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COWLES FOUNDATION PAPER NO. 1038

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
AT YALE UNIVERSITY
Box 208281
New Haven, Connecticut 06520-8281
2002

Econometrica, Vol. 70, No. 3 (May, 2002), 1007–1033

INFORMATION ACQUISITION AND EFFICIENT MECHANISM DESIGN

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We consider a general mechanism design setting where each agent can acquire (covert) information before participating in the mechanism. The central question is whether a mechanism exists that provides the efficient incentives for information acquisition ex-ante and implements the efficient allocation conditional on the private information ex-post.

It is shown that in every private value environment the Vickrey-Clark-Groves mechanism guarantees both ex-ante as well as ex-post efficiency. In contrast, with common values, ex-ante and ex-post efficiency cannot be reconciled in general. Sufficient conditions in terms of sub- and supermodularity are provided when (all) ex-post efficient mechanisms lead to private under- or over-acquisition of information.

KEYWORDS: Auctions, mechanism design, information acquisition, ex-ante and ex-post efficiency.

1. INTRODUCTION

1.1. *Motivation*

IN MOST OF THE LITERATURE on mechanism design, the model assumes that a number of economic agents possess a piece of information that is relevant for the efficient allocation of resources. The task of the mechanism designer is to find a game form that induces the agents to reveal their private information. An efficient mechanism is one where the final allocation is efficient given all the private information available in the economy.

In this paper, we take this analysis one step further. We assume that before participating in the mechanism each agent can covertly obtain additional private information at a cost. After the information has been acquired, the mechanism is executed. Hence the primitive notion in our model is an information gathering technology rather than a fixed informational type for each player. It is clear that the properties of the mechanism to be played in the second stage affect the players' incentives to acquire information in the ex ante stage.

The main results in this paper characterize information acquisition in ex post efficient mechanisms. Efficiency of a mechanism in this paper is understood in the same sense as in the original contributions by Vickrey, Clarke, and Groves. In particular, we do not impose balanced budget or individual rationality constraints

[1] The authors thank Sandeep Baliga, Jeff Ely, Steve Matthews, Stephen Morris, Joe Ostroy, Nicola Persico, Martin Pesendorfer, Phil Reny, Bill Zame, and particularly Jon Levin for several helpful discussions. We are especially grateful for suggestions from two anonymous referees and a co-editor. Comments from seminar participants at Minnesota, U.C.L.A., and Yale are greatly appreciated. Financial support from NSF Grant SBR 9709887 and 9709340, respectively, is acknowledged.

on the mechanism designer. In the independent private values case, we show that the Vickrey-Clarke-Groves (henceforth VCG) mechanism induces efficient information acquisition at the ex ante stage.

The common values case is much less straightforward to analyze. In light of the recent results by Dasgupta and Maskin (2000) and Jehiel and Moldovanu (2001), it is in general impossible to find mechanisms that would induce ex post efficient allocations. Adding an ex ante stage of information acquisition does not alleviate this problem. The two basic requirements for incentive compatibility of the efficient allocation rule are that the signals to the agents be single dimensional and that the allocation rule be monotonic in the signals. Even when these two conditions are met, we show that any efficient ex-post mechanism does not result in ex ante efficient information acquisition. We use ex post equilibrium as our solution concept. An attractive feature of this concept for problems with endogenously determined information is that the mechanisms do not depend on the distributions of the signals. By the revenue equivalence theorem, any allocation rule that can be supported in an ex post equilibrium results in the same expected payoffs to all of the players as the VCG mechanism, provided that the lowest type receives the same utility in the mechanisms. But the defining characteristic of the VCG mechanism is that an agent's payoff changes only when the allocation changes due to his announcement of the signal. As a result, the payoffs cannot reflect the direct informational effects on other agents, and hence the private and social incentives will differ in general.

We also investigate the direction in which the incentives to acquire information are distorted. We restrict our attention to the case where the efficient allocation rule can be implemented in an ex post equilibrium and derive new necessary and sufficient conditions for the ex post implementability. It turns out that under our sufficient conditions for implementability, the information acquisition problem also satisfies the conditions for the appropriate multi-agent generalization of a monotone environment as defined in Karlin and Rubin (1956) and Lehmann (1988). As a result, we can expand the scope of our theory beyond signal structures that satisfy Blackwell's order of informativeness to the much larger class of signals ordered according to their effectiveness as defined in Lehmann (1988). We show that in settings with conflicting interests between agent i and all other agents, as expressed by their marginal utilities, every ex post efficient mechanism results in excessive information acquisition by agent i. With congruent interests between agent i and agents $-i$, there is too little investment in information by agent i at the ex ante stage.

The paper is organized as follows. The model is laid out in the next section. Section 3 presents the case of a single unit auction as an example of the general theory. The analysis of the independent private values case is given in Section 4. Results on efficient ex post implementation are presented in Section 5. Section 6 deals with ex ante efficiency in the common values case and Section 7 concludes.

1.2. *Literature*

This paper is related to two strands of literature in mechanism design. It extends the ideas of efficient mechanism design pioneered by Vickrey (1961), Clarke (1971), and Groves (1973) in an environment with fixed private information to an environment with information acquisition.

Our results on ex-post efficient mechanisms in common values environments complement recent work by Dasgupta and Maskin (2000) and Jehiel and Moldovanu (2001). Dasgupta and Maskin (2000) suggest a generalization of the VCG mechanism to obtain an efficient allocation in the context of multi-unit auctions with common values. Jehiel and Moldovanu analyze the efficient design in a linear setting with multidimensional signals and interdependent allocations. We give necessary conditions as well as weaker sufficient conditions for the efficient design in a general nonlinear environment. The results here are valid for general allocation problems and not only for single or multi-unit auctions.

The existing literature on information acquisition in mechanism design is restricted almost entirely to the study of auctions.[2] In the private values setting, Tan (1992) considers a procurement model where firms invest in R&D expenditure prior to the bidding stage. In the symmetric equilibrium with decreasing returns to scale, he observes that revenue equivalence holds between first and second price auction. Stegeman (1996) shows that the second price auction induces efficient information acquisition in the single unit independent private values case. Our results in the private values case can thus be seen as extensions of these earlier results. We show that analogous results hold for a much larger class of models and that as long as the conditions of the revenue equivalence theorem are satisfied, there is no need to analyze separately different indirect mechanisms that result in efficient allocation. Matthews (1977 and 1984) considers endogenous information acquisition in a pure common values auction and analyzes the convergence of the winning bid to the true value of the object when the number of bidders increases. Those papers are different from our papers in at least two respects. First, Matthews compares given auction forms for a single unit auction rather than taking a mechanism design approach to general allocation problems. Second, as Matthews considers the pure common value model, the efficient level of information acquisition is always identical to zero. Persico (2000) compares the equilibrium incentives of the bidders to acquire information in first and second price auctions within a model of affiliated values. Persico (2000) also uses the same notion of informativeness of information structures as we use. Again the main difference between his approach and ours is that we take the mechanism design approach rather than compare given auction formats for a single-unit auction.

[2] The notable exceptions are Rogerson (1992), who analyzes an n-person investment and subsequent allocation problem in Bayesian-Nash equilibrium; Crémer, Khalil, and Rochet (1998a and 1998b), who study information acquisition in a Baron-Myerson adverse selection model; and Auriol and Gary-Bobo (1999), who consider decentralized sampling in a collective decision model for a public good.

2. MODEL

2.1. *Payoffs*

Consider a setting with I agents, indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$. The agents have to make a collective choice x from a compact set X of possible alternatives. Uncertainty is represented by a set of possible states of the world, $\Omega = \times_{i=1}^{I} \Omega_i$, where Ω_i is assumed to be a finite set for every i.[3] An element $\omega \in \Omega$ is a vector $\omega = (\omega_i, \omega_{-i}) = (\omega_1, \ldots, \omega_i, \ldots, \omega_I)$. The prior distribution $q(\omega)$ is common knowledge among the players. The marginal distribution over ω_i is denoted by $q_i(\omega_i)$ and we assume that the prior distribution $q(\omega)$ satisfies independence across i, or

$$q(\omega) = \prod_{i=1}^{I} q_i(\omega_i).$$

We assume that agent i's preferences depend on the choice x, the state of the world ω, and a transfer payment t_i in a quasilinear manner:

$$u_i(x, \omega) - t_i.$$

We also assume that u_i is continuous for all i. The mechanism designer is denoted with a subscript 0, and her utility is assumed to be

$$\sum_{i=1}^{I} t_i + u_0(x).$$

The model is said to be a *private value model* if, for all ω, ω',

$$\omega_i = \omega_i' \Rightarrow u_i(x, \omega) = u_i(x, \omega'). \tag{1}$$

If condition (1) is violated, then the model displays *common values*.

2.2. *Signals and Posteriors*

Agent i can acquire additional information by receiving a noisy signal about the true state of the world. Let S_i be a compact set of possible signal realizations that agent i may observe. Agent i acquires information by choosing a distribution from a family of joint distributions over the space $S_i \times \Omega_i$:

$$\{F^{\alpha_i}(s_i, \omega_i)\}_{\alpha_i \in A_i}, \tag{2}$$

parameterized by $\alpha_i \in A_i$. We refer to $F^{\alpha_i}(s_i, \omega_i)$ as the signal and s_i as the signal realization. Since the conditional distribution of s_i depends only on Ω_i and since the prior on Ω satisfies independence across i, s_i is independent of

[3] The extension to a compact, but not necessarily finite, state space would only change sums to integrals in the appropriate formulae.

s_j for all $i \neq j$.[4] Each A_i is assumed to be a compact interval in $\mathbb{R}$. The cost of information acquisition is captured by a cost function $c_i(\alpha_i)$ and $c_i(\cdot)$ is assumed to be continuous in α_i for all i. We endow $\Delta(S_i \times \Omega_i)$ with the topology of weak convergence and assume that $F^{\alpha_i}(s_i, \omega_i)$ is continuous in α_i in that topology. This ensures that the marginal distributions on S_i are continuous in α_i as well.

Agent i acquires information by choosing α_i. Each fixed α_i corresponds to a statistical experiment, and observing a signal realization $s_i \in S_i$ leads agent i to update his prior belief on ω_i according to Bayes' rule. The resulting posterior belief, $p_i(\omega_i|s_i, \alpha_i)$ summarizes the information contained in the signal realization s_i, with

$$p_i(\omega_i|s_i, \alpha_i) = \frac{f^{\alpha_i}(s_i, \omega_i)}{\sum_{\omega_i' \in \Omega_i} f^{\alpha_i}(s_i, \omega_i')}.$$

Considered as a family of distributions on Ω_i parameterized by s_i, we assume that $p_i(\omega_i|s_i, \alpha_i)$ is continuous in s_i in the weak topology on Ω_i.[5]

A profile of signal realizations $s = (s_1, \ldots, s_I)$ leads to a posterior belief $p(\omega|s, \alpha)$, which can be written by the independence of the prior belief and the signals as

$$p(\omega|s, \alpha) = \prod_{i=1}^{I} p_i(\omega_i|s_i, \alpha_i).$$

In many instances, it is convenient to let the signal realization s_i be directly a posterior belief $p_i(\cdot)$. The experiment α_i can then be represented directly by a joint distribution over ω_i and p_i.

2.3. *Efficiency*

The ex-ante efficient allocation requires each individual agent i to acquire the efficient amount of information and the allocation x to be optimal conditional on the posterior beliefs of all agents. Since the model has quasilinear utilities, Pareto efficiency is equivalent to surplus maximization.[6] The social utility is defined by

$$u(x, \omega) \triangleq \sum_{i=0}^{I} u_i(x, \omega).$$

The expected social surplus of an allocation x conditional on the posterior belief $p(\omega)$ is given by

$$u(x, p) \triangleq \sum_{\omega \in \Omega} u(x, \omega) p(\omega). \tag{3}$$

[4] Since we focus on ex post equilibria, this independence is not needed for the results on efficient implementation. If we wanted to extend the analysis to Bayesian implementation, then this assumption would have real strength. The independence assumptions allow us to give conditions on the economic fundamentals that lead to over- and underacquisition of information in the ex ante stage.

[5] The continuity and compactness assumptions made above are sufficient to guarantee that the choice set of each agent is compact and that the objective function is continuous in the choice variable.

[6] Recall that the mechanism designer collects all the payments and receives utility from them.

The *ex-post efficient* allocation $x(p)$ maximizes $u(x, p)$ for a given p. Given the assumptions made in the previous subsection, it is clear that a maximizer exists for all p.

Similarly, denote by p_{-i} the information held by all agents but i, with $p_{-i}(\omega) = q_i(\omega_i)\prod_{j\neq i} p_j(\omega_j)$ and let $x_{-i}(p_{-i})$ be the allocation that maximizes the expected social value $u_{-i}(x, p_{-i})$ of all agents excluding i, with

$$(4) \qquad u_{-i}(x,\omega) \triangleq \sum_{j\neq i} u_j(x,\omega)$$

and

$$(5) \qquad u_{-i}(x,p_{-i}) \triangleq \sum_{\omega\in\Omega} u_{-i}(x,\omega)p_{-i}(\omega).$$

Let $F^\alpha(p)$ be the distribution induced on posteriors by the vector of experiments, where $\alpha = (\alpha_1, \ldots, \alpha_I)$ and let $c(\alpha) = \sum_i c_i(\alpha_i)$. An *ex-ante efficient* allocation is a vector of experiments, α^*, and an ex-post efficient allocation $x(p)$, such that α^* solves

$$(6) \qquad \max_{\alpha\in A} \int u(x(p), p)\, dF^\alpha(p) - c(\alpha).$$

Observe that since we have used the posterior probabilities as arguments in the choice rule, the optimal allocation $x(p)$ does not depend on α. Again, given the continuity and compactness assumptions made in the previous subsection, a solution is guaranteed to exist.

3. ILLUSTRATING EXAMPLES

In this section, we present an example of a single unit auction with two bidders. It is meant to introduce the basic arguments for the private and common values results and to indicate how to extend the logic of the arguments to any number of agents and allocations. A similar example is discussed in Maskin (1992) with a signal space but without an underlying state space. After presenting the example, we briefly discuss the role of the independence assumptions of ω_i across i by arguing how it could arise in the auction setting and then in a different environment, namely procurement.

3.1. *Information Acquisition in an Auction*

The set of allocations is the set of possible assignments of the object to bidders, or $X = \{x_1, x_2\}$, where x_i denotes the decision to allocate the object to bidder $i \in \{1, 2\}$. The state space of agent i is given by $\Omega_i = \{0, 1\}$. We begin with a private value model, where the value of the object for bidder i is $u_i(x_i, \omega) = 2\omega_i$

and $u_i(x, \omega) = 0$ for $x \neq x_i$. We let the signal of agent i be simply his posterior belief $p_i = \Pr(\omega_i = 1)$. The expected (ex-post) utility for agent i depends on p_i and p_j: $u_i(x_i, p_i, p_j) = 2p_i$.[7] The direct VCG mechanism in this setting is the second price auction where bidder i pays the reported valuation of bidder j conditional on obtaining the object. Ex post efficiency implies that i gets the object if $u_i(x_i, p_i, p_j) \geq u_j(x_j, p_i, p_j)$, i.e. if $p_i \geq p_j$. It follows that the equilibrium utility of bidder i, conditional on obtaining the object, is $u_i(x_i, p_i, p_j) - u_j(x_j, p_i, p_j)$. For an arbitrary fixed realization $p_j = \hat{p}$, the valuations by i and j are depicted in Figure 1a as functions of p_i. The equilibrium net utility of bidder i has the same slope in p_i as the social utility, as displayed in Figure 1b.

Consider next information acquisition within this auction. With a binary state structure, a signal is more informative if the posteriors are more concentrated around 0 and 1. Around $\hat{p}$, a local increase in informativeness can be represented as a lottery (with equal probability) over $\hat{p} - \varepsilon$ and $\hat{p} + \varepsilon$ for some $\varepsilon > 0$. The convexity of the equilibrium net utility (see Figure 1b) implies that information has a positive value. More importantly, the private marginal value of the lottery coincides with the social marginal value. As a result each agent acquires the socially efficient level of information. The logic of this argument extends to all private value problems as the utility $u_{-i}(x, p)$ of all agents but i is constant in p_i.

To extend the example to a common values environment, let $u_i(x_i, \omega) = 2\omega_i + \omega_j$. The expected valuation is then $u_i(x_i, p_i, p_j) = 2p_i + p_j$ and under an efficient allocation rule i gets the object when $p_i \geq p_j$. For a given $p_j = \hat{p}$, the utilities are displayed as functions of p_i in Figure 2a. The valuation of bidder j now varies with p_i, even though it is less responsive to p_i than the valuation of i. The valuations therefore satisfy a familiar single-crossing condition. However, as the valuation of bidder j varies with p_i, the original VCG mechanism does not induce truth telling in ex post equilibrium. If we were to apply the mechanism, the equilibrium utility of agent i would be $u_i(x_i, p_i, \hat{p}) - u_j(x_j, p_i, \hat{p})$, but for any $p_i > \hat{p}$, there is an $\varepsilon > 0$ such that bidder i could lower his report to $p_i - \varepsilon$, still get the object, but receive $u_i(x_i, p_i, \hat{p}) - u_j(x_j, p_i - \varepsilon, \hat{p}) > u_i(x_i, p_i, \hat{p}) - u_i(x_i, p_i, \hat{p})$. The above argument remains valid until $p_i = \hat{p}$, where a lower report would induce an undesirable change in the allocation. Thus by asking bidder i to pay $u_j(x_j, \hat{p}, \hat{p})$, incentive compatibility is preserved. The equilibrium utility of agent i is then $u_i(x_i, p_i, \hat{p}) - u_j(x_j, \hat{p}, \hat{p})$. When we now compare the slopes of individual payoffs and social payoffs locally at $\hat{p}$, we find that the equilibrium utility of agent i has a sharper kink than the social utility as depicted in Figure 2b.

As before, more information can be represented locally as a randomization over posteriors around $\hat{p}$. In equilibrium bidder i has excessive incentives to acquire information relative to the socially optimal level as his objective function

[7] The notation in this section is in minor conflict with the general notation presented in the previous section to take advantage of the binary structure of the example: (i) p_i in this section is simply a scalar rather than a probability distribution and (ii) the expected gross utility is written as a function of p_i and p_j rather than the implied probability vector p over the state space Ω, which is here simply: $\Omega = \{0, 1\} \times \{0, 1\}$.

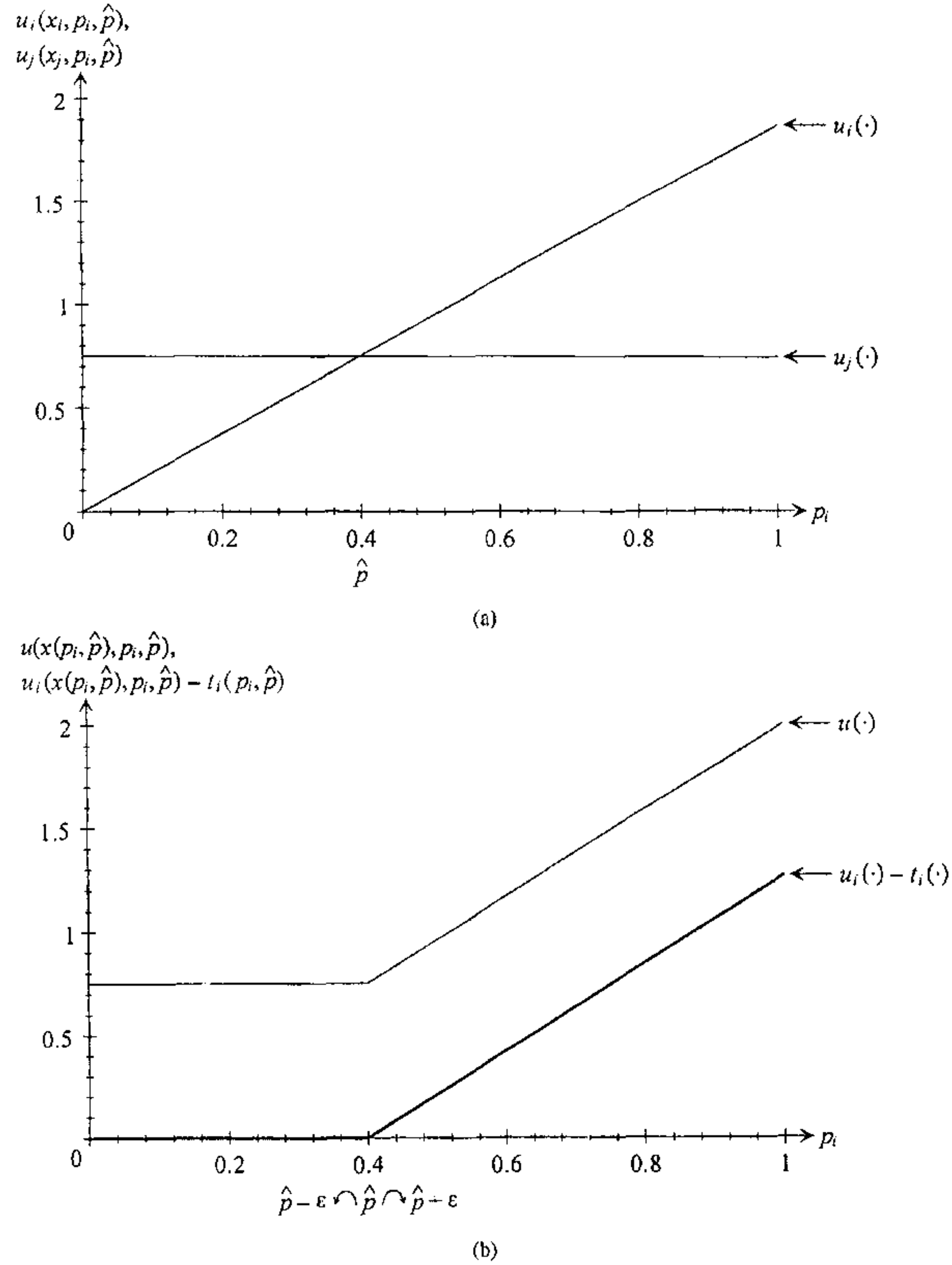

FIGURE 1.— (a) Private value utilities $u_i(x_i, p_i, \hat{p})$ and $u_j(x_j, p_i, \hat{p})$ for $p_j = \hat{p} = 3/8$. (b) Social value $u(x(p_i, \hat{p}), p_i, \hat{p})$ and equilibrium utility $u_i(x(p_i, \hat{p}), p_i, \hat{p}) - t_i(p_i, \hat{p})$.

is (locally) more convex. Conversely, agent i has insufficient incentives to acquire information if $\partial u_j(x_j, p_i, p_j)/\partial p_i < 0$.

Next we briefly sketch how these insights generalize beyond the current example. The (single) crossing of the utilities at $p_i = \hat{p}$ has two important implications.

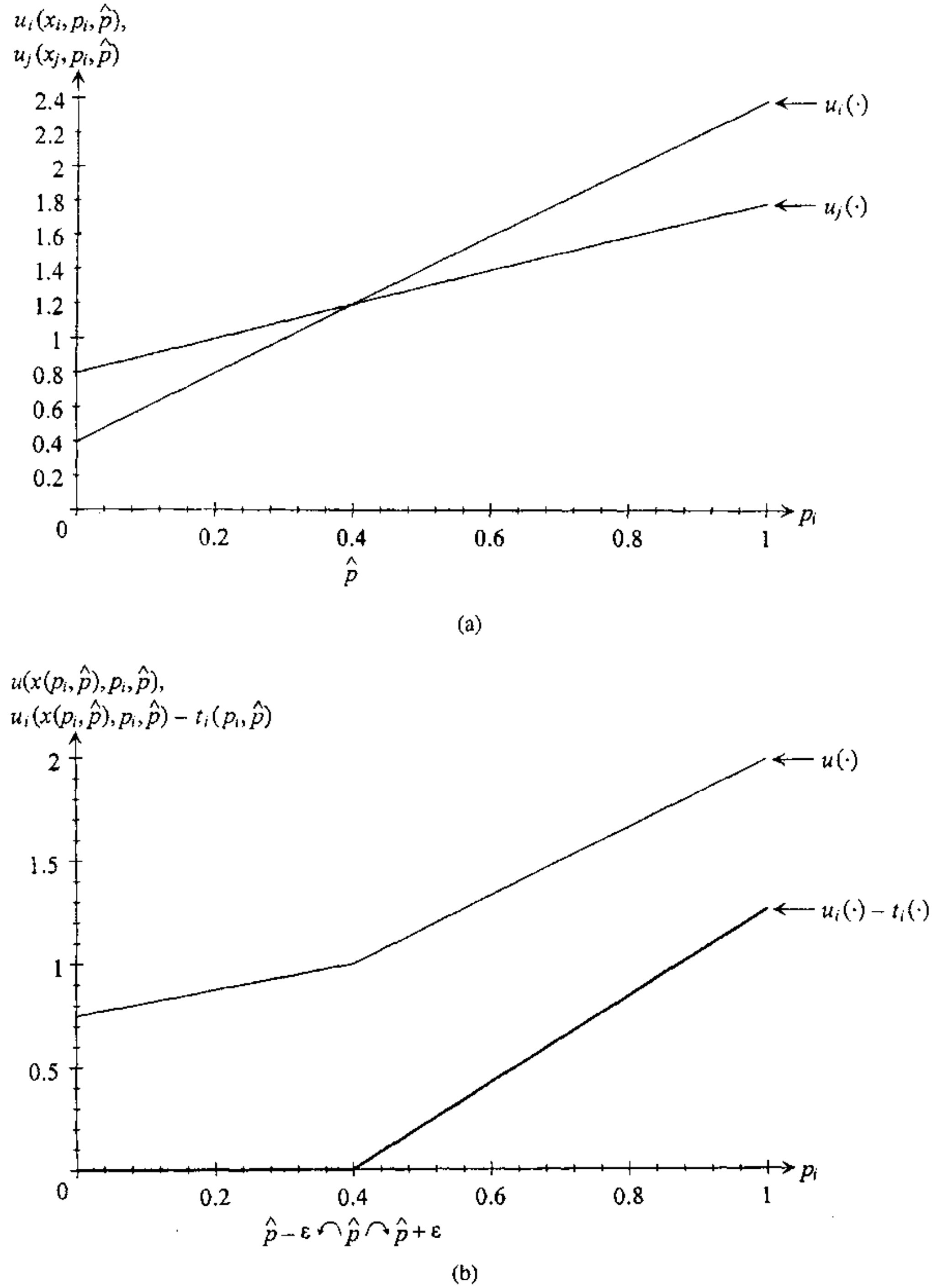

FIGURE 2.— (a) Common value utilities $u_i(x_i, p_i, \hat{p})$ and $u_j(x_j, p_i, \hat{p})$ for $p_j = \hat{p} = 3/8$. (b) Social value $u(x(p_i, \hat{p}), p_i, \hat{p})$ and equilibrium utility $u_i(x(p_i, \hat{p}), p_i, \hat{p}) - t_i(p_i, \hat{p})$.

First, it indicates that it is socially efficient to change the assignment from agent j to agent i at $p_i = \hat{p}$. Consequently the social utility satisfies at $p_i = p_j = \hat{p}$:

$$\frac{\partial u(x_i, p_i, p_j)}{\partial p_i} - \frac{\partial u(x_j, p_i, p_j)}{\partial p_i} \geq 0. \tag{7}$$

Consider an order $\succ$ on X, such that $x_i \succ x_j$. If the local condition (7) holds for all $p_i, p_j \in [0,1]$, then $u(x, p_i, p_j)$ is supermodular in (x, p_i). The second implication of the single crossing condition is that at $p_i = p_j = \hat{p}$

$$\frac{\partial u_i(x_i, p_i, p_j)}{\partial p_i} - \frac{\partial u_i(x_j, p_i, p_j)}{\partial p_i} \geq 0, \tag{8}$$

where the latter condition is necessary for truth telling by agent i. (The partial derivative of the second term is naturally equal to zero for all (p_i, p_j) in auctions without externalities.) If $u_i(x, p_i, p_j)$ is supermodular in (x_i, p_i), i.e., if we require (8) to hold globally for all $p_i, p_j \in [0,1]$, we obtain a sufficient condition for truth telling by agent i. Finally, the condition for under- or overacquisition of information by bidder i was related to the responsiveness of the utility of agent j, $u_j(x, p_i, p_j)$ to p_i. We can now restate the conditions for overacquisition of information by agent i in terms of

$$\frac{\partial u_j(x_i, p_i, p_j)}{\partial p_i} - \frac{\partial u_j(x_j, p_i, p_j)}{\partial p_i} \leq 0, \tag{9}$$

or equivalently that $u_j(x, p_i, p_j)$ is *submodular* in (x, p_i). We shall see in the subsequent sections that the supermodularity conditions for $u(x, p_i, p_j)$ and $u_i(x, p_i, p_j)$ are sufficient and almost necessary conditions for efficient implementation and that sub- or supermodularity conditions for $u_{-i}(x, p_i, p_{-i})$, provide sufficient conditions for over- and underacquisition of information in ex-post efficient mechanism.

3.2. *Independence*

The example just discussed is a linear version of an auction model introduced by Maskin (1992) and Dasgupta and Maskin (2000) to analyze privatization and related asset sale problems. A very similar model with common values and bidders with independent private information appears in Bulow, Huang, and Klemperer (1999) to analyze takeovers (with toeholds). The sale of a company is a fine example to see how the independence of ω_i across i might arise in substantive economic problems. Consider the sale of a company whose primary value is its client list. The state ω_i would then describe the extent to which the client list of the target company overlaps with the acquiring firm. The acquiring firm could either value differing client lists (to gain access to new clients) or overlapping client lists (to enhance the cross-selling of products). For competitive reasons, it also matters for firm j how the client list of the target compares with the client list of firm j, which is represented by ω_j. This introduces the common value aspect into the takeover contest. The single crossing condition in this environment simply states that the marginal value of information about the extent of the intersection between the client list of firm i and the target is larger for firm i than for firm j. The independence of ω_i across i then amounts to assuming that the clients are distributed independently across the acquiring firms i and j.

The general interpretation in the context of an asset sale, where the value of the asset could arise from a proprietary technology, a marketing strategy, a specific product or market niche or alike, is then that value of the asset for the acquiring firm is determined by the match of the asset with the characteristics of the acquiring firm. The independence assumption requires that the characteristics of the acquiring firms are distributed independently across firms.

Yet a different environment where independence of the state variable ω_i arises quite naturally, is in the context of procurement and R&D. The following is a stylized version of a model recently analyzed by D'Aspremont, Bhattacharya, and Gerard-Varet (2000). Their analysis focuses on bargaining with information sharing, whereas we adopt a procurement interpretation. Suppose there are two firms who compete for a contract by the government, regarding a project, say a weapons or software system, which has a social value v. The two firms, $i \in \{1, 2\}$, are pursuing the realization of the same project, but follow different design routes or approaches. Uncertainty in the model is described by, for simplicity, a binary state space $\Omega_i = \{0, 1\}$, where ω_i represents the probability that the research of firm i will eventually be successful. If there are many possible ways of pursuing the same goal, it is sensible to assume that the priors on Ω_i satisfy independence. Denote the (expected) cost of firm i of completing the project, independent of eventual failure or success, by γ_i. The government agency designs a game form that decides which of the firms should pursue the project. The space of possible decisions is then $X = \{0, 1, 2, 12\}$ where $x = i$ stands for the case where firm i continues with its research, $x = 12$ denotes the case where both firms continue in the race, and $x = 0$ denotes the case where neither firm continues with research in the second stage. In the first stage, each firm can obtains information about its research project by observing a signal realization s_i generated by an information structure α_i. Each choice of α_i and realization of s_i result in a posterior belief $p_i(\omega_i|s_i, \alpha_i)$. We may assume that if $\omega_1 = \omega_2 = 1$, then firm i wins the race with probability π_i and firm j with probability π_j. Assuming quasilinearity of the payoffs in the cost, we can write the ex-post utilities $u_i(x, \omega_i, \omega_j)$ for the case that both projects receive support by

$$u_i(12, 1, 0) = v - \gamma_i, \quad u_i(12, 1, 1) = \pi_i v - \gamma_i,$$
$$u_i(12, 0, 0) = u_i(12, 0, 1) = -\gamma_i,$$

and for the case that only a single project receives support by

$$u_i(i, \omega_i, \cdot) = \omega_i v - \gamma_i,$$
$$u_i(j, \cdot, \cdot) = 0.$$

Since $u_i(12, \omega_i, \omega_j)$ depends nontrivially on ω_j when $\omega_i = 1$, this model is one with common values. Let $p_i \triangleq p_i(\omega_i = 1|s_i, \alpha_i)$ and we observe that the socially optimal decision (conditional on the posterior beliefs) is to have firm i engage in the race (possibly jointly with firm j) whenever

$$p_i v - \gamma_i > 0 \quad \text{and} \quad p_i v - \gamma_i > p_j v - \gamma_j.$$

It is efficient to have both firms doing research in the second stage if

$$p_i(1-p_j)v > \gamma_i \quad \text{and} \quad p_j(1-p_i)v > \gamma_j.$$

In words, only the superior firm should continue with the research if its success is sufficiently certain (this is to avoid duplication of costly effort). If the posterior probability of success is in the intermediate range for both firms, then they should engage in further research jointly.[8] The independence here reflects the fact that each firm is pursuing an independent research program.

4. PRIVATE VALUES

This section considers information acquisition in the context of independent private values. For this environment Vickrey (1961), Clarke (1971), and Groves (1973) showed in increasing generality that the ex-post efficient allocation can be implemented in a direct revelation mechanism.

DEFINITION 1: A *direct revelation mechanism* is defined by a pair (x, t), where x is an outcome function, $x: S \to X$, and t is a transfer scheme, $t: S \to \mathbb{R}^I$.

In the private value environment we may consider without loss of generality the set of signal realizations S to be the probability simplex Δ over the state space Ω. The efficient allocation is implemented in dominant strategies if the transfer function has the following form: For all $i \in \mathcal{I}$,

$$t_i(p) = h_i(p_{-i}) - u_{-i}(x(p), p_{-i}), \tag{10}$$

where $h_i(p_{-i})$ is an arbitrary function of p_{-i}. We refer to the class of mechanisms that implement the efficient allocation with a transfer function of the form displayed in (10) as *the Vickrey-Clark-Groves (VCG) mechanism*.

DEFINITION 2: A vector of experiments, α, is a *local social optimum* if for every i, α_i solves

$$\alpha_i \in \underset{\alpha_i' \in A_i}{\arg\max} \left\{ \int u(x(p), p)\, dF^{(\alpha_i', \alpha_{-i})}(p) - c(\alpha_i', \alpha_{-i}) \right\}.$$

Notice that local here refers to the property that α solves the maximization problem for each agent separately, or Nash locality. In consequence, a local social optimum may not necessarily be a solution to the problem when the experiments of all agents are jointly maximized.

THEOREM 1 (Private Values): *With independent private values, every local social optimum can be achieved by the VCG mechanism.*

[8] It can also be verified that the ex-post efficient allocation satisfies the required single crossing properties and hence our results apply to this example as well.

PROOF: See Appendix.

With the VCG mechanism the equilibrium net utility of agent i behaves as the social utility up to $h_i(p_{-i})$, which does not depend on p_i. It therefore follows that the decision problem of agent i with respect to the information acquisition in terms of the posterior belief p_i is equivalent to the problem faced by the social planner. An immediate consequence of Theorem 1 is the following corollary.

COROLLARY 1: *The ex-ante efficient allocation can be implemented by the VCG mechanism.*

PROOF: See Appendix.

The ex-ante stage has many equilibria if there are multiple local social optima.[9] It follows that the VCG mechanism uniquely implements the ex-ante efficient allocation only if there is a unique local and hence global optimum in the information acquisition stage. This efficiency result can also be generalized to environments where each agent can invest ex ante in technologies that increase their private payoffs. The ex-ante efficiency result derived for the VCG mechanism can also be extended to any ex-post efficient mechanism by the revenue equivalence theorem.

In the current model, information is acquired by all agents simultaneously. However, it is well known in statistical decision theory that a sequential decision procedure may dominate any simultaneous procedure as it economizes on the cost of information acquisition. This observation is valid in the current model as well. An important consequence of a sequential version of the VCG mechanism is that the efficient allocation is now strongly implementable as every agent acts at every node as if he were maximizing the social value function (for a more detailed argument see Bergemann and Välimäki (2000)).

The essential property that allows us to prove ex-ante efficiency with independent private values is the restriction that only agent i can (efficiently) invest in information about his own utility associated with various allocations. The logical next step is therefore to ask whether efficiency can be maintained in environments where the information of agent i is relevant to the utility calculus of agent j. We pursue this question in the context of the interdependent values model investigated recently by Dasgupta and Maskin (2000) and Jehiel and Moldovanu (2001). Before we analyze the information acquisition per se, we give a complete characterization of the ex-post efficient allocation and associated equilibrium utilities for each agent in the following section.

5. COMMON VALUES: EX POST EFFICIENCY

We adopt the model of Dasgupta and Maskin (2000) to our environment with uncertainty about the true state of nature in subsection 5.1, where we present

[9] Tan (1992) makes a similar observation in the context of ex-ante R&D investments in procurement auctions.

necessary and sufficient conditions for efficient implementation with a direct revelation mechanism.[10] Similar results are briefly stated for a continuous allocation space in subsection 5.2.

5.1. *Finite Allocation Space*

We start by considering a set of finitely many allocations: $X = \{x^0, x^1, \dots, x^N\}$. For a fixed α_i, player i's expected utility from an allocation x^n after observing signal s is, using independence, given by

$$u_i(x^n, s) = \sum_{\omega \in \Omega} u_i(x^n, \omega) \prod_{j \in \mathcal{J}} p_j(\omega_j | s_j, \alpha_j).$$

In anticipation of the requirements for the implementability of the efficient allocation rule, we restrict our attention to an arbitrary class of one-dimensional signal realizations $S_i = [\underline{s}_i, \bar{s}_i] \subset \mathbb{R}$ with the result that the associated posterior beliefs $p_i(\cdot | s_i, \alpha_i)$ form a one-dimensional manifold in $\Delta(\Omega_i)$. In this section the allocation problem is analyzed exclusively at the ex-post stage. The utilities are therefore written as functions of (x, s) rather than (x, ω) and consequently $x(s)$ is *an ex-post efficient allocation rule* conditional on the signal s. We assume $u_i(x, s)$ to be continuously differentiable in s for all i.

Next we present necessary and sufficient conditions for efficient implementation in an ex-post equilibrium. By the revelation principle, we can restrict ourselves to direct mechanisms and truth telling strategies.

DEFINITION 3: A direct revelation mechanism (x, t) permits *implementation in an ex-post equilibrium* if $\forall i, \forall s \in S$:

$$u_i(x(s), s) - t_i(s) \geq u_i(x(\hat{s}_i, s_{-i}), s) - t_i(\hat{s}_i, s_{-i}), \quad \forall \hat{s}_i \in S_i.$$

An ex-post equilibrium, while not requiring dominant strategies, remains a Bayesian equilibrium for any prior distribution over types. For the rest of this subsection, we fix the realization of the signals s_{-i} and focus on truth telling conditions for agent i. Let the set S_i^n be defined as the subset of S_i for which x^n is an efficient allocation:

$$S_i^n = \{s_i \in S_i \mid u(x^n, s_i, s_{-i}) \geq u(x^k, s_i, s_{-i}), \forall x_k \neq x_n\}.$$

For any two sets S_i^k and S_i^l with a nonempty intersection, we call a point $s_i^{kl} \in S_i^k \cap S_i^l$ a *k to l change point* if there exists an $\varepsilon > 0$ such that either:[11]

$$(11) \qquad s_i \in [s_i^{kl} - \varepsilon, s_i^{kl}) \Rightarrow s_i \in S_i^k, s_i \notin S_i^l$$

[10] Dasgupta and Maskin (2000) actually restrict attention to multi-object auctions and achieve implementation through an indirect mechanism in which the bidders report their valuations contingent on the reports by the other bidders, but not directly their signals. Jehiel and Moldovanu (2001) present sufficient conditions in a linear model for general allocation problems with a direct revelation mechanism.

[11] The condition is written as an either/or condition as the social utility may display the same partial derivative with respect to s_i for the alternatives x^k and x^l over an interval where the social values of the alternatives x^k and x^l are equal.

or

(12) $\quad \forall s_i \in (s_i^{kl}, s_i+\varepsilon] \Rightarrow s_i \notin S_i^k, s_i \in S_i^l.$

Symmetrically, we can define $s_i^{lk} \in S_i^k \cap S_i^l$ to be an *l to k change point*. By extension, let $s^{kl} \triangleq (s_i^{kl}, s_{-i})$. Every change point s^{kl} has the property that at $s = s^{kl}$:

$$\frac{\partial u(x^k, s)}{\partial s_i} \leq \frac{\partial u(x^l, s)}{\partial s_i}.$$

Consider next the ex-post truthtelling condition for agent i:

$$u_i(x(s), s) - t_i(s) \geq u_i(x(\hat{s}_i, s_{-i}), (s_i, s_{-i})) - t_i(\hat{s}_i, s_{-i}), \quad \forall \hat{s}_i \in S_i.$$

It follows that the transfer payment of agent i has to be constant conditional on the allocation $x(s) = x^n$ and we denoted it by t_i^n.

PROPOSITION 1: *A necessary condition for ex-post implementation is that for* $\forall k, \forall l$, *at* $s = s^{kl}$

$$\frac{\partial u_i(x^k, s)}{\partial s_i} \leq \frac{\partial u_i(x^l, s)}{\partial s_i}. \tag{13}$$

PROOF: See Appendix.

The inequality (13) is a familiar local sorting condition and implies that the incentive compatible transfers are uniquely determined (up to a common constant) at the change point s^{kl} by

$$t_i^k - t_i^l = u_{-i}(x^l, s^{kl}) - u_{-i}(x^k, s^{kl}). \tag{14}$$

As the transfer payments t_i^n for every allocation x^n are necessarily determined at the change points, it follows that (generically) every pair of sets S_i^k and S_i^l must have an intersection that forms a connected set, as otherwise t_i^k and t_i^l would be overdetermined. The latter condition can be rephrased as follows:

DEFINITION 4: The collection $\{S_i^n\}_{n=0}^N$ satisfies *monotonicity* if for every n:

$$s_i, s_i' \in S_i^n \Rightarrow \lambda s_i + (1-\lambda)s_i' \in S_i^n, \quad \forall \lambda \in [0, 1].$$

A sufficient condition for monotonicity is that the social value $u(x^n, s)$ be single-crossing in (x^n, s_i). If monotonicity is satisfied, then there exists an optimal policy $x(s)$ such that x^n is chosen on a connected subset $R_i^n \subseteq S_i^n$ and nowhere else.[12] After possibly relabeling the indices, we can endow the allocation space X with the following order, denoted by $\prec$:

$$x^0 \prec x^1 \prec \cdots \prec x^N, \tag{15}$$

[12] The socially optimal policy $x(s)$ is not unique as any (randomized) allocation over the set $\{x^k, x^l\}$ is optimal for all $s_i \in S_i^k \cap S_i^l$, and in particular at the change points. Moreover for some x^k the corresponding set R_i^k may be empty and, in consequence, the associated optimal allocation policy would use only a strict subset of the feasible allocations. Naturally, the order defined in (15) and (16) would then extend only over the subset of allocations selected by the allocation rule $x(s)$.

such that for all $s_i \in R_i^k$ and $s_i' \in R_i^l$, with

$$s_i < s_i' \Rightarrow x(s_i, s_{-i}) = x^k \prec x^l = x(s_i', s_{-i}). \tag{16}$$

For the remainder of this section we continue to work with the order defined by (15) and (16).

PROPOSITION 2: *A generically necessary condition for ex-post implementation is monotonicity.*

PROOF: See Appendix.

The class of mechanisms that implement the efficient allocation with the transfers determined by (14) is referred to as *the generalized Vickrey Clark Groves mechanism*, where we initialize t_i^0 by

$$t_i^0 \triangleq h_i(s_{-i}) - u_{-i}(x_0, (\underline{s}_i, s_{-i})), \tag{17}$$

for some arbitrary $h_i(s_{-i})$. Next we strengthen the local sorting condition to obtain sufficient conditions for ex-post implementation by extending the local to a global sorting condition.

PROPOSITION 3: *Sufficient conditions for ex post implementation are:*
(i) *monotonicity is satisfied for all i and s;*
(ii) *for all i, s and n,*

$$\frac{\partial u_i(x^{n-1}, s)}{\partial s_i} \leq \frac{\partial u_i(x^n, s)}{\partial s_i}. \tag{18}$$

PROOF: See Appendix.

Thus if the utility of every agent i displays supermodularity in (x^n, s_i) and monotonicity is satisfied, then an ex-post implementation exists. We wish to emphasize that the particular order imposed on the allocation space X may depend on i and s_{-i}, and all that is required is that for every s_{-i}, an order on X can be constructed such that the conditions above for necessity and sufficiency can be met.

It may be noted that monotonicity and supermodularity are strictly weaker than the conditions suggested by Dasgupta and Maskin (2000) in the context of a multi-unit auction. In the linear (in the signals) version of the model that is investigated by Jehiel and Moldovanu (2001), where

$$u_i(x^n) = \sum_{j=1}^{I} u_{ij}(x^n) s_j,$$

the necessary and sufficient conditions coincide. For details we refer the reader to an earlier version of the paper (Bergemann and Valimaki (2000)).

5.2. *Continuum of Allocations*

The sorting and monotonicity conditions naturally extend to the case of a continuum of allocations. Let $X \subset \mathbb{R}$ be a compact interval of the real line. As before, fix the realization of signal s_{-i} and let $x(s)$ denote the efficient allocation rule. We also assume that $u_i(s, x)$ is twice continuously differentiable, and thus $x(s)$ is differentiable almost everywhere. If $x(s_i, s_{-i})$ is monotonic in s_i, we can impose a complete order, denoted by $\prec$, on the allocation space X such that the order on X mirrors the order of the signal space by requiring that for all s_i, s_i' and $x(s_i, s_{-i}) \neq x(s_i', s_{-i})$,

$$s_i < s_i' \Rightarrow x(s_i, s_{-i}) \prec x(s_i', s_{-i}). \tag{19}$$

We endow X with the complete order defined by (19).

PROPOSITION 4: *Sufficient conditions for ex-post implementation are*:
(i) *monotonicity*;
(ii) *global sorting condition*:

$$\frac{u_i(x,s)}{\partial s_i \, \partial x} \geq 0, \quad \forall i, \forall s, \forall x.$$

PROOF. See Appendix.

As in the discrete case, monotonicity is generically necessary. If the global sorting condition is weakened to a local sorting condition at $x = x(s)$,

$$\frac{u_i(x(s),s)}{\partial s_i \, \partial x} \geq 0, \quad \forall i, \forall s,$$

then we obtain the corresponding necessary conditions for implementation. Proposition 4 generalizes an earlier proposition by Jehiel and Moldovanu (2001) from a linear to a nonlinear environment with one-dimensional signals. The transfer payments in the generalized VCG mechanism can be represented as

$$t_i(s) = -\int_{\underline{s}_i}^{s_i} \frac{\partial u_{-i}(x(v_i, s_{-i}), (v_i, s_{-i}))}{\partial x} \frac{\partial x(v_i, s_{-i})}{\partial v_i} \, dv_i + t_i(\underline{s}_i, s_{-i}), \tag{20}$$

where $t_i(\underline{s}_i, s_{-i}) = h_i(s_{-i})$ for some arbitrary $h_i(s_{-i})$.

6. COMMON VALUES: EX-ANTE INEFFICIENCY

In this section, we analyze the implications of ex-post efficient mechanisms for the ex-ante decisions of the agents to acquire information. This problem is addressed by extending the monotone environment for a single decision-maker defined by Karlin and Rubin (1956) to a multiple agent decision environment. The monotone environment is introduced first in subsection 6.1 and the informational inefficiency is analyzed in subsection 6.2.

6.1. *Monotone Environment*

We investigate the possibility of achieving efficient ex-ante decisions, while requiring efficiency in the second stage mechanism for an arbitrary choice of signals by the agents. This implementation requirement imposes certain restrictions on the posteriors and the utility functions. As we consider the ex-ante decision problem, the appropriate sorting and monotonicity conditions have to be formulated for the state space Ω, rather than the signal space S.

The first restriction concerns the implementability of the efficient allocation. Jehiel and Moldovanu (2001) show that it is generically impossible to implement the efficient allocation if the players must submit multi-dimensional reports. Hence we assume that for each player i, there exists a one-dimensional submanifold M_i, with $M_i \subset \Delta(\Omega_i)$, and a function $\lambda_i : S_i \times A_i \to M_i$ such that the ex-post efficient allocation $x(\cdot)$ can be determined by $x(\lambda(s, \alpha))$ rather than $x(p(\omega|s, \alpha))$, where

$$\lambda(s, \alpha) = (\lambda_1(s_1, \alpha_1), \ldots, \lambda_I(s_I, \alpha_I)).$$

In the most straightforward case λ_i is an invertible mapping that associates to every posterior p_i in M_i exactly one signal realization s_i that generates the posterior. Then (M_i, λ_i) can be thought of as a direct dimensionality restriction on the posterior beliefs that can be generated by a family of signals. However, λ_i does not have to be invertible and then λ_i can be thought of as a sufficient statistic relative to the social allocation problem. We briefly present illustrations for this condition below.

First, observe that for a binary state space $\Omega_i = \{\omega_i^0, \omega_i^1\}$ the dimensionality assumption is satisfied for every class of signals, as the posterior on Ω_i can be represented by a single number in the unit interval, say $p_i(s_i, \alpha_i) \triangleq p_i(\omega_i^0|s_i, \alpha_i)$. Since $p_i(s_i, \alpha_i) \in [0, 1]$, we can take $M_i = [0, 1]$ for all i.

A second class of information structures satisfying the assumption is given by the following model, sometimes referred to as the "hard news" model in the literature. Suppose $\Omega_i \subset \mathbb{R}$. The signal realization is with probability α_i perfectly informative or $s_i \in \Omega_i$, where the conditional probability of $s_i = \omega_i$ being realized is given by the prior distribution $q_i(\omega_i)$; and with probability $1 - \alpha_i$, the signal realization is completely uninformative and hence the agent i maintains his prior as his posterior. In this class of models, the choice of α_i determines the probability of observing a completely informative signal of the state.[13] This class of models can be further extended to the case where conditional on receiving information, the signal is not perfectly informative, or $s_i = \omega_i + \varepsilon_i$, where ε_i can be distributed arbitrarily and possibly dependent on ω_i, or $\varepsilon_i \sim g_i(\cdot; \omega_i)$. As long as the distribution of the error term is assumed to be independent of the choice of α_i, the posterior $p(\omega_i|s_i, \alpha_i)$ is independent of $\alpha_i \in A_i$ and as long as s_i is one-dimensional, the posterior will lie on a one-dimensional submanifold of $\Delta(\Omega_i)$ and the requirement is satisfied.

[13] We thank the co-editor for suggesting this class of information structures.

A third class of models where the dimensionality condition holds for arbitrary families of information structures is one where the payoffs are linear in the states. In this case, the payoff from allocation x in state ω to agent i is given by

$$(21) \qquad u_i(\omega, x) \triangleq \sum_{j=1}^{I} u_{ij}(x)\omega_j.$$

The linear setting is the one investigated in Jehiel and Moldovanu (2001) as well. In our model, the state ω is the primitive notion rather than the vector of signals s as in Jehiel and Moldovanu (2001), and as a result, the linearity is assumed here with respect to the state rather than the signal. The reason why the dimensionality condition holds in the linear model, is that the posterior expectation of ω_i, expressed by

$$\pi_i(s_i, \alpha_i) = \mathbb{E}[\omega_i | s_i, \alpha_i],$$

is sufficient for the determination and implementation of the efficient choice rule. Thus we let the $\lambda_i : S_i \times A_i \to M_i$ define a set of equivalent signal realizations and information acquisition decisions in the sense that each element in the equivalence class generates the same posterior expectation, or

$$\forall (s_i, \alpha_i), (s_i', \alpha_i') \in S_i, \quad \pi_i(s_i, \alpha_i) = \pi_i(s_i', \alpha_i') \Leftrightarrow \lambda_i(s_i, \alpha_i) = \lambda_i(s_i', \alpha_i').$$

As π_i is a real number, we can find a one-dimensional manifold M_i such that $\lambda_i(s_i, \alpha_i) \in M_i$ for all $(s_i, \alpha_i) \in S_i \times A_i$.[14] Thus with linear preferences, the dimensionality restriction on the signals is satisfied for every family A_i of signals. This argument also shows that the basic intuition developed in the introductory example for a binary state space extends to an arbitrary state space Ω_i.

With this dimensionality restriction in place, we can assume without loss of generality that every signal realization s_i leads to a fixed posterior belief $p_i(\omega_i | s_i, \alpha_i)$ independent of the choice of signal α_i. Two distinct signal choices α_i and α_i' differ in the frequency by which signal realizations s_i are observed.

Next we consider the appropriate sorting and monotonicity conditions in the state space Ω. We require that for every i, the allocation space X can be endowed with a complete order, denoted by $\prec$, such that $u_i(x, \omega_i, \omega_{-i})$ and $u(x, \omega_i, \omega_{-i})$ are supermodular in (x, ω_i) for all ω_{-i}.[15] Observe that the ranking of the allocations is allowed to vary with i as in the previous section, but here the ranking has to be invariant with respect to ω_{-i}. In addition we require that for all i, the posterior probabilities satisfy the monotone likelihood ratio property: for

[14] Observe that M_i can always be embedded in $\Delta(\Omega_i)$ so that the restriction $M_i \subset \Delta(\Omega_i)$ is satisfied as well.

[15] In the monotone environment of Karlin and Rubin (1956), the utility function of the decision maker was only assumed to be single-crossing in (x, ω_i). The stronger condition of supermodularity is imposed here as we consider a multi-dimensional signal space and when taking the expectations over ω_{-i}, supermodularity in (x, ω_i) is preserved while the single-crossing property is not.

all $s_i' > s_i$ and $\omega_i' > \omega_i$, $p_i(\omega_i'|s_i')p_i(\omega_i|s_i) - p_i(\omega_i'|s_i)p_i(\omega_i|s_i') \geq 0$. The supermodularity of $u_i(x, \omega)$ in (x, ω_i) guarantees (globally) the sorting condition, whereas the supermodularity of $u(x, \omega_i, \omega_{-i})$ guarantees the monotonicity of the efficient allocation. The monotone likelihood ratio condition implies that supermodularity in (x, ω_i) translates into supermodularity in (x, s_i) when taking expectations with respect to the posterior beliefs based on the signal realizations.

PROPOSITION 5: *Suppose the monotone likelihood ratio and supermodularity conditions hold for all i; then*:

(i) *$u_i(x, s_i, s_{-i})$ and $u(x, s_i, s_{-i})$ are supermodular in (x, s_i)*;

(ii) *for all s_i, s_i' with $x(s_i, s_{-i}) \neq x(s_i', s_{-i})$*,

$$s_i < s_i' \Rightarrow x(s_i, s_{-i}) \prec x(s_i', s_{-i}).$$

PROOF. See Appendix.

6.2. *Inefficiency*

In order to determine whether agent i has the socially correct incentives to acquire information, we must compare the returns from information acquisition for the social planner and agent i. The information provided by a signal realization s_i affects both the valuation of any particular allocation x as well as the choice of the (socially) optimal allocation $x(s)$. By incentive compatibility, the generalized VCG mechanism guarantees that the social utility and the private utility of agent i are evaluated at $x(s)$ for every s. In the private values environment, the congruence between social and private utilities go even further since as functions of s_i they are identical up to a constant, possibly dependent on s_{-i}.

In the common value environment, in contrast, the marginal utility of s_i is in general different for the social utility and the private utility of agent i. This discrepancy is due to the fact that with common values, the utility of all agents but i, $u_{-i}(x, s)$, is responsive to the signal realization s_i. As a result, the social planner's preferences for information are in general different from the individual preferences. In order to determine how this discrepancy affects the incentives to acquire information, we make use of a characterization of the transfer function associated with the generalized VCG mechanism.

THEOREM 2 (Inefficiency in Mechanisms): *Every ex-post efficient mechanism*:

(i) *leads (weakly) to underacquisition of information by agent i if $u_{-i}(x, \omega_i, \omega_{-i})$ is supermodular in (x, ω_i).*

(ii) *leads (weakly) to overacquisition of information by agent i if $u_{-i}(x, \omega_i, \omega_{-i})$ is submodular in (x, ω_i).*

PROOF: See Appendix.

We emphasize that the inefficiency result above is only a local result in the sense that we compare the decision of agent i with the planner's decision for agent i, when both take the decisions of the remaining agents as given. In particular, the theorem is not a statement about the (Nash) equilibrium decisions of the agents. The interaction between the information structures chosen by the agents may conceivably lead to an equilibrium outcome in which all agents acquire too much information relative to the social optimum even though the local prediction, based on $u_{-i}(x, \omega)$ being, say, supermodular in (x, ω_i) for all i, is that all agents acquire too little information. In this case, the theorem would still tell us that relative to the equilibrium information structure α_{-i}, the social planner would like agent i to acquire more information than i chooses to acquire in equilibrium. Observe, however, that for an important class of models, the result above is also global. This is the case where a single player has the opportunity to acquire (additional) information.

Next we give a brief outline of the proof. The key function in the proof is the difference between the social utility function and agent i's private utility function. We show that this difference (i) has a global maximum at $x(s)$ and (ii) is supermodular in (x, s_i). The first attribute holds locally in the generalized VCG mechanism and can be suitably extended to a global property. The difference between social and private utility is composed of the gross utility of all agents but i and the transfer payment of agent i. The latter is constant in s_i conditional on x in the generalized VCG mechanism and hence the second attribute follows by the hypothesis of supermodularity of $u_{-i}(x, \omega)$ in (x, ω_i) after using Proposition 5. Finally, since the difference satisfies the same supermodularity conditions as $u(x, \omega)$ and $u_i(x, \omega)$, we can order signal structures in their informativeness according to the criterion of effectiveness suggested by Lehmann (1988). As the difference is increasing in the effectiveness order, we know that the marginal value of information is larger to the social utility than to agent i's private utility.

The inefficiency results in Theorem 2 are stated simply in terms of the marginal utility of the remaining agents after excluding agent i. If the marginal preferences of the complement set to i are congruent (in their direction) with agent i, then i has insufficient incentives to acquire information. With congruent marginal preferences, the ex-post efficient mechanism induces positive informational externalities that lead agent i to underinvest in information. If the marginal utilities of agent i and all remaining agents move in opposite directions, then the resulting negative informational externality leads the agent to overinvest in information.

We conclude this section with a generalization of the single unit auction model presented earlier to a finite number of bidders and a finite state space. In a single unit auction, the feasible allocations are simply the assignments of the object to the various bidders, and we denote by x_j the assignment of the object to agent j. We restrict attention to symmetric environments where for all $\omega = (\omega_i, \omega_{-i})$ and $\omega' = (\omega_i', \omega_{-i})$ such that $x(\omega), x(\omega') \neq x_i$, we have $x(\omega) = x(\omega')$. In other words, information about ω_i is never pivotal for an allocative decision between x_j and x_k. The nature of the inefficiency in the information acquisition can then be decided on the basis of the properties of the utility function of each bidder at x_j: $u_j(x_j, \omega)$. The utility of agent j is trivially zero for $u_j(x_k, \omega)$ for all $x_k \neq x_j$.

THEOREM 3 (Inefficiency in Auctions): *Every ex-post efficient single-unit auction:*

(i) *leads (weakly) to underacquisition of information by agent i if $u_j(x_j, \omega_i, \omega_{-i})$ is nonincreasing in ω_i for all $j \neq i$;*

(ii) *leads (weakly) to overacquisition of information by agent i if $u_j(x_j, \omega_i, \omega_{-i})$ is nondecreasing in ω_i for all $j \neq i$.*

PROOF: See Appendix.

7. CONCLUSION

This paper considers the efficiency of information acquisition in a mechanism design context. In the private values world, any mechanism that implements the efficient allocation, also leads to an efficient level of information acquisition by the agents ex-ante. The efficiency results with private values also extend to a setting where the information is acquired sequentially before a final social allocation is implemented.

The common value model we investigated here is one where the components ω_i of the state of the world $\omega = (\omega_1, \ldots, \omega_I)$ are distributed independently. The results in this paper can be generalized to settings including ones where the signals are single dimensional and independent conditional on the state of the world as long as we make the appropriate assumptions on utilities in terms of allocations and signals directly. If we move away from ex-post implementation to Bayesian implementation, the mechanisms suggested by Cremer and McLean (1985, 1988) can be adapted to our environment to induce efficient information acquisition in models with correlated signals.

Finally, this paper considered information acquisition with a fixed number of agents. It may be of interest to investigate the limiting model as the number of agents gets large. Intuitively, one might expect that the problem of each individual agent might be closer to the private value model. If the responsiveness of the marginal utility of all other agents to the signal of agent i declines, then the sub- or supermodularity of $u_{-i}(x, s)$ in (x, s_i) may vanish and yield efficiency in the limit.

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Manuscript received February, 2000; final revision received May, 2001.

APPENDIX

The appendix collects the proofs to the propositions and theorems in the main body of the text.

PROOF OF THEOREM 1: A necessary and sufficient condition for a local social optimum α is that for all i, α_i solves

$$\alpha_i \in \arg\max_{\alpha_i' \in A_i} \left\{ \int u(x(p), p)\, dF^{(\alpha_i', \alpha_{-i})}(p) - c(\alpha_i', \alpha_{-i}) \right\}. \tag{22}$$

In contrast, the expected equilibrium utility of agent i under the VCG mechanism is maximized by α_i where

$$\alpha_i \in \arg\max_{\alpha_i' \in A_i} \left\{ \int (u_i(x(p), p_i) + u_{-i}(x(p), p_{-i}) - h(p_{-i}))\, dF^{(\alpha_i', \alpha_{-i})}(p) - c_i(\alpha_i') \right\},$$

or

$$\alpha_i \in \arg\max_{\alpha_i' \in A_i} \left\{ \int u(x(p), p)\, dF^{(\alpha_i', \alpha_{-i})}(p) - c_i(\alpha_i') \right\}, \tag{23}$$

where $h(p_{-i})$ can be omitted from the objective function using independence: $F^{(\alpha_i', \alpha_{-i})}(p) = F^{\alpha_i'}(p_i) F^{(\alpha_{-i})}(p_{-i})$. The equivalence of (22) and (23) follows from the additive separability of the cost function $c(\alpha)$. *Q.E.D.*

PROOF OF PROPOSITION 1: The argument is by contradiction. We suppose that condition (11) for the change point s^{kl} is met; a similar argument would apply if instead (12) would hold. Suppose that (13) doesn't hold; then there exist some $\epsilon > 0$ such that

$$u_i(x^k, (s_i^{kl} - \epsilon, s_{-i})) - u_i(x^l, (s_i^{kl} - \epsilon, s_{-i})) < u_i(x^k, s^{kl}) - u_i(x^l, s^{kl}). \tag{24}$$

But at the same time we require implementation, or

$$u_i(x^k, (s_i^{kl} - \epsilon, s_{-i})) - t_i^k \geq u_i(x^l, (s_i^{kl} - \epsilon, s_{-i})) - t_i^l,$$

and

$$u_i(x^k, s^{kl}) - t_i^k \leq u_i(x^l, s^{kl}) - t_i^l,$$

which jointly imply that

$$u_i(x^k, (s_i^{kl} - \epsilon, s_{-i})) - u_i(x^l, (s_i^{kl} - \epsilon, s_{-i})) \geq u_i(x^k, s^{kl}) - u_i(x^l, s^{kl}),$$

which leads immediately to a contradiction with (24). *Q.E.D.*

PROOF OF PROPOSITION 2: Suppose monotonicity fails to hold. Then there exists at least one set S_i^k such that for $s_i, s_i' \in S_i^k$ and for some $\lambda \in (0, 1)$, $\lambda s_i + (1-\lambda) s_i' \notin S_i^k$, but $\lambda s_i + (1-\lambda) s_i' \in S_i^l$. By Proposition 1, the differences $t_i^k - t_i^l$ are uniquely determined by the change points. It follows that if a set S_i^k is not connected, then there are more equations (as defined by the incentive compatibility conditions at the change points) than variables, t_i^n's, and generically, in the payoffs of $u_i(x, s)$, the system of equations has no solution. *Q.E.D.*

PROOF OF PROPOSITION 3: By Proposition 1, the transfers are uniquely determined up to a common constant. Consider any adjacent sets R_i^{n-1} and R_i^n:

$$\forall s_i \in R_i^{n-1}: \quad u_i(x^n, s) - u_i(x^{n-1}, s) \leq t_i^n - t_i^{n-1},$$

and

$$\forall s \in R_i^n: \quad u_i(x^n, s) - u_i(x^{n-1}, s) \geq t_i^n - t_i^{n-1}.$$

Now consider any arbitrary pair S_i^k and S_i^m ordered so that $x_k \prec x_m$. We want to show that

$$\forall x_k \prec x_m, \forall s \in R_i^k: \quad u_i(x^k, s) - u_i(x^m, s) \geq t_i^k - t_i^m, \tag{25}$$

as well as

$$\forall x_m \succ x_k, \forall s_i \in R_i^m: \quad u_i(x^m, s) - u_i(x^k, s) \geq t_i^m - t_i^k.$$

Consider (25). We can expand the difference on the right-hand side to

$$u_i(x^k, s) - u_i(x^m, s) \geq \sum_{l=k}^{m-1} t_i^l - t_i^{l+1}. \tag{26}$$

Consider the uppermost element of the sum:

$$t_i^{m-1} - t_i^m = u_i(x^{m-1}, s^m) - u_i(x^m, s^m),$$

and for all $s < s^m$,

$$t_i^{m-1} - t_i^m \leq u_i(x^{m-1}, s) - u_i(x^m, s),$$

or

$$u_i(x^m, s) - t_i^m \leq u_i(x^{m-1}, s) - t_i^{m-1}, \tag{27}$$

by (18). Replacing the left-hand side by the right-hand side of (27) in the inequality (26), the modified inequality becomes a priori harder to satisfy. Doing so leads to

$$u_i(x^k, s) - u_i(x^{m-1}, s) \geq \sum_{l=k}^{m-2} t_i^l - t_i^{l+1},$$

and by repeatedly using the argument in (27), (26) is eventually reduced to

$$u_i(x^k, s) - u_i(x^{k+1}, s) \geq t_i^k - t_i^{k+1},$$

which is satisfied by (18), when the transfers are as in (14). *Q.E.D.*

PROOF OF PROPOSITION 4: The sufficient conditions with a continuum of allocations can be obtained directly by considering the conditions of the discrete allocation model in the limit as the set of discrete allocations converges to the set of a continuum of allocations. The details are omitted. *Q.E.D.*

PROOF OF PROPOSITION 5: By assumption, $u(x, \omega_i, \omega_{-i})$ is supermodular in (x, ω_i) for every ω_{-i}. The supermodularity property is preserved under expectations:

$$u(x, \omega_i, s_{-i}) = \sum_{\Omega_{-i}} u(x, \omega_i, \omega_{-i}) \prod_{j \neq i} p_j(\omega_j | s_j),$$

and a fortiori $u(x, \omega_i, s_{-i})$ satisfies the single crossing property in (x, ω_i). By Lemma 1 of Karlin and Rubin (1956), it follows that $u(x, s_i, s_{-i})$ satisfies the single crossing property in (x, s_i). A similar argument applies to $u_i(x, \omega_i, s_{-i})$. Furthermore, by Theorem 1 of Karlin and Rubin (1956), it follows that an optimal strategy which is monotone in s_i exists. This proves the first part of the theorem.

If $u(x, \omega_i, s_{-i})$ is supermodular in (x, ω_i) for every s_{-i}, then $u(x, s_i, s_{-i})$, defined as

$$u(x, s_i, s_{-i}) = \sum_{\Omega_i} u(x, \omega_i, s_{-i}) p_i(\omega_i | s_i),$$

is also supermodular in (x, s_i) by Theorem 3.10.1 in Topkis (1998) since $p_i(s_i, \omega_i)$ satisfies the monotone likelihood ratio. *Q.E.D.*

PROOF OF THEOREM 2: The following proof is written for a continuum of allocations, but all arguments go through with the obvious notational modifications for a finite set of allocations. We start with the net utility of agent i under the generalized VCG mechanism, which is given by

$$u_i(x,s)-t_i(s),$$

where $s=(s_i,s_{-i})$ is the true signal by truthtelling under the VCG mechanism. For a fixed s_{-i}, we can rewrite the transfer $t_i(s_i,s_{-i})$ to be determined directly by x rather than (s_i,s_{-i}). This is without loss of generality as we recall that $t_i(s_i,s_{-i})$ is constant in s_i conditional on x. The net utility of agent i can now be written directly as

$$v_i(x,s)\triangleq u_i(x,s)-t_i(x). \tag{28}$$

The transfer function $t_i(x)$ is given by analogy with (20) as

$$t_i(x)=-\int_{\underline{x}}^{x}\frac{\partial u_{-i}(z,s(z))}{\partial z}\,dz+t_i(\underline{x}), \tag{29}$$

where $s(x)$ is the inverse function of $x(s)$ for a fixed s_{-i}, or $s(z)=x^{-1}(z)$. The function $s(x)$ is well defined if $x(s)$ is strictly increasing in s_i. If x has 'flats' in s_i, then the integral would have to be modified in the obvious way. It follows directly from (28) that $v_i(x,s)$ is supermodular in (x,s_i) if and only if $u_i(x,s)$ is supermodular in (x,s_i), which in turn is guaranteed by the supermodularity of $u_i(x,\omega)$ in (x,ω_i), as shown in Proposition 5.

Next we show that $u(x,s)-v_i(x,s)$ is (i) supermodular in (x,s_i) and (ii) achieves a global maximum at $x=x(s)$ for all s. The first property is guaranteed by the same argument as before if $u_{-i}(x,s)$ is supermodular in (x,s_i) as

$$u(x,s)-v_i(x,s)=u_{-i}(x,s)+t_i(x). \tag{30}$$

Observe next that $u_{-i}(x,s)+t_i(x)$ has a stationary point at $x=x(s)$ for all s by (29):

$$\frac{\partial u_{-i}(x,s)}{\partial x}+\frac{\partial t_i(x)}{\partial x}=\frac{\partial u_{-i}(x,s)}{\partial x}-\frac{\partial u_{-i}(x,s(x))}{\partial x}=0.$$

Notice also that locally at $x=x(s)$ the function is concave in x as the second derivative with respect to x is given by

$$\frac{\partial^2 u_{-i}(x,s)}{\partial x^2}-\frac{\partial^2 u_{-i}(x,s(x))}{\partial x^2}-\frac{\partial^2 u_{-i}(x,s(x))}{\partial x\,\partial s}\frac{ds(x)}{dx},$$

as the first two terms cancel at $s=s(x)$, and

$$\frac{\partial^2 u_{-i}(x,s(x))}{\partial x\,\partial s_i}\frac{ds(x)}{dx}\geq 0$$

by the supermodularity of $u_{-i}(x,s)$ and $u(x,s)$ in (x,s_i). However our standing assumptions don't allow us to conclude that the local maximum is also a global maximum. This final obstacle can be removed by modifying the objective function $u_{-i}(x,s)+t_i(x)$ through the addition of a new function $g(x,s)$ with

$$G(x,s)\triangleq u_{-i}(x,s)+t_i(x)+g(x,s),$$

such that the following properties are satisfied:

(a) $g(x(s),s)=0$, for all s;

(b) $G(x,s)$ is supermodular in (x,s_i);

and

$$\text{(c)} \quad G(x(s),s) \geq G(x,s), \quad \forall s, x.$$

If a function $g(x,s)$ exists such that $G(x,s)$ satisfies the properties (a)–(c), then $G(x,s)$ satisfies assumption (i) and (ii) of Lehmann's theorem. Moreover the expected value of $G(x,s)$ evaluated at $x = x(s)$ is equal to $u_{-i}(x,s) + t_i(x)$ evaluated at $x = x(s)$. To accomplish this define an auxiliary function $b(s)$ by

$$b(s) \triangleq u(x(s),s) - u_{-i}(x(s),s) - t_i(x(s)),$$

and define $g(x,s)$ to be

$$g(x,s) \triangleq u(x,s) - u_{-i}(x,s) - t_i(x) - b(s).$$

It is now easy to verify that $G(x,s)$ shares the supermodularity properties of $u(x,s)$, has a global maximum at $x = x(s)$ for every s, and indeed $g(x(s),s) = 0$. It remains to take expectations. We maintain s_{-i} to be fixed. We take the expectation with respect to the distribution F^{α_i} and $F^{\alpha_i'}$ and denote

$$G(\alpha_i, s_{-i}) \triangleq \mathbb{E}_{s_i}[G(x(s_i,s_{-i}),(s_i,s_{-i}))|F^{\alpha_i}]$$

and

$$G(\alpha_i', s_{-i}) \triangleq \mathbb{E}_{s_i}[G(x(s_i,s_{-i}),(s_i,s_{-i}))|F^{\alpha_i'}].$$

It then follows by Lehmann's theorem that if α_i is more effective than α_i', we have

$$G(\alpha_i, s_{-i}) \geq G(\alpha_i', s_{-i}).$$

From (a)–(c), we can then conclude that

$$u_{-i}(\alpha_i, s_{-i}) + t_i(\alpha_i, s_{-i}) \geq u_{-i}(\alpha_i', s_{-i}) + t_i(\alpha_i', s_{-i}),$$

adopting again the notation that

$$u_{-i}(\alpha_i, s_{-i}) + t_i(\alpha_i, s_{-i}) \triangleq \mathbb{E}_{s_i}[u_{-i}(x(s_i,s_{-i}),(s_i,s_{-i})) + t_i(x(s_i,s_{-i}))|F^{\alpha_i}],$$

and similarly for α_i'. By (30), this is equivalent to

$$u(\alpha_i, s_{-i}) - u(\alpha_i', s_{-i}) \geq v_i(\alpha_i, s_{-i}) - v_i(\alpha_i', s_{-i}). \tag{31}$$

As the inequality holds for every s_{-i}, it remains to hold after taking expectation over the realization of s_{-i}, which concludes the proof. The corresponding result for submodularity can be obtained by simply reversing the inequalities. *Q.E.D.*

PROOF OF THEOREM 3: This theorem is a special case of Theorem 2 after introducing the following ranking for the allocations. With a single unit auction, the set of allocations is simply the assignment of the object to a particular bidder. For every i, partition the set of allocations X into x_i and x_{-i} and order the assignments such that $x_i \succ x_{-i}$. (The order among the remaining bidders is irrelevant.) By definition of the single object auction

$$u_i(x_{-i}, \omega) = 0.$$

To verify the supermodularity property, it is therefore sufficient to examine the behavior of

$$u_i(x_i, \omega) - u_i(x_{-i}, \omega),$$

as a function of ω_i. Similarly for $u_{-i}(x,\omega)$. The result is now a direct consequence of Theorem 2. *Q.E.D.*

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