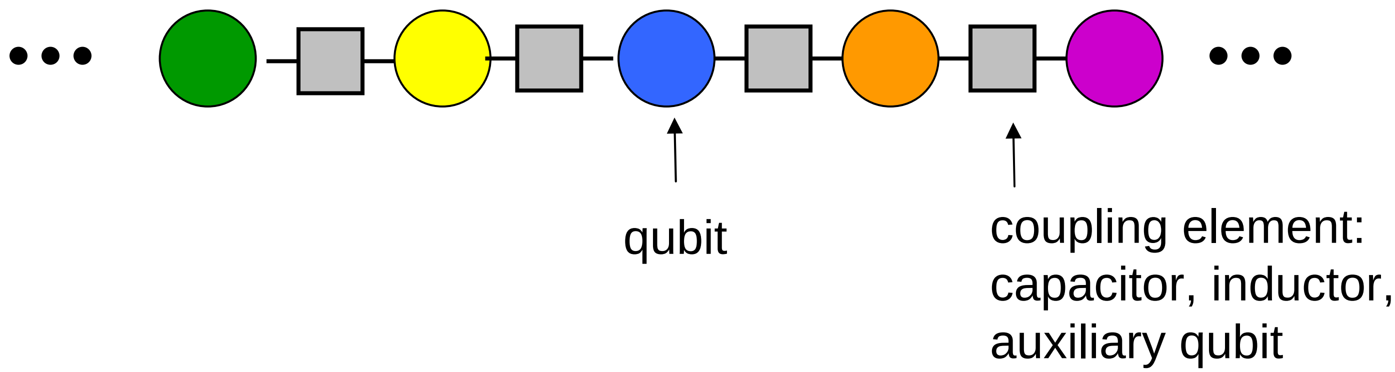
adjustment of gate duration time: $$\tau = \frac{\pi\hbar}{4g_{\parallel}}$$

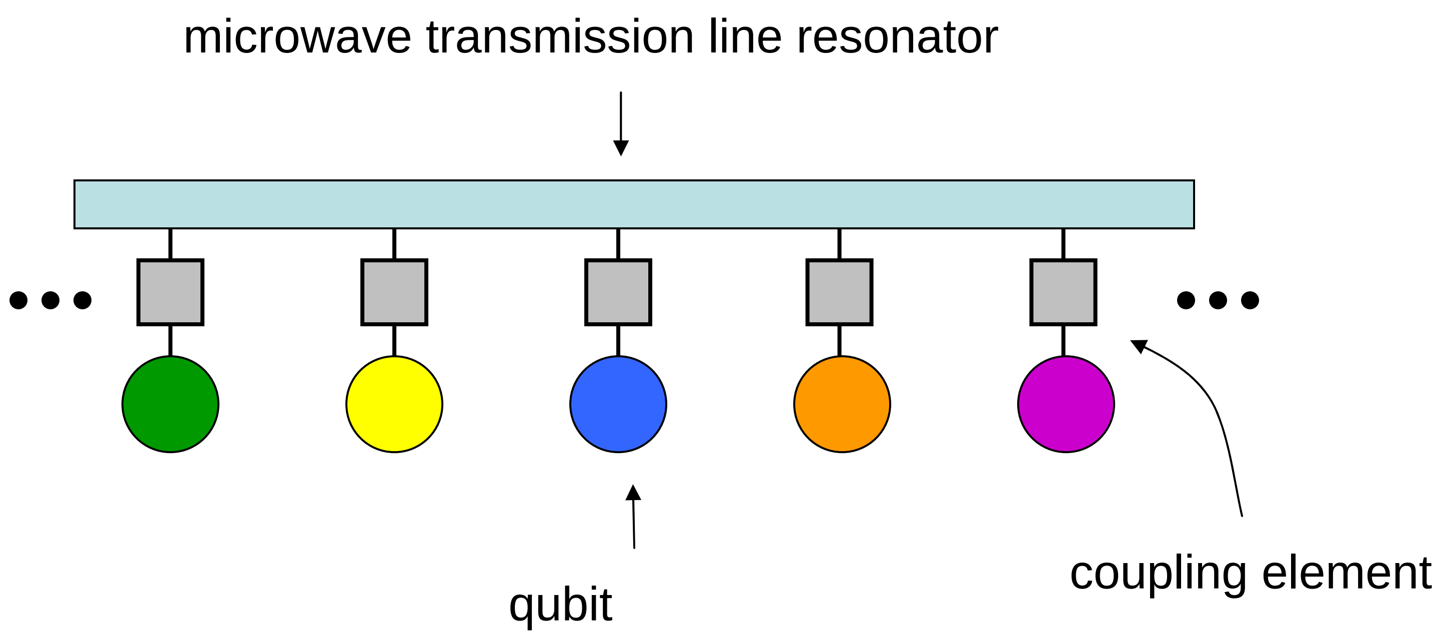
Flip-flop interaction: $$\hat{H}_{\text{int}} = g_{\perp}\sigma_{1+}\sigma_{2-} + h.c.$$

$$= g_{\perp}\left(\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y}\right)$$

$$\longrightarrow [XX]^{1/2}[YY]^{1/2}$$

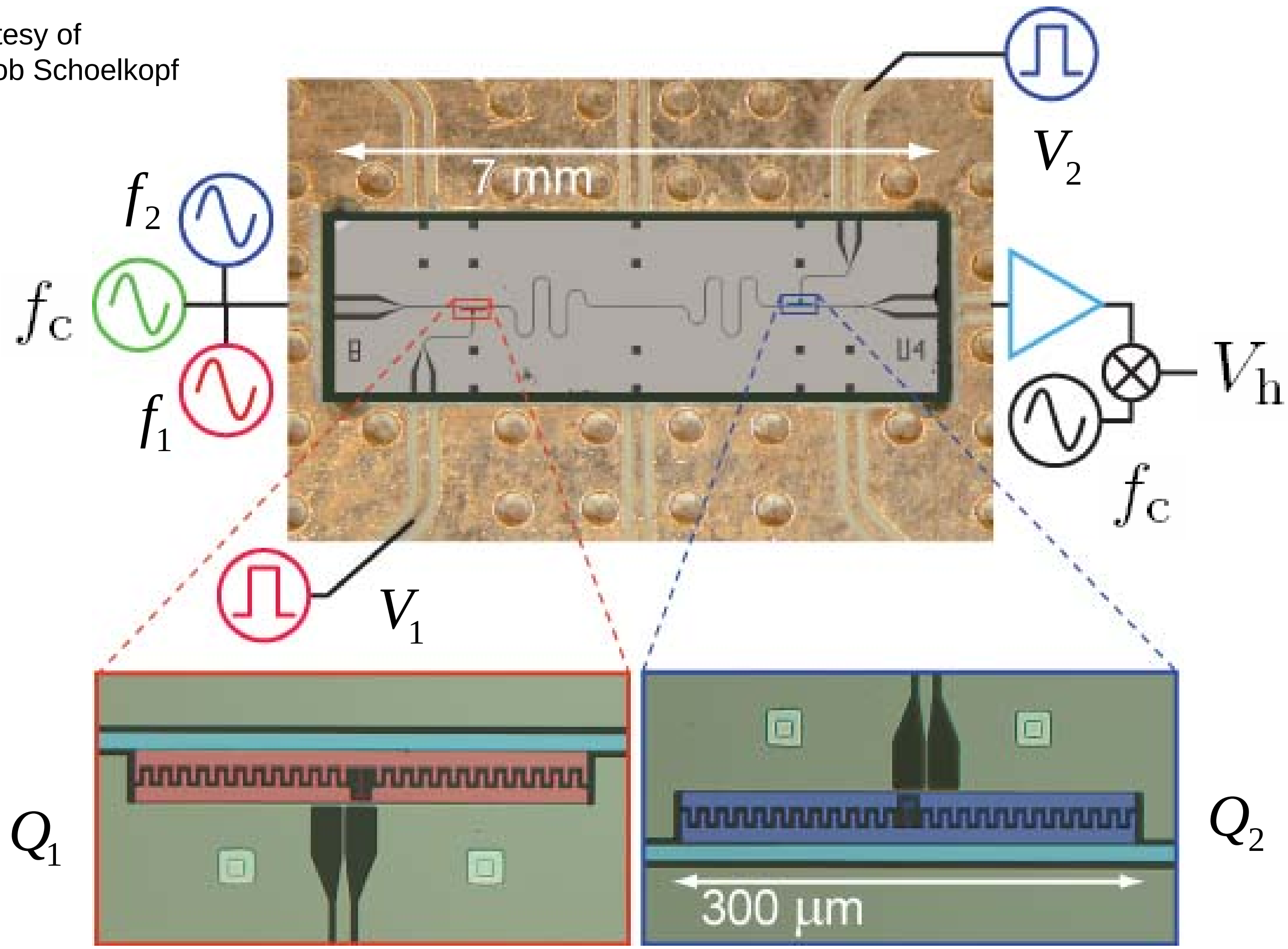
PAIRWISE COUPLING v.s. BUS COUPLING





TWO-QUBIT QUANTUM PROCESSOR

slide courtesy of
Leo DiCarlo & Rob Schoelkopf



T29, V26, Y26

see 1 qubit and 2 cavities: B. Johnson et al. , 3 qubits and 1 cavity L. DiCarlo et al.: **T26, W6**

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